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Effect of correct statistical description of fatigue crackpropagation data on the time to first inspection

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Abstract: Each maintenance strategy demands for the definition of an inspection threshold and further inspection intervals. A general criterion for the calculation of the time to first inspection is high probability of detection of a certain crack size and low failure probability in case a predicted crack size was not detected. The proposed method demonstrates that a top down analysis of crack development from critical sizes to detectable sizes reveals an economic benefit with respect to the frequency of inspections. The dispersion of fatigue stress cycles at rupture obtained from component tests at riveted lap joints is transformed to the distribution of time to predicted detectable crack sizes using an analogy between the structure of Wöhlers- and Paris equation. The distribution function of stress cycles at various crack stages is then derived from integration of the governing stochastic limit state equation.

keyword: Damage tolerance, probabilistic analysis, statistical description

1 Introduction

Two approaches of probabilistic damage tolerant analysis are available:

1. Probabilistic analysis based on an initial flaw size distribution together with the description of the long crack propagation phase.

2. Use of quantile values of Wöhler test data for the time to fatigue failure and the evaluation of correct distribution functions of the crack propagation phase. Both methods are in line with each other if the design basis corresponds to the same statistical characteristics used. But the first method strongly depends on the assumption of an initial micro- crack size which in turn is responsible for the mean initial crack size distribution and the corresponding stress cycles.As the scatter of the time to crack initiation increases rapidly with the dispersion of an assumed micro- crack size the fatigue analysis becomes very uncertain. In order to illustrate this statement the dispersion of the micro-crack formation phase is compared to that of the long crack phase using a particle flow model [Konietzky, Kamp, Bertrand (2000)].

In the following a detailed description of the proposed second method is given. It circumvents the necessity to establish mechanical models to describe the complex micro-crack initiation- and small crack development phase. Instead a top down procedure is established which is based on fatigue data from component tests and simple open hole long crack propagation data starting from an artificially introduced initial through-crack.

Design of a maintenance strategy by probabilistic analysis has to solve two tasks in order to be reliable.

Choosing an appropriate mechanical model which fits fatigue and crack propagation test data as well

Correct description of the scatter of test data by a refined statistical analysis.

It is shown that the use of the simple Wöhler formula for the calculation of the time to fatigue rupture together with the Paris equation for the long crack propagation are suitable to define more realistic the time to first inspection (threshold) and further inspection intervals than it is done by conventional deterministic analysis using scatter factors.

To focus again on the problem :

An economic maintenance strategy must be based on an inspection threshold which guarantees with high probability detection of predicted crack sizes. Therefore the main target is to evaluate the density/distribution function of time to detectable crack sizes. This is done by a back calculation scheme starting from distribution of time to fatigue collapse. The distribution function of stress cycles at detectable sizes is then found by subtraction the long crack propagation phase from the total time to fatigue rupture.

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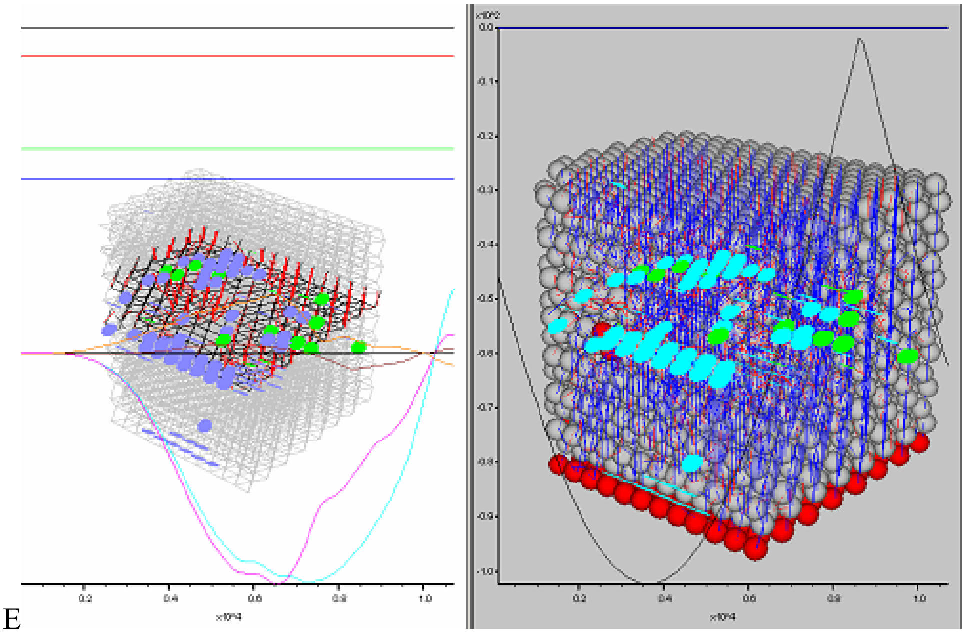

