

Supplementary material on the manuscript entitled: Elite Population-Driven Backtracking Search Algorithm

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We discussed the performance of the Improved Backtracking Search Optimization Algorithm (IBSA) on the CEC 2017 test set ¹ in the supplementary materials and compared it with the classical gaining-sharing knowledge based algorithm (GSK) ². The experimental results are presented in Supplementary Tables S1 and S2. In Supplementary Tables S1 and S2, Mean solution(MEAN) and BEST represent the average value and the optimal value, respectively. In Supplementary Tables 1 and 2, GSK's results were directly extracted from the original literature, and the experimental results of IBSA are obtained through simulation, and the population size and maximum number of function evaluations in the simulation parameters are the same as those in the original GSK literature.

By comparing the experimental results of IBSA and GSK on the four dimensions of the Congress on Evolutionary Computation (CEC) 2017 test set shown in Supplementary Tables S1 and S2, the following conclusions can be drawn:

- 10-dimensional. GSK can get better BEST than IBSA in 13 functions, which are F₁, F₃, F₄, F₆, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₆, F₁₈, and F₁₉, F₃₀. In addition, GSK has the same BEST with IBSA in four functions including F₉, F₂₁, and F₂₅, F₂₇. GSK outperforms IBSA in 14 functions in terms of MEAN, which are F₁, F₃, F₄, F₆, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₈, F₁₉, F₂₇, F₂₈, and F₂₉. In addition, GSK can obtain the same MEAN with IBSA in F₉ and F₂₆.
- 30-dimensional. GSK is superior to IBSA in 15 functions in terms of BEST, which are F₁, F₃, F₄, F₆, F₉, F₁₁, F₁₂, F₁₃, F₁₅, F₁₈, F₁₉, F₂₀, F₂₃, F₂₇, and F₃₀. In addition, GSK can achieve the same performance with IBSA in terms of BEST in F₂₂ and F₂₈. GSK can beat IBSA in 18 functions in terms of MEAN, which are F₁, F₃, F₄, F₆, F₉, F₁₁, F₁₃, F₁₄, F₁₅, F₁₈, F₁₉, F₂₀, F₂₂, F₂₅, F₂₆, F₂₇, F₂₈, and F₃₀.
- 50-dimensional. GSK shows better performance than IBSA in terms of BEST, which can beat IBSA in 17 functions including F₃, F₄, F₆, F₉, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₆, F₁₈, F₁₉, F₂₃, F₂₄, F₂₇, F₂₉, and F₃₀. In addition, GSK can get the same BEST with IBSA in F₂₂, F₂₅, and F₂₈. GSK can obtain better MEAN than IBSA in 17 functions F₁, F₃, F₄, F₆, F₉, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₈, F₁₉, F₂₃, F₂₆, F₂₇, F₂₉, and F₃₀. In addition, GSK and IBSA have the same MEAN in F₂₈.
- 100-dimensional. GSK can achieve better BEST than IBSA in 24 functions including F₁, F₃, F₄, F₅, F₆, F₇, F₈, F₉, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₆, F₁₇, F₁₈, F₁₉, F₂₃, F₂₄, F₂₅, F₂₇, F₂₈, F₂₉, and F₃₀. GSK can beat IBSA in 23 functions in terms of MEAN, which are F₁, F₃, F₄, F₅, F₆, F₉, F₁₁, F₁₂, F₁₃, F₁₄, F₁₅, F₁₆, F₁₇, F₁₈, F₁₉, F₂₁, F₂₃, F₂₄, F₂₅, F₂₆, F₂₇, F₂₈, F₂₉, and F₃₀.

¹ The detailed information of CEC 2017 test suite can refer to the link :<https://github.com/P-N-Suganthan/CEC2017-BoundConstrained>

² Mohamed A W, Hadi A A, Mohamed A K. Gaining-sharing knowledge based algorithm for solving optimization problems: a novel nature-inspired algorithm[J]. International Journal of Machine Learning and Cybernetics, 2020, 11(7): 1501-1529.

Supplementary Table S1: The experimental results of IBSA and GSK in solving CEC 2017 test suite with 10dimensional and 30-dimensional.

