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A Nonlinear Viscoelastic Finite Element Model of Polyethylene

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Abstract: A nonlinear viscoelastic finite element model of ultra-high molecular weight polyethylene (UHMWPE) was developed in this study. Eight cylindrical specimens were machined from ram extruded UHMWPE bar stock (GUR 1020) and tested under constant compression at 7% strain for 100 sec. The stress strain data during the initial ramp up to 7% strain was utilized to model the "instantaneous" stress-strain response using a Mooney-Rivlin material model. The viscoelastic behavior was modeled using the time-dependent relaxation in stress seen after the initial maximum stress was achieved using a stored energy formulation. A cylindrical model of similar dimensions was created using a finite element analysis software program. The cylinder was made up of hexahedral elements, which were given the material properties utilizing the "instantaneous" stress-strain curve and the energy-relaxation curve obtained from the experimental data. The cylinder was compressed between two flat rigid bodies that simulated the fixtures of the testing machine. Experimental stress-relaxation, creep and dynamic testing data were then used to validate the model. The mean error for predicted versus experimental data for stress relaxation at different strain levels was 4.2%. The mean error for the creep test was 7% and for dynamic test was 5.4%. Finally, dynamic loading in a hip arthroplasty was modeled and validated experimentally with an error of 8%. This study establishes a working finite element material model of UHMWPE that can be utilized to simulate a variety of postoperative arthroplasty conditions.

Keywords: Polyethylene, UHMWPE, finite element model, finite element analysis, total joint arthroplasty, hip arthroplasty, knee arthroplasty

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1 Introduction

High density polyethylene was first used as a bearing surface in total hip arthroplasty by John Charnley in 1962 [3]. Today, UHMWPE is the most common bearing surface being used in hip and knee arthroplasty. UHMWPE is a complex material with a nonlinear stress-strain relationship and inelastic behavior [9,13,16]. However, in computer simulations, the material has often been simulated with simple linear elastic properties [1,8,21].

Several investigators have attempted to characterize the complex behavior of UHMWPE. Some material models have incorporated the nonlinear elasticity by using piecewise linear [2,7], polynomial [17] or exponential [11] elastic properties. Elastoplastic formulations have been presented to capture the inelastic property either with perfectly plastic behavior [19] or with strain-hardening [1,10]. Time-dependent properties such as strain-rate dependence [9], creep [6,12,14] and stress-relaxation [20] have also been studied. While these reports have predicted various aspects of the time-dependent behavior of UHMWPE, there have been no reports of a model that can accurately predict viscoelastic behavior under a variety of loading conditions.

It is necessary to accurately model the material behavior of UHMWPE in computer simulations of clinically relevant in vitro and in vivo conditions. Investigations into the long-term performance of UHMWPE require accurate representation of time-dependent properties. In this study, we developed a nonlinear viscoelastic finite element model of UHMWPE. Stress-relaxation, creep, and dynamic experimental data were used to validate the model.

1.1 Methods

Cylindrical specimens 38.1 mm (1.5") long and 12.7 mm (0.5") in diameter were machined from ram-extruded UHMWPE bar stock (GUR 1020). All specimens were packaged in aluminum foil in vacuum and were sterilized by gamma irradiation (40KGy). For stress-relaxation experiments, 8 specimens were tested under constant compression of 7% strain for 100 sec in an Instron 8511 servohydraulic testing machine at 37^o C (Fig. 1A). The mean stress response against strain and time in these specimens was monitored. The time taken for the initial strain to reach 7% was approximately 0.07 sec. This initial stress-strain data (the first 0.07 sec) was assumed to be representative of the instantaneous UHMWPE elastic response (Fig. 2A). The subsequent stress-relaxation data (Fig. 2B) were assumed to be representative of the viscoelastic behavior of UHMWPE.

A cylindrical model was constructed using a finite element analysis software program (MARC, MSC Corporation, Santa Ana, CA). The cylinder had the same dimensions as the test specimens and was made up of 1260 8-noded hexahedral elements. The elements comprising the cylinder were assigned elastic material properties utilizing the "instantaneous" stress-strain data: An elastomeric model (Mooney-Rivlin) was used to model the instantaneous nonlinear elastic behavior of polyethylene. A stored energy equation, based on the extension of an energy formulation from small strain to large strain by Simo[18], was employed to represent nonlinear viscoelasticity. A third-order Mooney-Rivlin formulation was fitted to the experimental nonlinear stress-strain data, which also provided the initial stored energy value (the short-term variable). The viscoelastic behavior was modeled using stress relaxation experimental data at 7% strain from the experiment described above. To determine the relaxation parameters, the experimental stress relaxation data were approximated using appropriate values of the multipliers, time constants, as well as the amount of stored energy after nearly all of the relaxation has occurred.

