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Vibration Fatigue Analysis of Cylinder Head of a New Two-Stroke Free Poston Engine Using Finite Element Approach

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Abstract: The focus of this paper is to design a new two-stroke linear generator engine. This paper describes the finite element based vibration fatigue analysis techniques that can be used to predict fatigue life using total life approach. Fatigue damage in traditionally determined from time signals of loading, usually in the form of stress and strain. However, there are scenarios when a spectral form of loading is more appropriate. In this case the loading is defined in terms of its magnitude at different frequencies in the form of a power spectral density (PSD) plot. A power spectral density function is the most common way to representing the loading in the frequency domain. The PSD simply shows the frequency content of the time signal and is an alternative way of specifying the time signal. It is obtained by utilizing the Fast Fourier Transform. A frequency domain fatigue calculation can be utilized where the random loading and response are categorized using power spectral density functions and the dynamic structure is modeled as a linear transfer function. This paper describes how this technique can be implemented in the finite element environment to rapidly identify critical areas in the structure. This significantly reduces cost, time to market; improve product reliability and customer confidence consequences of premature produce failure.

keyword: Fatigue, Fast Fourier Transform, vibration, power spectral density function, frequency response.

1 Introduction

Structures and mechanical components are frequently subjected to oscillating loads of random in nature. Random vibration theory has been introduced for more then three decades to deal with all kinds of vibration behaviour when random is concerned. Since fatigue is one of the major causes when component failure is considered, fatigue life prediction has become a major subject in almost any random vibration [Bolotin (1984)], [Soong and Grigoriu (1993)], [Newland (1993)]. Nearly all structures or components have traditionally been designed using time based structural and fatigue analysis methods. However, by developing a frequency based fatigue analysis approach, the true composition of the random stress or strain responses can be retained within a much optimized fatigue design process.

The free piston linear generation is an energy conversion device that integrated a combustion engine and an electrical generator into a single unit. Thereby the intermediary crankshaft stage present in conventional hybrid topologies is eliminated. This has benefits in efficiency, weight reduction, robustness, variable compression operation and multi-fuel possibilities. The hybrid vehicle concept is environmental friendly, highly efficient and is gaining popularity by the day [Arshad, Bäckströom and Sadarangani (2002)]. Fatigue durability has long been important issues in the design of free piston linear generator engine structures [Arshad, Bäckströom and Sadarangani (2002], [Rahman, Ariffin, Jamaludin and Haron (2005a)]. From the viewpoint of engineering applications, the purpose of fatigue research consists of predicting the fatigue life of structures, increasing fatigue life and simplifying fatigue tests especially fatigue tests of full-scale structures under a random load spectrum. The fatigue life of an engineering structure principally depends upon that of its critical structure members. There is an increasing interest within the internal combustion engine industry in the ability to produce designs that are strong, reliable and safe, whilst also light in weight, economic and easy to produce. These opposing requirements can be satisfied by analytically optimizing components of linear generator engine. In the free piston engine system, the cylinder head is the most important component affecting the performance of engine. The main purpose of the present paper is to derive ready-to-

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use formulas for the prediction of fatigue damage and fatigue life when a component is subjected to statistically defined random stresses.

2 Theoretical Basis

The equation of motion of a linear structural system is expressed in matrix format in Eq. 1. The system of time domain differential equations can be solved directly in the physical coordinate system.

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{p(t)\} \quad (1)$$

where $\{x(t)\}$ is a system displacement vector, $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively, {p(t)}is an applied load vector.

When loads are in random in nature, a matrix of the loading power spectral density (PSD) functions $[S_p(\omega)]$ can be generated by employing Fourier transform of load vector $\{p(t)\}$.

$$[S_p(\omega)]_{m\times m} = \begin{bmatrix} S_{11}(\omega) & \Lambda & S_{1i}(\omega) & \Lambda & S_{1m}(\omega) \\ M & 0 & & \Lambda & M \\ S_{i1}(\omega) & & S_{ii}(\omega) & & S_{im}(\omega) \\ M & \Lambda & & 0 & M \\ S_{m1}(\omega) & \Lambda & S_{mi}(\omega) & \Lambda & S_{mm}(\omega) \end{bmatrix} \quad (2)$$

where m is the number of input loads. The diagonal term $S_{ii}(\omega)$ is the auto-correlation function of load$p_i(t)$, and the off-diagonal term $S_{ij}(\omega)$ is the cross- correlation function between loads $p_i(t)$ and $p_j(t)$. from the properties of the cross PSDs, it can be shown that the multiple input PSD matrix $[S_p(\omega)]$ is a Hermitian matrix.

