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The Theory of Critical Distances Applied to the Prediction of Brittle Fracture in Metallic Materials

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Abstract: The Theory of Critical Distances (TCD) is a general term for any of those methods of analysis which use continuum mechanics in conjunction with a characteristic material length constant, L. This paper discusses the use of two simple versions of the TCD: a point-stress approach which we call the Point Method (PM) and a line-average approach: the Line Method (LM). It is shown that they are able to predict the onset of unstable, brittle fracture in specimens of metallic materials containing notches of varying root radii. The approach was successful whatever the micromechanism of crack growth (cleavage or ductile tearing); values of L determined from experimental data were found to be broadly similar to microstructural quantities (e.g. grain size) but an understanding of the micromechanism of failure is not necessary since the TCD is a continuum-mechanics approach. The TCD, in this form, can be thought of as an extension of linear elastic fracture mechanics (LEFM). Whereas LEFM requires one characteristic parameter (K_c), the TCD requires two parameters: K_c and L. The TCD is subject to many of the same limitations as LEFM: in particular it is shown here that the value of L varies with the level of constraint at the notch. However the use of the TCD greatly extends the applications of LEFM, allowing predictions to be made for notches and stress concentration features of any geometry for which an elastic stress analysis can be obtained.

keyword: Theory of critical distances, brittle fracture, metals, constraint.

1 Introduction

Ever since the Greek philosopher Leucippus (5^{th} century BC) proposed the existence of the atom, we have known that matter is not continuous; it is composed of discreet units at various size scales: atoms, molecules, precipitates, grains, etc. Despite this, many problems in mechanics, and other branches of physics, can be solved by assuming that matter behaves as if it were a continuum. We can expect, however, that errors will arise in the use of continuum mechanics when dealing with size scales which are similar to those of the microstructural features which directly affect the phenomena concerned.

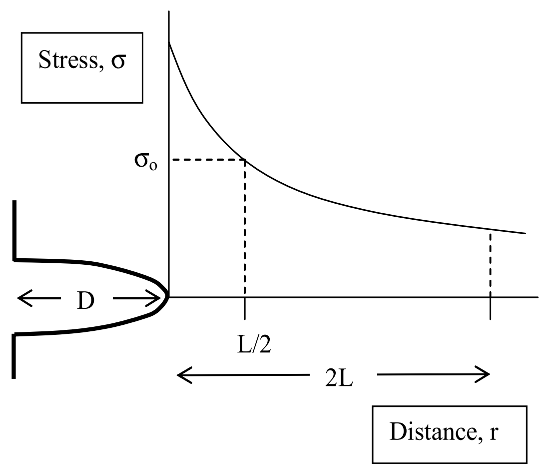

Figure 1 : A typical notch, with depth D and root radius ρ TCD methods use the stress/distance curve: for the Point Method (PM), failure occurs when the stress at a distance L/2 is equal to the characteristic strength σ_o; for the Line Method (LM), failure occurs when the average stress over a distance 2L is equal to σ_0.

This paper is concerned with the fracture behaviour of bodies containing cracks and notches. Fracture parameters such as strength and toughness are strongly affected both by microstructural features (e.g. grains, precipitates) and local plasticity (the plastic zone). Consider an edge notch of length D and root radius ρ (fig.1); if $\rho = 0$ we have a crack. If D is relatively large, and if the applied stress is relatively low, then the behaviour of this crack can be predicted using the well-known continuum-mechanics theory Linear Elastic Fracture Mechanics (LEFM). However, if D is small, LEFM predictions tend to be inaccurate, usually non-conservative. As regards the value of ρ, this is assumed to be zero in LEFM theory;

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in practice of course it must be finite, and the experimental evidence (some of which is presented below) suggests that there is a critical value ρ_c below which LEFM predictions are accurate, i.e. failure occurs when the stress intensity, K, is equal to the fracture toughness K_c. Currently there is no universally accepted method for predicting ρ_c, nor for predicting the behaviour of notches which have $\rho>\rho_c$. One clue to this behaviour is the fact that the value of ρ_c in many materials is often of the same order of magnitude as microstructural features such as grains [Cottrell 1963, Usami, Kimoto, Takahashi and Shida (1986), Yokobori and Konosu (1977)].

