

A POD Coupled Adaptive DEIM (POD-ADEIM) Reduced-Order Model for Incompressible Multiphase Flow in Porous Media

Jingfa Li^{1,2}, Shuyu Sun^{2,*}, Bo Yu¹, Yang Liu² and Tao Zhang²

¹School of Mechanical Engineering, Beijing Key Laboratory of Pipeline Critical Technology and Equipment for Deepwater Oil & Gas Development, Beijing Institute of Petrochemical Technology, Beijing, 102617, China.

²Computational Transport Phenomena Laboratory, Division of Physical Science and Engineering, King Abdullah University of Science and Technology, Thuwal, 23955-6900, Saudi Arabia.

*Corresponding Author: Shuyu Sun. Email: shuyu.sun@kaust.edu.sa.

Extended Abstract. The multiphase fluid flow in porous media is one of the most fundamental phenomena in various physical processes, such as oil/gas flow in reservoir, subsurface contamination dispersion, chemical separation, etc. Due to its importance, the efficient and accurate solution and prediction of multiphase flow in porous media is highly required in engineering applications and mechanism studies, which has been a research hot spot with increasing interest in recent years. However, the strong nonlinearity implicated in the multiphase flow model has brought great challenges for the computation and analysis. In addition, the permeability in Darcy-type pressure equation is always represented as a high contrast coefficient, which further complicates the simulation difficulty especially for high fidelity prediction and decision-making process.

In this study, we attempt to apply one popular global model reduction method, namely Proper Orthogonal Decomposition (POD), to accelerate the simulation of incompressible multiphase flow in porous media. The cornerstone of POD is processing information from a sequence of snapshots (or instantaneous solutions) to identify a set of basis functions to construct a low-dimensional space, the most energetic part of these basis functions is then selected to represent the solution of the original system. In our work, one key point is how to establish the accurate POD reduced order model (ROM) for incompressible multiphase porous flow problems. According to Li et al. [1], the traditional Galerkin projection method (directly project the original governing equation into the low-dimensional space spanned by basis functions) is incapable of introducing the fluid saturation and permeability located on the cell face in POD-ROM for incompressible multiphase flow whether in homogeneous or heterogeneous porous media, thus we adopt the matrix operation method (project the discrete governing equation instead of the original governing equation into the low-dimensional space spanned by basis functions) to build the POD-ROM in this study. Based on the proposed POD-ROM framework, we use the implicit method to solve the (non-) wetting-phase pressure and saturation equations.

However, in the implicit solution of the pressure and saturation equations, the standard POD-ROM alone sometimes cannot yield an expected acceleration owing to two main reasons: (1) the nonlinearity between pressure and saturation is sensitive, thus the saturation would change largely even with slight pressure change. And the small error of pressure results would lead to a large deviation of saturation, which in turn exerts negative impacts on the computation of pressure; (2) the complexity for computing a projected nonlinear term in POD-ROM still depends on the dimension of the original full-order system, it is unavoidable in conventional POD-ROM. Therefore, more POD modes are needed to be used in order to achieve the prescribed numerical accuracy, which could worsen the

speed up efficiency of proposed POD-ROM.

To further reduce the computational complexity of POD-ROM to speed up the implicit solution of pressure and saturation equations, another model reduction method called Discrete Empirical Interpolation Method (DEIM) [2] is applied to treat the projected nonlinear term in the proposed incompressible multiphase porous flow POD-ROM. Different from previous studies, we develop an adaptive DEIM (ADEIM) in this work to achieve a more flexible selection of the interpolation points. Combined the POD-ROM with ADEIM, a POD-ADEIM-ROM for incompressible multiphase flow in porous media is established. The computational efficiency and numerical accuracy of the proposed model is validated through several representative numerical examples. Numerical results indicate that an outstanding acceleration (2~3 orders of magnitude faster) without sacrificing numerical accuracy significantly is obtained from the proposed model when comparing to the traditional solution method that without any acceleration technique. In addition, the effects of POD mode number, the distribution of permeability field, and the mesh density on the overall performances of the proposed model are investigated in detail.

Keywords: POD; incompressible multiphase flow; porous media; adaptive DEIM; numerical simulation

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