Kalman Filter Dynamic Mode Decomposition for Data Assimilation

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Abstract: In this presentation, a family of Kalman filter dynamic mode decomposition, which consists of algorithms of the linear Kalman filter DMD method which identify the linear system and the extended Kalman filter DMD method which simultaneously identify the system and estimates state variable, is introduced. Then, the application of the extended Kalman filter DMD to data assimilation is discussed.

Keywords: Computational geometry; graph theory; Hamilton cycles

1 Family of Kalman Filter DMD

In recent years, “data assimilation” has been developed in order to obtain simulation results along the observation data, mainly in meteorological fields and other fields. In data assimilation, the model equation is considered to be the constraint condition and the optimum condition, and the objective function is set to make the observation data and the simulation results consistent with each other. Then, optimization and probability theory are applied to this problem and the most consistent simulation results with the observation data are obtained.

On the other hand, “data science” has been intensively investigated in recent years. In the “data-driven science” framework, an algorithm that creates self-propelled models from data alone without constructing and using model equations and predicts what will happen next is now being studied in areas such as “machine learning”. In this paper, the trial of data assimilation without using the model equation using this data-driven science is described.

Important issues in data science are to breakdown the prior knowledge of what kind of characteristics the target data possesses from a more general “meta” viewpoint. In the case of fluid field analysis, “meta” prior knowledge often used in recent researches is the following 2 points: 1) it is possible to express data in low dimensions, and that 2) the linear approximation accuracy of the system is not bad. Of course, it is necessary to examine how this prior knowledge can be used for complicated turbulence fields, but it is expected that those knowledges work for large scale structure of flow field such as alternating vortex discharge such as Karman vortex. 1) In the fluid field, the point 1) above is corresponding to effectiveness of proper orthogonal decomposition (called principal component analysis etc. in the other field) originated by the research of Lamley et al. [1], and the point 2) is corresponding to the fact that the dynamic mode decomposition (DMD method), which was proposed by Schmid[4], effectively works. Here, the DMD method is a method of obtaining a transition matrix of a linear system and performing eigendecomposition, which is consistent with finding eigenvalues and eigenvectors of an approximated linear system from data. Based on this dynamic mode decomposition, we introduce a data assimilation method without using model equations.

The author proposes a family of Kalman filter dynamic mode decomposition. A family of Kalman filter DMD consists of algorithms of the linear Kalman filter DMD method [2] which identify the linear system and the extended Kalman filter DMD method [3] which simultaneously identify the system and estimates state variable, as reported in [2,3] In this presentation, A family of Kalman filter DMD is explained, and then the application of the extended Kalman filter DMD to data assimilation is discussed.

References

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