The system of time domain differential equation of motion of the structure in Eq. 1, is then reduced to a system of frequency domain algebra equations

$$[S_x(\omega)]_{n\times n} = [H(\omega)]_{n\times m}\,[S_p(\omega)]_{m\times m}\,[H(\omega)]^T_{m\times n} \quad (3)$$

where n is the number of output response variables. The Tdenotes the transpose of a matrix. $[H(\omega)]$ is the transfer function matrix between the input loadings and output response variables.

$$[H(\omega)] = \left(-[M]\,\omega^2 + i\,[C]\,\omega + [K]\right)^{-1} \quad (4)$$

The response variables $[S_p(\omega)]$ such as displacement, acceleration and stress response in terms of PSD functions are obtained by solving the system of the linear algebra equations in Eq. 3.

The stress power spectra density represents the frequency domain approach input into the fatigue. This is a scalar function that describes how the power of the time signal is distributed among frequencies [Bendat (1964)]. Mathematically this function can be obtained by using a Fourier transform of the stress time history's auto-correlation function, and its area represents the signal's standard deviation. It is clear that PSD is the most complete and concise representation of a random process. There are a many important correlations between the time domain and frequency domain representations [Bishop (1988)], [Bishop and Sherratt (2000)] of a random process. These are highlighted by the spectral moments that are particular PSD functions. The expression of the moment of nth order is given by Eq. 5. The detail derivation of Eq. 5 has been done by Rahman, Ariffin, Jamaludin and Haron [2005b] and will not be repeated here.

$$M_n = \int_0^\infty f^n G(f)\,df \quad (5)$$

where $f$is the frequency and $G(f)$ is the one sided PSD.

A method for computing these moments is shown in Fig. 1. Some very important statistical parameters can be computed from these moments. These parameters are root mean square (rms), number of zero crossing with positive slope($E[0]$), number of peaks per second ($E[P]$),. The formulas in Eq. 6 highlight these properties of the spectral moments.

$$rms = \sqrt{m_0}\,; \quad E[0] = \sqrt{\frac{m_2}{m_0}}; \quad E[P] = \sqrt{\frac{m_4}{m_2}} \quad (6)$$

where m_0, m_1, m_2 and m_a are the zeroth, 1st, 2nd and 4th order moment of area of PSD, respectively.

Another important property of spectral moments is the fact that it is possible to express the irregularity factor as

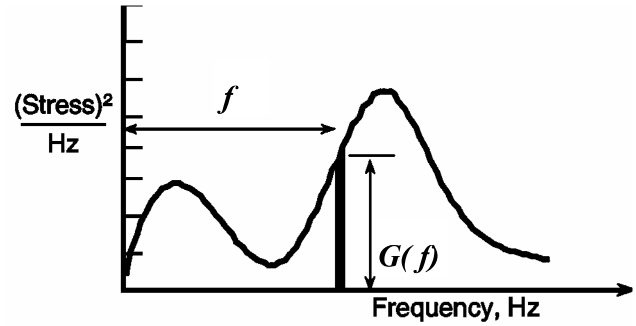

Figure 1 : Calculating moments from a PSD

a function of the zero, second and fourth order spectral moments, as shown in Eq. 7.

$$\gamma = \frac{E[0]}{E[P]} = \frac{m_2}{\sqrt{m_0 m_4}} \tag{7}$$

The irregularity factor γ is an important parameter that can be used to evaluate how concentrated near a central frequency the process is. So it can be used to determinate whether or not the process is narrow band or wide band.

A narrow band process (γ →1) is characterized by only one predominant central frequency meaning that the number of peaks per second is very similar to the number of zero crossings of the signal. This assumption leads to the fact that the pdf of the fatigue cycles range is the same as the pdf of the peaks in the signal [Bendat (1964)]. In this case fatigue life is easy to estimate. In contrast, the same property is not true for wide bend process (γ →0). Fig. 2 shows different type of time histories and its corresponding PSD function. In Fig. 2(a), a sinusoidal time history appears as a single spike on the PSD plot. The spike is centered at the frequency of the sine wave and the area of the spike represents the mean square amplitude of the wave. In theory this spike should be infinitely tall and infinitely narrow for a pure sine wave. However, because any sine wave used is, by definition, finite in length, the spike always has finite height and finite width. With PSD plots it is the area under the graph that is of interest and not the height of the graph. In Fig. 2(b), a narrow band process is shown which is built up of sine waves covering only a narrow range of frequencies. A narrow band process is typically recognized in a time history by amplitude modulation, often referred to as a beat envelope. In Fig. 2(c), a broad band processes is shown which is made up of sine waves over a broad range of frequencies. These are shown in the PSD plot as either a number of separate response is usually more difficult to identify from the time history but is typically characterized by positive valleys (troughs in the signal above the mean level) and negative peaks. In Fig. 2(d), a white noise process is shown. This is a special time history, which is built up of sine wave over the whole frequency range.

