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Probability Methods for Estimation of Cleavage Fracture Toughness from Small Data Sets

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Abstract: Consideration of the structural integrity is one of the inputs when evaluating potential solutions to plant problems. Structural integrity assessments of components forming the pressure boundaries of nuclear plant evaluate safety margins against cleavage fracture. These assessments consider the reserve factors between the applied stress and fracture toughness of the material as well as temperature margins between the operating temperature and the temperature at which the steel is ductile as defined by upper shelf behaviour. To carry out these structural integrity assessments, estimates of cleavage fracture toughness are required. The approach presented in this paper allows for differences between cleavage fracture toughness properties of different materials associated with material to material variability. Mean cleavage fracture toughness properties are described as a function of temperature, section thickness and ductile crack growth. The random scatter is described by standard statistical probability distributions with variance a constant percentage of the mean cleavage fracture toughness. Micromechanisms of cleavage fracture and numerical modelling show that fracture toughness is determined by the microstructure of the steel which also defines the work hardening characteristics, the yield stress and its temperature dependence. On this basis, temperature and thickness dependence, and scatter of cleavage fracture toughness are specific to a given type of material. This paper presents methods for estimation of cleavage fracture toughness from small data sets using simple statistical tools and cleavage fracture toughness curves fitted to large databases for materials with the specification similar to the small data set. Applications of this methodology are presented. The methodology is validated by comparing the predicted values with data that were not used in the analyses. One of the applications illustrated involves predictions of cleavage fracture toughness for the neutron irradiated condition. This is validated by comparing predictions with data measured on specimens removed from ex-service reactors.

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1 Introduction

Assessments of components forming a pressure boundary, certainly for nuclear power generating plant, need to consider safety margins against brittle fracture [Dowling and Flewitt (2003)]. It is well recognised that cleavage fracture toughness of ferritic steels is dependent on a number of variables the most important of which are test temperature, yield stress, crack length which is governed by component thickness, component width and propensity of the steel to work hardening [Knott (2003)]. In order to predict cleavage fracture toughness, it would be necessary to develop a functional relationship for cleavage fracture toughness and the variables referred to earlier. A large database would be required to evaluate all the variables that affect cleavage fracture toughness to the required level of confidence.

To provide a direct and simply interpreted procedure for the assessment of cleavage fracture toughness the so-called 'Master Curve' approach has been implemented in the ASTM E1921 standard [ASTM Standard E 1921-97 (1997)]. The standard is based on the approach developed by Wallin where the behaviour of cleavage fracture toughness is considered in terms of the weakest link concept [Wallin (1984)]. This requires the random variability of cleavage fracture toughness to be represented by a Weibull distribution. The cumulative probability of cleavage failure, P_{fi}, for fracture toughness tests conducted at a single test temperature is given by:

$$P_{fi} = 1 - \exp\left\{ - \left(\frac{K_{JCi} - K_{\min}}{K_0 - K_{\min}} \right)^b \right\} \tag{1}$$

where K_{min} is the threshold value of cleavage fracture toughness below which the probability of cleavage is zero, K_{JCi} is the measured cleavage fracture toughness value associated with P_{fi}, K_0 is a scale parameter which corresponds to 63.2% cumulative failure probability and b is the shape parameter [Lawless (1982)]. In equation (1), the value of b is fixed to be 4, K_{min} is 20 MPa$\sqrt{}$m and K_0 needs to be estimated by statistical analysis. To combine fracture toughness data obtained from specimens of different thicknesses, B, into a single probability distribution the data are thickness adjusted to a reference specimen thickness, B_0, equal to 25mm, as follows:

$$K_{0(25mm)} = K_{\min} + (K_{JCi} - K_{min}) \left(\frac{B_i}{B_0}\right)^{1/4} \tag{2}$$

It is noteworthy that the parameter for thickness correction is set at $1/4$ rather than estimated by statistical analysis.

In the Master Curve concept described by Wallin et al [Wallin, Törrönen, Ahlstrand, Timofeev, Rybin, Nikolaev and Morozov (1992)], the temperature dependence of cleavage fracture toughness is assumed to be the same for all low alloy ferritic steels irrespective of composition or thermo-mechanical history. The relationship for the temperature dependence is given by:

$$K_0 = 31 + 77 \exp\{0.019 (T - T_0)\} \tag{3}$$

where 0.019 is the parameter for the temperature dependence of cleavage fracture toughness. The temperature at which the median value of cleavage fracture toughness is 100MPa$\sqrt{}$m is denoted T_0, the reference temperature. Therefore this approach for describing of cleavage fracture toughness requires four parameters, three, of which are fixed: (i) shape parameter, (ii) thickness correction factor and (iii) the temperature dependence parameter. The fourth, a scale parameter equivalent to T_0, needs to be estimated. The procedure for this estimation of T_0 is given in the ASTM standard E1921 [ASTM Standard E 1921-97 (1997)] and requires a minimum of six valid cleavage fracture toughness test measurements to be undertaken on the specific material. These data are used to estimate the value of T_0, or its equivalent K_0, by employing the likelihood principle to maximise the value of T_0.

