Hierarchical Geographically Weighted Regression Model

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Abstract: In spatial analysis, two problems of the scale effect and the spatial dependence have been plagued scholars, the first law of geography presented to solve the spatial dependence has played a good role in the guidelines, forming the Geographical Weighted Regression (GWR). Based on classic statistical techniques, GWR model has ascertain significance in solving spatial dependence and spatial non-uniform problems, but it has no impact on the integration of the scale effect. It does not consider the interaction between the various factors of the sampling scale observations and the numerous factors of possible scale effects, so there is a loss of information. Crossing a two-stage analysis of "return of regression" to establish the model of Hierarchical Geographically Weighted Regression (HGWR), the first layer of regression analysis reflects the spatial dependence of space samples and the second layer of the regression reflects the spatial relationships scaling. The combination of both solves the spatial scale effect analysis, spatial dependence and spatial heterogeneity of the combined effects.

Keywords: Geographic information, regression analysis, scale effect, spatial dependence.

1 Introduction

In scientific research, many research questions are reflected as multi-level and multi-layer data structure. For example, middle school students in educational research embedded in the class, and the class is embedded in the school, where students represent the first level of data, classes or schools represent a second layer of data [Lei and Zhang (2002)]. Here students can be studied as individuals, and the information on the class or the school, students can be regarded as background information on the basis of class or school group. Based on grouping students in classes or schools, individual students' information can be combined with the class or school's information to build the research information. The above nesting phenomenon of data is widespread in the geospatial information; the nested relation to spatial dates taken into account in the data structure hierarchy of spatial data and spatial scale effect relationship. In solving practical problems, spatial information of a space problem involves not only from multiple levels, but also includes a multi-level

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spatial scale. If this information were applied without distinction, on the one hand, there is a loss of information; on the other hand, it may also contain a theoretical paradox.

Scale spatial information dependent feature makes the spatial information polymorphic changes that reflect the spatial state values with scale changes i.e., a value reflecting the state of the space changes with size. Under certain conditions, this change will lead to changes in the structure of spatial information; at the same time, in some cases, the complex interaction between the research object may cause spatial dependence and spatial heterogeneity exist, so coupling relationship between multi-source and multi-dimensional spatial information performance cross-scaling and up scaling relationship of multi-scale spatial elements information and knowledge.

The scale dependence of spatial information, spatial dependence and spatial heterogeneity determine the essential characteristics of spatial information that it is more complex analysis of spatial information scale, multi-dimensional spatial data processing problem. In the spatial analysis, scale effect and spatial dependence has been the two main challenges of scholars, the first law of geography proposed to solve the spatial dependence has played a good role in the guidelines, forming the Geographical Weighted Regression (GWR) [Brundson, Fotheringham and Chalton (1996)]. Based on classic statistical techniques, GWR model has certain significance in solving spatial dependence and spatial non-uniform problems, but it has no impact on the integration of the scale effect. It does not consider the interaction between the various factors of the sampling scale observations and the numerous factors of possible scale effects [Tran, Thai and Nguyen-Xuan (2018)]. Scale effect widely exists in the spatial information, in geospatial analysis, data processing scaling relationship usually using the following two methods: ① all variables are divided into lower level variables, data analysis on the lower variable; ② the lower level set of variables for variable data analysis on the level variables [Li and Wu (2005)]. These two methods are aimed to analyze certain level, and the emphasis is among the group effect or group of effects data, ignoring the data between groups effect (background effect, scale effect) and within group effects (spatial dependencies) common effect, thus there is a loss of information.

For an overall statistical, in the situation regarding to the background effect, each group has a different data regression model, and the model has its own statistical parameters, so the background effect (group effect) on the survey results can be seen by impact on the statistical parameters established for each set of data regression model reflected. This idea formed the data “return of regression” two-stage analysis: regression analysis of the data at the individual level, forming the individual layers’ regression parameters, and then forming comprehensive regression coefficients through another regression putting these statistics and the second layer of the variables (background variables). Comprehensive regression coefficient reflects the combined effect of the relationship between the group and the effects of background data, and spatial analysis can be used to solve spatial dependence (relationship within the group) and the spatial scale effect (background effect) interaction problems [Jin, Wang, Ze et al. (2018)].