Figure 1 : Three-dimensional ball assembly models a micro-structure: Shear and tension bonds are released (circles). Arrows represent contact forces.

2 Statistical properties of micro crack development Phase I and long crack propagation Phase II

2.1 Coefficient of variation of time to crack intiation.

The physics of Micro- crack propagation is governed by several uncertain parameter: The most important are the following:

- initial micro- crack defect size
- grain size
- material parameter of the micro- lattice structure

This fact explains the big scatter of test data of time to small crack sizes.

In order to derive a figure for the coefficient of variation of the distribution function of time to formation and coalescing of micro- crack sizes the particle flow code method is used [Konietzky, Kamp, Bertrand (2000)].

The behaviour of micro-crack development was simulated by this code. The micro structure is represented by balls of 1-5 μm diameter. The micro-structural properties are defined by tension-, compression- and shear contact

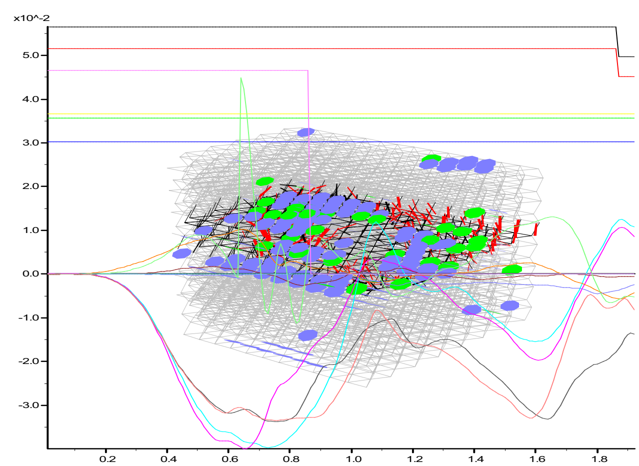

Figure 2 : Progress of crack formation and transient amplitude of contact forces

forces. Results of a two- and a three dimensional model are shown below.

Figures of the three dimensional model (s. Fig. 1-2) show how micro-cracks develop and coalesce

spreading from stochastically distributed normal and shear contact strength acting between grains (balls).