The Master Curve approach imposes a restriction on data that can be used in the analyses. The restriction is based on the validity limit for J controlled fracture, K_{LIM}, that can be evaluated from:

$$K_{LIM} \leq \sqrt{\frac{b_0 R_{p0.2} E}{M (1 - \upsilon)}} \tag{4}$$

where $R_{p0.2}$ is the yield stress appropriate to the test temperature, E is the Young modulus of elasticity, b_0 is the initial size of the unbroken ligament, M is the size criterion constant which has a value of 30 derived from finite element analysis and υ is the Poisson ratio. All data that are greater than the limiting value given by equation (4) are set to this limiting value and treated as censored data [ASTM Standard E 1921-97 (1997)].

The estimates of cleavage fracture toughness obtained from the Master Curve analyses of measurements undertaken on ferritic steel are compared by Heerens et al [Heerens, Ainsworth, Moskovic and Wallin] with results of other methods of analysis. One of the methods used in this comparison employed the competing risk methodology described by Moskovic [Moskovic (1992); Moskovic and Crowder (1995); Moskovic (1995)]. Although the basis of the competing risk statistical approach is very similar to that for the Master Curve method, it is much more flexible. This flexibility arises for several reasons:

- All the constants in the equations for cleavage fracture toughness can be estimated by statistical analysis.
- Data that are both smaller and greater than the J validity limit can be included in the analysis. Only those data obtained from tests terminated by unloading the specimens are treated as censored values.
- The approach can provide a description of cleavage fracture toughness across the whole ductile to brittle transition temperature range because the competing risk approach can include data for cleavage instability after prior ductile crack growth. By comparison, the Master Curve approach is essentially restricted to the lower transition temperature associated with cleavage instability at the initiation of cracking.
- In the Master Curve approach, physical interpretation of cleavage crack initiation is based on the concept of weakest link and this, in turn, invokes a Weibull distribution to describe random scatter in cleavage fracture toughness data. Competing risks

can select the best fit distribution from a range of lifetime distributions.

In Section 2.0 we describe briefly the competing risk methodology. The experimental procedures and fracture toughness data obtained for C-Mn steel plate and forging material are presented in Sections 3.0 and 4.0. The analyses of these data is based on both a reference curve method and a binomial model and these are described and discussed in Section 5.0. Finally, concluding comments are made in Section 6.

2 Analysis Methodology

2.1 Competing Risks

In the ductile to brittle transition temperature region, fracture toughness tests can be terminated by either the onset of cleavage instability or by unloading the specimens [Moskovic and Crowder (1995)]. The former generates random values of cleavage fracture toughness and the latter censored values of fracture toughness that represent survivor probabilities. Cleavage instability can occur either at the initiation of cracking or after some prior ductile crack growth. Cleavage crack initiation is a low temperature cracking mechanism which is progressively replaced by the ductile crack initiation mechanism as the test temperature is increased [Moskovic and Crowder (1995)]. Below a certain critical temperature the probability of cleavage initiation is 1. Over a narrow temperature range above this critical temperature, the probability of cleavage crack initiation decreases from 1 to 0 as the temperature increases. Ductile crack initiation is followed by crack growth. The amount of pre-cleavage ductile crack growth increases with increasing temperature until the condition is reached when this is 100% fully ductile. The experimentally observed values of both cleavage fracture toughness and pre-cleavage ductile crack growth are random variables and exhibit an appreciable amount of scatter. Furthermore, the relationship between the fracture toughness and ductile crack growth is prescribed by a crack growth resistance curve.

The distributions of cleavage fracture toughness and pre-cleavage ductile crack growth can be described using log linear regression models by defining two parameters: one linked to the mean and referred to as the location parameter and the other that describes the scatter of the data and is referred to as the scale parameter. The magnitude of the mean values, location parameter, for both cleavage fracture toughness and pre-cleavage ductile crack growth are dependent, through a functional relationship, on the experimental variables. The variables of particular importance are test temperature and specimen thickness and, as mentioned earlier, the magnitude of cleavage fracture toughness varies with pre-cleavage ductile crack growth. The observed values of cleavage fracture toughness and pre-cleavage ductile crack growth fluctuate about their mean values, due to microstructural inhomogeneities. These fluctuating components are random variables which are statistically dependent on each other. The pairs of values of fracture toughness and ductile crack growth associated with cleavage instability form a joint probability distribution which is represented by the second term in the relationship given below:

$$F_{cj}(K,\Delta a) = Pa\int_{o}^{K} f_{cK}(Kc|0)dK_c + (1-P_a)\int_{0}^{K_{\Delta a}}\int_{0}^{\Delta a} f_{cK}(K_c|\Delta a_c)f_{ca})dK_c d\Delta a_c \tag{5}$$

where P_a is the probability of cleavage at the initiation of cracking, i.e. $(\Delta a_c = 0)$, $f_{cK}(K_c|0)$ and $f_{cK}(K_c|\Delta a_c)$ are the conditional probability density functions for K_c given that $\Delta a_c = 0$ and Δa_c, respectively and $f_{ca}(\Delta a_c)$ is the marginal probability density function for Δa_c given that $\Delta a_c > 0$.

The main objective of fracture toughness testing in the brittle to ductile transition temperature region is to determine the dependence of cleavage fracture toughness in the joint distribution, K_{cj}, on test temperature and specimen thickness. Statistical analysis of data can achieve this objective by following one of two routes:

Evaluation of the temperature and thickness dependence of K_{cj} directly.

Evaluation of the joint distribution from the conditional and marginal density functions for $K_c|\Delta a_c$ and Δa_c respectively.

These procedures have been illustrated by Moskovic [Moskovic (1992); Moskovic (1995)] for C-Mn submerged arc weld metal, silicon killed C-Mn plate steels and A508 class 3 forging steel, respectively. The relationship derived for the cleavage fracture toughness of the C-Mn submerged arc weld metal [Moskovic (1992)] is:

Below 0°C

$$K_c = 306.87B^{-0.179}\exp(0.00916T + 0.3134U_p) \quad (6)$$

At $T \geq 0$°C

$$K_c = 945.93B^{-0.391}\exp(0.0305T)[-1n(1-P)]^{-0.03233} \quad (7)$$

where B is thickness in mm, K_c is the cleavage fracture toughness in MPa$\sqrt{}$m, T is temperature in °C, U_p is the standard normal deviate for the required probability of cleavage fracture (for 5% $U_p = -1.64$, for 50% $U_p = 0$ and for 95% $U_p = 1.64$) and P, a random value between 0 and 1, is the cumulative probability for the Weibull distribution. The probability P_a that $\Delta a = 0$ can be calculated from equations (6) and (7) as a function of thickness by deriving the temperature at which the two values for K_c are equal.

The fracture toughness, in the brittle to ductile transition temperature region, of silicon killed C-Mn plate steels is given by [Moskovic]:

$$\begin{aligned} K_{cj} =&774.33\exp(0.0160T)B^{-0.279} \\ &[\ln(1-p)/1n(0.5)]^{\Omega} \quad \text{for} \quad T < T_{c_1} \end{aligned} \quad (8)$$

$$\begin{aligned} K_{cj} =&928.90(0.0258T)B^{-0.216} \\ &[\ln(1-p)/1n(0.5)]^{\Omega} \quad \text{for} \quad T_{c_1} \leq T < T_{c_2} \end{aligned} \quad (9)$$

$$\begin{aligned} K_{cj} =&928.90(0.0228T)B^{-0.208} \\ &[\ln(1-p)/1n(0.5)]^{\Omega} \quad \text{for} \quad T_{c_2} \leq T \end{aligned} \quad (10)$$

The estimates of the critical temperatures T_{c_1} and T_{c_2} for the 25mm thickness, are -45°C and 1°C, respectively. Note that the equations for cleavage fracture toughness of submerged are weld metal were derived from weld joints different from those used for validation to be described later.

2.2 Data Used in Analysis

In order to illustrate the procedures proposed in this paper for small data sets, two different sets of data will be considered.

1. The first set of data is, in fact, a large data base obtained from measurements on precracked Charpy geometry specimens extracted from C-Mn submerged are weld metal in the neutron irradiated condition removed from a decommissioned Magnox reactor [Bolton,Bischler, Wootton, Moskovic, Morri, Pegg, Haines, Smith and Woodman (2002)]. The cleavage fracture toughness equation was derived [Moskovic (1992)] from tests performed on test pieces including 10mm thick precracked Charpy specimens, 25mm 75mm and 100mm thick compact tension specimens. This provides a strong test to validate the approach.

2. The second set of data was measured, using 25mm thick compact tension specimens on modern C-Mn silicon killed plate and forging steels and the measurement of the experimental data is described in section 3. In this case, the analytical equations were derived from an analysis performed on specimens 25mm to 100mm thick.

The latter provides experimental measurements of fracture toughness over the brittle to ductile transition temperature range. The aim of these tests is two fold; a) to define a temperature at which the fracture initiation mechanism is fully ductile; b) establish the lower bound values of cleavage fracture toughness. Since the number of test results generated is too small to derive a cleavage fracture toughness curve, the data are compared to a reference fracture toughness curve for the same type of material. A statistical procedure for undertaking this task is described.