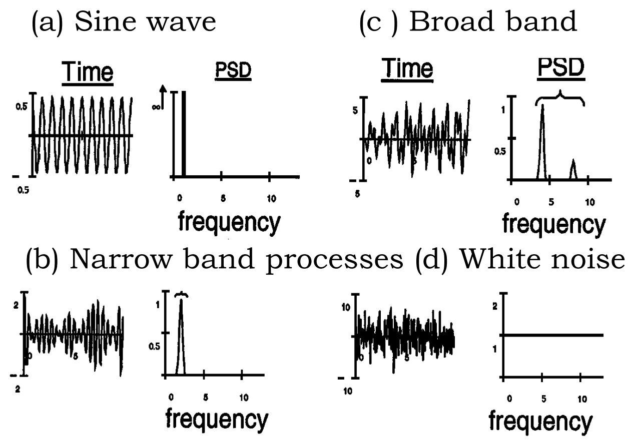

Figure 2 : Equivalent time histories and PSD's

3 Probability Density Functions (Pdf's)

The most convenient way, mathematically, of storing stress range histogram information is in the form of a probability density function (pdf) of stress ranges [Bendat (1964)], [Bishop (1988)], [Bishop and Sherratt (2000)], [Rahman, Ariffin, Jamaludin and Haron (2005b)]. A typical representation of this function is shown in Fig. 3. It is very easy to transform from a stress range histogram to a pdf, or back. The bin widths used, and the total number of cycles recorded in the histogram are the only additional pieces of information required. To get a pdf from a rainflow histogram each bin in the rainflow count has to be multiplied by $S_t \times dS$. where $S_t$is the total number of cycles in histogram; dS is the interval width. The probability of the stress range occurring between $S_i - dS/2$ and $S_i + dS/2$ is given by $p(S_i)dS$.

The actual counted number of cycles, $n_i = p(s)\,dS\,S_t$

The allowable number of cycles, $N(S_i) = \frac{k}{S^b}$

$$\text{Damage, } E[D] = \sum_i \frac{n_i}{N(S_i)} = \frac{S_t}{k} \int S^b p(s) dS \tag{8}$$

Failure occurs, $D \geq 1.0$.

In order to compute fatigue damage over the lifetime of the structure in seconds the form of materials S-N data must also be defined using the parameters k and b. In addition, the total number of cycles in time T must be determined from the number of peaks per seconds $E[P]$. If the damage caused in time T is greater than 1.0 then the structure is assumed to have failed or alternatively the fatigue life can be obtained by setting $E[D]$ =1.0 and then finding the fatigue life T in seconds from the fatigue damage is given by Eq. 8 .

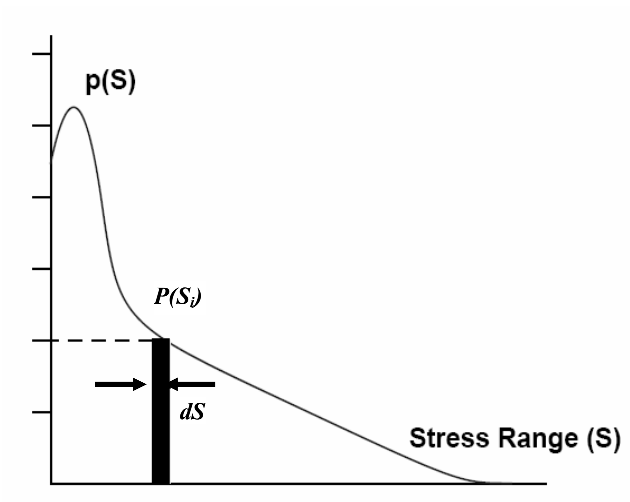

Figure 3 : Probability density functions.

