MCB, vol.2, no.2, pp.77-86, 2005

On Eulerian Constitutive Equations for Modeling Growth and Residual Stresses in Arteries

K.Y. Volokh[1]

Abstract: Recently Volokh and Lev (2005) argued that residual stresses could appear in growing arteries because of the arterial anisotropy. This conclusion emerged from a continuum mechanics theory of growth of soft biological tissues proposed by the authors. This theory included Lagrangian constitutive equations, which were formulated directly with respect to the reference configuration. Alternatively, it is possible to formulate Eulerian constitutive equations with respect to the current configuration and to 'pull them back' to the reference configuration. Such possibility is examined in the present work. The Eulerian formulation of the constitutive equations is used for a study of arterial growth. It is shown, particularly, that bending resultants are developed in the ring cross-section of the artery. These resultants may cause the ring opening or closing after cutting the artery *in vitro* as it is observed in experiments. It is remarkable that the results of the present study, based on the Eulerian constitutive equations, are very similar to the results of Volokh and Lev (2005), based on the Lagrangian constitutive equations. This strengthens the authors' argument that anisotropy is a possible reason for accumulation of residual stresses in arteries. This argument appears to be invariant with respect to the mathematical description.

keyword: Constitutive theory, Growth, Tissue, Residual stress; Artery

1 Introduction

A ring, extracted from an artery in vitro, opens up (or closes) after a radial cut (Chuong and Fung, 1986; Vaishnav and Vossoughi, 1987). This proves the existence of residual stresses in arteries, which provide the arterial compatibility and integrity. The residual stresses are formed during growth of arteries. According to the traditional point of view (Fung, 1993), the residual stresses are a consequence of the differential growth of arteries.

Recently Volokh and Lev (2005) argued that the arterial anisotropy combined with volumetric growth could be another source of residual stresses. They showed, particularly, that the ring opening or closing mode depends on elastic anisotropy of the artery. The authors' argument was based on a continuum mechanics theory of tissue growth, which included full-scale mass and momentum balance equations, constitutive equations, and boundary conditions. The constitutive equations related stresses, mass flux, and volumetric mass supply with the deformation gradient, mass density, and the mass density gradient. It is important that the constitutive equations were formulated directly with respect to the reference configuration and, thus, they can be called the Lagrangian constitutive equations.

As an alternative to the Lagrangian constitutive equations formulated in the reference configuration, it is possible to formulate Eulerian constitutive equations in the current configuration (what is not a 'push-forward' of the Lagrangian equations!). The latter is done in this note. The Eulerian equations are set and used for a study of arterial growth and formation of residual stresses. Numerical examples considered in Volokh and Lev (2005) are re-considered in the present work by using the modified, Eulerian, constitutive equations. This reveals the effect of the alternative formulation of constitutive equations on the results and conclusions of the consideration of the problem of growth and residual stresses in arteries.

2 Methods

2.1 Governing equations

Equations of mechanics of tissue growth can be motivated by a microstructural model of tissue growth as it is discussed in length in Volokh (2004) and Volokh and Lev (2005). In this section, we do not revisit the microstructural growth model given in the mentioned works and write down the main equations without delay

[1] Department of Mechanical Engineering, Johns Hopkins University, Baltimore, MD
on leave of absence from the Technion

$$\mathrm{Div}\boldsymbol{\psi}_R + \xi_R = 0, \tag{1}$$

$$\mathrm{Div}\,\mathbf{P} = \mathbf{0}, \tag{2}$$

$$\mathbf{P}\mathbf{F}^T = \mathbf{F}\mathbf{P}^T, \tag{3}$$

$$\begin{cases} \mathbf{P} = \mathbf{F}\partial W/\partial\mathbf{E} - \mathbf{F}\boldsymbol{\eta}\rho \\ \boldsymbol{\psi}_R = \beta\,\mathrm{Grad}\,\rho \\ \xi_R = \omega + f - \gamma\rho \end{cases}, \tag{4}$$

$$\begin{cases} \rho_R = \overline{\rho} \quad \text{on } \partial\Omega_\rho \\ \phi_R = \boldsymbol{\psi}_R \cdot \mathbf{N} = \overline{\phi} \quad \text{on } \partial\Omega_\phi \end{cases}, \tag{5}$$

$$\begin{cases} \boldsymbol{\chi} = \overline{\boldsymbol{\chi}} \quad \text{on } \partial\Omega_{\boldsymbol{\chi}} \\ \mathbf{t} = \overline{\mathbf{t}} \quad \text{on } \partial\Omega_{\mathbf{t}} \end{cases}. \tag{6}$$

Here subscript "R" designates the referential description of body Ω; ρ_R is mass density; $\boldsymbol{\psi}_R$is a vector of mass flux; ξ_R is volumetric mass supply; $\mathbf{P}$ is the 1^{st} Piola-Kirchhoff stress tensor; $\mathbf{F} = \mathrm{Grad}\boldsymbol{\chi}(\mathbf{X})$ is the deformation gradient; $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X})$ is a current position of point $\mathbf{X}$; W is a strain potential; $\rho = \rho_R - \overline{\rho}_0$ is the density increment; $\overline{\rho}_0$ is the initial distribution of mass density; $\mathbf{E} = (\mathbf{F}^T\mathbf{F} - \mathbf{1})/2$ is the Green strain tensor; and $\mathbf{1}$ is the identity tensor; $\boldsymbol{\eta} = \boldsymbol{\eta}^T$ is a tensor of material growth parameters; $\beta > 0$ is a coefficient of mass conductivity of the tissue and it counts for how much the mass supply changes for the spatially varying increment of mass density; $\omega > 0$ is the genetic mass supply, which is determined by the genetically controlled production of new cells and the extracellular matrix proteins by the existing cells; $f(\mathbf{F}, \mathbf{P})$ is the epigenetic mass supply, which should depend on stress and/or strain measures (its correct expression is a key problem when tissue remodeling is considered); $\gamma > 0$ is a coefficient of tissue resistance, which defines the resistance of the tissue to accommodate new mass for the increasing mass density; $\mathbf{t}$ is a surface traction; $\phi_R = \psi_R \cdot \mathbf{N}$ is mass supply through a surface with an outward unit normal $\mathbf{N}$ in the reference configuration; and the barred quantities are prescribed.