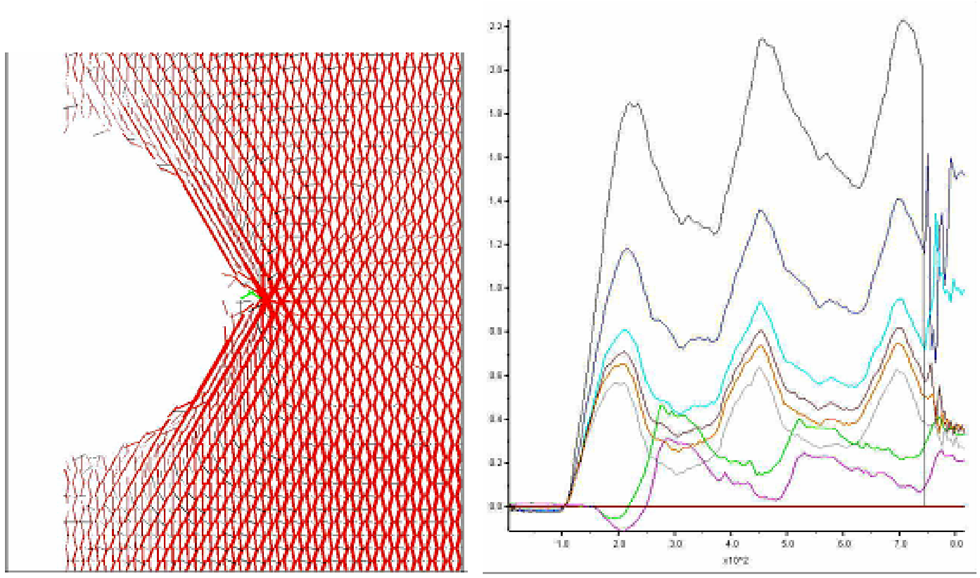

Figure 3 : Initial crack and monitoring of decreasing contact forces with increasing distance from crack tip

Contact forces representing friction, shear and tension/compression are monitored during dynamically imposed stress cycles. Strength degradation is imposed according to fatigue ageing.

The two-dimensional model represents the long crack behavior starting from initial through cracks at rivet holes (s. Fig.3-4)

Simulation runs of cycles to failure are performed for both models. Corresponding histograms show expected big difference of the coefficient of variation (s.Fig 5):

The result of this investigation demonstrates that the coefficient of variation of fatigue stress cycles (N-cycle) in the micro crack phase is much greater than that of the stress cycles in the long crack phase. An issue which can simply be confirmed by evaluation of the distribution function of stress cycles from Paris equation F(Np) on the basis of eq. 1:

$$\tilde{N}p = \frac{1}{\tilde{C}p\,\pi^{\frac{\tilde{m}p}{2}} \cdot E(\Delta S^{\tilde{m}p})} \int_{ao}^{ac} \frac{da}{f(a)^{\tilde{m}p} \cdot a^{\tilde{m}p}/2} \qquad (1)$$

Cp- Paris constant ; mp- Paris constant ; f(a) Geometry correction ; a_o- initial crack size ; a_c- critical crack size

$$\Psi(\tilde{N}p) = \int_{a}\int_{\tilde{c}}\int_{\tilde{m}p} f(\tilde{N}p)dC \cdot dm_p da \qquad (2)$$

The initiation phase is governed by an assumption of scatter of the initial crack size (a0)which has a coefficient of variation of $\approx$ 50 %. Four- time integration of eq. (2) with initial crack density function (f(a0) and uncertain Paris constant Cp and uncertain exponent m results in the following :

the coefficient of variation of stress cycles N taking into account the uncertainity of the initial crack size distribution is cov $\cong$ 45%

in the long crack case a fixed initial crack size (mean value) is used: the cov is now $\cong$ 15%

2.2 *Dispersion of fatigue cycles Nf at critical crack sizes.*

The classical method uses regression analysis of S-N data to determine the median of stress initiation cycles to fatigue failure [Sindel (1998)]

$$\mu n = Cw \left(\frac{1}{\Delta S}\right)^{mw} \qquad (3)$$

Assuming ΔS being lognormal the distribution of N_f is normal with dispersion δ:

$$F(\tilde{N}) = \phi\left(\frac{\ln\frac{\Delta S^{mw}}{Cw}}{\delta}\right) \qquad (4)$$