4 Narrow Band Solution

Bendat [1964] presented the theoretical basis for the first of these of these frequency domain fatigue models, so called Narrow band solution. This expression was defined solely in terms of the spectral moments up to m_4.However, the fact that this solution was suitable only for a specific class of response conditions was an unhelpful limitations for the practical engineer. The narrow band formula [Bishop and Sherratt (2000)],[Rahman, Ariffin, Jamaludin and Haron (2005)] is given by the Eq. 9.

$$E[D] = \sum_i \frac{n_i}{N(S_i)} = \frac{S_t}{K} \int S^b p(S) dS \, N(S)$$

$$= \frac{E[P]T}{K} \int S^b [\frac{S}{4.m_0} e^{\frac{s^2}{8m_0}}] \, dS = E[P]T \left\{ \frac{S}{4m_0} e^{\frac{s^2}{8m_0}} \right\} \quad (9)$$

This is the first frequency domain method for predicting fatigue damage from PSDs and it assumes that the pdf of peaks is equal to the pdf of stress amplitudes. The narrow band solution was then obtained by substitutions the Rayleigh pdf of peaks with the pdf of stress ranges. The full equation is obtained by noting that S_t is equal to $E[P].T$, where T is the life of the structure in seconds. The basis of the narrow band solution is shown in Fig. 4.

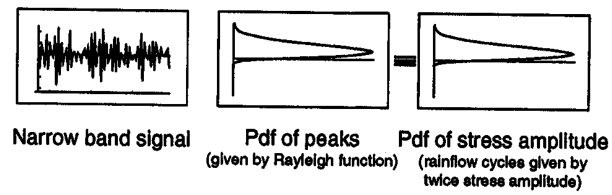

Figure 4 : The Basis of the narrow band solution

5 Results and discussion

A three dimensional model of the cylinder head of the free piston engine is developed using CATIA© V5R13 software. First model is imported to finite element software and has created fine mesh using Tetra10 elements. Sensitivity analysis was performed to obtain the optimum element size. These analyses were performed iteratively at different element lengths until the solution obtained appropriate accuracy. Convergence of stresses was observed, as the mesh size was successfully refined. The element size of 0.20 mm was finally considered. A total of 35415 elements and 66209 nodes were generated at 0.20 mm element length. In the finite element model of the cylinder head of free piston engine, there are several contact areas for examples: cylinder block, hole for bolt, etc concerning multi-point constraints. Therefore, constraints are employed for the following purposes: to specify the prescribed enforce displacements; to simulate the continuous behavior of displacement in the interface area, and to enforce rest condition in the specified directions at grid points of reaction. Because of the complexity of geometrical design and load path of the engine, it is not easy to model the complicated stiffness distribution of cylinder block structure using simply analytical model. Compressive loads were applied as pressure (4 MPa) on the surface of combustion chamber of cylinder head and preload were applied as pressure (0.2 MPa) on the bolt hole surfaces, and preload

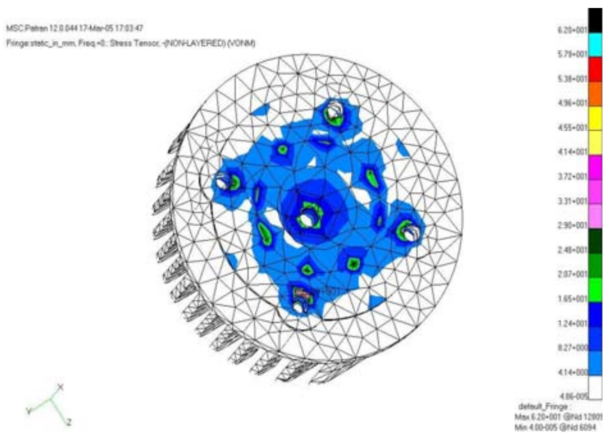

Figure 5 : von Mises stresses at Zero Hz

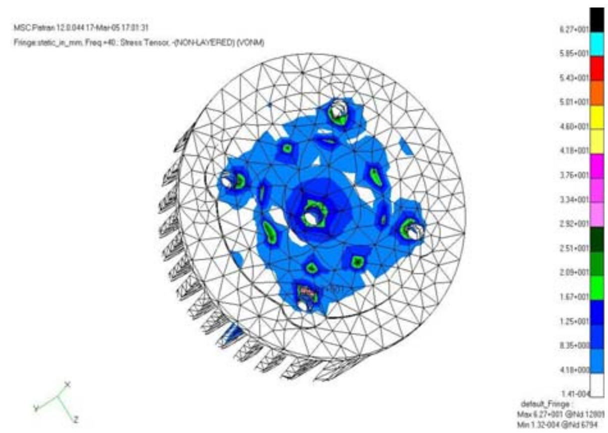

Figure 6 : von Mises stresses at 40 Hz

were also applied on the gasket surface as pressure (0.3 MPa). The constraints were applied on the bolt hole for all six degree of freedom [Rahman, Ariffin, Jamaludin and Haron (2005d)]. The frequency response analyses are performed using MSC.NASTRAN© finite element software. The frequency response analysis used a damping ratio of 5% of critical. The result of frequency response finite element analysis i.e. the von Mises stresses of the cylinder head is presented in Fig. 5 and Fig. 6 for zero Hz and 40 Hz respectively. From the results, maximum von Mises stresses of 62.0 MPa and 62.7 MPa were obtained at node 12809 at zero Hz and 40 Hz, respectively.