Various theoretical solutions have been suggested in order to overcome this problem, to develop an approach which is capable of predicting the behaviour of notches as well as that of cracks. One school of thought, typified by the RKR Model [Ritchie, Knott and Rice (1973)], considers microstructural features explicitly and attempts to describe the actual mechanism of the failure process (e.g. brittle fracture in steels initiated from a cracked carbide in a grain boundary). These types of models we can call 'micromechanical models'. A second school of thought, exemplified by the work of Pluvinage (1998), uses continuum mechanics and includes the effects of local plasticity, the assumption being that the errors made by LEFM are due to its neglect of plastic deformation. A third approach, which is the one used in the present paper, retains the elastic, continuum-mechanics philosphy of LEFM but modifies it by introducing a parameter, denoted L, which has units of length and which represents a characteristic distance in the material. This parameter is not identified directly with any particular microstructural feature, rather it is calculated using theoretical arguments at the continuum level. We call this approach the Theory of Critical Distances (TCD); the present paper concerns the application of the TCD to the prediction of brittle fracture in metallic materials containing notches. The structure of the paper is as follows:

i) A brief description will be given of the background to TCD, especially its simplest forms: the Point Method (PM) and Line Method (LM), which have already been used for solving other problems in materials failure.

ii) Using data from the literature, it will be shown that the PM and LM can be used to predict notch fracture behaviour in various metallic materials.

iii) It will be shown that constraint (i.e. the degree of plane stress or plane strain) affects the length constant L as well as the fracture toughness K_c.

iv) Whilst the underlying theoretical justification for the approach will not be considered in any detail, some possible explanations for the success of the TCD will be discussed.

2 The Theory of Critical Distances

The theory of critical distances (TCD) is the suggested name for a class of theories which predict the effect of notches and other stress concentration features by considering the stress field in the region close to the notch tip. Two parameters are required: a characteristic distance L and a characteristic stress σ_o (or, in some cases, a characteristic strain). Normally the condition for failure (by either brittle fracture or fatigue) is that σ_o becomes equal to some function of the stress field, evaluated over a distance which is a function of L, which may be considered to be a material constant. The simplest possible version of the TCD is a method which we call the Point Method (PM), in which an elastic stress analysis is used and the criterion for failure is simply that the stress will be equal to σ_o at a given distance from the notch root. If the loading is tensile and the notch lies perpendicular to the loading axis, σ_o is expressed in terms of the notch-opening stress, equal to the maximum principal stress, and the critical distance is located along a line forming the extension of the notch, in the direction of expected crack growth (fig.1). Other conditions are used for mixed mode loading but this subject is beyond the scope of the present paper. In a second version of the TCD, which we call the Line Method (LM), failure is assumed to occur when σ_o is equal to the average elastic stress along a line of a given length; the line is drawn in the same direction as that for the PM (fig.1). Early examples of the use of PM and LM are found in the work of Peterson (1959) and Neuber (1958), respectively, who used these methods to predict high-cycle fatigue behaviour in metallic materials.

More recently the theoretical basis of these methods has been improved by linking them to fracture mechanics concepts [Tanaka (1983), Kfouri (1997), Taylor (1999)]. The link is made as follows: consider the case of a long sharp crack, of length a, loaded with a nominal stress σ which is much less than the yield strength of the material σ_y. The elastic stress in the crack-opening direction $\sigma(r)$, as a function of distance from the crack tip r and the applied stress intensity K, is (assuming $r \ll a$):

$$\sigma(r) = K/(2\pi r)^{1/2} \tag{1}$$

Failure by crack propagation, occurs when K reaches a critical value, which will be denoted K_c, the fracture toughness. Note that K_c is a material parameter, though its value varies according to the degree of constraint. The plain-strain value of K_c, in Mode I tensile loading, is usually denoted K_{Ic}, however we will use the term K_c throughout in this paper to mean the critical value of K, whatever the degree of constraint. We define a characteristic distance L as follows:

$$L = (1/\pi)(K_c/\sigma_0)^2 \tag{2}$$

Combining these equations with the failure criterion for the PM as stated above, we find the location of the critical point as r=L/2; similarly for the LM, the length of the critical line can be shown to be 2L. However, this analysis does not give us any specific value for the critical strength, since here σ_0 can take any value (provided that L remains very much less than the crack length, *a*, so that equation 1 continues to be valid). Also, whilst this analysis is exact for sharp cracks there is no simple general derivation for notches, since different notches have very different stress fields. Further progress in this argument can only be made by comparing predictions with experimental data. To this end, Taylor and co-workers [Taylor (1999), Taylor and Wang (2000)] showed that fatigue limits for notched specimens could be predicted using both the PM and the LM, using the value of L given by equation 2, replacing K_c by the fatigue crack propagation threshold ΔK_{th} and taking σ_0 to be equal to the cyclic stress amplitude at the fatigue limit of plain (i.e. unnotched) specimens. The same assumption could also be used to predict the behaviour of short fatigue cracks, which deviated from LEFM predictions when *a* was of the same order of magnitude as L.

Recently, we have shown that the same approach can be used to predict brittle fracture under monotonic loading in ceramic materials, this time taking σ_0 to be the strength of plain, defect-free specimens [Taylor (2004)]. Kinloch and Williams (1980) used the PM to predict failure from blunted cracks and notches in brittle polymer materials such as epoxies. However, they found that the value of σ_o (the optimum value of which was discovered by fitting to the experimental data) was significantly larger than the plain-specimen tensile strength. This was recently confirmed also for PMMA (Perspex) [Taylor, Merlo, Pegley and Cavatorta (2004)].