The cylinder was compressed between two flat rigid bodies that simulated the fixtures of the Instron testing machine (Fig. 1B). To verify the model formulation, several stress-relaxation tests at 2%, 4% and 6% strain were performed experimentally on similar cylindrical specimens of UHMWPE and simulated using the same material parameters. Next, UHMWPE behavior in creep was measured after compressing experimental specimens under a constant load of 2000 N (Fig. 3A). Dynamic stress-strain response was also monitored during cyclic testing at 3% strain at 3, 0.3, and 0.03 Hz (Fig. 3B). These different loading conditions were simulated in the finite element model for each of the experiments. Stress and strain data were compared between experimental data and predicted data generated by the model.

To determine whether the finite element model could predict clinically relevant geometry and boundary conditions, polyethylene acetabular liners were tested under physiologic loading conditions. Acetabular metal-backed shells were potted into custom fixtures at an angle of 23^o and UHMWPE liners were assembled into corresponding shells. Femoral heads were mounted on adapters centered over the liners and vertical load was applied through the neck (Fig. 4A). The displacement of the femoral head was monitored during cyclic dynamic loading (peak 2500N) at 1 Hz. A finite element model of the assembly was constructed in MARC (Fig. 4B). The liner was modeled as a hemisphere with inner and outer diameters matching the nominal dimensions of the acetabular components using 8-noded solid elements. The metal backing and femoral head were modeled as rigid bodies. Contact was simulated between the inner surface liner elements and the femoral head, and between the outer surface liner elements and the metal backing. The metal backing was rigidly constrained, while the femoral head was allowed to translate in the direction of loading. No boundary conditions (other than contact) were applied to the liner. A load was applied through the femoral head similar to experimental condition and the displacement of the femoral head was calculated.

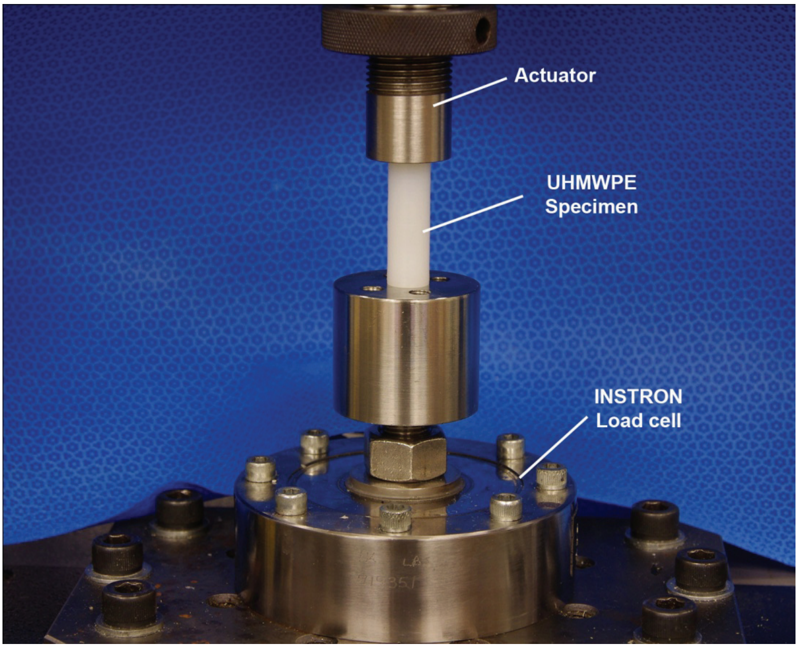

Figure 1A: Cylindrical polyethylene specimen used for material properties testing.

2 Results

2.1 Stress relaxation

There was very close agreement between the experimental data used to develop the model and the computed results obtained after simulating the model in MARC. The average absolute error at each time point for the stress relaxation over time was less than 1%. When stress-relaxation at different magnitudes of strain were simulated, the finite element model predicted the stress-relaxation in the experimental data (at 2%, 4% and 6% strains) with an average absolute error of 4.2% at each time point.