Dynamic processes of mechanical deformation as well as the inertia effects are ignored in the field equations above. Besides, it is assumed that the growth process can be considered quasi-statically because of its slowness: $\partial\rho_R/\partial t = 0$, i.e. every increment of the volumetric material supply inside a *material point* is in equilibrium with the material coming through the boundaries of this point. It is crucial to make a distinction between the real material particles and the mathematical concept of the *material point*. Physically, the material point is a finite and very small volume comprising few material particles. During growth, the number of real material particles is changing within the material point, while the 'number' of material points is preserved. The latter allows using continuum mechanics where a one-to-one mapping of material points before and after growth-deformation is assumed.

The proposed constitutive equations have a simple mathematical form and they are written directly in terms of the referential quantities. They are objective in the sense that they are not affected by a superposed rigid body motion. Alternatively, it is possible to define constitutive equations in terms of the current configuration – Eulerian form – first and then to transform them into the referential description – Lagrangian form. Thus, first we re-define the simple constitutive equations in the current configuration

$$\begin{cases} \boldsymbol{\sigma} = J^{-1}\mathbf{F}(\partial W/\partial\mathbf{E})\mathbf{F}^T - \tilde{\boldsymbol{\eta}}\tilde{\rho} \\ \boldsymbol{\psi} = \tilde{\beta}\,\mathrm{grad}\,\tilde{\rho} \\ \xi = \tilde{\omega} + \tilde{f} - \tilde{\gamma}\tilde{\rho} \end{cases}, \tag{7}$$

where $\tilde{\boldsymbol{\eta}}$, $\tilde{\beta}$, $\tilde{\omega}$, $\tilde{f}$, $\tilde{\gamma}$, being defined with respect to the current configuration, have the same meaning as $\boldsymbol{\eta}$, β, ω, f, γ. Other current quantities are related with the referential ones as follows

$$\boldsymbol{\sigma} = J^{-1}\mathbf{P}\mathbf{F}^T, \tag{8}$$

$$\tilde{\rho} = J^{-1}\rho, \tag{9}$$

$$\boldsymbol{\psi} = J^{-1}\mathbf{F}\psi_R, \tag{10}$$

$$\text{grad} = \mathbf{F}^{-T}\,\text{Grad}, \tag{11}$$

$$\xi = J^{-1}\xi_R. \tag{12}$$

Here $\boldsymbol{\sigma}$ is the Cauchy stress tensor; and $J = \det \mathbf{F}$. Substituting Eqs. (8) - (12) in Eq. (7) we have

$$\begin{cases} \mathbf{P} = \mathbf{F}\partial W/\partial \mathbf{E} - J\tilde{\boldsymbol{\eta}}\mathbf{F}^{-T}\tilde{\rho} \\ \boldsymbol{\psi}_R = J(\mathbf{F}^T\mathbf{F})^{-1}\tilde{\beta}\,\text{Grad}\tilde{\rho} \\ \xi_R = J\tilde{\omega} + J\tilde{f} - J\tilde{\gamma}\tilde{\rho} \end{cases} . \tag{13}$$

Comparing formulations based on Eq. (4) and Eq. (13) it easy to recognize the following substitutes, which should be made in the former equation in order to arrive at the latter one

$$\boldsymbol{\eta} \rightarrow J\mathbf{F}^{-1}\tilde{\boldsymbol{\eta}}\mathbf{F}^{-T}, \tag{14}$$

$$\rho \rightarrow \tilde{\rho} = \rho/J, \tag{15}$$

$$\mathbf{1}\beta \rightarrow J(\mathbf{F}^T\mathbf{F})^{-1}\tilde{\beta}, \tag{16}$$

$$\omega \rightarrow J\tilde{\omega}, \tag{17}$$

$$f \rightarrow J\tilde{f}, \tag{18}$$

$$\xi_R = J\tilde{\omega} + J\tilde{f} - J\tilde{\gamma}(\rho/J). \tag{19}$$

As expected, all the parameters defined in the current configuration are 'pulled back' to the reference configuration. There is a subtle question of what formulation, based on Eq. (4) or Eq. (13), is more appealing physically. We do not have a strong opinion about that. There is no doubt, however, that the constitutive formulation based on Eq. (4) is simpler mathematically.

2.2 Artery growth

Focusing on the arterial growth, we introduce the following form of the growth tensor for an orthotropic artery

$$\begin{cases} \tilde{\eta}_{11} = c(c_1\alpha_1 + c_4\alpha_2 + c_6\alpha_3) \\ \tilde{\eta}_{22} = c(c_4\alpha_1 + c_2\alpha_2 + c_5\alpha_3) \\ \tilde{\eta}_{33} = c(c_6\alpha_1 + c_5\alpha_2 + c_3\alpha_3) \end{cases}, \quad \tilde{\eta}_{ij} = 0, \quad i \neq j, \tag{20}$$

where the coefficients of growth expansion $\alpha_i > 0$ define how much the relative volume changes for the given increment of mass density. These coefficients are analogous to the coefficients of thermal expansion in the classical small-strain thermoelasticity. Other parameters (c and c_is) were chosen in accordance with the Fung pseudo-strain energy expression

$$W = ce^Q/2,$$

$$Q = c_1E_{11}^2 + c_2E_{22}^2 + c_3E_{33}^2 + 2c_4E_{11}E_{22} + 2c_5E_{33}E_{22} + 2c_6E_{11}E_{33}, \tag{21}$$

where c has the unit of stresses and c_is are unitless.