To recap, previous work has shown that simple TCD methods such as the PM and LM can be used to predict the behaviour of specimens containing cracks and notches, using values of L and σ_0 linked to K_c through equation 2. In problems with very limited plasticity - high-cycle fatigue in metals and brittle fracture in ceramics - the value of σ_0 in this equation coincides with the relevant plain-specimen strength, but in polymers, where significant amounts of plasticity (or other non-linear deformation such as crazing) preceded failure, the same theory could be used but only with a higher value of σ_0. From this information it can be hypothesised that the TCD will also be able to predict brittle fracture in metals, but that the appropriate value of σ_0 will, as with polymers, not coincide with the material's tensile strength.

3 Application of the TCD to Brittle Fracture in Metals

Fig.2 shows a typical set of experimental data recording the effect of notch root radius on the measured fracture toughness of steel. In this and subsequent figures, the square root of the notch radius is used for the horizontal axis. This is common practice among workers in the field, because it has been noted that, above a certain critical radius, the data so plotted tend to lie approximately on a straight line. This data is due to Wilshaw, Rau and Tetelman (1968), who tested a mild steel at a temperature of -196^oC; failure occurred by brittle cleavage. Bars were used, of dimensions 10 x 10 x 60mm, containing a 2mm-deep notch with an included angle of 45^o, loaded in three-point bending. At ρ=0 we have a crack and the measured K_c value is equal to the fracture toughness of the material. Beyond a critical root radius the measured K_c increases, and of course this is no longer a valid measurement of the material's toughness.

Approximate predictions of the stress field near the notch root can be made using a modified form of equation 1, developed by Creager and Paris (1967), for notches of length D and root radius ρ:

$$\sigma(r) = \frac{K}{\sqrt{\pi}}\frac{2(r+\rho)}{(2r+\rho)^{3/2}} \tag{3}$$

Here K is the stress intensity factor for a crack of the

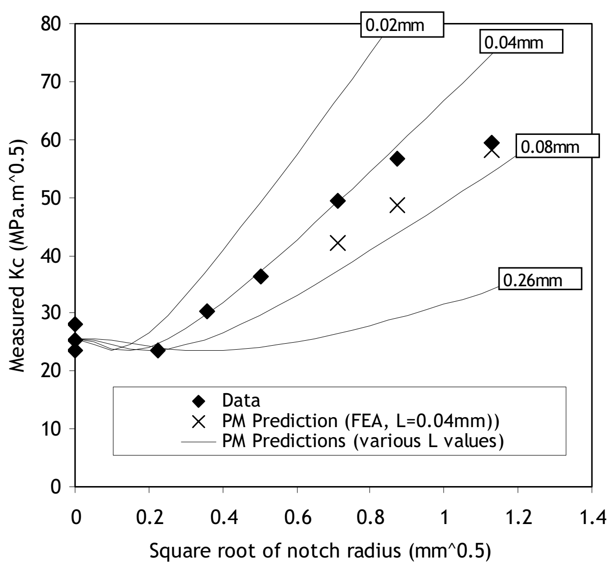

Figure 2 : Experimental data [Wilshaw et al (1968)] showing measured fracture toughness as a function of the square root of notch radius. PM predictions using stress values predicted by eqn 3 with various values of L. Further PM predictions for L=0.04mm using stress results from FEA rather than eqn 3.

same length as the notch. Using the PM and combining equations 1-3, we obtain a value for the measured toughness of a notch:

$$MeasuredToughness = K_c \frac{(1+\rho/L)^{3/2}}{(1+2\rho/L)} \quad (4)$$

Here K_c is the fracture toughness measured from cracked specimens, which was 25.7MPa(m)$^{1/2}$. The yield strength of the material was 829MPa; the UTS was not given in the paper – we assume a value of 900MPa. Using these parameters in equation 2 gives a value for L of 0.26mm. Fig.2 shows prediction lines using equation 4, choosing various values of L; in each case σ_o was then calculated using equation 2. It is clear that this approach can make reasonable predictions of the data, but that the value of L required is of the order of 0.04mm: much smaller than 0.26mm. Even for this L value there is some deviation at the larger values of ρ, but this error is due to inaccuracies in equation 3, which is valid only when $\rho \ll D$ and assumes infinite specimen dimensions. As fig.2 shows, calculations made using finite element analysis to obtain the stress field give, with the same value L=0.04mm, significantly lower predictions for the blunter notches. By further trial and error it was found that the best fit to the data is obtained for an L value of 0.035mm as shown in fig.3. It is interesting to note that this is exactly equal to the measured grain size in this material. The corresponding value of σ_o (from equation 1) is 2447MPa, which is higher than the yield strength of the material by a factor of 2.95.