2.2 Validation of creep test

The strain at each time point during the experimental creep test was compared with the data generated by the model. The average absolute error was less than 7% of the experimental strain (Fig. 3A).

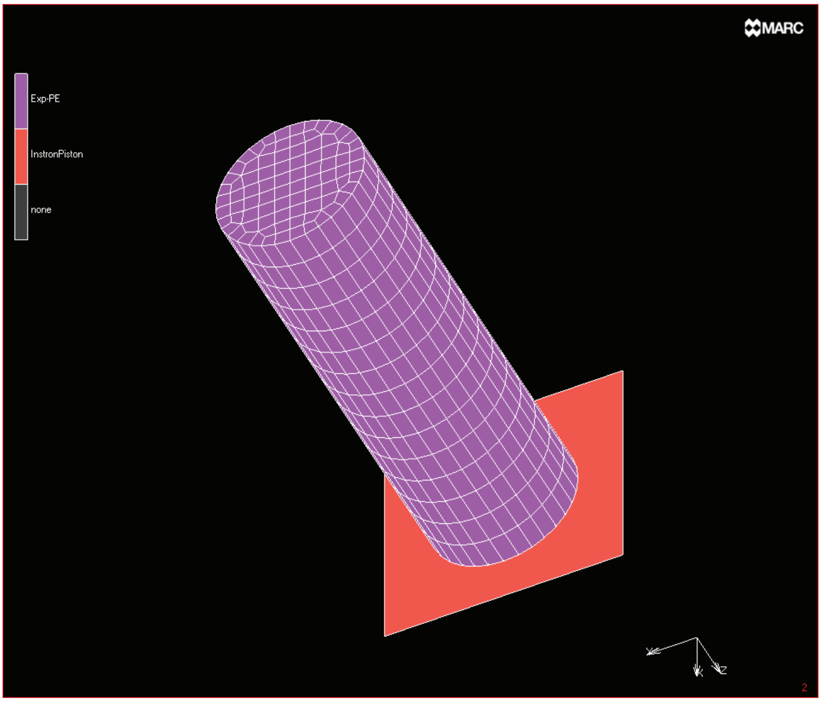

Figure 1B: FEA model of test specimen. A cylindrical model was constructed in MARC made up of 1260 8-noded elements. The cylinder was compressed between two rigid bodies simulating the platen and piston of the materials testing machine.

2.3 Validation of cyclic test

The maximum and minimum stress generated during each dynamic cycle was compared between experimental and modeled data. The mean error between the two for all the cycles at all frequencies tested was 5.4%. Figure 3B demonstrates a representative dynamic test result and the corresponding finite element prediction.

2.4 Dynamic testing of acetabular liners

Displacement of the femoral head was monitored during dynamic ramp cycling between 50 and 2500N (Fig. 4C). The predicted values were within 8% of measured values. The finite element model also predicted the cumulative creep with each dynamic cycle.

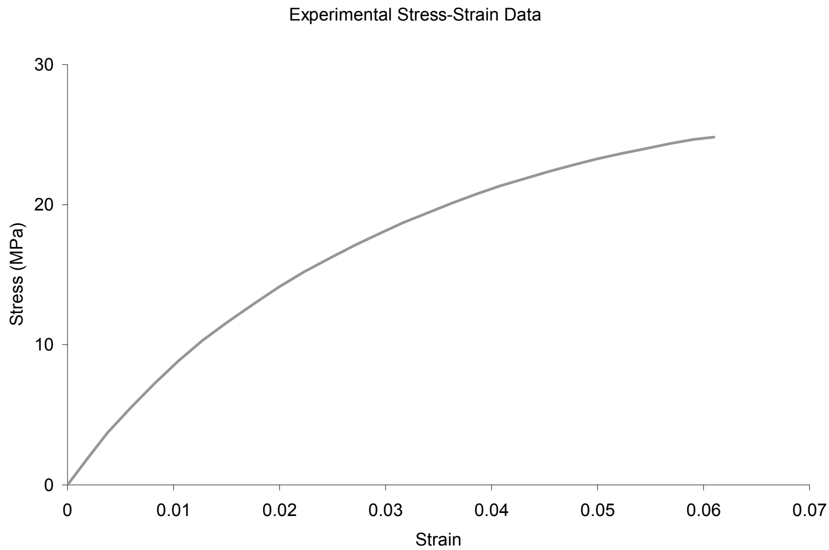

Figure 2A: Stress-strain data was obtained as the strain increased from 0% to 7% in 70 milliseconds. This was assumed to be representative of the instantaneous polyethylene elastic response.