We consider artery growth as a radial growth of an infinite cylinder under the plane strain conditions where a point occupying position $(R,\ \Theta,\ Z)$ in the initial configuration is moving to position $(r,\ \theta,\ z)$ in the current configuration and the law of motion takes the following form

$$r = r(R), \quad \theta = \Theta, \quad z = Z. \tag{22}$$

Then the deformation gradient is

$$\mathbf{F} = (dr/dR)\mathbf{k}_r \otimes \mathbf{K}_R + (r/R)\,\mathbf{k}_\theta \otimes \mathbf{K}_\Theta + \mathbf{k}_z \otimes \mathbf{K}_Z, \tag{23}$$

where $\{\mathbf{K}_R,\ \mathbf{K}_\Theta,\ \mathbf{K}_Z\}$ and $\{\mathbf{k}_r,\ \mathbf{k}_\theta,\ \mathbf{k}_z\}$ form the orthonormal bases[2] in cylindrical coordinates at the reference and

[2] $\mathbf{K}_R = (\cos\Theta, \sin\Theta, 0)^T$; $\mathbf{K}_\Theta = (-\sin\Theta, \cos\Theta, 0)^T$; $\mathbf{K}_Z = (0,0,1)^T$ and $\mathbf{K}_M \otimes \mathbf{K}_N = \mathbf{K}_M\mathbf{K}_N^T$ $\mathbf{k}_r = (\cos\theta, \sin\theta, 0)^T$; $\mathbf{k}_\theta = (-\sin\theta, \cos\theta, 0)^T$; $\mathbf{k}_z = (0,0,1)^T$ and $\mathbf{k}_m \otimes \mathbf{k}_n = \mathbf{k}_m\mathbf{k}_n^T$

current configurations accordingly. Because of the deformation assumption we have

$$\{F_{Rr}^{-1}, F_{\Theta\theta}^{-1}, F_{Zz}^{-1}\} = \{(dr/dR)^{-1},\ (r/R)^{-1},\ 1\}, \tag{24}$$

$$2\{E_{RR}, E_{\Theta\Theta}, E_{ZZ}\} = \{(dr/dR)^2 - 1,\ (r/R)^2 - 1,\ 0\}. \tag{25}$$

Mass balance (1) and momentum balance (2) take the following forms accordingly

$$\frac{d(\boldsymbol{\psi}_R)_R}{dR} + \frac{(\psi_R)_R}{R} + \xi = 0, \tag{26}$$

$$\frac{dP_{rR}}{dR} + \frac{P_{rR} - P_{\theta\Theta}}{R} = 0, \tag{27}$$

Constitutive equations (13) transform into

$$\left\{\begin{array}{l} (\boldsymbol{\psi}_R)_R = \tilde{\beta}\,\frac{r}{R}(\frac{dr}{dR})^{-1}\frac{d\tilde{\rho}}{dR} \\ \xi = (\tilde{\omega} - \tilde{\gamma}\tilde{\rho})\frac{r}{R}\frac{dr}{dR} \\ P_{rR} = c(e^{Q}(c_1 E_{RR} + c_4 E_{\Theta\Theta})\frac{dr}{dR} \\ \qquad -(c_1 + c_4 + c_6)\alpha\tilde{\rho}\frac{r}{R}) \\ P_{\theta\Theta} = c(e^{Q}(c_2 E_{\Theta\Theta} + c_4 E_{RR})\frac{r}{R} \\ \qquad -(c_2 + c_4 + c_5)\alpha\tilde{\rho}\frac{dr}{dR}) \end{array}\right. , \tag{28}$$

where $\alpha = \alpha_1 = \alpha_2 = \alpha_3$; $Q = c_1 E_{RR}^2 + c_2 E_{\Theta\Theta}^2 + 2c_4 E_{RR} E_{\Theta\Theta}$; and remodeling is ignored, $\tilde{f} = 0$.

Boundary conditions (5) and (6) are set as follows

$$\left\{\begin{array}{l} \phi_R(R = a) = 0 \\ \phi_R(R = b) = 0 \end{array}\right. , \tag{29}$$

$$\left\{\begin{array}{l} \sigma_{rr}(R = a) = (R/r)P_{rR}(R = a) = 0 \\ \sigma_{rr}(R = b) = (R/r)P_{rR}(R = b) = 0 \end{array}\right. . \tag{30}$$

We assume that new material is supplied uniformly. In this case

$$\tilde{\rho} = \tilde{\omega}/\tilde{\gamma} = \text{constant} \tag{31}$$

is a solution of Eqs. (26) and (29).

Substituting Eqs. (21) and ($28_{3,4}$) in the equilibrium equation (27) and boundary conditions (30), we have a two-point boundary value problem (BVP) in terms of $r(R)$. The two-point BVP is solved numerically employing the shooting procedure, where the Mathematica 'NDSolve' procedure (Wolfram, 2003) is used iteratively as a solver of the corresponding initial-value problem.

3 Results

Radial displacements, radial and circumferential Cauchy stresses (Figs. 1 and 2) were computed for two sets of material parameters with $a = 1.0$ and $b = 1.3$:

(1) Chuong and Fung (1986)

$$\begin{array}{ll} c_1 = 0.0499; & c_2 = 1.0672; \\ c_3 = 0.4775; & c_4 = 0.0042; \\ c_5 = 0.0903; & c_6 = 0.0585. \end{array} \tag{32}$$

(2) Chuong and Fung (1984)

$$\begin{array}{ll} c_1 = 1.744; & c_2 = 0.619; \\ c_3 = 0.0405; & c_4 = 0.004; \\ c_5 = 0.0667; & c_6 = 0.0019. \end{array} \tag{33}$$

Every set of material parameters was considered with four growth parameters varying by four orders of magnitude: $\alpha\tilde{\omega}/\tilde{\gamma}$ = 0.001; 0.01; 0.1; 1.