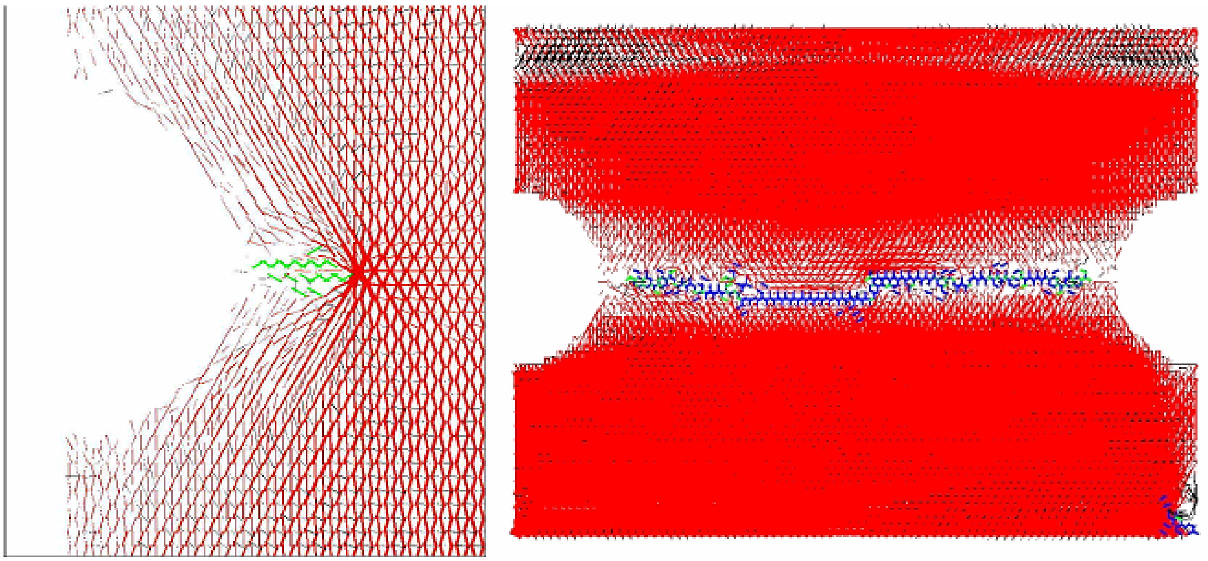

Figure 4 : Crack propagation and coalescing of adjacent cracks

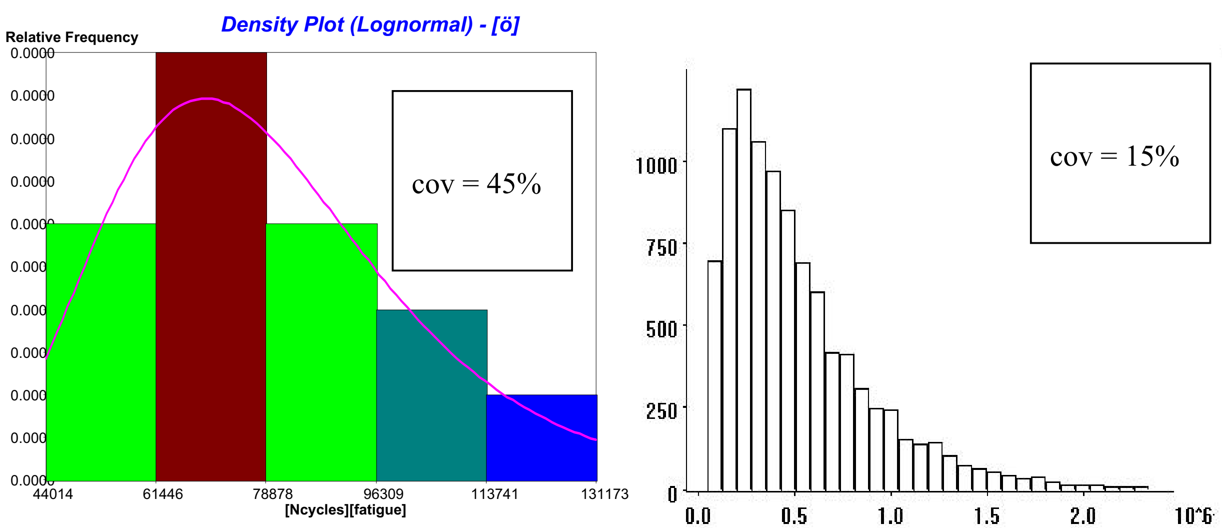

Figure 5 : Histogram of micro-crack stress cycles (cov=45%) and cycles to failure of the long crack phase (cov = 15%)

