

Mathematical Foundation of the Collocation Boundary Element Method: Consistent Formulation, Convergence Theorem and Accurate Numerical Quadrature

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Abstract: The use of boundary integral equations as an attempt to solve general problems of elasticity and potential has largely preceded the use of domain-related developments, which only became feasible (and conceivable) with the advent of powerful computational devices. On the other hand, the present-day matrix, computational-ready outline of the boundary element method (including its nowadays prevalent name) has borrowed – in part correctly and in part wrongly – much from the finite element concepts and formulation. We propose a revisit of the method, including, as for elasticity problems: a) conceptual reformulation in terms of weighted residuals with a consistent derivation of the single-layer potential matrix; b) an original convergence theorem for general curved boundaries and high order elements (2D and 3D problems); c) a unified numerical evaluation technique of regular, improper, quasi-singular and singular integrals that only uses Gauss-Legendre quadrature (and no interval subdivisions) plus locally defined corrections for general curved boundaries – thus leading to arbitrarily high accurate results independently from singularity or quasi-singularity intensity. Several numerical examples illustrate the applicability of the proposed procedure to elasticity and potential problems of complicated topology and including the evaluation of results at internal points arbitrarily close to the boundary.

Keywords: Boundary elements; convergence theorem; accurate numerical integration