3 Experimental Procedure

3.1 Materials

A C-Mn steel plate and a C-Mn forging steel manufactured to BS1501 part 1 223 490B and BS1503 224 430E respectively were selected for this investigation. The chemical concentration of the main alloying elements, in weight %, are given in Table 1. The room temperature tensile properties, obtained from the mill certificates, for the forging are: $R_{p0.2} = 304$MPa, $R_m = 503$MPa, elongation = 30% and reduction of area = 70.7% and those for the plate are: $R_{p0.2}$ = 378MPa, R_m = 545MPa, and elongation = 31%. Both forging and plate were subjected to a thermo-mechanical treatment that comprised:

Table 1 : Chemical Composition (wt.%) of C-Mn Steel Plate and Forging

Forging										
C	Si	Mn	P	S	Cr	Mo	Ni	A1	Cu	Fe
0.23	0.22	1.14	0.009	0.006	0.13	0.020	0.08	0.02	0.011	bal.
Plate										
0.18	0.35	1.13	0.013	0.002	0.14	0.025	0.044	0.052		bal.

Plate: Hot pressed in two operations, each after soaking at a temperature in the range from 870 to 900°C for 40 minutes and air cooled (AC), normalised for 40 minutes at temperature of 860 to 890°C and AC, stress relieved for 3.6h at 600 to 615°C and cooled at a maximum rate of 200°C/h.

Forging: Forged from 1230 $\pm 10^{o}$C to a minimum temperature of 980°C and AC, normalised for 9h at a temperature between 890 to 930°C and AC, stress relieved for 3h at 610$\pm 10^{o}$C and cooled to 300°C at a maximum rate of 100°C/h.

The microstructure of both steels, shown in Figure 1, comprises fine equiaxed grained ferrite and pearlite (ferrite plus carbide) with very little evidence of non-metallic inclusions. The latter is consistent with the low sulphur content of these two steels, Table 1.

3.2 *Fracture Toughness Testing*

All fracture toughness tests were carried out using standard 25mm thick compact tension geometry specimens. Forging and plate specimens were notched in CR and TL orientations, respectively. In the two letter code, the first letter indicates the direction perpendicular to the crack plane and the second the direction of crack front movement. The letters C and R represent the circumferential and radial directions in the forging, respectively and the letters T and L represent the transverse and longitudinal rolling directions in the plate, respectively.

Fracture toughness testing followed the guidance given in ESIS P2 procedure [European Structural Integrity Society (1992)]. Specimens were instrumented with a LVDT gauge mounted on the loading rams and with a clip gauge mounted across the open mouth of the test piece between the knife edges. The test specimens and the loading shackles were enclosed in an environmental chamber in which the test temperature was controlled to within ±1°C. Prior to testing, the specimen was held within ±2°C of the test temperature to ensure a uniform temperature within the specimen. Loading of specimens was carried out under the displacement control at a rate of increase of stress intensity factor, in the elastic regime, of approximately 1 MPa$\sqrt{\text{m}}$s^{-1}. Nine tests were performed on each material comprising three tests at each −46°C, −20°C and 0°C test temperature. Specimens which had not failed by a cleavage mechanism were interrupted after a certain amount of ductile crack growth and heat tinted for one hour a 300°C before being reloaded to failure. The extent of ductile crack growth, in heat tinted specimens, was measured using a shadow-graph microscope. The prior ductile crack growth in the three specimens that failed by cleavage instability was measured at a higher magnification in a JEOL 840 scanning electron microscope using the secondary electron imaging mode. An average value of ductile crack extension was calculated from eight values comprising the mean of the two surface measurements and seven equally spaced measurements across the crack width.

Values of J_c appropriate to cleavage instability or values of J appropriate to the final load point for the interrupted tests were calculated from load versus displacement records using equations 11, 12 and 13 given by Neale et al [Neale, Curry, Green, Haigh and Akhurst (1985)].

$$J = \eta\, U/B(W - a_o) \quad (11)$$

The value η is given by:

$$\eta = 1.97 + 0.815 \quad (12)$$

where a_o is the initial crack length, U is the area under the load displacement curve appropriate to the final point, B is the specimen thickness and W is the specimen width. In the ductile to brittle transition temperature region, fracture toughness is analysed using stress intensity, K. The values of J obtained from equation 11 were used to calculate the equivalent K values from the relationship:

$$K = (EJ/(1-\nu^2))^{1/2} \quad (13)$$

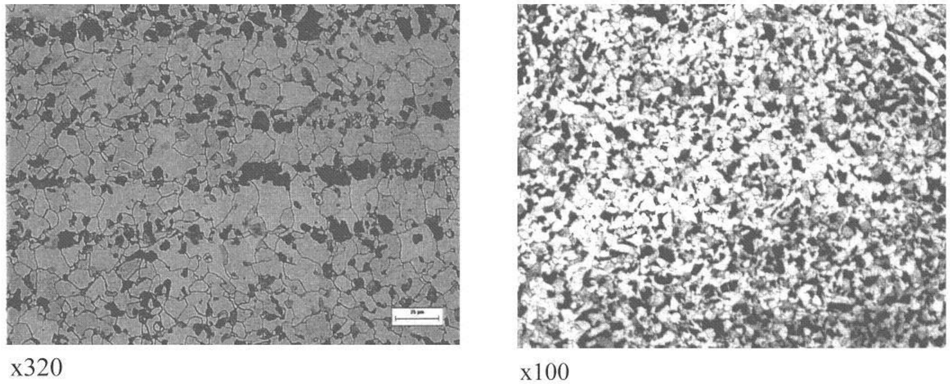

Figure 1 : Optical Micrographs of C-Mn Steel a) Forging and b) Plate showing Ferrite and Pearlite (etched in Nital)

where E is the Young modulus of elasticity and ν is the Poisson ratio. Values of E in GPa were taken from R51 Materials Data Handbook [Lamb and Wootton (2002)] and the Poisson ratio was assumed to be equal to 0.3. Below a temperature of 20°C, values for the Young modulus were calculated from the relationship $E = 210 - 0.05T(°C)$ which gives the same values as the tabulated data in reference [Lamb and Wootton (2002)].

4 Results

An example of a force versus displacement curve, obtained by testing a forging specimen at a temperature of -46°C, is presented in Figure 2. This curve shows a characteristic non-linear behaviour and a rising force as the displacement increases. The behaviour is typical of modern ferritic steels that contain a small volume fraction of non-metallic inclusions. The results of fracture toughness tests are presented in Table 2. All but one plate specimen tested at -46°C showed a significant amount of plastic displacement. Except for two plate and two forging specimens tested at a temperature of -46°C that failed by cleavage instability, the tests were terminated by unloading the specimens. Apart from one plate specimen, cleavage instability occurred prior to 0.2mm of ductile crack growth. Figure 3 shows a plot of the measured J values as a function of ductile crack growth and a mean line fitted by linear regression analysis using the method of least squares. Within the scatter of the data, the difference between the data for the forging and the plate was not discernible. Hence, the data for the two materials were analysed together giving the relationship for the mean for $\Delta a \geq 0.2$mm:

$$J = 178.2 + 829.1\Delta a \tag{14}$$

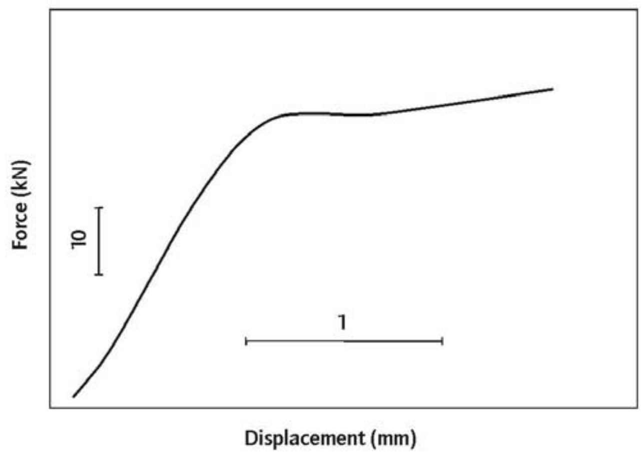

Figure 2 : Example of a Force Displacement Curve Obtained on a Forging Specimen at -46°C.

where J is N/mm and Δa, ductile crack growth, in millimetres.

5 Analysis of the Test Data

In this section we consider two methods of analysis of fracture toughness data spanning the lower to upper shelf of the ductile to brittle transition curve. The first is based upon the direct use of a reference curve when there is a sufficiently populated data set and the second is based upon a binomial model to address sparse fracture toughness data.

5.1 Reference Curve Method

The reference curve method uses a similar principle to that adopted for the application of the Master Curve [Wallin (1984); Wallin, Törrönen, Ahlstrand, Timofeev,