When plot higher frequencies, it will be seen a small divergence. This is due to dynamic influences of the first mode shape. This divergence is shown in Fig. 7 as a function of frequency at high stress of interest for node 12809. The transfer function contains frequencies from 0 to 50 Hz. The transfer function is very important in order to obtain accurate fatigue results.

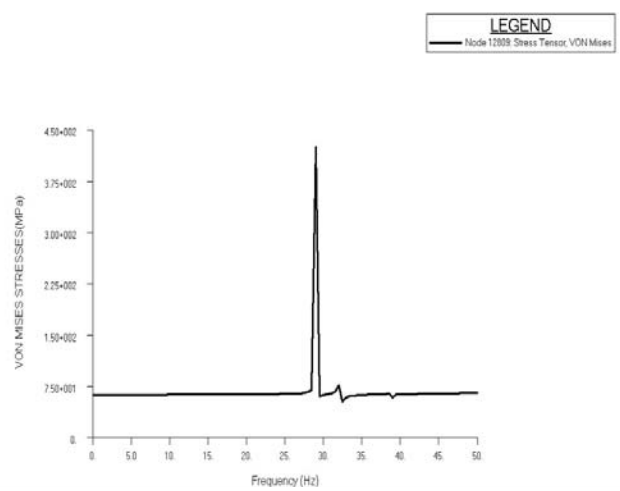

Figure 7 : Variation of von Misses stresses with frequency

The fatigue life contour at zero Hz using the SAETRN loading histories is shown in Fig. 8. The full set of comparison results for most critical damage location (node 12809) is given in Tab. 1 at different loading conditions. Dirlik method [Bishop (1988)], [Bishop and Sherratt (2000)], [Rahman, Ariffin, Jamaludin and Haron (2005b)] with mean stress correction is considered in this study. The near the circular hole shows the position of the shortest life and the white areas the longest fatigue lives. It can be seen that the Goodman method has been found to give the best conservative results when compared with the Gerber and no mean stress correction methods. It is also observed that AA5083-87-CF have been given the highest lives for all mean stress correction methods.

There are several types of loading histories were selected for the simulation from the SAE and ASTM profiles. The detailed information about these histories was contained in the literature [MSC.FATIGUE User's guide, (2004)]. Raw time loading histories are shown in Fig. 9 and the corresponding PSD plot are also shown in Fig. 10.