3 Discussion

UHMWPE possesses complex nonlinear elastic and viscoelastic behavior and is currently the bearing surface of choice in joint arthroplasty [9,13,16]. For accurate simulation of clinically relevant behavior, the complex elastic and time-dependent behavior has to be accurately simulated. Although there have been several material models of UHMWPE none have captured the nonlinear elasticity and viscoelasticity under a variety of loading conditions[1,6–9,11–14,16,19–21]. Our study utilized a nonlinear viscoelastic material property to model UHMWPE using the finite element method. Incorporating nonlinear viscoelasticity predicted experimental tests under differing conditions (differing strain levels, creep and dynamic loading) within a reasonable error.

Early finite element models of UHMWPE have used linear elastic models to simulate its behavior[1,21]. This simplifying assumption may not hold up under realistic in vitro and in vivo experiments. We used a nonlinear elastomeric material property to model UHMWPE. The nonlinear elastic property of UHMWPE is an

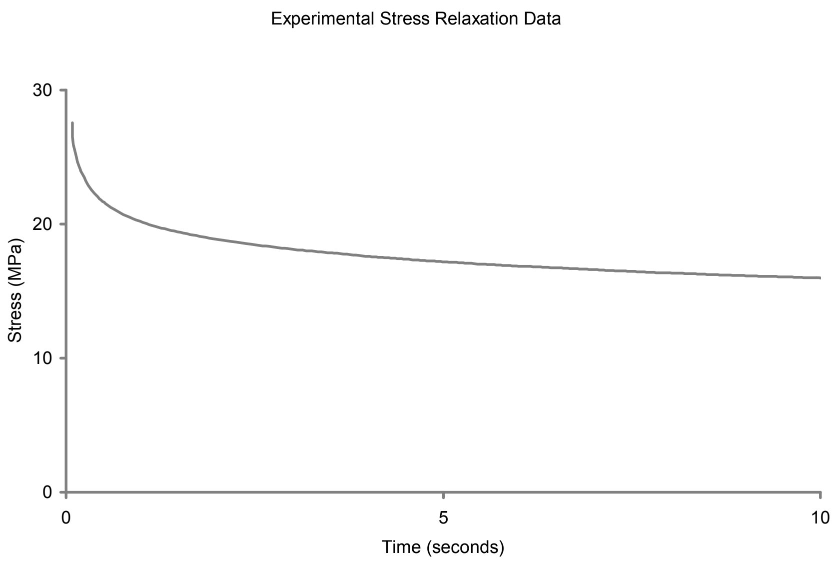

Figure 2B: Stress-relaxation data at a constant 7% strain for 10 seconds.

important model parameter since the tangent modulus of elasticity decreases significantly with increasing strain. This behavior can be beneficial since it tends to increase contact area and to reduce contact stress concentration at high loads under physiologic conditions.

3.1 Nonlinear Elasticity

Under uniaxial compression UHMWPE has a distinct nonlinear elastic response. The tangent modulus reduced from over 1 GPa at low stresses to less than 100 MPa at stresses over 20 MPa. This behavior is consistent with previous reports[4,10,11]. Experimental measurements of uniaxial compression of cylindrical UHMWPE specimens revealed a nonlinear elastic response. The compliance of UHMWPE increased at higher stresses. Since contact stresses are directly related to the stiffness of the bearing material, reduction of stiffness can significantly reduce contact stresses. This reduction in contact stresses explains why most polyethylene knee inserts do not undergo dramatic failure even though the predicted contact stresses based on linear elastic UHMWPE properties are significantly higher than the reported yield stress for UHMWPE.