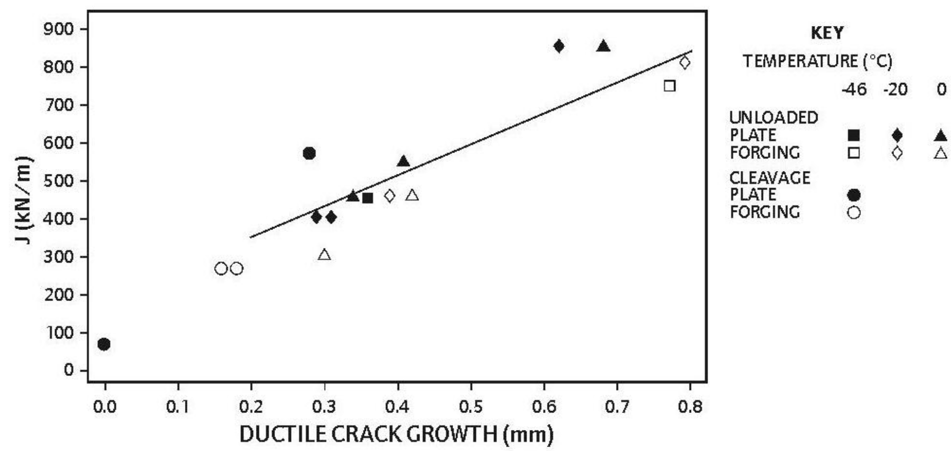

Figure 3 : Fracture Toughness, J, of Plate and Forging as a Function of Ductile Crack Growth

Rybin, Nikolaev and Morozov (1992)] but it is more flexible. In this case, it uses reference curves established for the particular composition of steel and the thermo-mechanical history: including neutron irradiation in the case of reactor pressure vessel steels. Following decommissioning of the Magnox reactor at Trawsfynydd, an extensive sampling programme was undertaken for a region of the steel pressure vessel exposed to neutron irradiation at low temperature, 187 ° C, and high for neutron dose, 350×10^{-5} dpa. Fracture toughness measurements were made on the extracted C-Mn submerged arc weld metal using Charpy geometry fracture toughness specimens tested in quasi-static three point bend. This produced a significant number of fracture toughness data. The results have been described elsewhere [Bolton, Bischler, Wootton, Moskovic, Morri, Pegg, Haines, Smith and Woodman (2002); Flewitt, Bolton and Edens (1998); Flewitt and Moskovic (2004)] and are summarised in Figure 4. To compare the measured fracture toughness with reference cleavage fracture toughness derived from tests performed on unirradiated weld metal, it is necessary to apply an irradiation shift in ductile to brittle transition temperature to the fracture toughness curve for this unirradiated condition. The shift was obtained from a trend curve derived from Charpy impact energy data [Moskovic, Jordinson, Stephens and Smith (2000)]. The reference fracture toughness curve is also dependent on specimen thickness. Hence, a curve appropriate for 10mm thickness was calculated, shifted for neutron irradiation dose and compared with the measured values obtained on 10mm thick specimens. Figure 4 shows that there is an excellent agreement between the fracture toughness results and the predictions with the results distributed evenly about the median prediction. In addition, there is the expected proportion of data above and below the 5 and 95 percentiles. The measured data used in Figure 4 has not been used in statistical analysis to derive the equations used to predict the curves. Hence, Figure 4 validates the premise that the curve referred to is generally applicable to submerged arc weld metal.

It should be noted, however, that consideration of this reference curve method for the case where the data set is small indicates that judgement has to be invoked by allowing the use of the constants in equation to displace the curve along the temperatures axis. Such a procedure is similar to that adopted for the application of the Master Curve. This is apparent if for the abundant data given in Figure 4 sub-sets are selected of between six to ten data points. Such data can lie above, below or be equally distributed about the reference curve. The form of the distribution may be simply due to sampling or a bias in these data. Hence judgement has to be applied to establish the origin of the displacement of the data from the reference curve. In the next section we consider an alternative approach for interrogating and evaluating sparse data sets.

5.2 Binomial Model

A qualitative assessment of resistance to cleavage fracture can be made by inspection of the force versus displacement records obtained from the fracture toughness tests. Due to the high ductile crack initiation and crack growth resistance of the two C-Mn (plate and forging) steels, there is a significant amount of plastic deforma-

Table 2 : Fracture Toughness Results Obtained for C-Mn Steel Plate and Forging

Spec	Test Temp. (˚C)	Crack Ext. (mm)	J (N/mm)	K (MPa$\sqrt{m}$)	Termin
Plate					
P1	0	0.34	458	325	Unload
P2	0	0.68	859	445	Unload
P3	0	0.41	549	356	Unload
P4	-20	0.29	401	305	Unload
P5	-20	0.31	400	305	Unload
P6	-20	0.62	857	446	Unload
P7	-46	0.36	452	325	Unload
P8	-46	0	60	118	Cleavage
P9	-46	0.28	571	365	Cleavage
Forging					
F1	0	0.30	302	263	Unload
F2	0	4.42	461	326	Unload
F3	0	0.86	867	447	Unload
F4	-20	0.39	457	326	Unload
F5	-20	0.79	815	435	Unload
F6	-20	0.89	862	447	Unload
F7	-46	0.18	265	249	Cleavage
F8	-46	0.77	754	419	Unload
F9	-46	0.16	265	249	Cleavage

tion that occurs prior to cleavage instability without giving rise to a large amount of prior ductile crack growth. Clearly, these materials accommodate a large amount of plastic deformation without bringing about plastic collapse of the specimens. Despite the small proportion of ductile crack growth observed when undertaking these fracture toughness tests the amount of plastic deformation observed, in all but one plate specimen, is typical of upper shelf fracture toughness behaviour.