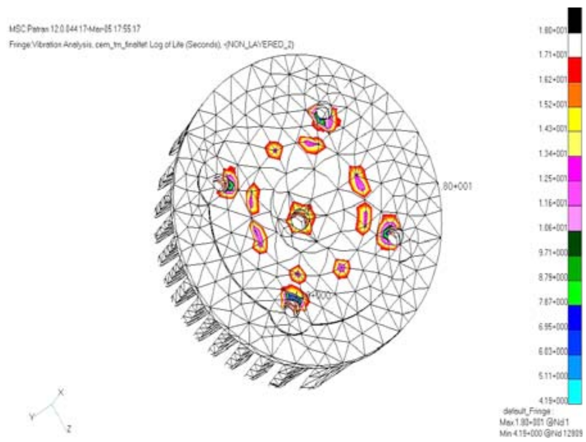

Figure 8 : Vibration fatigue life contour plotted at Zero Hz

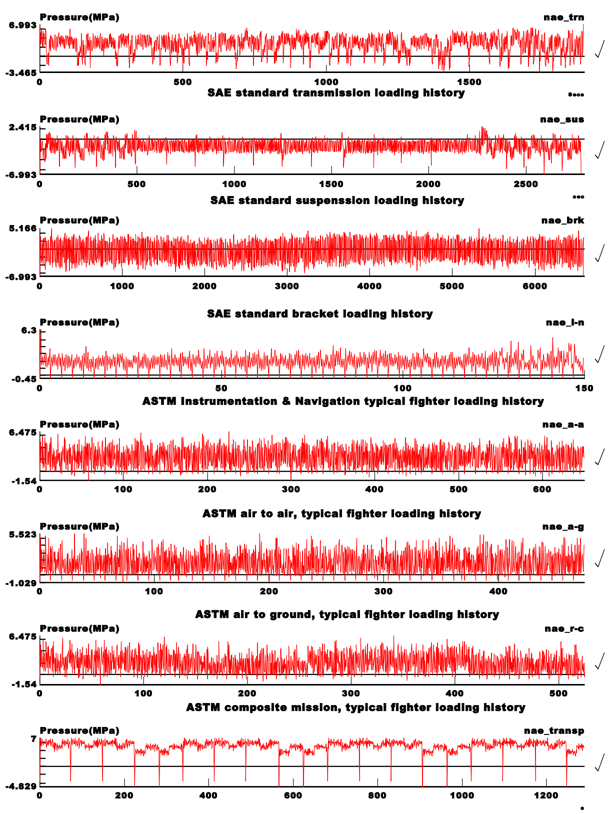

Figure 9 : Time loading histories

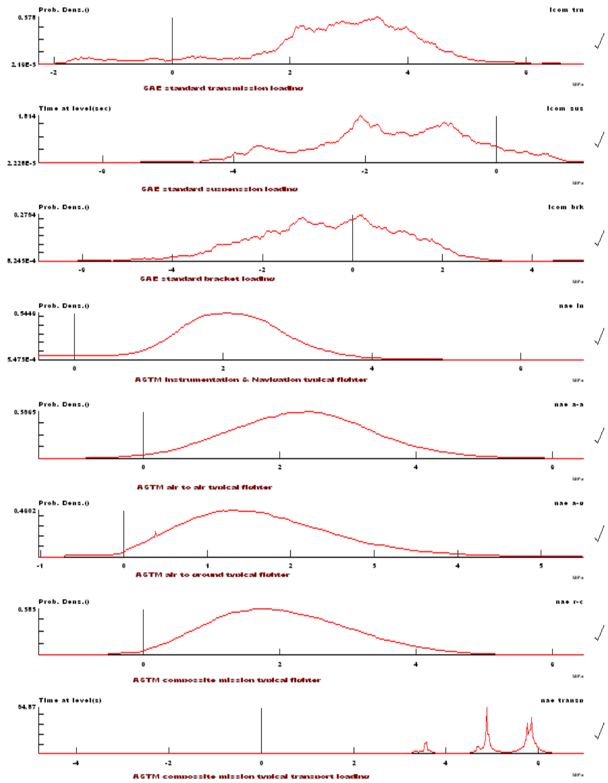

Figure 10 : Corresponding PSD's Response

The SAETRN, SAESUS, and SAEBRAKT in the figure represent the SAE's load-time history obtained from the transmission, suspension, and bracket respectively. I-N, A-A, A-G, R-C, and TRANSP are the ASTM instrumentation & navigation typical fighter, ASTM air-to-air typical fighter, ASTM air to ground typical fighter, ASTM composite mission typical fighter, and ASTM composite mission typical transport loading history, respectively.

Frequency resolution of the transfer function is also influence to capture input PSD. The influence of frequency resolutions of I-N loading histories are also shown in Fig. 11 and Fig. 12 used FFT buffer size of 1024: 0.9766 Hz width and 2048: 0.4883 Hz width, respectively. The dominant factor is the frequency resolution of the transfer function in the important areas of the input PSD. Because transfer function has evenly incremented frequency steps of 2 Hz, may skip over certain peaks or valleys in the input PSD. Interpolation only occurs within the input PSD at frequency points found in the transfer function and not those found in the input PSD. Figures 13-16 show the applied time histories, PSD's of narrow band signal (SAESUS), corresponding probability density function (PDF)

Table 1 : Predicted Fatigue life in seconds using the Dirlik method with mean stress correction

Loading conditions	Predicted vibration fatigue life in seconds					
	AA5083-87-CF			AA6061-T6-80-HF		
	None	Goodman	Gerber	None	Goodman	Gerber
SAETRN	1.55E4	1.20E4	1.54E4	1.20E4	9.13E3	1.19E4
SAESUS	3.71E5	3.09E5	3.70E5	2.61E5	2.13E5	2.60E5
SAEBKT	5342	4065	5305	4273	3164	4237
I-N	2.56E6	2.27E6	2.56E6	1.624E6	1.420E6	1.620E6
A-A	1.38E5	1.17E5	1.37E5	9.61E4	8.03E4	9.58E4
A-G	3.94E5	3.40E5	3.93E5	2.65E5	2.25E5	2.64E5
R-C	1.533E5	1.303E5	1.53E5	1.066E5	8.91E4	1.062E5
TRANSP	4.78E6	4.22E6	4.77E6	3.06E6	2.66E6	3.05E6