Others have approximated the nonlinear elastic behavior of UHMWPE with reasonable success. Bartel et al reported on one early finite element model of UHMWPE

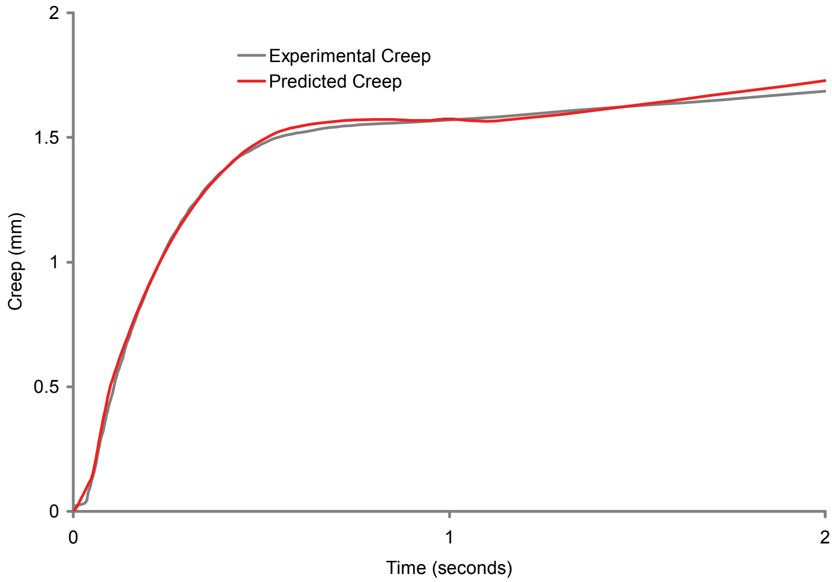

Figure 3A: Comparison between experimental data and finite element model predictions for creep.

for knee arthroplasty and defined the material properties of UHMWPE as a bilinear material with two moduli of elasticity (an initial elastic modulus of 514 MPa, a yield stress of 12.7 MPa and a post-yield modulus of 214 MPa) [1]. In a later study, the authors used a quadrilinear stress-strain relationship to describe the material properties of polyethylene [2]. The authors also noted that the nonlinear model predicted UHMWPE could withstand stresses much greater than its yield stress. Kurtz et al described a nonlinear exponential model that successfully predicted the stress-strain behavior of non-irradiated, irradiated, and oxidatively degraded UHMWPE based on the density of the material [11]. In a previous study, using the nonlinear elastoplastic material from Kurtz et al, in a finite element model, we found in vivo contact stresses to be lower than the reported yield of UHMWPE after total knee arthroplasty[5].

3.2 Stress Relaxation

Waldman and Bryant described the stress relaxation behavior of irradiated UHMWPE in uniaxial compression[20]. The authors used a modified superposition principle to approximate the nonlinear viscoelastic behavior[15]. A 1-dimensional stress-relaxation

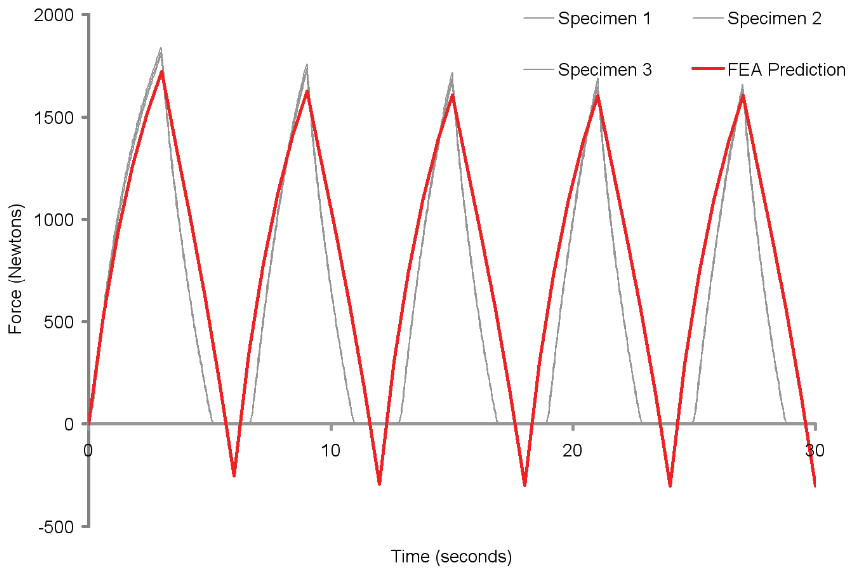

Figure 3B: Dynamic force response under 3% compressive strain. The dynamic compressive cycles were strain-controlled, and the specimen did not fully recover between cycles. Therefore, with progressive cycles, zero force was registered by the load cell when the machine actuator lost contact with top of the specimen.

constitutive relation for an arbitrary strain history was defined using kernel functions. Several step strain experiments were performed and the observed stress-relaxation data were used to determine the kernel functions. An optimum fit between experimental data to that predicted by the stress-relaxation constitutive relation was reported using three kernel functions. The authors acknowledged the limited application of the constitutive relation to uniaxial conditions. An additional limitation was the lack of validation with experimental data measuring creep under constant load.

