

Efficient Computation of the Green's Function and Its Derivatives for Three-Dimensional Piezoelectricity

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Abstract: Efficient three-dimensional infinite Green's function and its first- and second-order derivatives for materials with piezoelectric coupling are studied in this paper. The procedure is based on an explicit solution recently introduced by the authors which presents three valuable characteristics: (i) it is explicit in terms of the Stroh's eigenvalues, (ii) it remains well-defined when some Stroh's eigenvalues are repeated (mathematical degeneracy) or nearly equal (quasi-mathematical degeneracy), and (iii) it is exact. Then, this solution is used to compute coefficients for a double Fourier series representation of the Green's function and its derivatives. These Fourier expansion representations are realvariable which is an important feature for numerical applications. In the past, double Fourier series representation have been shown a high performance for anisotropic elasticity. Its attractiveness stems from the fact that the information of the material properties is contained only in the Fourier coefficients, while the information of the dependence of the evaluation point position is contained in simple trigonometric functions. This results in two advantages. First, any order derivatives can be determined by simple spatial differentiation of the trigonometric functions. Second, Fourier coefficients need to be obtained only once for a given material so a library of materials can be easily constructed, leading to a very efficient methodology for numerical applications in piezoelectric structures. Higher-order derivatives can be straightforwardly built upon this methodology if required. In this work, after present the solutions we study the dependence of various parameters on the computational efficiency.

Keywords: Explicit expressions; mathematical degenerate materials; Stroh formalism; Green's functions; Fourier series representation