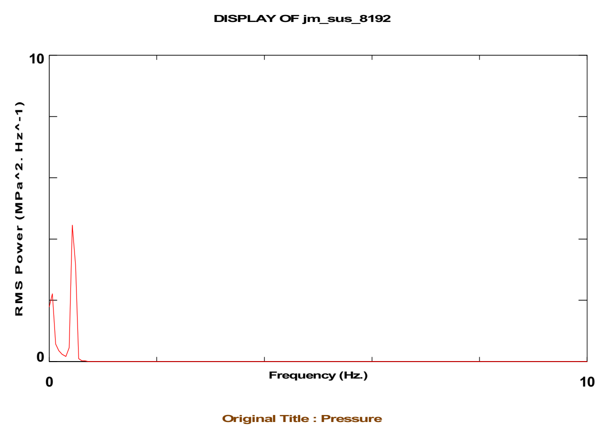

Figure 11 : Power spectral density at FFT buffer size of 8192:0.06104 Hz width.

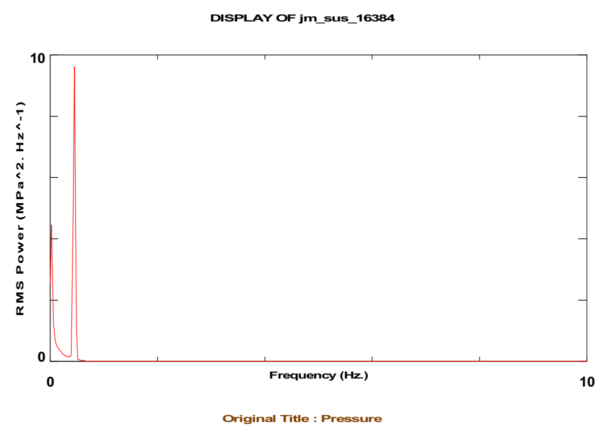

Figure 12 : Power spectral density at FFT buffer size of 16384:0.03052 Hz width.

and cycle histogram, respectively.

6 Conclusions

The concept of vibration fatigue analysis has been presented, where the random loading and response are categorized using PSD functions. A state of art of vibration fatigue techniques has been presented. Frequency response fatigue analysis has been applied to a typical cylinder head of free piston engine. From the results, it can be concluded that Goodman mean stress correction method has been found to give the most conservative for all loading conditions and both materials. According to the results, it is clearly say that AA5083-87-CH is a superior material using all mean stress methods. Life predicted from the vibration fatigue analysis is consistently

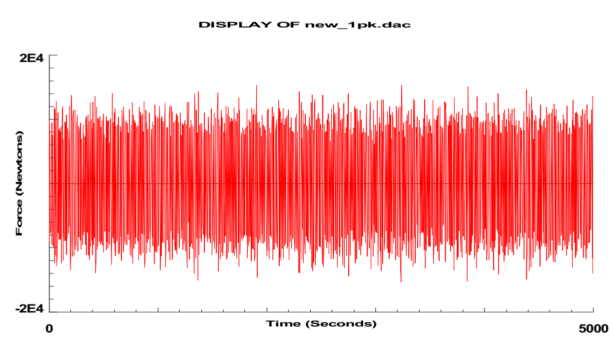

Figure 13 : Time-load histories at critical location

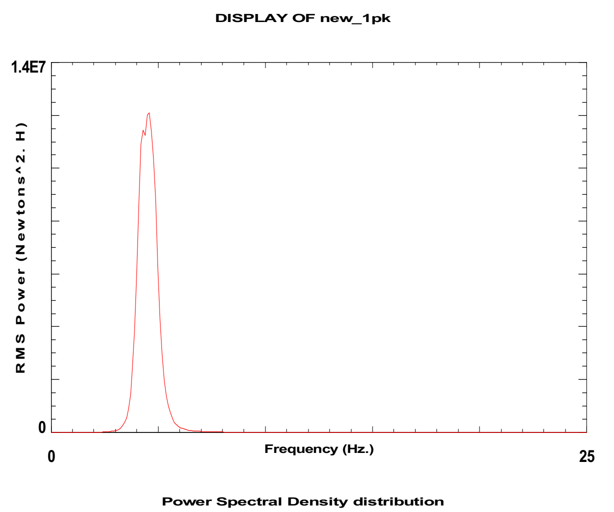

Figure 14 : Power spectral density.