3.3 Creep

Creep is an important clinical parameter and can contribute significantly to radiographic "wear rates" based on measured penetration of the femoral head in the acetabular component. Little reported on the linear compressive creep behavior of UHMWPE when plotted on log-log scale[14]. The isochronous (100 second) secant modulus at multiple stress levels was also linear on a log-log scale suggesting linear

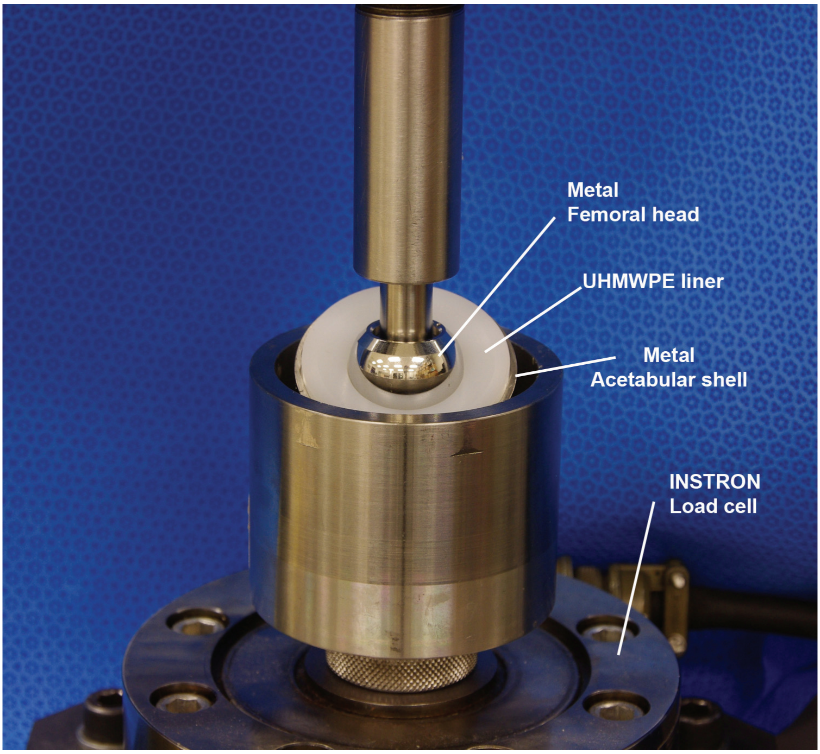

Figure 4A: Experimental setup of acetabular liner testing.

viscoelastic behavior up to a stress of 3 MPa. Lee and Pienkowski also reported linear variation of creep strains and creep strain rates with stress levels ranging from 2 to 8 MPa [12]. Deng et al found that the long-term UHMWPE creep was not linear even on a logarithmic plot and found an empirical formula [creep compliance = $c_1(\log t)^b$] best predicted both the short-term and long-term creep behavior[6]. These reports did not attempt to use the material models (derived from experimental creep data) to predict stress relaxation or dynamic loading. In the present study, although experimental stress-relaxation data was used to model the viscoelastic properties of UHMWPE, creep was also predicted with reasonable accuracy. A finite element model that can accurately predict and incorporate creep in contact analysis can be extremely valuable in providing insight in understanding the long-term behavior of UHMWPE in vivo.