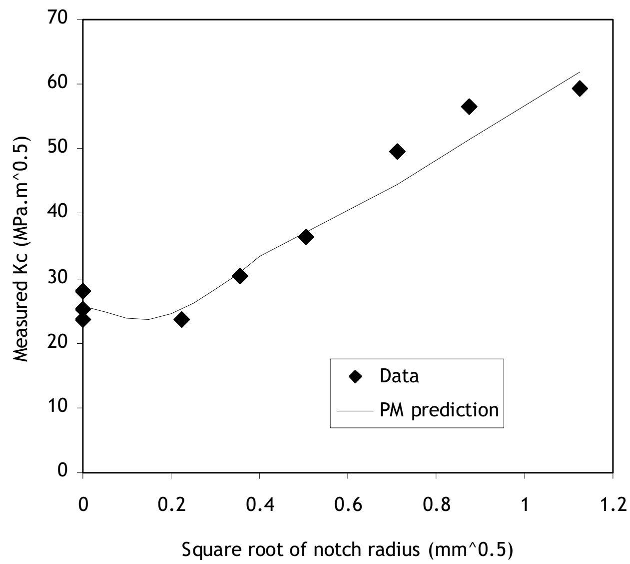

Figure 3 : The same data as fig.2, with PM predictions using the optimum value of L (0.035mm). Results from FEA were used for the higher notch radii where they differed significantly from those of equation 4. The largest difference between data and predictions is 10.1%.

Using the LM with equation 3 results in the following prediction for measured toughness:

$$MeasuredToughness = K_c \left(\frac{\rho}{4L}+1\right)^{1/2} \quad (5)$$

The main difference between the two predictions occurs at low values of ρ: the LM line increases monotonically with ρ whilst the PM line is almost constant at low ρ, showing an increase only when ρ rises above a critical value, which accurately reflects the experimental data. In fact the PM curve shows a shallow minimum point. It is not clear whether this minimum value really exists, though there is some suggestion of it in the experimental data here and elsewhere. In any case one can show from equation 4 that the minimum value is only 8% below

K_c, occurring when ρ=L/2. For the data on figs 2 and 3 the PM gives a better prediction than the LM; however there are cases where there is no obvious plateau and critical radius, such as fig.4, which presents data on an aluminium alloy, DISPAL-2, tested by Srinivas and Kamat (2000) at four different temperatures. Here the LM, which always predicts a monotonically increasing curve, modelled the data more accurately. Crack propagation occurred by ductile void growth, so the difference in behaviour may be related to the mechanism of failure. The optimum value of L was constant at 0.045mm for the three lower temperatures, rising to 0.075mm at 350^{o}C. The corresponding σ_o values were again of the order of 3 times the relevant yield strength.

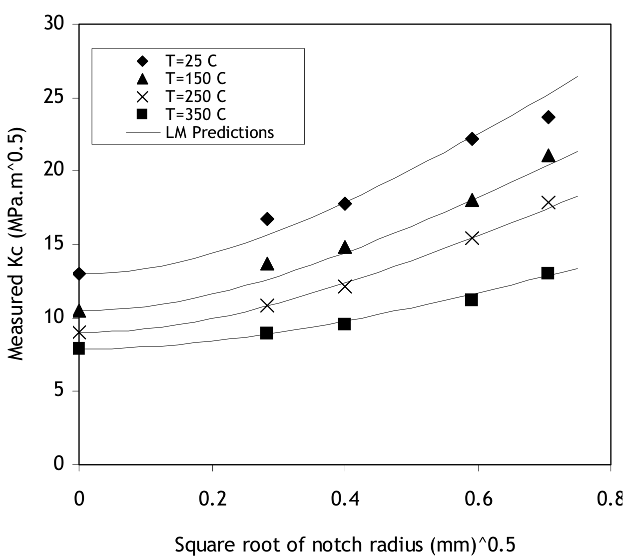

Figure 4 : Experimental data [Srinivas and Kamat (2000)] and predictions using LM, for an aluminium alloy tested at various temperatures.

In addition to the data shown in figs 3 and 4, a further eleven sets of data taken from the literature were analysed, covering a wide range of metallic materials. These results are not shown here for the sake of brevity; suffice it to say that similar predictive accuracy was obtained in all these cases. In these examples, failure invariably occurred by the rapid, unstable propagation of a crack and, crucially, the dimensions of the specimens and cracks was large enough to ensure small-scale yielding and plane strain conditions throughout. In the following section we will consider failures which occur under conditions of reduced constraint.