No.	CEC 2017 with 10-dimensional				CEC 2017 with 30-dimensional			
	IBSA		GSK		IBSA		GSK	
	MEAN	BEST	MEAN	BEST	MEAN	BEST	MEAN	BEST
F ₁	1.99E+02	1.51E-01	0.00E+00	0.00E+00	1.36E+02	3.05E-01	0.00E+00	0.00E+00
F ₃	4.50E-03	3.40E-06	0.00E+00	0.00E+00	3.43E+03	1.01E+03	7.07E-07	0.00E+00
F ₄	2.35E+00	1.73E-02	0.00E+00	0.00E+00	5.41E+01	5.50E-02	1.11E+01	9.28E-03
F ₅	7.18E+00	1.58E+00	2.03E+01	1.53E+01	8.02E+01	3.18E+01	1.60E+02	1.36E+02
F ₆	1.63E-10	7.96E-12	0.00E+00	0.00E+00	3.79E-02	1.43E-07	1.52E-06	0.00E+00
F ₇	1.83E+01	9.74E+00	3.07E+01	2.46E+01	9.77E+01	7.04E+01	1.87E+02	1.64E+02
F ₈	6.30E+00	1.19E+00	2.02E+01	1.45E+01	7.16E+01	3.28E+01	1.55E+02	1.24E+02
F ₉	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.87E+00	1.14E-13	0.00E+00	0.00E+00
F ₁₀	4.33E+02	1.51E+02	1.06E+03	7.54E+02	3.38E+03	1.49E+03	6.69E+03	5.84E+03
F ₁₁	3.33E+00	1.25E-02	0.00E+00	0.00E+00	7.07E+01	1.69E+01	3.30E+01	6.00E-01
F ₁₂	1.11E+04	6.92E+02	8.93E+01	1.42E+01	3.00E+04	4.64E+03	6.62E+03	1.03E+03
F ₁₃	1.67E+01	7.39E+00	6.56E+00	9.19E-01	1.22E+04	6.10E+01	9.83E+01	4.61E+01
F ₁₄	9.49E+00	1.29E+00	5.88E+00	5.19E-03	5.86E+01	3.92E+01	5.69E+01	4.72E+01
F ₁₅	2.69E+00	8.84E-01	2.22E-01	2.96E-03	4.00E+03	2.91E+01	1.45E+01	2.13E+00
F ₁₆	1.73E+00	4.29E-01	4.27E+00	3.29E-01	5.06E+02	5.98E+01	7.96E+02	4.35E+02
F ₁₇	1.46E+01	4.73E-01	1.17E+01	9.76E-01	1.38E+02	4.23E+01	1.89E+02	6.28E+01
F ₁₈	2.28E+01	1.18E+01	3.20E-01	2.05E-02	1.09E+04	5.24E+02	3.68E+01	2.25E+01
F ₁₉	2.47E+00	1.43E+00	1.55E-01	2.81E-02	4.62E+03	4.13E+01	1.29E+01	3.42E+00
F ₂₀	7.11E-01	6.82E-13	1.19E+00	3.12E-01	1.45E+02	3.56E+01	1.08E+02	1.34E+00
F ₂₁	1.37E+02	1.00E+02	1.93E+02	1.00E+02	2.66E+02	2.38E+02	3.46E+02	3.20E+02
F ₂₂	9.17E+01	4.00E-10	1.00E+02	1.00E+02	1.75E+02	1.00E+02	1.00E+02	1.00E+02
F ₂₃	3.08E+02	3.00E+02	3.18E+02	3.11E+02	3.99E+02	3.77E+02	4.70E+02	3.50E+02
F ₂₄	2.87E+02	1.00E+02	3.44E+02	2.78E+02	4.68E+02	4.41E+02	5.68E+02	5.29E+02
F ₂₅	4.20E+02	3.98E+02	4.27E+02	3.98E+02	3.88E+02	3.83E+02	3.87E+02	3.87E+02
F ₂₆	3.00E+02	2.00E+02	3.00E+02	3.00E+02	1.43E+03	2.00E+02	9.87E+02	6.93E+02
F ₂₇	3.90E+02	3.89E+02	3.89E+02	3.89E+02	5.12E+02	4.91E+02	4.93E+02	4.80E+02
F ₂₈	3.42E+02	2.37E+00	3.12E+02	3.00E+02	3.61E+02	3.00E+02	3.21E+02	3.00E+02
F ₂₉	2.59E+02	2.32E+02	2.48E+02	2.39E+02	5.47E+02	4.20E+02	5.77E+02	4.23E+02
F ₃₀	2.05E+04	6.74E+02	4.56E+02	3.96E+02	4.48E+03	2.11E+03	2.08E+03	1.94E+03

Abb: IBSA, improved backtracking search algorithm; CEC, Congress on Evolutionary Computation; MEAN, mean solution; GSK, gaining-sharing knowledge based algorithm.

Overall, at low dimensions (dimensions below 50), there is no significant difference in performance between IBSA and GSK. When the dimension is 10, the overall performance of IBSA is slightly better than that of GSK, while when the dimensions are 30 and 50, GSK's overall performance is slightly better than that of IBSA. When the dimension is 100, GSK's overall performance is significantly better than that of IBSA. This indicates that IBSA remains competitive in solving low-dimensional optimization problems, but this competitiveness diminishes as the dimension increases.

Supplementary Table S2: The experimental results of IBSA and GSK in solving CEC 2017 test suite with 50-dimensional and 100-dimensional.