Resulting displacements vary almost linearly along the radius (Figs. 1a and 2a). Absolute magnitudes of the radial stresses increase towards the mid-surface of the wall (Figs.1b and 2b), while the absolute magnitudes of the circumferential stresses approach zero at the mid-surface and they vary almost linearly along the radius (Fig. 1c and 2c). It should be noted that circumferential stresses are larger than the radial stresses by an order of magnitude in both cases of material parameters. It is interesting that the directions of the stresses are different for the two sets of material parameters. This is, particularly, critical for the circumferential stresses because it means that different bending resultants appear in the ring. If the ring is cut radially, then it opens as shown in Fig. 3a (right) for the first set of elastic parameters. A ring with the second set of material parameters behaves differently: it closes (Fig. 3b) after the cut, i.e. its edges overlap.

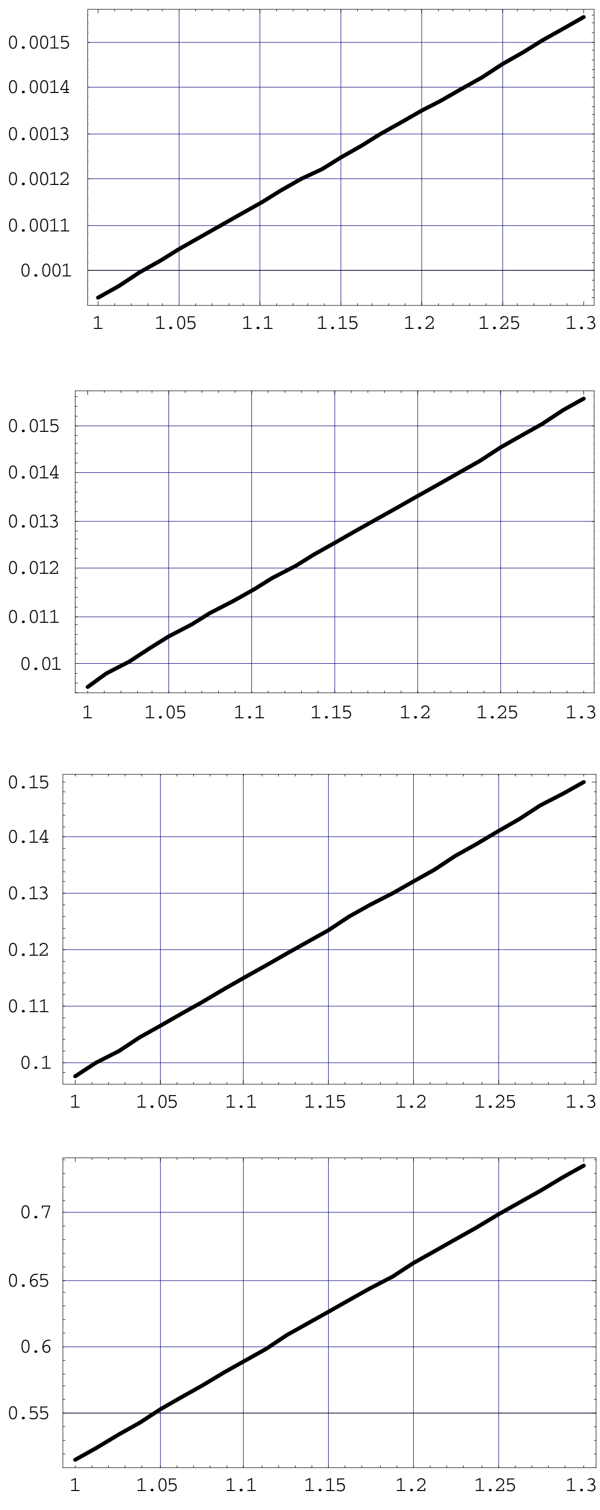

Figure 1 : (a) Normalized radial displacements $(r-R)/a$ (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (32).