4 The Effect of Constraint

It is well known that the value of K_c depends on the level of constraint, being lowest under conditions of plane strain and increasing considerably as constraint is reduced towards plane stress conditions. This causes difficulties for the measurement and use of fracture toughness, especially in relatively tough, metallic materials. Despite considerable research in this field, there is no agreed method for predicting the effect of constraint on K_c, and even the assessment of the level of constraint in a given situation is not a trivial matter. We can anticipate that the material constants used in the TCD will also be affected by constraint: the aim of the present section is to investigate these changes. For this we will use some simple, approximate methods to estimate the degree of constraint. The specimen dimensions required to ensure plane strain conditions are specified by various national and international standards (e.g. BS 7448-1:1991). A typical requirement is that the specimen thickness B shall be larger than some critical value B_c, a function of K_c and the yield strength σ_y:

$$B_c = 2.5(K_c/\sigma_y)^2 \tag{6}$$

The same restriction applies to other dimensions: crack length a and remaining ligament width (W-a). In the data presented below, these other dimensions were always large enough to ensure conformance, so we are concerned only with the effect of B. Rearranging this equation gives us a value for K_c which we will refer to as the 'plane strain limit':

$$K_c[\text{plane strain limit}] = \sigma_y(B/2.5)^{1/2} \tag{7}$$

This condition is designed to be a conservative one, so we can say that if K_c is less than the value given by equation 7, then we certainly have conditions of plane strain, but cases in which K_c is slightly larger may still give a valid plane strain result in practice.

Constraint is reduced through the specimen thickness by the spread of plasticity. As thickness is decreased (or applied load increased) the plane stress regions at the two surfaces occupy an increasing fraction of the thickness until a point is reached when no part of the crack front experiences plane strain. Many workers have attempted to estimate this point, either analytically or experimentally (e.g. Irwin (1964), Knott (1973), Ando, Mogami and Tuji (1992)).

Irwin (1964) estimated the plane-stress plastic zone size r_y as:

$$r_y = (1/\pi)(K/\sigma_y)^2 \quad (8)$$

He noted from experimental results that, if r_y=B, specimens showed 50% or more of slanted fracture, which is associated with plane stress, and that this increased to almost 100% if r_y=2B. Knott (1973) pointed out that the measurement of slant fracture will tend to underestimate the amount of plane stress, since some plane stress fracture will produce flat surfaces. Given this, we will use the condition r_y=B to indicate the 'plane stress onset', i.e. to give a value of K_c above which plane stress conditions will begin to dominate:

$$K_c[\text{plane stress onset}] = \sigma_y(\pi B)^{1/2} \quad (9)$$

A similar condition is required to predict constraint limits for notches. To do this we note that equation 8 has the same form as equation 1, so we can use a variation of the PM in which the critical stress is σ_y and the critical distance is B/2. This will be an exact prediction of the size of the plane-stress plastic zone for a crack, and an approximate prediction in the case of a notch. Tsuji, Iwase and Ando (1999) used a slightly different approach based on matching areas under the stress/distance curves for elastic and plastic conditions. Their method is probably more accurate than the one used here but we found that it gave very similar predictions (within 10%) for the data used here.

For the data of Wilshaw, Rau and Tetelman (1968) shown above, all fractures occurred at K values below the plane strain limit (eqn.7). Fig.5 shows further data on low-temperature cleavage fracture of steel, in this case from Tsuji, Iwase and Ando (1999). There is considerable scatter but a PM prediction fits the data reasonably well using an L value of 0.05mm. The plain strain limit occurs at 54MPa$(m)^{1/2}$; some of the data points lie above this limit, but all lie below the plane stress onset value (not shown). Fig.6 shows data from Yokobori and Konosu (1977) who tested a steel which was similar to that of Tsuji et al, but used thinner specimens. When we attempted to use the TCD we found that there was no single value of L which would fit this data, using either the PM or LM. However, the data for root radius values up to about 1mm could be predicted using the PM with a value of L identical to that used for the Tsuji data (0.05mm).

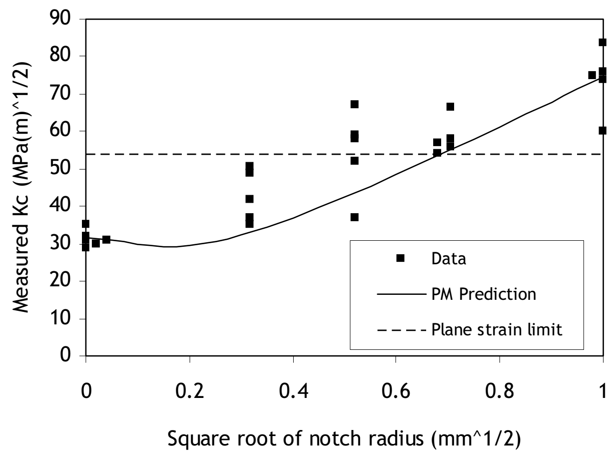

Figure 5 : Data from Tsuji et al (1999) on low-temperature cleavage fracture of steel. Predictions using the PM (with L=0.05mm); plane strain limit value calculated using equation 7.