Procedures such as the Master Curve [Wallin, Törrönen, Ahlstrand, Timofeev, Rybin, Nikolaev and Morozov (1992)] seek to provide a common curve to describe the fracture toughness behaviour of a range of ferritic steels in the ductile to brittle transition region. However, the temperature dependence of the fracture toughness of ferritic steels within this region of the ductile to brittle transition curve depends upon the plasticity work hardening rate and hardening capacity of the specific material. Moreover, these parameters will depend upon the specific composition of the steel and the thermo-mechanical history [Knott (2003)]. As a consequence it is unrealistic to expect steels even within a broad specification range to necessarily obey a common trend curve. It is to accommodate these differences in behaviour that alternative procedures to describe data in the transition region based upon statistical analysis have been developed [Moskovic and Crowder (1995); Moskovic (1995)].

The test programme described in Section 4 was intended to provide a sample of fracture toughness data for the ductile to brittle transition temperature region. Hence, for each material, there are nine test results of which two values at -46°C for each material are for cleavage instability, Table 2. For these test results, it would be difficult to derive a relationship for cleavage fracture toughness as a function of temperature by statistical analysis. However, the information obtained from these tests can be used to make a judgement whether the cleavage fracture toughness of plate and forging is bounded by recommendations given for the silicon killed plate steels [Moskovic]. Fracture toughness values at cleavage instability or at a point when a specimen was unloaded are either within the scatter or, in most cases, above the 95% probability limit for cleavage fracture toughness of silicon killed plate steels in the database [Moskovic]. Indeed, equations by Moskovic [Moskovic] and Windle and Moskovic [Windle and Moskovic (1989); Windle

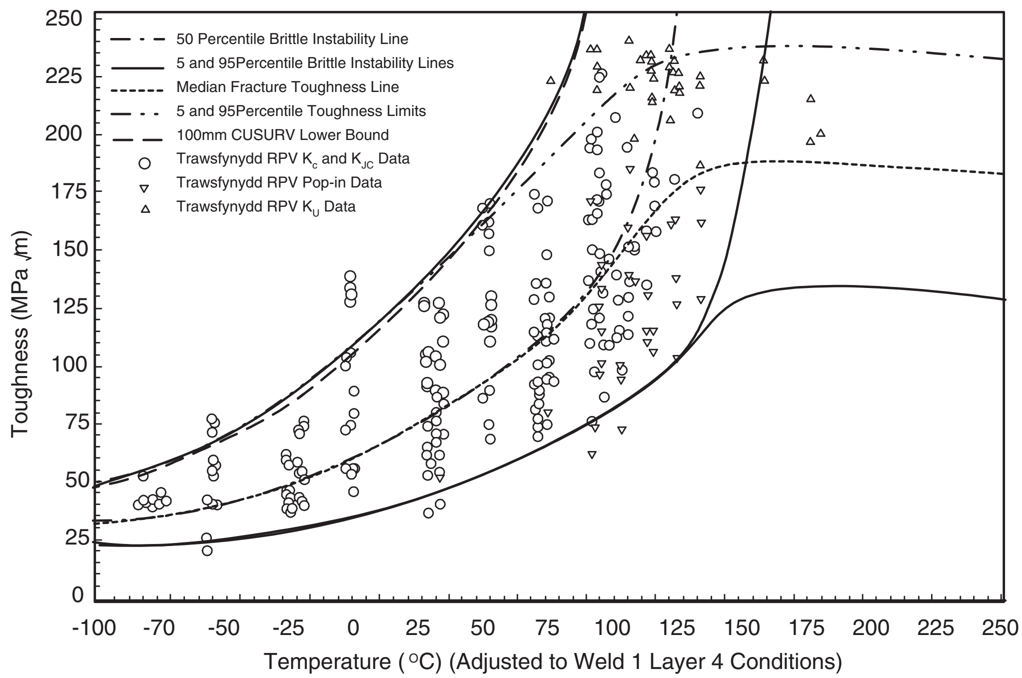

Figure 4 : Fracture toughness data, K_c or K_{Ic} obtained for C-Mn submerged arc weld metal obtained from a decommissioned Trawsfynydd pressure vessel as function of temperature compared with the CUSURV prediction.