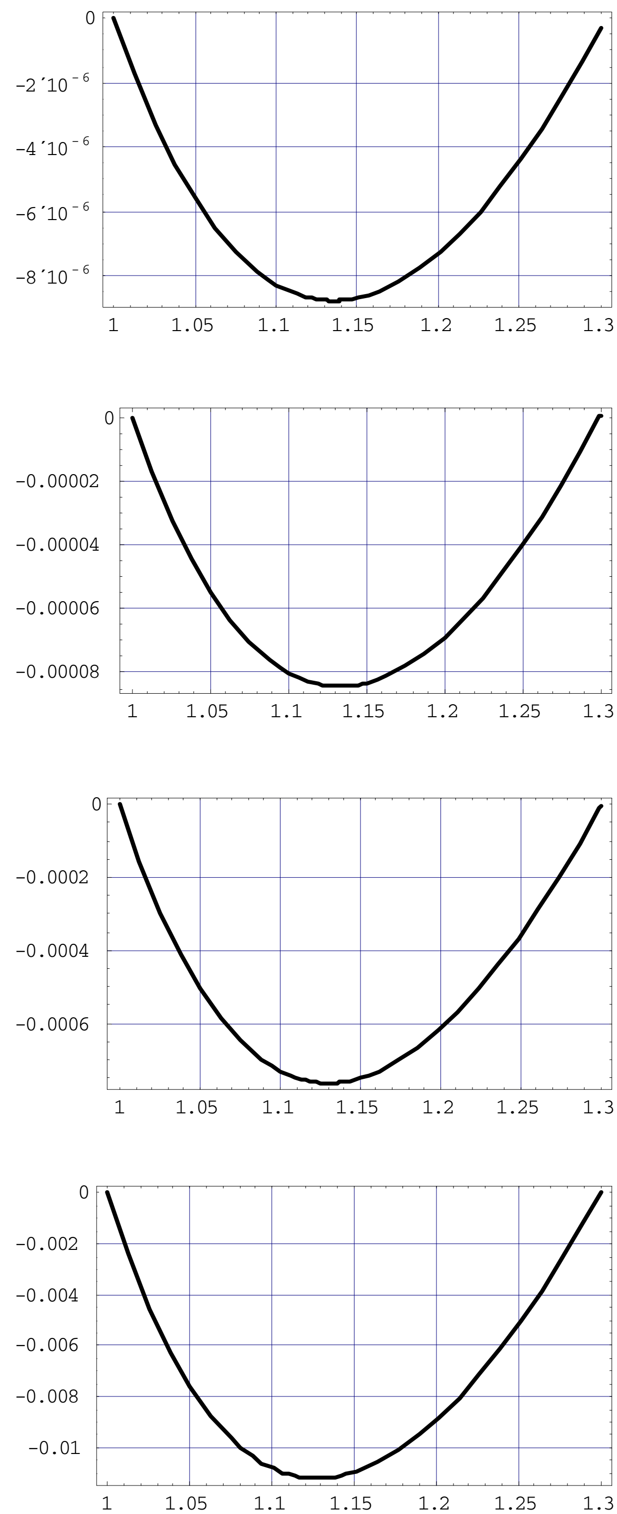

Figure 1 : (b) Normalized radial stress σ_{rr}/c (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (32).

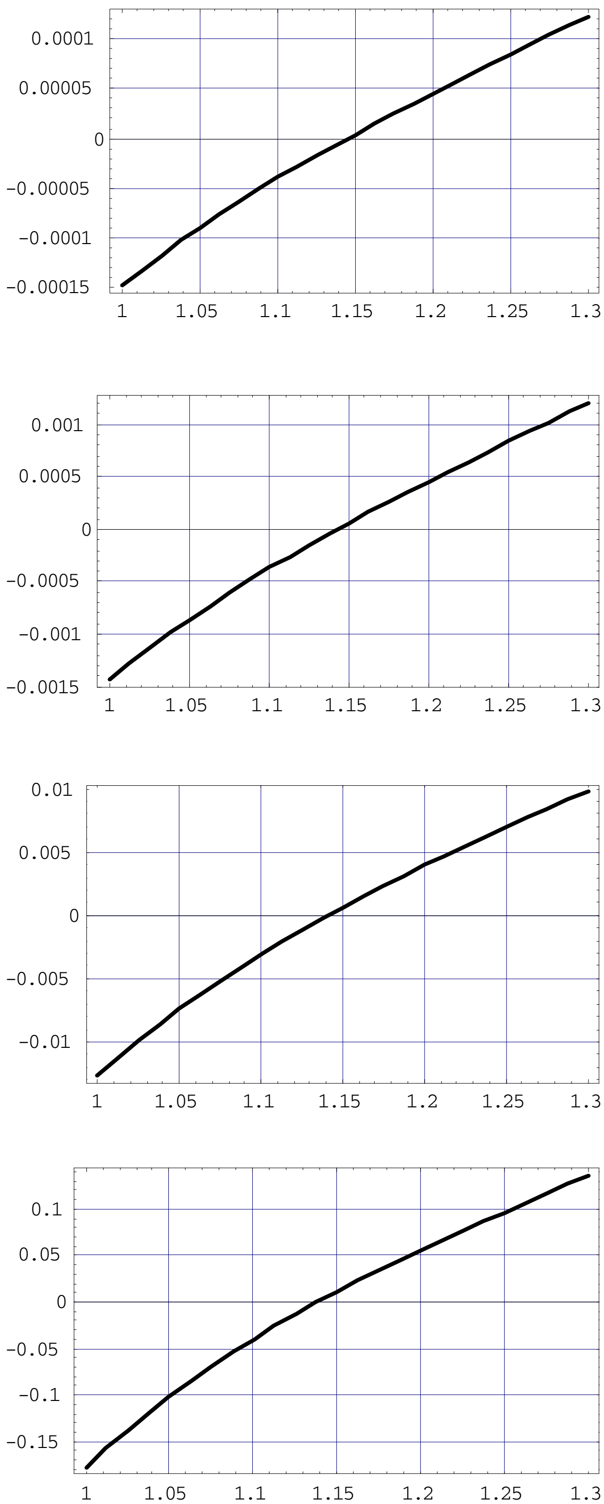

Figure 1 : (c) Normalized circumferential stress $\sigma_{\theta\theta}/c$ (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (32).