In this case all results lie above the plane strain limit line, and the data points begin to deviate from the PM prediction line at the point of plane stress onset. We conclude from this that the PM prediction works for these notches provided a high level of constraint is maintained. Also shown on the graph is a line corresponding to general yield in these specimens, indicating that the failures in the blunter notches occurred under conditions of full plasticity. It is interesting to note that, around the transition point ($(\rho)^{1/2}$ values of 0.9mm$^{1/2}$ and 1mm$^{1/2}$) there is more scatter in the data than elsewhere, perhaps indicating a change in fracture mechanism with some specimens failing under plane strain conditions and others being affected by reduced constraint and therefore failing at higher stress levels.

Fig.7 shows a similar situation occurring in data on a high strength steel [Irwin (1964)] tested at room temperature, which had a K_c value of 29.6MPa$(m)^{1/2}$. Again there was no single value of L which could predict all the data: a value of 0.0023mm was successful at low notch radii and the data shifted to values above the prediction line in

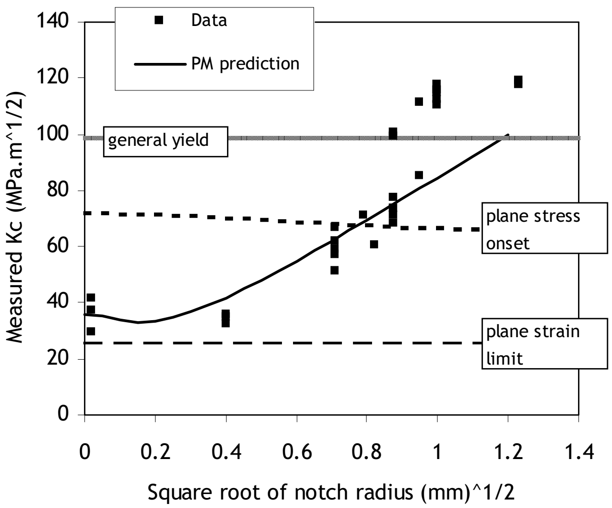

Figure 6 : Data from Yokobori et al (1977). Predictions using the PM; estimated values for the plane strain limit, plane stress onset and general yield.

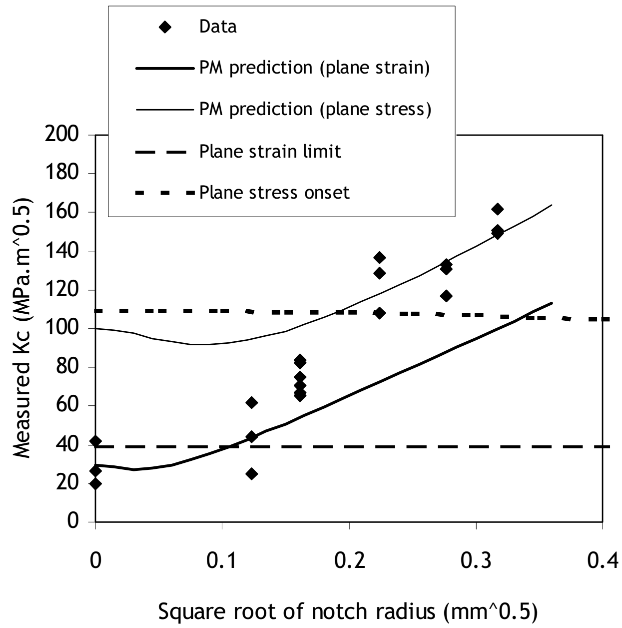

Figure 7 : Data on a high-strength steel tested at room temperature [Irwin (1964)], showing the plane strain limit, plane stress onset and some possible PM predictions for plane strain and plane stress.

the region between the plane strain and plane stress limit lines. This time a second prediction line has been drawn, also using the PM, which passes through the data points for the blunter notches and may represent fully plane stress conditions. This prediction is a very tentative one, since we do not know the value of K_c for plane stress so it was necessary to choose values for both K_c and L. The resulting values were K_c=100MPa(m)$^{1/2}$, L=0.015mm; this K_c value is at least plausible given that the measured fracture toughness in metals in plane stress is typically three times higher than in plane strain [Knott (1973)]. The very small plane-strain value of L, only 2.3μm, probably reflects the fact that the relevant microstructural parameter in this quenched and tempered steel will be the lath width, rather than the grain size. The plane-strain value of σ_o was 11,010MPa, which is much greater than three times the yield strength (σ_y=1,590MPa), showing that there is no fixed relationship between σ_o and σ_y. The plane-stress value of σ_o was even higher, at 14,570MPa.