2 Build hierarchical geographical weighted regression model

2.1 The basic principles of hierarchical linear model

2.1.1 Hierarchical linear model overview

Statistical data were stratified for studying originated in the social sciences, the basic assumptions of social science research is influenced by both individual behaviors of its own individual characteristics, but also by the impact of the environment in which it lives, researchers have been trying to distinguish between individual effect and group effect (background effect). In the exploration process, polymerization and decomposition of these two concepts were first proposed, namely, the researcher must make a choice between the individual effects and group effects, but just focus on individual effects neglected group effects or environmental effects when the individual that the correlation coefficient obtained from the data layer may be wrong; on the other hand when the researchers put data together in a group so that it only effects the role of this layer, which results in the loss of important information of the individual layer data.

Further development of research on these questions is to propose a so-called "in-group analysis and inter-group analysis" is a technical concept; the basic logic is: when the data reflect the two layers of information, why just focus on one? Group between analysis and group within analysis of the same data for three times calculation: first at the individual level within the group performed, is called the group effect; the second is drawn between the two groups of data by averaging the second layer or integration, which layer analysis known as the between group effect; finally neglected as the group of all the characteristics of the data summarized, which is referred to the overall effect. Analysis between the group and the group can form cross-level analysis of the relevant statistics, which reflect the proportion of variance between groups and within groups' variance, but cannot explain these variances.

On the basis of the above research has formed through the hierarchical research of data [Stephen and Anthony (2007)], the basic idea is that, for a statistical whole, the situation in regard to the background effect, each set of data has a different regression model, the model has its own intercept and slope (regression coefficient), so background effect (group effect) on the survey results can be viewed by the regression model for each set of data intercept and slope impact reflected. This idea formed the data "return of regression" two-stage analysis: at the individual level regression analysis of the data, the preservation of regression coefficients, and these statistics are coupled together with a second layer of variables to another return. This is the basic operation principle of hierarchical linear model, the theoretical implications of this principle clearly related cross-level statistics. Although hierarchical linear regression model and two are similar in concept, but their statistical estimation methods are different, hierarchical linear model is calculated using a contraction than using ordinary least squares estimation method as two regression models more stable and precise [Xue, Victor, Bin et al. (2018)].

2.1.2 The basic form of hierarchical linear model

Ordinary least squares regression equation is as follows:

$$Y_i = \beta_0 + \beta_1 X_i + r_i \tag{1}$$

β_0 is the intercept, characterizing Y value when X equals 0; β_1 is linear regression coefficients, characterizing the change of Y with a unit change in X; r_i is residuals, is assumed to be independent, normality and constant variance (Var (r_i) $=\sigma^2$) and has nothing to do with the predictor variables.

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij} \tag{2}$$

$$\beta_{0j} = \gamma_{00} + \mu_{0j} \tag{3}$$

$$\beta_{1j} = \gamma_{10} + \mu_{1j} \tag{4}$$

In the above equation, the subscript *j* represents the second layer of the first layer of the individual units (such as the village) belongs, such as townships.

γ_{00} and γ_{10} are the average value β_{0j} and β_{1j}, and they are constant between the second layer unit, they are fixed portion β_{0j} and the β_{1j};

μ_{0j} and μ_{1j} are random part of sum, and they represent a variation of the second layer.

Replaced the corresponding item with β_{0j} and β_{1j}, you will get:

$$Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \mu_{0j} + \mu_{1j} X_{ij} + r_{ij} \tag{5}$$

($\mu_{0j} + \mu_{1j} X_{ij} + r_{ij}$) is the residuals, because all individuals in each units of the second layer have the same sum. Therefore, a second layer in the same unit of similarity between individuals, it is higher than the similarity between individuals of different units within the second layer, and due to the different values of the residuals from the different units on the second layer, there may have different variances, are also included in the residuals between the error term is thus associated, unequal variance, and there is associated with. If the difference between the second layer of the unit does not exist, and is equal to 0, the equation will be reduced to ordinary least squares regression.

The above equation contains only the impact of the second layer of information for the regression coefficients of random error, not considered predictors of the second layer. When the second layer includes the predictor variables, it will form a complete model of multi-linear model:

First layer:

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij} \tag{6}$$

Second layer:

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + \mu_{0j} \tag{7}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + \mu_{1j} \tag{8}$$

γ_{01}, γ_{11} are the second layer equation regression coefficients, W_j is second layer equations predictors (i.e., variable background variables between the two groups). A complete model for the hierarchical linear model can be understood as follows: the dependent variable (Y_i) affected individual variables (X_i), but this effect is due to the individual which background vary, due to individual that is nested into the background, and therefore cannot be directly X_i and W_j as independent variables to predict dependent variable Y_i, $W_j$role by affecting X_i and Yi of the intercept of the regression equation Yi and slope to achieve.