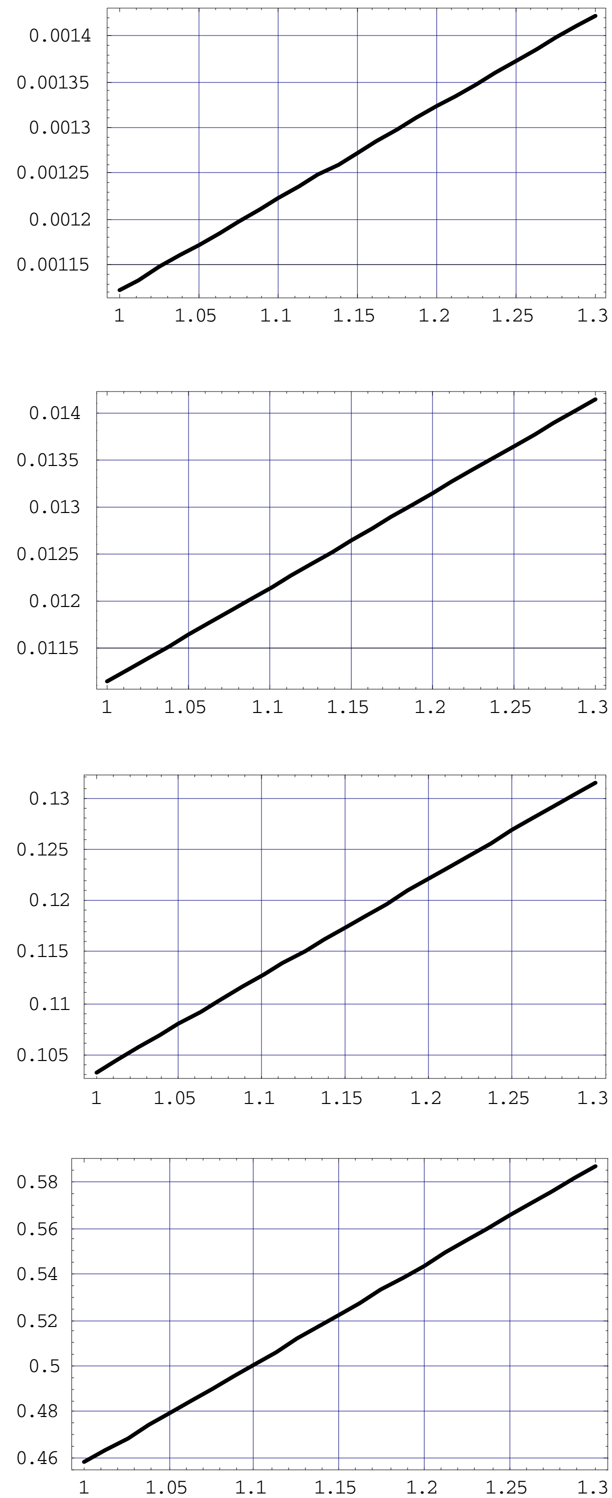

Figure 2 : (a) Normalized radial displacements $(r-R)/a$ (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (33).

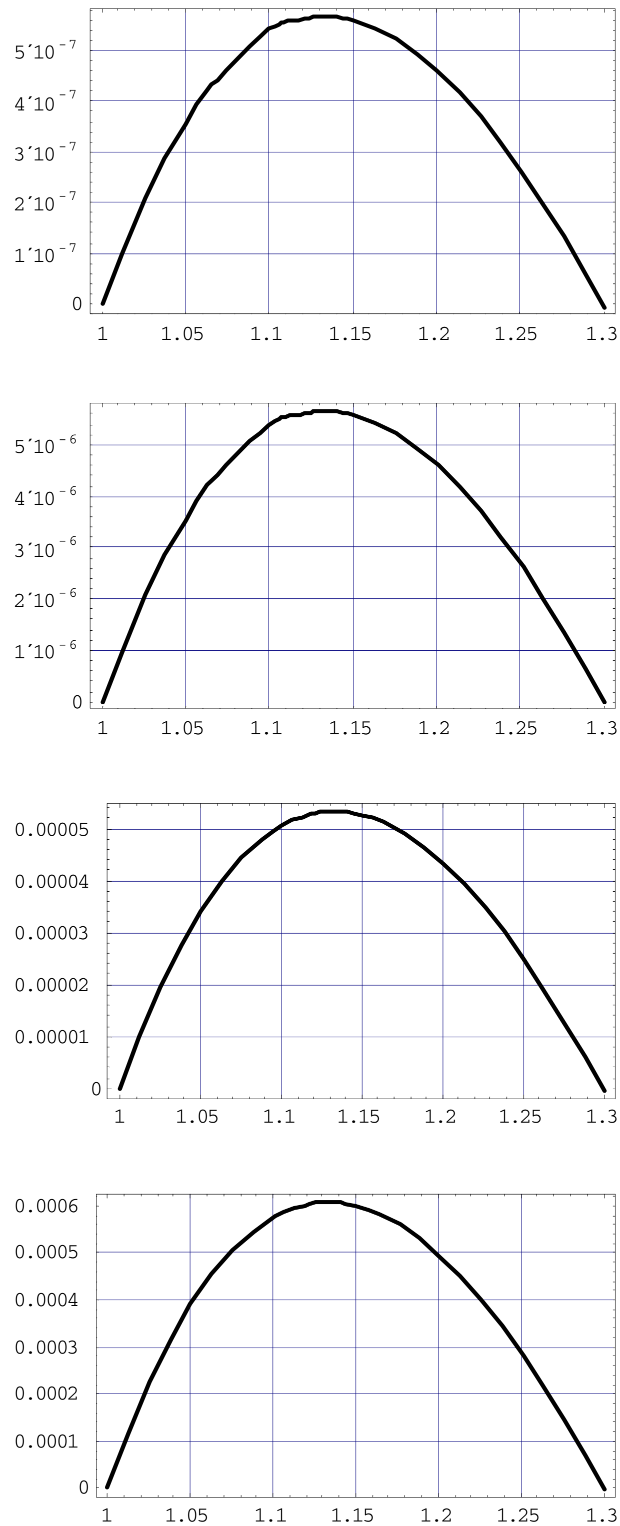

Figure 2 : (b) Normalized radial stress σ_{rr}/c (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (33).