Finally, fig.8 shows data obtained under fully plane stress conditions, using relatively thin specimens of aluminium alloy 7075-T6 [Mulherin, Armiento and Marcus (1963)] with a yield strength of 498MPa. Good predictions were obtained using the PM with a K_c value of 77MPa(m)$^{1/2}$ and an L value of 0.07mm, giving σ_o=5,190MPa.

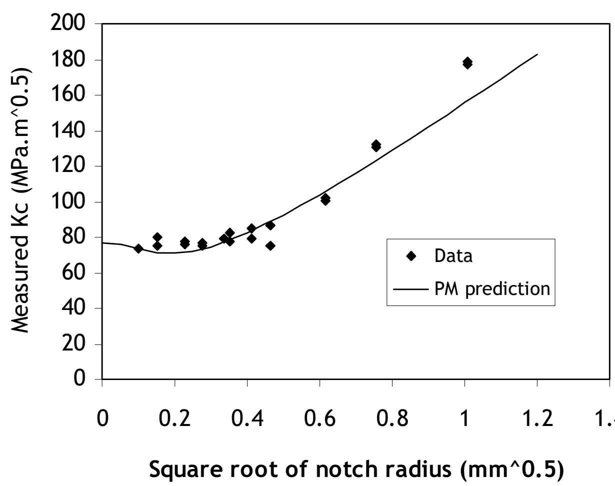

Figure 8 : Data [Mulherin, Armiento and Marcus (1963)] and PM predictions for an aluminium alloy tested under plane stress conditions.

5 Discussion

This analysis has shown that simple versions of the Theory of Critical Distances, such as the PM and LM, are capable of accurate predictions of the effect of notch radius on the fracture strength of metallic materials. The approach can be used to predict failures which occur by unstable crack propagation, whether the micromechanism is cleavage or a ductile process. One difficulty, compared to previous work on high-cycle fatigue in metals and on brittle fracture in ceramics, is that the value of the critical distance L cannot be predicted analytically but must be found from experimental data. However, once L is known the method has considerable predictive capacity over a range of notch radii. It has been noted by a number of workers that the measured toughness increases approximately as the square root of notch radius (for radii above the critical value): this result can be predicted using the present approach, because as ρ increases and becomes much larger than L, the predicted K_c value becomes proportional to the stress near the notch root, which (by eqn.3) becomes proportional to $(\rho)^{1/2}$. However, whilst this relationship is a useful approximate one it is not exact; in general the result will depend not only on root radius but also on notch length and specimen dimensions: the TCD can still be used but FEA will be needed to provide an accurate description of the stress field (as shown in fig.2).

A second difficulty, which is to be expected when dealing with metals, is the effect of constraint. In notched specimens the problem is complicated by the fact that a specimen which may fail under plane strain conditions when it contains a crack or sharp notch, may no longer do so when the root radius is increased because the higher load before failure will generate more plasticity. Using some approximate estimates for the limit of plane strain and onset of plane stress, it was demonstrated that the TCD can give good predictions whenever plane strain dominates. It may also be capable of predicting behaviour under plane stress and intermediate constraint levels, though further analysis of data is required before this can be confirmed. Just as we know that K_c changes with constraint, so we can expect that L and σ_o will also change. It should be emphasised that the evidence presented here in regard to plane stress is very limited and that the whole question of predicting toughness under conditions of reduced constraint is a much more difficult one. We have considered only the particular case of out-of-plane constraint: the level of in-plane constraint can also vary (for example due to variations in the T stress) and this can also be shown to affect toughness.

Under conditions of low constraint the LEFM requirement of contained yielding is often lost, but even when it is maintained fracture is often characterised by considerable amounts of stable crack growth before final fracture. During this stable crack growth the apparent toughness of the material increases, creating the so-called resistance curve (or R-curve) which is often used as a predictive tool. Recently, however, the R-curve philosophy has been criticised [Sumpter (1999)]; in any case it has been noted that the amount of stable crack growth before unstable fracture is much smaller for notches of radius larger than ρ_c than it is for cracks [Irwin (1964)], so an analysis such as the TCD, which is based only on initial conditions, may still be valid in these circumstances.

As stated in the Introduction, the TCD is advocated as a continuum mechanics theory, essentially a modified version of LEFM, and (like LEFM) it functions using material parameters obtained from experimental testing, which characterise the material at a continuum level. The introduction of only one new parameter – the characteristic distance L – is sufficient to extend the use of LEFM very widely, to allow predictions to be made from notches of varying root radius and (as we showed in previous papers on fatigue and on brittle materials) to predict the effect of short cracks and small notches. Because only an elastic stress analysis is needed the predictions can be made using simple analytical equations or computer models, thus providing a practical tool for the prediction of failure in complex geometries such as engineering components.