2.1.3 Hierarchical linear model parameter estimation

With most estimated linear analysis of different estimates using ordinary least squares (OLS), are used to estimate the parameters of hierarchical linear model. It is a method of shrinkage estimation (Shrinkage Estimates). Shrinkage estimates the reliability λ using weighted estimate of two parts: the first part is the use of the first layer of variable parameter estimation, and the second part is the use of a second layer of variable parameter estimation. Reliability hierarchical linear model parameter estimation relates to the reliability or accuracy, is a statistical value. Reliability parameter λ represents the earliest layer of the unit on the estimated what proportion of the variation is due to the second layer unit of the "real" difference (or real differences caused by group *j*, instead of the estimated error).

(1) Variable parameters estimated by the first layer

The first layer of the equation:

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + r_{ij} \tag{9}$$

We can get estimates of β_{0j} and β_{1j} using OLS estimate:

$$\hat{\beta}_{0j} = \overline{Y} - \hat{\beta}_{1j} \overline{X}_{\cdot j} \tag{10}$$

$$\hat{\beta}_{1j} = \frac{\sum (X_{ij} - \overline{X}_{\cdot j})(Y_{ij} - \overline{Y}_{\cdot j})}{\sum (X_{ij} - \overline{X}_{\cdot j})^2} \tag{11}$$

$\overline{X}_{\cdot j}$, $\overline{Y}_{\cdot j}$ the expected value of the first layer of individuals in the j-th unit of Second layer.

For the second layer of the equation:

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + \mu_{0j} \tag{12}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + \mu_{1j} \tag{13}$$

With estimated values $\hat{\beta}_{0j}$ and $\hat{\beta}_{1j}$ of the first layer and to replace β_{0j} and β_{1j} , it becomes the following equation:

$$\hat{\beta}_{0j} = \gamma_{00} + \gamma_{01} W_j + e_{0j} \tag{14}$$

$$\hat{\beta}_{1j} = \gamma_{10} + \gamma_{11} W_j + e_{1j} \tag{15}$$

Among them,

$$e_{0j} = \mu_{0j} + (\hat{\beta}_{0j} - \beta_{0j}) \tag{16}$$

$$e_{1j} = \mu_{1j} + (\hat{\beta}_{1j} - \beta_{1j}) \tag{17}$$

As can be seen from the above equation, e_{0j} contains two components, one is truly random parameters μ_{0j}, and the other （$\hat{\beta}_{0j}$ - β_{0j}） is the error estimate-that is, the difference between the estimated value of β_{0j} and its real value. Similarly, e_{1j} also contains two components, i.e., a true random parameters u_{1j} and estimation error ($\hat{\beta}_{1j}$ - β_{1j}).

The variance of e_{0j} and e_{1j} has two components:

$$V_{ar}(\mu_{0j}) = \tau_{00} \tag{18}$$

$$V_{ar}(\mu_{1j}) = \tau_{11} \tag{19}$$

Parameter estimation variance parameter estimation error variance and the variance of these two components:

$$V_{ar}(e_{0j}) = \Delta_{0j} = [\mu_{0j} + (\hat{\beta}_{0j} - \beta_{0j})] = \tau_{00} + v_{0j} \tag{20}$$

$$V_{ar}(e_{1j}) = \Delta_{1j} = [\mu_{1j} + (\hat{\beta}_{1j} - \beta_{1j})] = \tau_{11} + v_{1j} \tag{21}$$

Among them,

$$v_{0j} = \sigma^2 / n_j \tag{22}$$

$$v_{1j} = \sigma^2 / \sum X_{ij}{}^2 \tag{23}$$

v_{0j} , v_{1j} the estimated error variance of the first layer

(2) Parameter estimation using a second layer of variable

For the second layer of the equation:

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + \mu_{0j} \tag{24}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + \mu_{1j} \tag{25}$$