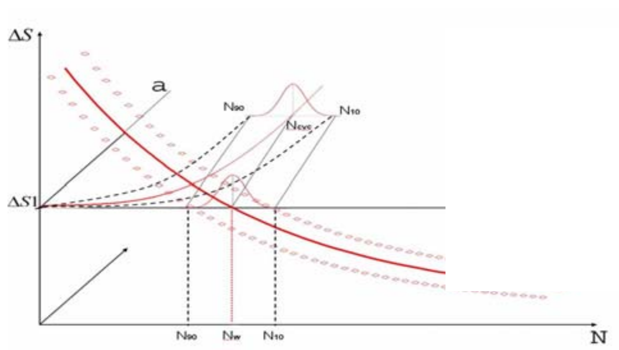

Figure 6 : Correspondence between scatter of crack propagation cycles at critical crack size and scatter of time to fatigue rupture

The dispersion δ is taken from fatigue test at coupon level. As a consequence the bandwidth between 10% and 90 % quantile values of cycles N is very large accompanied by big scatter of stress cycles. A very promising alternative solution is proposed: From fatigue tests at component level an observation was made called system scale effect occurring at larger scale level, which is repeated here [Bertrand (2005)]:

The scatter of stress cycles in the long crack region which was deduced from large scale test is smaller than that derived from fatigue coupon test specimen. It is suggested that fatigue rupture of large riveted panels starts always from clusters of small cracks developing from sizes between 0,5 to 1 mm which coalesce before the component collapses.

The development of clusters of small cracks (which initiate coalescing of adjacent cracks) at stress cycles N near the mean of fatigue rupture is much more probable than starting very early from cycles in the tail region of the distribution function of time to fatigue collapse. The dispersion of stress cycles at critical crack sizes can therefore be projected to time of detectable crack sizes N_{Det} (s. Fig. 6). The affinity in structure of equation (3) and the Paris equation (s.eq. 1) reflects this observation. Both are factored by (ΔS^{-m}):

$$N_F = \tilde{C}_w \cdot \Delta s^{-m_w} \quad (5a)$$

$$\tilde{N}_P = (\tilde{C}_{P-mp}) \cdot \Delta s^{-m_P} \quad (5b)$$

The dispersion δ occurring in equation (3) is therefore adjusted to that of equation (5b) by an appropriate transformation.

2.3 *Statistical properties of long crack propagation (phase II)*

The statistical characteristics of Paris constant $C_p$and exponent m_p are calculated from da/ dN crack propagation data s.fig.8. Mean values are obtained by regression analysis. As $C_p$and m_p are statistically dependent their standard deviation has to be evaluated by their joint density function. The distribution function Ψ (N_p) at defined crack sizes (a) can be derived from integration over uncertain C_p and m using their joint density function (s. Fig. 7). The procedure is explained in [Caracciolo and Bertrand].

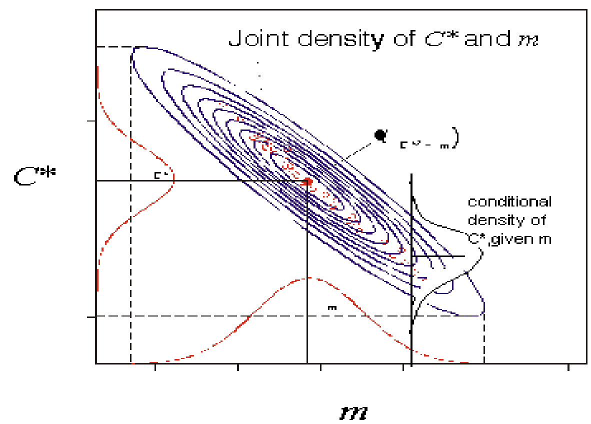

Figure 7 : Joint distribution of Paris constants Cp and exponent mp and correlation between both s. [Caracciolo and Bertrand]

For the correct definition of quantiles of stress cycles N related to certain crack sizes (a) it is of utmost importance to account for the correlation between C_p and m_p.

Modeling the joint distribution exactly reduces considerably the error in the estimate of the standard deviation of the predicted stress cycles N at various crack sizes (a).