and Moskovic (1989)] for fracture toughness properties have been used to predict the probability (percentage of cleavage), π, for C-Mn silicon killed plate steels at temperatures of −46°C, −20°C and 0°C. The main analysis adopted takes into account the recognised competition between cleavage and ductile fracture modes in the ductile to brittle transition region. For this, the well established competing risk statistical procedure described in Section 2 can be adopted. The respective values, derived using the standard computer program CUSURV as described by Doig and Moskovic [Doig and Moskovic (1993] are: 58.7%, 17.4% and 5.9%, Table 3. These values can be used to calculate the probability, P, of getting the number of cleavage and ductile values that have been obtained for each material at each test temperature. The observed data can be classified as either ductile or brittle. The results were classified as brittle if cleavage instability occurred prior to 0.2mm of ductile cracking and ductile if 0.2mm of ductile growth was achieved. The probability of different outcomes can be modeled by the binomial distribution [Clarke and Cooke (1992)]:

$$P(y|\pi) = \frac{n!}{x!y!}\pi^x (1-\pi)^y \tag{15}$$

where $P(y/\pi)$ is the probability of x number of cleavage failures and y number of ductile termination outcomes conditional on probability of cleavage failures, π, n is the total number of tests and ! denotes factorial. At temperatures of 0°C and -20°C all three outcomes are ductile terminations. The predicted probabilities of different types of outcomes are given in Table 3 and the experimental outcome in Table 4.

As shown in Table 3 the probable outcome of a given contribution of fracture at the three testing temperatures conditional upon the probability of cleavage fracture gives a wide range of values. These values show that there is a high probability of three ductile failures at a temperature of 0°C for both the plate and forging steel, whereas the probability of cleavage for the three specimens is extremely low. However, at a temperature of -46°C the most probable outcome is one ductile and two cleavage failures with the next most probable be-

Table 3 : Predicted Probability of the Outcomes based upon Competing Risk Analysis

Outcomes		3 Ductile	2 Ductile and 1 Cleavage	1 Ductile and 2 Cleavage	3 Cleavage
Probability (percentage) of cleavage	Test temp. (°C)	P/B, Probability of outcome conditional on probability of cleavage			
58.7	-46	0.07	0.30	0.43	0.20
17.4	-20	0.56	0.35	0.075	0.005
5.9	0	0.83	0.16	0.01	0.0002

ing two ductile and one cleavage. These predictions are to be compared with the experimentally observed outcomes given in Table 4 where at a temperature of -46°C for 0.2mm of ductile crack growth there is one ductile outcome for the forging steel and two for plate steel. The respective probabilities of having more ductile failures are 0.37 and 0.07. At temperatures of -20°C and 0°C the outcome is three ductile values with respective values of having less than three ductile values of 0.43 and 0.17.

6 Concluding Comments

In section 5, we have described two methods of the analysis of fracture toughness measurements of ferritic steels that can be applied to both a very large database and a sparse data set. The first, Reference, method offers a flexible alternative to the Master curve approach and the second, Binomial, method can be used when most of the cleavage fracture toughness data are censored. In both cases, cleavage fracture toughness values with a prior ductile crack growth can be used although the Master curve approach could not be used.

Sample of nine fracture toughness values were obtained for both plate and forging at three different temperatures to assess whether the constants in the relationships for cleavage fracture toughness in [Moskovic] can be adjusted to derive cleavage fracture toughness for the plate and forging. Since most of the tests were terminated by unloading the specimens the associated values of fracture toughness are censored and cannot be used to modify the constants [Moskovic]. To assess whether the relationships in [Moskovic, Windle and Moskovic (1989); Windle and Moskovic (1989)] predict conservative values of cleavage fracture toughness for the plate and forging, the probabilities of cleavage fracture for the reference curve were predicted by competing risks (Moskovic and Crowder (1995); Moskovic]. Conditionally on these probabilities, the probabilities of different test outcomes for plate and forging were computed based on a binomial distribution. This shows that the reference cleavage fracture toughness relationship in [Moskovic] provides a conservative description of cleavage fracture toughness behaviour of the plate and forging.

The application of the reference curve method showed that a thickness correction, developed from data measured on specimens with thickness in the range from 25 to 100mm, can be used to predict the fracture toughness of precracked Charpy specimens. This is in contrast with Master curve approach where it was found that the use of thickness correction under predicts the cleavage fracture toughness of precracked Charpy specimens due to the loss of constraint in these specimens.

Overall we consider that the two methods for analysing fracture toughness data across the brittle to ductile transition temperature range of ferritic steels provide a powerful framework as an alternative to the Master Curve approach.

Table 4 : The Experimentally Measured Outcomes at 0.2mm of Ductile Crack Growth (d = ductile, c = cleavage)

Test Temp °C	Forging	Plate
-46	1d + 2c	2d + 1c
-20	3d	3d
0	3d	3d

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