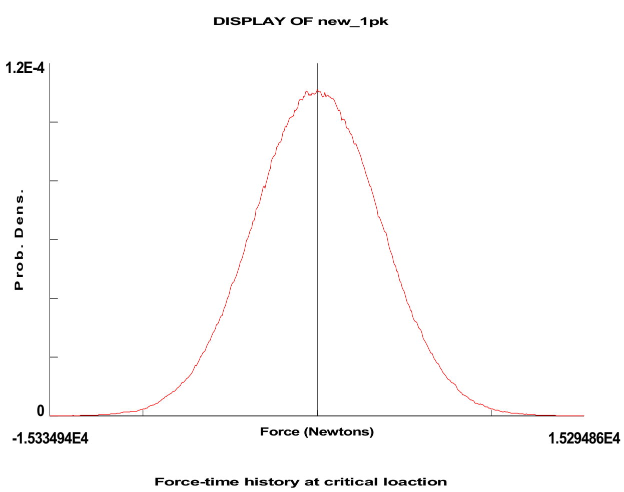

Figure 15 : Probability density function

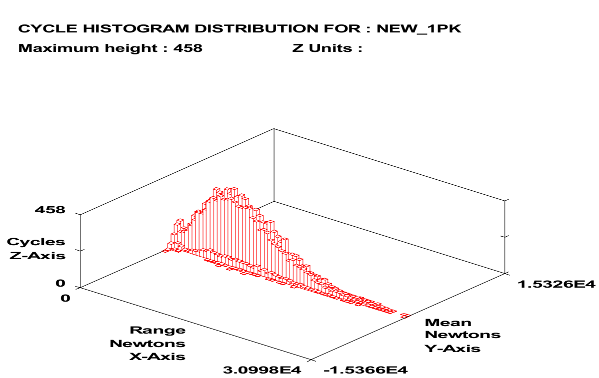

Figure 16 : Rainflow Cycle Counting histogram

higher except bracket loading condition. In addition, the vibration fatigue analysis can improve understanding of the system behaviors in terms of frequency characteristics of both structures and loads and their couplings.

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References

Arshad, W. M.; Bäckströom, T.; Sadarangani, C. (2002): *Integrated Free-piston Generator: An overview*, Proc. in the NORPIE,Stockholm, Sweden.

Bendat, J. S. (1964): Probability Functions for Random Responses, NASA report on Contract NASA-5-4590,1964.

Bishop N. W. M.; Sherratt, F. (2000): *Finite element based fatigue calculations*, NAFEMS Ltd. UK.

Bishop, N. W. M. (1988): The use of Frequency Domain Parameters to Predict Structural Fatigue, Ph.D. Thesis, University of Warwick, UK.

Bolotin, V. V. (1984): *Random Vibrations of Elastic Systems*, The Hague, Martinus Nijhoff , The Netherlands.

MSC/FATIGUE user's guide, vol. 1& 2. version (2004): MSC/Corporation. CANADA, 2004.

Newland, D. E. (1993): *An Introduction to Random Vibrations, Spectral and Wavelet Analysis*, 3^{rd} ed. Longman Scientific and Technical, Essex, UK.

Rahman, M. M.; Ariffin, A. K.; Jamaludin N.; Haron, C. H. C. (2005a): Modeling and Analysis of Cylinder Block of Linear Generator Engine for Fatigue Durability. In the Proc. of the 4^{th} International Conference on Numerical Analysis in Engineering (NAE-2005), Hotel, Yogjakarta, Indonesia 8-9 April, 2005.

Rahman, M. M.; Ariffin, A. K.; Jamaludin N.; Haron, C. H. C. (2005b): Analytical and Finite Element Based Fatigue Life Assessment of Vibration Induced Fatigue Damage, Proc. in the Second International Conference on Research and Education in Mathematics (ICREM2), Residence Hotel, Putrajaya, Malaysia, 25-27 May, 2005, pp. 331-345.

Rahman, M. M.; Ariffin, A. K.; Jamaludin N.; Haron, C. H. C. (2005c): Vibration Fatigue Analysis of Linear Generator Engine using Frequency Response Approach, Proc. in the Seminar on Advances in Malaysian Noise, Vibration and Comfort (NVC2005), ESSET, Bangi, Malaysia, 17-18 May, 2005, pp. 277-286.

Rahman, M. M.; Ariffin, A. K.; Jamaludin N.; Haron, C. H. C. (2005d): Prediction of Lifetime of Cylinder head in the Two-stroke Free Piston Engine, In the Proc. of the 2nd International Conference on Mechatronics 2005 (ICOM05), Hotel Pan Pacific, Kuala Lumpur, Malaysia, 10-12 May 2005, pp. 677-684.

Soong, T. T.; Grigoriu, M. (1993): *Random Vibration of Mechanical and Structural Systems*, Prentice Hall International Ltd, London.