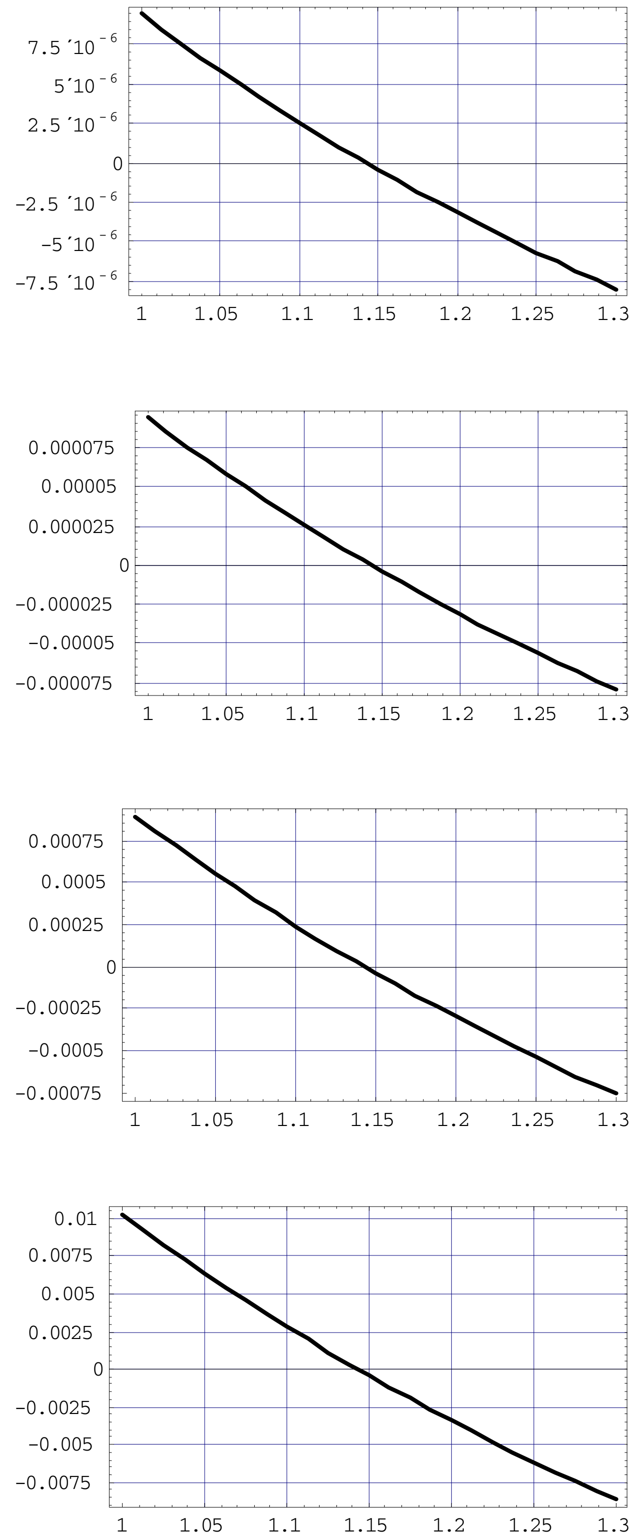

Figure 2 : (c) Normalized circumferential stress $\sigma_{\theta\theta}/c$ (vertical axis) for the dimensionless growth parameter $\alpha\tilde{\omega}/\tilde{\gamma}$ equal to 0.001; 0.01; 0.1; 1.0 (from the top to the bottom accordingly) for free volumetric growth of the cylinder: the first set of material parameters Eq. (33).

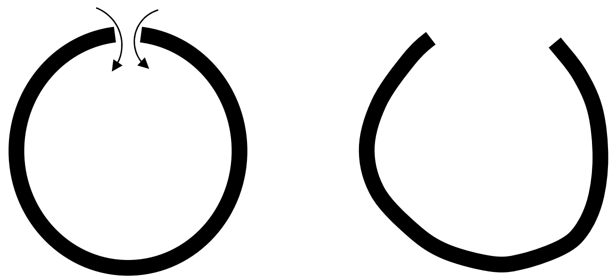

Figure 3 : (a) Bending moment (left) provides compatibility of the ring, which opens when cut (right).

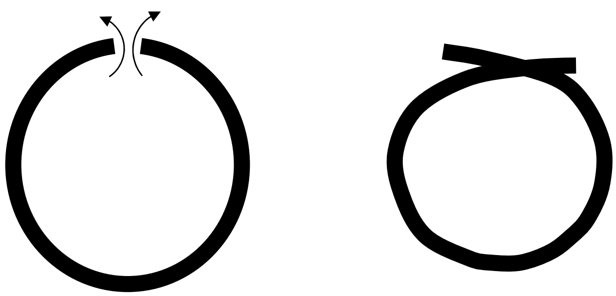

Figure 3 : (b) Bending moment (left) provides compatibility of the ring, which closes when cut (right).

It is remarkable that the results of the computation are qualitatively similar for the essentially varying growth parameter even though the deformation is large – up to 50%.

4 Discussion

A simple phenomenological theory of tissue growth has been used for explaining the phenomenon of the residual stresses in arteries qualitatively. Material anisotropy was included in the theoretical setting in accordance with the experimental data and the Eulerian constitutive framework was employed. The theory was applied to the problem of free and uniform radial growth of a cylindrical blood vessel. Displacement and stress fields were computed for the experimentally obtained values of the elasticity parameters. Computations give evidence of the appearance of the circumferential stresses resulting in the bending moments, which provide the compatibility of the grown arterial cross-section. The radial cut of the arterial ring will lead to the disappearance of the bending moments and opening or closing of the ring as it is observed in experiments. Circumferential stresses, which are accumulated in the residual stresses during the long-term growth, appear due to the arterial anisotropy. This conclusion is different from the traditional point of view stating that the differential growth is the main sources of the residual stresses (Fung, 1993). It is very likely that the material anisotropy is a complementary factor to the differential growth in causing the residual stresses. It is also interesting that depending on the mutual ratio of the parameters of anisotropy various scenarios of the ring opening in the artery-cutting experiments are available. The ring can open up after cutting – resulting in a positive opening angle; or the ring can close after the radial cut – resulting in a negative opening angle. Both these scenarios are in agreement with the experimental data (Fung, 1984; 1993; Rachev and Greenwald, 2003; Saini et al., 1995; Vaishnav and Vossoughi, 1987). It should not be missed that also radial stresses appear in the considered arterial growth. The magnitude of these stresses is of lower order of magnitude as compared to the circumferential stresses. Nonetheless, the radial stresses can play a role in forming the global residual stresses. Particularly, the radial stresses are a good candidate for the explanation of Vossoughi et al. (1993) experiments. These authors cut the opened artery ring along the midline and found that the inside segment opened more while the outside segment closed more. Probably, this happened because the radial residual stresses had been relieved partially.

