Damage Modeling of Heterogeneous Materials Using Multiscale Approach

Jurica Sorić*, Tomislav Lesičar, Filip Putar and Zdenko Tonković

Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, I. Lučića 5, 10000 Zagreb, Croatia.
*Corresponding Author: Jurica Sorić. Email: jurica.soric@fsb.hr.

Abstract: The paper deals with the novel multiscale approaches for modelling of both quasi-brittle and ductile damage responses of heterogeneous materials. The damage is induced at the microstructural level and, after the homogenization procedure, it is included in the constitutive stiffness of the material point at macrolevel. The derived algorithms are implemented into the finite element software ABAQUS. The new two-scale transition procedures have been verified on the standard benchmark examples.

Keywords: Heterogeneous material, multiscale approach, quasi-brittle damage, ductile damage.

1 Introduction

Damage phenomena, macroscopically characterized by decrease in material stiffness or so-called softening, are common in engineering materials and can decrease structural load-carrying capacity, and lead to loss of mechanical integrity. A lot of engineering materials can be treated as heterogeneous, particularly if they are observed at microscale. Therefore, in order to assess structural integrity and to predict structural lifetime, an analysis evolving microstructure is necessary. Derivation of an efficient multiscale approach is still an important challenge in the computational mechanics community.

In the present contribution the damage responses of both quasi-brittle and ductile materials are considered using two-scale computational procedures. To model quasi- brittle damage, a second-order computational homogenization approach is applied. The C^1 continuous triangular finite element formulation based on the nonlocal continuum theory is used for the discretization at both micro- and macroscale. The damage enhanced constitutive relations are employed at the microlevel, where an appropriate representative volume element (RVE), representing a sample of heterogeneous material, is considered. The ductile damage is modelled using the first-order computational homogenization scheme. The macrolevel discretization is performed by means of the regular displacement finite element formulation, while the RVE is discretized by the newly developed mixed finite element. Herein, the microstructural boundary value problem is solved by employing the gradient-enhanced elastoplasticity. According to the multiscale computation technique, all homogenized variables are mapped to the macrolevel. All developed algorithms are implemented into the finite element software ABAQUS via user subroutines. The proposed computational models are verified by means of several benchmark examples.

2 Numerical Formulation

The multiscale approach is based on the transition of state variables between two or more scales. Here a two-scale transition is considered, which employs the solution of the two boundary value problems, one at the macroscopic and one at the microscopic scale. In this framework the constitutive relation in a material point at the macrolevel is obtained by the simulation of a microscopic representative sample of material, i.e., an RVE. The state variables computed at the material point at the macrolevel are transferred to the microlevel using the RVE boundary conditions. After solution of the RVE boundary value problem, the homogenized microstructural variables forming the constitutive relation are upscaled at the macroscopic material point. The transition scheme is displayed in Fig. 1. The algorithms derived are implemented into the ABAQUS which significantly contributes to numerical efficiency.

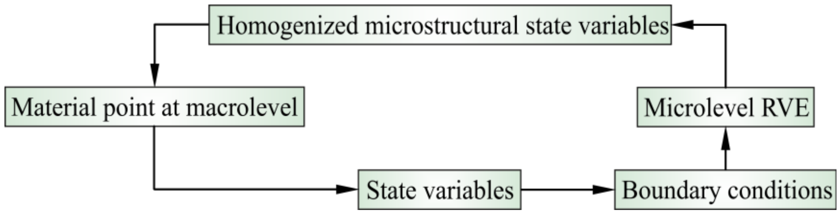

Figure 1: Two-scale computational scheme

2.1 Modeling of Quasi-Brittle Damage

As mentioned above, the C^1 continuous plane strain triangular finite element is used for the discretization at both micro- and macrolevel. It consists of three nodes, each having 12 degrees of freedom which are two displacement components and their first- and second-order derivatives. The element is based on the nonlocal continuum theory under assumption of small strain. The constitutive relations at the macrolevel and the finite element derivation are presented in [1]. The state variables computed at the macrostructural material point are the strain and the strain gradient which are transformed to the RVE boundary displacement. The constitutive relations at the microscale employ the damage variable as presented in the following incremental expressions

$$\begin{aligned}
\Delta\boldsymbol{\sigma} &= (1-D)\mathbf{C}\,\Delta\boldsymbol{\varepsilon} - \mathbf{C}\,\boldsymbol{\varepsilon}\,\Delta D, \\
\Delta\boldsymbol{\mu}_{x_1} &= l^2(1-D)\mathbf{C}\,\Delta\boldsymbol{\varepsilon}_{x_1} - l^2\mathbf{C}\,\boldsymbol{\varepsilon}_{x_1}\,\Delta D, \\
\Delta\boldsymbol{\mu}_{x_2} &= l^2(1-D)\mathbf{C}\,\Delta\boldsymbol{\varepsilon}_{x_2} - l^2\mathbf{C}\,\boldsymbol{\varepsilon}_{x_2}\,\Delta D.
\end{aligned} \tag{1}$$