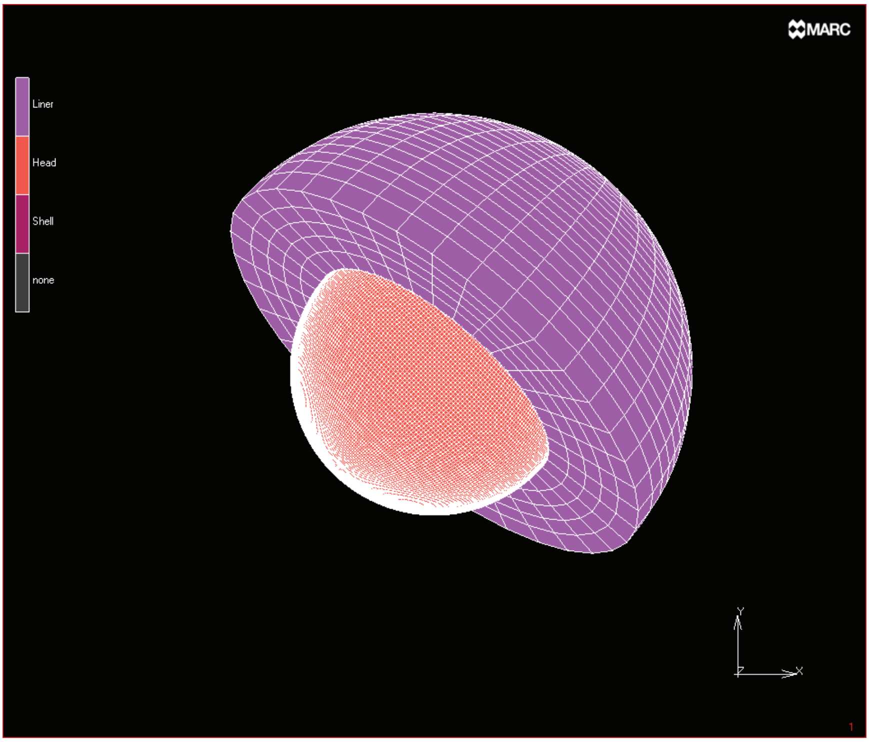

Figure 4B: Finite element model of acetabular component.

3.4 Study Limitations

Material properties of UHMWPE were obtained at only one temperature (37^{o}C), since this was thought to be most clinically relevant. The temperature dependency of UHMWPE material properties is well known and the behavior at different temperatures would probably yield very different results. Another limitation is that the plastic behavior of UHMWPE was not modeled. Previous studies have used the 0.2 % offset yield to define the threshold for plasticity behavior [10]. However, the plastic behavior of UHMWPE is not clearly defined and does not occur at a specific stress or strain level. Also a significant fraction of the so-called "plastic deformation" has been shown to recover over time. The range of strains studied experimentally was found to be below any measurable plastic behavior. In addition, almost all the compressive creep strain recovered within a finite time interval after testing. Since physiologic loading of UHMWPE in vivo after hip arthroplasty is rarely expected to exceed these strain levels, we elected to ignore the plastic behavior. Only

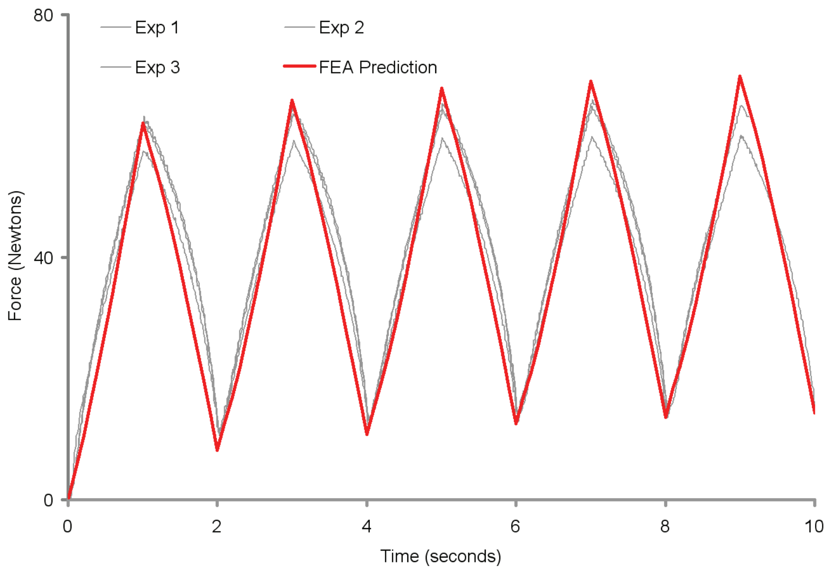

Figure 4C: Comparison between experimental and predicted data.

one type of polyethylene (ram-extruded GUR 1020 bar stock irradiated at 40 kGy) was tested. Several factors can affect the material properties of UHMWPE including material source, manufacturing process, and sterilization process. For accurate representation, fitting nonlinear viscoelastic parameters to experimental data from each different type of polyethylene may be necessary. Within these limitations, this study establishes a working finite element material model of UHMWPE that can be utilized to simulate clinical conditions with reasonable accuracy.

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