The values of L found in this work were of the same order of magnitude as microstructural features such as grains or bainite laths. To further illustrate this point, fig.9 shows L values calculated from the data of Yokobori and Konusu (1977) who tested their material in different heat treated conditions to produce six different grain sizes, including the one shown above in fig. 6. There is a clear relationship between grain size and L in this case.

Thus, in the future, if the relevant micromechanism of failure is well understood in a particular case, it may be possible to estimate L from microstructural distances. On the other hand, the strength parameter σ_o is unlikely to have any physical meaning. Its values for cleavage in steels are considerably higher than mea-

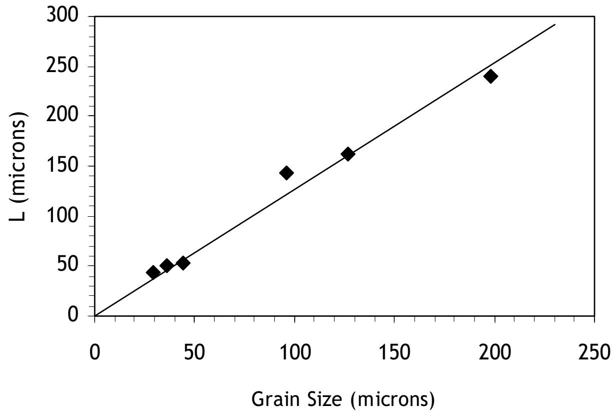

Figure 9 : L values calculated from the data of Yokobori and Konusu (1977), plotted as a function of the grain size of the material.

sured values of the fracture stress, which are typically of the order of 1000MPa [Ritchie, Knott and Rice (1973), Wilshaw, C.A.Rau and A.S.Tetelman (1968)]. Kinloch and Williams (1980), working with polymers, suggested that, because σ_o is approximately equal to three times the yield strength, it may be related to the peak stress value ahead of a crack or notch in plane strain. However the peak stress occurs at a distance different from L/2 and its magnitude is a feature of the elastic/plastic stress distribution. In any case we found that in some materials σ_o is much larger than $3\sigma_y$, even exceeding $10\sigma_y$.

The situation is different when using the TCD to predict high cycle fatigue, because there the value of σ_o is equal to the material's fatigue limit and actually exists at the distance L/2 because the elastic analysis is valid at that distance, the plastic zone being considerably smaller. The same may be true for brittle fracture in ceramics, but not for brittle fracture in metals because the point L/2 will lie inside the plastic zone. The theoretical justification for using the PM and LM, despite the existence of local plasticity, is the same justification that we invoke when using LEFM: there is a one-to-one relationship between conditions inside the plastic zone and conditions in the surrounding elastic zone. Therefore we can use a purely elastic analysis to predict failure. LEFM can be interpreted as specifying that failure will occur when the elastic stress field near a crack has certain characteristics (specifically that the proportionality constant K attains a certain value). The PM and LM also make use of the elastic stress field, but use different specifications for failure: that the elastic stress should reach a certain value at a distance L/2 or that its average should reach a certain value over a distance 2L.

It is expected that the other limitations which normally apply to LEFM will also apply to the TCD; for example the approach is unlikely to be successful in situations of extensive plasticity where the small-scale yielding condition is violated. It may be possible to modify the theory to take account of the T stress, and it should certainly be possible to include general multiaxial fracture laws as part of the theory, in the same way that they have been introduced into LEFM in the past.

6 Conclusions

1) Simple versions of the Theory of Critical Distances (TCD), such as the Point Method (PM) and Line Method (LM), can be used to predict the onset of unstable, brittle fracture in metallic materials containing cracks and notches.

2) Predictions are successful whether the micromechanism of crack growth is cleavage (e.g. in steels at low temperature) or ductile tearing (e.g. in aluminium alloys); the micromechanism is not of importance because the TCD is a continuum mechanics method. It differs from LEFM only in that it makes use of a material constant, L, which has the units of length and can be thought of as a characteristic distance.

3) The value of L for a particular material and failure mechanism can obtained from experimental data. Knowing L, and the fracture toughness K_c, a characteristic strength value σ_o can be determined for use in the PM and LM calculations. L is typically of the same order of magnitude as microstructural features: the value of σ_o, being derived from an elastic analysis, is probably not of any fundamental significance.

4) Like K_c, the value of L changes with the level of constraint experienced at the crack or notch. Under fully plane-strain conditions, unique values of these parameters are capable of determining the effect of root radius on the stress to failure. However, as notch root radius increases, the degree of constraint prior to failure may decrease, necessitating changes to the parameter values.

5) Since the TCD can be applied to any body under stress for which the elastic stress field can be predicted (e.g. by computer methods such as FEA), it can very easily be used to assess the behaviour of stress concentration

features of complex shape, such as exist in engineering components.

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