

# Three-Dimensional J-Integral Based on a Domain Integral Method for Non-Homogeneous Solid with Residual Stresses Undergoing Large Deformation

Hiroshi Okada\*, Tatsuro Ishizaka, Akira Takahashi, Koichiro Arai and Yasunori Yusa

Tokyo University of Science, 2641 Yamazaki, Noda 278-8510, Japan.

\*Corresponding Author: Hiroshi Okada. Email: hokada@rs.noda.tus.ac.jp.

**Abstract:** In this paper, a new three-dimensional J-integral for non-homogeneous solids undergoing large deformation and associated with residual stresses is presented. The formulation of J-integral involves the strain energy density  $W$  that is generally defined by the integral  $W = \int_0^t \tau_{ij} \dot{\epsilon}_{ij} d\bar{t}$  over the entire deformation history of a material point where  $t_{ij}$  and  $\dot{\epsilon}_{ij}$  are the components of the Kirchhoff stress and those of velocity strain.  $t$  and  $\bar{t}$  represents the time. It is assumed that at  $t=0$  the body is free from any deformation and therefore the stresses are zeros.

Residual stresses are induced by plastic deformation occurring during the prior material processing, such as welding, plastic forming, shot peening, etc. They induce the plastic deformation in the material. Hence, material non-homogeneities are also induced by the material processings. Typically, such processes enhance the hardness. Hence, the yield stress increases. Then, the structure is subject to service loads.

We may assume that the histories of the service loads are completely known to us. However, the deformation histories during the prior processes are not known in general. Some of the information is often missing. The residual stresses may be measured by the experiment or are determined by some empirical formula. We do not have the complete information on the deformation histories of material for the computation of J-integral.

On the other hand, an engineering structure sometimes undergoes a large deformation prior to its failure. Therefore, the evaluation of J-integral needs to account for the effects of the large deformation. The J-integral evaluations have often been carried out under the assumption of small deformation in which the Cauchy stress was used as the stress measure. The assumption of proportional loading was also often introduced. When the J-integral formulation is extended to the finite deformation, the Cauchy stress is replaced by the first Piola-Kirchhoff stress or by the nominal stress. The strain energy density is defined on the original undeformed configuration. However, Koshima and Okada (T. Koshima and H. Okada, "Three-dimensional J-integral evaluation for finite strain elastic-plastic solid using the quadratic tetrahedral finite element and automatic meshing methodology", *Engineering Fracture Mechanics*, Vol. 135, pp. 34-63, 2015) pointed out that the so-called path independent property was lost due to the effect of finite rotation. To retain the path-independent property, an additional term must be included. Such formulation was presented as the  $T^*$

integral by Atluri, Nishioka and Nakagaki (S. N. Atluri, T. Nishioka, and M. Nakagaki, "Incremental path-independent integrals in inelastic and dynamic fracture mechanics" *Engineering Fracture Mechanics*, Vol. 20, pp. 209-244, 1984). Arai, Okada and Yusa (K. Arai, H. Okada and Y. Yusa, "A new three-dimensional J-integral formulation for arbitrary load history and finite deformation", *Transaction of JSME (in Japanese)*, Vol. 84, No. 863, pp. 18-00115, 2018) recently proposed a three-dimensional J-integral formulation considering the virtual crack extension. They also included the additional term. The result was shown to be unconditionally independent of the size and the shape of the integral domain. In other word, it is unconditionally path-independent.

In this research, we sought for a method to compute the three-dimensional J-integral for non-homogeneous solids whose complete deformation histories are not known. The solid undergoes a large deformation due to the applied service load. We assume that the residual stresses may be known because they may be measured or evaluated by the finite element analysis without assuming the crack. Then, we carefully examine the order of singularity in the vicinity of crack front to find the convergence of each term arising in the derivation for the J-integral. Some terms converge to zero and some remain to have finite values. Finally, some numerical examples in which the residual stresses were produced in a controlled manner in the finite element analyses are presented and some concluding remarks are presented.