Natural Boundary Element Method and Hypersingular Integrals

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Summary

In many fields of scientific and engineering computing the Artificial Boundary Method has been widely applied to solve boundary value problems of partial differential equations, especially it is a very important method for solving problems on unbounded domains. This method is first suggested by K. Feng and D. Yu, called the natural boundary element method, and then also called DtN method by J.B. Keller and D. Givoli. The exact artificial boundary condition is the natural boundary integral equation on the artificial boundary, that is just the Dirichlet to Neumann mapping.

The natural boundary integral equation is hypersingular integral equation. The hypersingular integrals have some quite different properties from regular and weak singular integrals. A very important topic is how to evaluate the hypersingular integrals efficiently, which should be understand in the Hadamard finite-part sense. The standard numerical integration is not effective for hypersingular integrals, and some special numerical integration should be developed.

The method using series expansion of the integral kernel was first suggested by Yu. He solved the harmonic and biharmonic natural boundary integral equations on circle successfully. Then numerous works have been devoted to this area. The method of subtracting the singularity, the method of regularization, the approximate integration formulas for finite-part integrals, and the indirect method are also developed.

References

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