Comparing the theory considered in the present work with the one considered in Volokh and Lev (2005) it is possible to conclude that both Lagrangian and Eulerian formulations of the constitutive equations give similar results. The results for the case of large growth-deformation ($\sim$50%) of the initial artery radius are presented in Figs. 4 and 5. It is clearly seen from these figures that the difference in the distribution of displacements and stresses is relatively small, i.e. all qualitative conclusions on the artery growth and residual stresses in arteries are invariant with respect to the mathematical formulation. The latter strengthens the argument that anisotropy is a possible reason for accumulation of residual stresses in arteries. This argument appears to be invariant with respect to the mathematical description.

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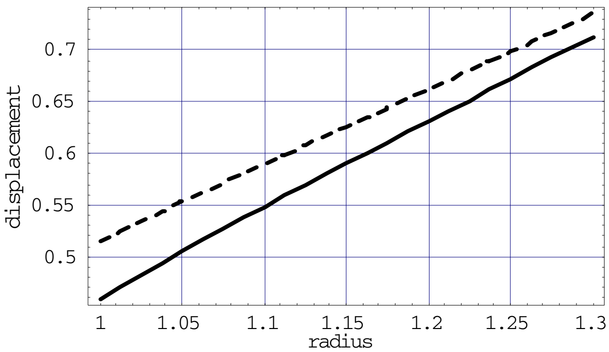

Figure 4 : (a) Normalized radial displacements $(r-R)/a$ (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the first set of material parameters Eq. (32).

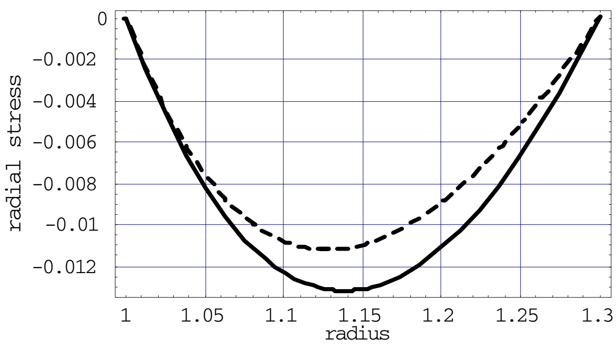

Figure 4 : (b) Normalized radial stress σ_{rr}/c (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the first set of material parameters Eq. (32).

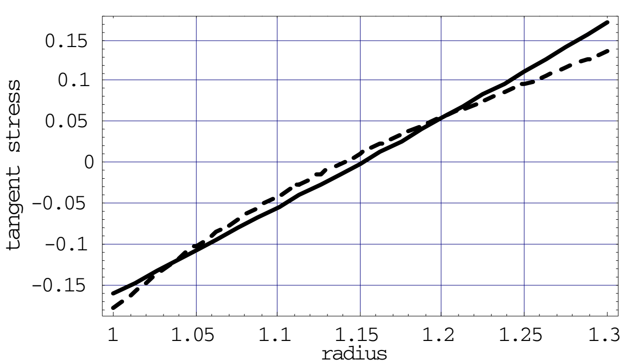

Figure 4 : (c) Normalized circumferential stress $\sigma_{\theta\theta}/c$ (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the first set of material parameters Eq. (32).

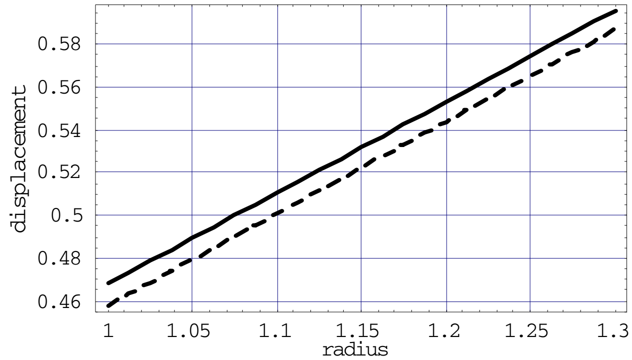

Figure 5 : (a) Normalized radial displacements $(r-R)/a$ (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the second set of material parameters Eq. (33).

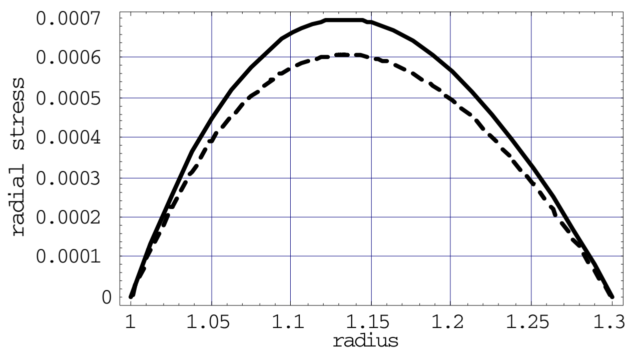

Figure 5 : (b) Normalized radial stress σ_{rr}/c (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the second set of material parameters Eq. (33).

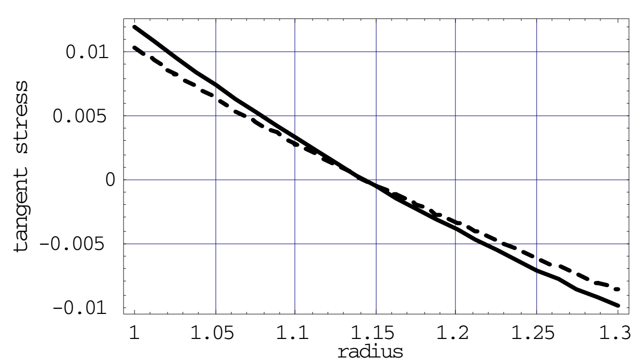

Figure 5 : (c) Normalized circumferential stress $\sigma_{\theta\theta}/c$ (vertical axis) for the Lagrangean (bold line, $\alpha\omega/\gamma = 1$) and Eulerian (dashed line, $\alpha\tilde{\omega}/\tilde{\gamma} = 1$) constitutive equations for the second set of material parameters Eq. (33).

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