The degree of accuracy to be in line between prediction and test results is shown for the density function of cycles N at crack size a=24mm in fig.9 . Deviations up to 500% of the predicted standard deviation of Ncyc are encountered as long as the nonlinear statistical correlation between Cp and m is neglected (s. fig.10)

The crack propagation phase is simulated exactly by the Paris law using correct mapping of the statistical dependence of Paris constant Cp and exponent mp(s.fig.10).

The probability distribution of cycles N between $P =$

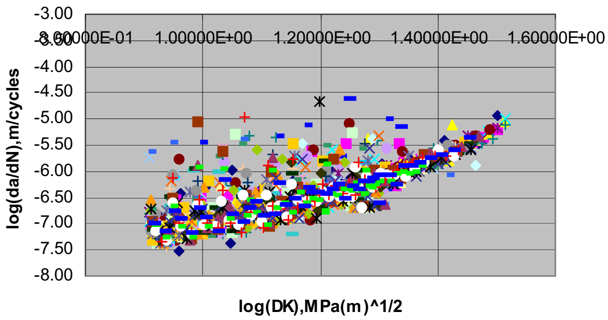

Figure 8 : da/dN data from open hole test

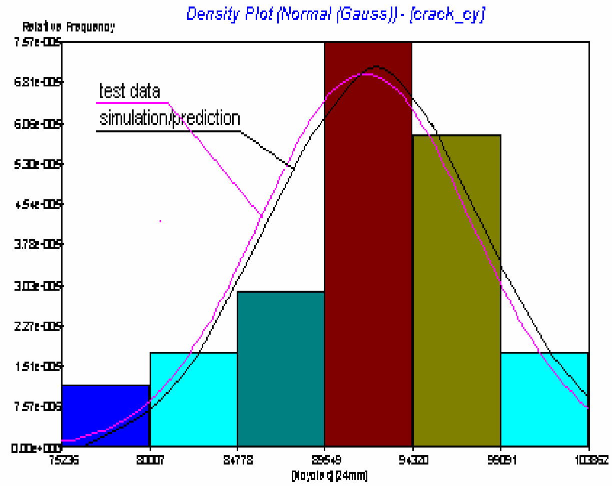

Figure 9 : Density function of test results and prediction of Ncyc at a=24mm (left); 5% different a in stand. dev.

0.01 and $P = 0.99$ as function of crack size a is compared to corresponding distribution functions from test results (s. Fig. 11). The prediction fits the test results very well.

3 Probabilistic sizing of time to first inspection

The distribution function $F(N_{th})$of stress cycles N_{th} to detectable crack sizes a_{Det} results from integration and subtraction of two density functions

$$f(N_{th}) = f(N_{FAT}) - f(N_{prop}) \tag{6}$$

with N_{th} - time to inspection threshold; N_{FAT} - fatigue stress cycles to rupture; $N_{prop}(a_{crit} - a_{Det})$ - stress cycles from detectable crack sizes a_{Det} to critical crack sizes a_{crit}

The fragility function of $\tilde{N}_{th}$ and its distribution is obtained with use of eq. 2 and eq.4 . A comparison of results from the initial flaw size method and the proposed method is shown in Fig. 12. The IFS –method demands an earlier first inspection than the proposed top-down procedure.

An updated inspection strategy uses detection results from first inspection. The failure probabilities of chosen limit states for first time to inspection $P(N_{Det})$ and of further intervals $P(N_{insp})$ accounting for detection events $E(N_{Det})$ are computed from:

$$P(E_{th}) = P(N_F - N_{prop}(a_{crit} - a_{Det}) - N_{thr} \leq 0)$$

Inspection Intervals Nint are derived from:

$$P(E_F \cap \overline{E}_{th} \cap \overline{E}_{Det}) =$$

$$P(E_F/\overline{E}_{Th} \cap \overline{E}_{Det}) \cdot P(E_{Det} \cap \overline{E}_{th})) \leq 10^{-9}$$