In the above formula, γ_{00}, γ_{01} and γ_{11} can be estimated using the weighted least squares (WSL). For example, γ_{10} and γ_{11} can be estimated to be:

$$\hat{\gamma}_{10} = \frac{\sum \Delta_{1j}{}^{-1} \overline{Y}_{\cdot j}}{\sum \Delta_{1j}{}^{-1}} \tag{26}$$

$$\hat{\gamma}_{11} = \frac{\sum \Delta_{1j}{}^{-1} W_j \hat{\beta}_{1j}}{\sum \Delta_{1j}{}^{-1} W_j{}^2} \tag{27}$$

And is $\Delta_{1j}{}^{-1}$ $\Delta_{1j}{}^{-1}$ seen as Eq. (20) estimates as weights obtained, according to $\hat{\gamma}_{10}$ and $\hat{\gamma}_{11}$, the second estimate of β_{1j} can be obtained, i.e., the second layer in the WSL estimation performed $\hat{\hat{\beta}}_{1j}$:

$$\hat{\hat{\beta}}_{1j} = \hat{\gamma}_{10} + \hat{\gamma}_{11} W_j \tag{28}$$

The same estimate of β_{0j} can be deduced from the WSL:

$$\hat{\hat{\beta}}_{0j} = \hat{\gamma}_{00} + \hat{\gamma}_{01} W_j \tag{29}$$

At this point, β_{0j} and β_{1j} have two estimates, a first layer using OLS estimated variables obtained, and the other is the use of a second layer on the estimated variables WSL, HLM and the final estimate of the estimated parameters is two comprehensive, namely the use of the reliability of these two estimates are weighted:

$$\hat{\hat{\hat{\beta}}}_{0j} = \lambda_{0j} \hat{\beta}_{0j} + (1 - \lambda_{0j}) \hat{\hat{\beta}}_{0j} \tag{30}$$

$$\hat{\hat{\hat{\beta}}}_{1j} = \lambda_{1j} \hat{\beta}_{1j} + (1 - \lambda_{1j}) \hat{\hat{\beta}}_{1j} \tag{31}$$

Reliability of λ can be obtained by the following equation:

$$\lambda_{0j} = \tau_{00} / (\tau_{00} + v_{0j}) \tag{32}$$

$$\lambda_{1j} = \tau_{11} / (\tau_{11} + v_{1j}) \tag{33}$$

2.2 Stratified geographical weighted regression model

2.2.1 The first layer of the model - the model spatial dependence

The GWR is introduced in the classical regression model to describe the spatial autocorrelation and space on the basis of non-stationary items, effectively overcome the classical regression model for spatial statistical analysis of defects, forming a deal with spatial heterogeneity and spatial dependence integrated model, the paper cited GWR model spatial information as a hierarchical model of the first layer of the composite analysis model-space-dependent model.

The general form of GWR model:

$$y_i = \sum_{j=1}^{p} \beta_{ij} (u_i, v_i) x_{ij} + \varepsilon_i \tag{34}$$

As the first layer model of Layered complex spatial information analysis model, where y_i

and x_{ij} is the dependent variable and the explanatory variables x y in position (u_i, v_i) observations; βj (u_i, v_i) is a spatial unit regression coefficient; it is about the spatial location of unknown function; ε_i is the error term, p is number of spatial sampling units. According to the least squares method, (u_i, v_i) of the parameter estimates as follows:

$$\hat{\beta}(u_i, v_i) = [X^t W(u_i, v_i) X]^{-1} X^t W(u_i, v_i) Y \tag{35}$$

In the formula, w_j (u_i, v_i) for the position (u_i, v_i) right at the weight, is at the position (u_i, v_i) to the function of the distance of other observation positions, W (u_i, v_i) for the weight matrix. Completed the spatial heterogeneity in the first layer model, and spatial is dependence of the local linear regression.