No.	CEC 2017 with 50-dimensional				CEC 2017 with 100-dimensional			
	IBSA		GSK		IBSA		GSK	
	MEAN	BEST	MEAN	BEST	MEAN	BEST	MEAN	BEST
F ₁	3.03E+03	8.69E-01	1.09E+03	1.71E+01	6.55E+03	1.52E+02	5.80E+03	3.96E-01
F ₃	2.55E+04	1.13E+04	3.85E+03	1.61E+03	1.19E+05	9.27E+04	1.15E+05	6.01E+04
F ₄	8.77E+01	1.02E+01	8.33E+01	1.33E-02	6.36E+02	5.31E+02	2.05E+02	8.46E+01
F ₅	1.90E+02	1.02E+02	3.20E+02	2.65E+02	1.03E+03	8.90E+02	5.31E+02	7.26E+01

F ₆	4.37E-01	3.75E-02	3.78E-06	3.11E-07	6.08E+02	6.03E+02	1.72E-03	1.25E-05
F ₇	1.91E+02	1.38E+02	3.70E+02	3.29E+02	1.28E+03	1.07E+03	8.75E+02	8.33E+02
F ₈	1.84E+02	1.28E+02	3.24E+02	2.90E+02	1.35E+03	1.03E+03	4.92E+02	5.90E+01
F ₉	2.61E+02	5.70E+00	1.07E-02	0.00E+00	7.67E+03	2.14E+03	8.46E+00	5.45E+00
F ₁₀	6.25E+03	3.80E+03	1.30E+04	1.21E+04	1.47E+04	1.19E+04	2.95E+04	2.87E+04
F ₁₁	1.55E+02	6.20E+01	3.45E+01	2.34E+01	1.82E+03	1.42E+03	2.78E+02	1.61E+02
F ₁₂	2.47E+05	5.77E+04	9.46E+03	2.16E+03	8.26E+05	1.80E+05	8.34E+04	2.25E+04
F ₁₃	4.06E+03	1.30E+02	1.49E+03	7.41E+01	7.28E+03	2.65E+03	3.20E+03	5.17E+01
F ₁₄	6.33E+02	1.12E+02	1.24E+02	5.76E+01	6.52E+04	1.01E+04	4.64E+03	3.30E+02
F ₁₅	6.00E+03	1.09E+02	4.20E+01	2.52E+01	4.13E+03	1.79E+03	7.33E+02	3.20E+01
F ₁₆	1.08E+03	4.62E+02	1.83E+03	1.30E+02	4.81E+03	3.33E+03	2.27E+03	2.87E+02
F ₁₇	7.76E+02	3.63E+02	1.35E+03	7.75E+02	4.20E+03	3.12E+03	3.91E+03	2.14E+03
F ₁₈	7.25E+04	2.49E+04	5.98E+02	1.78E+02	2.69E+05	5.65E+04	5.73E+04	1.93E+04
F ₁₉	9.17E+03	3.83E+02	3.05E+01	1.84E+01	4.22E+03	2.12E+03	1.00E+03	5.04E+01
F ₂₀	4.11E+02	1.45E+02	1.37E+03	1.17E+03	4.05E+03	3.39E+03	4.46E+03	3.97E+03
F ₂₁	3.53E+02	2.79E+02	5.21E+02	4.90E+02	2.69E+03	2.56E+03	6.07E+02	2.83E+02
F ₂₂	6.17E+03	1.00E+02	1.10E+04	1.00E+02	2.00E+04	1.46E+04	3.00E+04	2.86E+04
F ₂₃	5.48E+02	4.80E+02	5.42E+02	4.20E+02	3.14E+03	3.02E+03	6.11E+02	5.86E+02
F ₂₄	6.04E+02	5.60E+02	6.34E+02	5.01E+02	3.64E+03	3.48E+03	9.32E+02	8.89E+02
F ₂₅	5.51E+02	4.60E+02	5.56E+02	4.60E+02	3.26E+03	3.12E+03	8.21E+02	7.61E+02
F ₂₆	2.92E+03	3.00E+02	1.27E+03	1.06E+03	1.23E+04	2.90E+03	3.66E+03	3.33E+03
F ₂₇	6.39E+02	5.37E+02	5.92E+02	5.18E+02	3.51E+03	3.36E+03	6.57E+02	6.20E+02
F ₂₈	4.94E+02	4.59E+02	4.94E+02	4.59E+02	3.37E+03	3.32E+03	5.53E+02	4.99E+02
F ₂₉	7.88E+02	3.48E+02	3.60E+02	3.28E+02	6.16E+03	5.22E+03	1.21E+03	9.12E+02
F ₃₀	7.64E+05	5.88E+05	5.96E+05	5.79E+05	9.54E+03	5.99E+03	2.99E+03	2.41E+03

Abb: IBSA, improved backtracking search algorithm; CEC, Congress on Evolutionary Computation; MEAN, mean solution; GSK, gaining-sharing knowledge based algorithm.