Here, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the Cauchy stress and the strain tensors, respectively. The values $\boldsymbol{\varepsilon}_{x_1}$ and $\boldsymbol{\varepsilon}_{x_2}$ stand for the strain gradients with respect to the Cartesian coordinates, and $\boldsymbol{\mu}_{x_1}$ and $\boldsymbol{\mu}_{x_2}$ are their work conjugates. l represents the microstructural parameter, while $\mathbf{C}$ is the elasticity matrix which describes the stiffness behavior of the bulk material of RVE. D is the damage variable expressed by the exponential softening law [2]. After solution of the microscale boundary value problem and the homogenization procedure, the stress tensor components and stiffness dependent on the damage are mapped at the macrostructural level. In this way the RVE damage response is upscaled at the macrostructural finite element integration point, which yields the macrostructural material softening.

2.2 Modeling of Ductile Damage

According to the the first-order computational homogenization, the discretization at the macrolevel is performed by using a standard 4-node quadrilateral finite element formulation. Here the constitutive relation includes the damage variable and it is expressed as

$$\boldsymbol{\sigma} = (1-D)\mathbf{C}_{\mathrm{B}}\boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{C}_{\mathrm{B}}$ is the bulk material constitutive matrix of the heterogeneous RVE. Both the damage variable and the constitutive matrix are computed by the homogenization at the microstructural level. During the two-scale transition the macrostructural strain is mapped to the RVE, where the microstructural boundary value problem is solved. Here the damage variable, which is governed by the nonlocal equivalent plastic strain, is embedded into the Von Mises yield function. The exponential damage law is used. An implicit gradient approach driven by the nonlocal equivalent plastic strain is applied [3], which requires an additional differential equation employing the microstructural parameter, leading to additional degrees of freedom in the finite element framework. Therefore, the mixed finite element formulation is applied at the microlevel, where the nodal variables are the displacement and the nonlocal equivalent plastic strain. To model elastoplastic response, a standard well-known elastoplastic algorithm is applied. The new homogenization strategy is proposed in which the elastoplastic responses and the damage evolution are homogenized separately and upscaled at the macrostructural level.

3 Numerical Examples

3.1 Ductile Damage Responses of the Strip under Tensile Loading

To demonstrate ductile damage responses by means of the approach described, a strip with a weakened zone in the middle area subjected to the tensile loading is considered, as shown in Fig. 2.

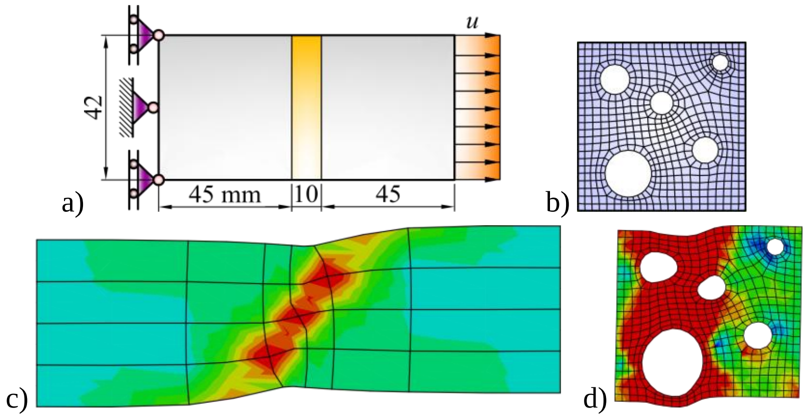

Figure 2: a) Geometry and loading of strip, b) heterogeneous RVE,
c) damage response of macromodel, d) damaged microstructural volume element

Fig. 2(c) displays the damage distribution over the deformed strip at the failure stage. The spreading of the softening zones over the RVE is shown in Fig. 2(d). As evident the realistic damage behaviour, typically observed in ductile materials, is obtained.

4 Conclusion

A two-scale algorithm for modelling of damage evolution at heterogeneous microstructure has been proposed. The softening in the quasi-brittle material response is captured by using the strain gradient continuum formulation, where the C^1 continuous triangular finite element discretization is applied. The ductile damage response is modelled by using the gradient-enhanced elastoplasticity, where the nonlocal elastoplastic variable is driven by the implicit formulation. Therein the discretization by means of the mixed C^0 finite elements is performed. It has been demonstrated that newly developed computational procedures are capable to capture evolution of the microstructural softening, which yields the realistic formations of the macrolevel localization zones.

References

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