2.2.2 The second layer model-spatial scale up scaling model

GWR model is not taken into account the differences in scale linear combination model, which does not consider the sampling scale observations of various factors, the scale effect of the interaction between the various factors that may exist, so there is a loss of information. In the second layer of the model is to reflect the scale spatial processes influencing factors, and thus inquire scale features affect the analysis of complex spatial information. The basic idea is in consideration of spatial scale effects, the influence of spatial scale effects on spatial analysis results can be seen through its effects on the intercept and slope of the regression model for each spatial unit of reflected, so you can make the "return regression". Two-stage analysis to establish the spatial dependence include spatial and spatial scale effects of local regression model:

i. In the space unit data, geographical weighted regression analysis, regression coefficient space gets dependence

ii. It will be included on the space unit of regression coefficients and spatial scale, spatial dependence of the predictor variables together to create a second layer of another regression model, the formation of scale effects regression coefficients;

iii. By shrinking the regression coefficients estimated weighted portfolio contains contains spatial scale effects of the regression coefficients depend on the relationship between the formation of the second layer of the model, the formation of a combination of regression coefficients space unit, which includes the spatial dependence coefficient and spatial scale effects common affected.

$\beta_j(u_i, v_i)$ of GWR model reflects the spatial dependence of spatial sampling values, due to their limited within a certain bandwidth, which is reflected in the interaction between local spatial elements, further reducing the resolution, can make them more overall

regional characteristics, thus reflecting scale spatial variables, the specific method is a weighted combination of a second, that the local effects of spatial elements of the relationship between the proposed and to form regional average $\overline{\beta}_j(u_i, v_i)$ of $\beta_j(u_i, v_i)$.

Construction $\beta_j(u_i, v_i)$ area average weighted average of the weights in the right model $w_i(u_i, v_i)$ using GWR weight, namely:

$$\overline{\beta}_j(u_i,v_i) = w_i(u_i,v_i)'\hat{\beta}_j(u_i,v_i) \quad (36)$$

$\overline{\beta}_j(u_i, v_i)$ and $\beta_j(u_i, v_i)$ have the same configuration mode, but they reflect the different spatial relationships: $\beta_j(u_i, v_i)$ reflects the value of the local spatial sampling spatial dependencies, $\overline{\beta}_j(u_i, v_i)$ is a weighted combination of the second $\beta_j(u_i, v_i)$, reflecting the scaling of spatial relationships, local-scale features will soon return to the model represented by the slope of the argument is converted to regional-scale features.

Based on the above ideas, the second layer of the model is:

$$\beta_0(u_i,v_i) = \lambda_0(u_i,v_i) + u_0 \quad (37)$$

$$\beta_1(u_i,v_i) = \lambda_1(u_i,v_i) + u_1 \quad (38)$$

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$$\beta_j(u_i,v_i) = \lambda_j(u_i,v_i) + u_j \quad (39)$$

$$\hat{\lambda}_j(u_i,v_i) = \overline{\beta}_j(u_i,v_i) = w_i(u_i,v_i)'\hat{\beta}_j(u_i,v_i) \quad (40)$$

Above constitute the basic model stratified geographical weighted regression model. After the addition of a second layer of information predictors influence a complete stratified geographically weighted regression model:

First layer:

$$y_i = \beta_0(u_i,v_i) + \sum_{j=1}^{p}\beta_j(u_i,v_i)x_{ij} + \varepsilon_i \quad (41)$$

Second layer:

$$\beta_0(u_i,v_i) = \lambda_0(u_i,v_i) + \sum_{P=1}^{n}\eta_{0P}(u_i,v_i)t_{iP} + u_0 \quad (42)$$

$$\beta_1(u_i,v_i) = \lambda_1(u_i,v_i) + \sum_{P=1}^{n}\eta_{1P}(u_i,v_i)t_{iP} + u_1 \quad (43)$$

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$$\beta_j(u_i,v_i) = \lambda_j(u_i,v_i) + \sum_{P=1}^{n}\eta_{jP}(u_i,v_i)t_{iP} + u_j \quad (44)$$

P is the number of the second layer of information predictor variables, T (tip) for the second layer of the predictor variables, their coefficients.

The second layer of the model reflects the large scale spatial interactions between elements, complements the analysis of complex spatial information in the large-scale information aggregation or decomposition to produce small-scale loss of information.