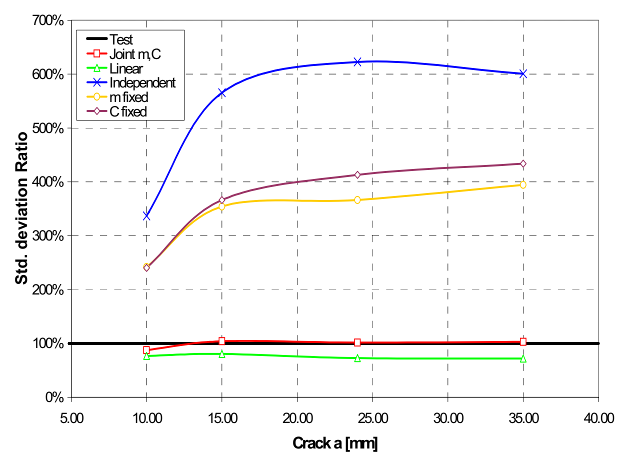

Figure 10 : Deviation of Ncyc with respect to exact solution (100%) by joint density of C and m

The strong influence of the detectability on reliability and of the chosen detection method is demonstrated in Fig. 13. A probabilistic scale is used to limit the failure probability Pf(shown as reliability index $\beta = \Phi^{-1}(pf)$) to allowable values, which are indicated by a safety objective (horizontal line).

4 Conclusion

On the basis of the dispersion of stress cycles from fatigue tests at component level a 'top down ' procedure is

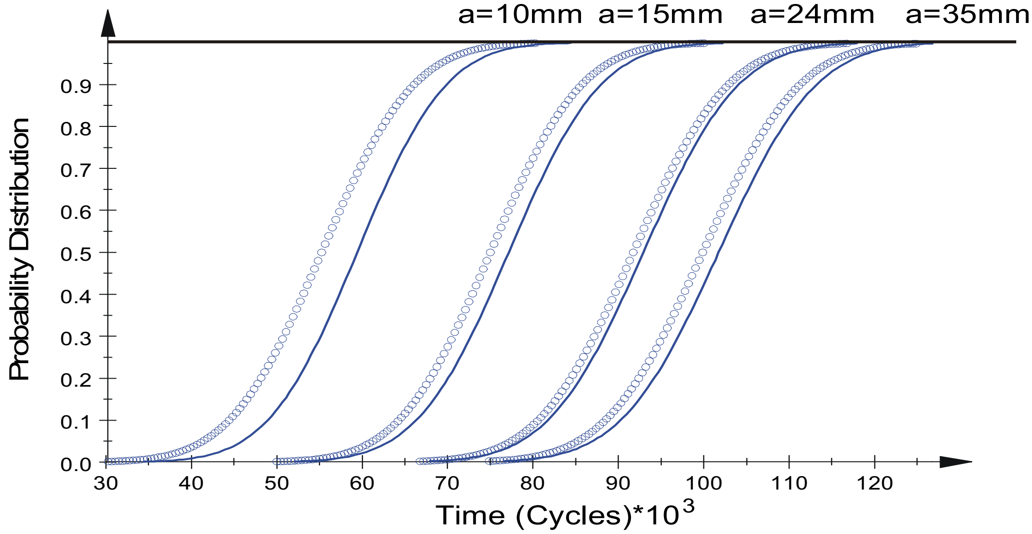

Figure 11 : Distribution function F(Ncyc) defining N%- Quantiles of crack propagation curves from test data and prediction for crack sizes between a=10mm to a=35mm

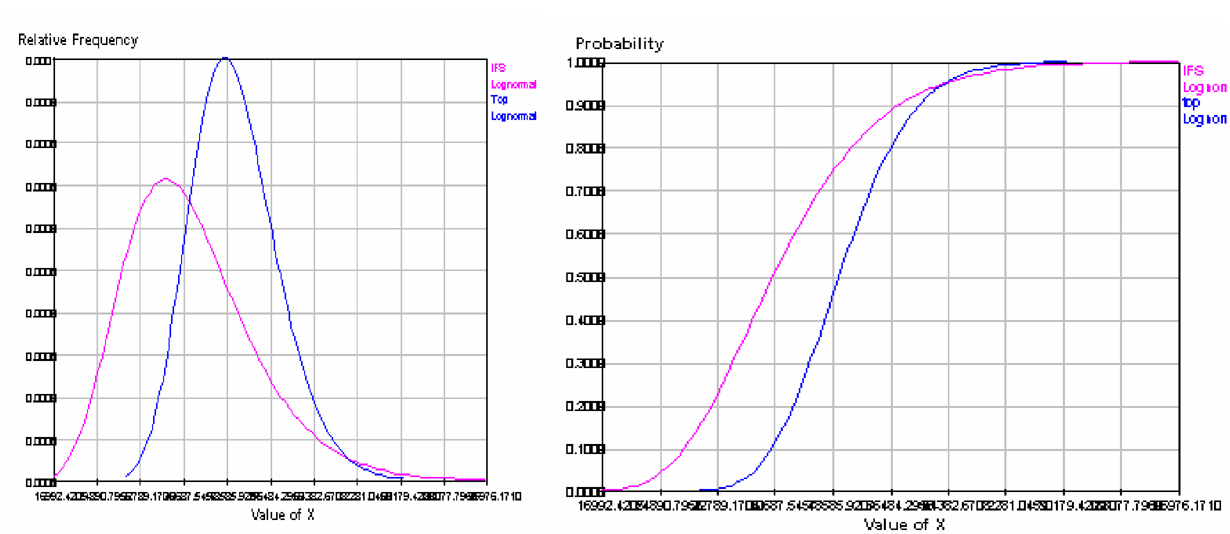

Figure 12 : Results of the density and distribution function f((N_{th}),F(N_{th}) compared with those from the IFS-method (left function)

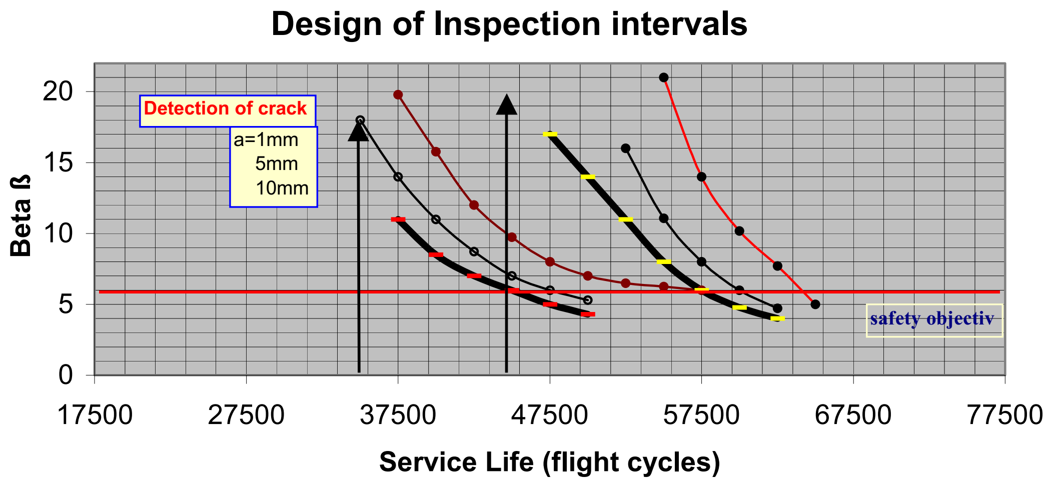

Figure 13 : Reliability index ß (as indicator of demand for inspection) as function of flight cycles : Fat line indicates decrease of reliability by IFS-method and detection probability of a 90% of a crack size of 10mm. Most upper line shows results of the top down method with a detectability of 90% of crack sizes of 1mm and 5mm.

developed which defines the distribution of time to detectable crack sizes which is the time to first inspection.

The proposed method is motivated by an investigation which compared the coefficient of variation of crack initiation time with that of the time to fatigue rupture.

Appropriate limit state functions in stochastic space are integrated from which the time to first inspection and further intervals are deduced. It turns out that a precise description of the statistical properties of the long crack development results in a realistic prediction of the coefficient of variation of time to detectable crack sizes. It is further shown that the chosen detection accuracy has a big impact on the frequency of planned inspections.

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