2.2.3 Integrated model-spatial heterogeneity, spatial dependence, and spatial scale effects combined effects model

The estimation $\hat{\beta}_j(u_i, v_i)$ of $\beta_j(u_i, v_i)$ can be obtained using GWR model estimation in the first layer:

$$\hat{\beta}(u_i, v_i) = [X^t W(u_i, v_i) X]^{-1} X^t W(u_i, v_i) Y \tag{45}$$

Thus, regional variation θ_j of $\hat{\beta}_j(u_i, v_i)$ can be obtained:

$$V_{ar}[\hat{\beta}_j(u_i, v_i)] = \theta_j \tag{46}$$

Using $\hat{\beta}_j(u_i, v_i)$, $\hat{\lambda}_j(u_i, v_i)$ to replace $\beta_j(u_i, v_i)$ and $\lambda_j(u_i, v_i)$ in the second layer of the equation, we can get:

$$\hat{\beta}_0(u_i, v_i) = \hat{\lambda}_0(u_i, v_i) + \sum_{P=1}^{n} \eta_{0P}(u_i, v_i) t_{iP} + \varsigma_0 \tag{47}$$

$$\hat{\beta}_1(u_i, v_i) = \hat{\lambda}_1(u_i, v_i) + \sum_{P=1}^{n} \eta_{1P}(u_i, v_i) t_{iP} + \varsigma_1 \tag{48}$$

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$$\hat{\beta}_j(u_i, v_i) = \hat{\lambda}_j(u_i, v_i) + \sum_{P=1}^{n} \eta_{jP}(u_i, v_i) t_{iP} + \varsigma_j \tag{49}$$

ς_j is the random error part when we use $\hat{\beta}_j(u_i, v_i)$, $\hat{\lambda}_j(u_i, v_i)$ to replace $\beta_j(u_i, v_i)$ and $\lambda_j(u_i, v_i)$.

Using $[\hat{\beta}_j(u_i, v_i) - \hat{\lambda}_j(u_i, v_i)]$ as an observation worth:

$$H = \hat{\beta}_j(u_i, v_i) - \hat{\lambda}_j(u_i, v_i) = \sum_{P=1}^{n} \eta_{jP}(u_i, v_i) t_{iP} + \varsigma_j \tag{50}$$

According to the principle of least squares can be:

$$\hat{\eta}_{jP}(u_i, v_i) = [T^t W_i(u_i, v_i) T]^{-1} T^t W\ (u_i, v_i) H \tag{51}$$

We can get the second estimation according to $\hat{\lambda}_j(u_i, v_i)$、$\hat{\eta}_{jP}(u_i, v_i)$:

$$\hat{\hat{\beta}}_j(u_i, v_i) = \hat{\lambda}_j(u_i, v_i) + \sum_{P=1}^{n} \hat{\eta}_{jP}(u_i, v_i) t_{iP} \tag{52}$$

So, we can get the regional variation ψ_j of $\hat{\hat{\beta}}_j(u_i, v_i)$:

$$V_{ar}[\hat{\hat{\beta}}_j(u_i, v_i)] = \psi_j \tag{53}$$

At this point there are two estimates, a first layer is the use of variable GWR estimates obtained, and the other is variable in conjunction with the second layer the first layer model parameter estimates obtained by estimation, the last two parameters were estimated, i.e., use reliability of these two estimates are weighted:

$$\hat{\hat{\hat{\beta}}}_j = \lambda_j \hat{\beta}_j + (1 - \lambda_j)\hat{\hat{\beta}}_j \tag{54}$$

Reliability λ can be obtained by:

$$\lambda_j = \frac{\theta_j}{(\theta_j + \psi_j)} \tag{55}$$

λ reflects the spatial dependence of the degree of influence of spatial scale effects on the overall variation (Y variation).

3 Conclusion

The first law of geography proposed to solve the spatial dependence has played a good role in the guidelines, forming a GWR model, classic GWR model comprehensive analysis model, spatial heterogeneity and spatial dependence, but not implemented to solve the problem of the role of the scale effect, that does not consider sampling scale observations of various factors, scale effects may exist between the various factors influencing each other, so there is a loss of information.

Stratified geographical weighted regression model contains spatial heterogeneity, spatial unit regression model, spatial dependence and spatial scale effect relationship. The first layer of the return reflects the value of the local spatial sampling by the "return of the regression of the" two-stage analysis. The heterogeneity and spatial dependence of the second return reflects the scaling of spatial relationships, local-scale features in the upcoming independent variable slope of the regression model represented by converting regional-scale features, so hierarchical linear model's complex spatial information of comprehensive analysis reflects the spatial heterogeneity, spatial dependence and spatial scale effects affecting the relationship between the spatial process which help to solve the spatial analysis scale effects involved in the multi-dimensional complexity of the data, the spatial dependence and spatial heterogeneity of the combined effects of the problem.

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