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An Improved Wheeler Model for Remaining Life Prediction of Cracked Plate Panels Under Tensile-Compressive Overloading

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Abstract: This paper presents an improved Wheeler residual stress model for remaining life prediction of the cracked structural components under variable amplitude loading. The improvement to the Wheeler residual stress model is in two folds. One is expressions for the shaping exponent, which are generally obtained through experiments. Another is calculation of effective plastic zone size to incorporate the sequent effects under tensile-compressive overloading. The remaining life prediction has been carried out by employing the linear elastic fracture mechanics (LEFM) principles. Studies on remaining life prediction of cracked plate panels subjected to tensile-compressive overloading have been carried out for validating the improved Wheeler model. It is observed from the studies that the predicted remaining life using the improved Wheeler model for these cracked plates are found to be in close agreement with the corresponding experimental values reported in the literature.

keyword: Fracture mechanics, crack growth model, fatigue loading, remaining life

1 Introduction

The structural components of offshore, airplanes, bridges and ships are often designed by using damage tolerant design concepts to ensure survival in the presence of growing cracks. The most effective approach to damage tolerant design is based on the fracture mechanics principles. Crack growth analysis is not only important in evaluating the remaining life of a given structural component, but also in establishing suitable inspection intervals. Structural inspections are often used to monitor the fatigue crack growth in structural components. Using inspection results, crack growth analyses can be performed to evaluate and update the structural integrity and component safety levels. Structural components are generally subjected to a wide spectrum of stresses over their lifetime. These components rarely experience loading amplitude that remains constant during length of the service. A major influencing parameter to be considered is the load history, which is usually variable. The assessment of the behaviour of structure subjected to variable amplitude loading (VAL) is more complex as compared to constant amplitude loading (CAL). Crack growth under VAL involves the sequence effects or interaction effects. The load interaction or sequence effects significantly affect the fatigue crack growth rate and consequently fatigue life. Reliable prediction of fatigue life of the structural components under VAL requires accurate representation of load interaction effects.

The effect of tensile overload has been reported by many investigators such as Gray and Gallagher (1976), Sheu et al. (1995), Dawicke (1997), Ramos et al. (2003), Kim and Shim (2003), Taheri et al. (2003), Evily et al. (2004), Rama Chandra Murthy et al. (2004). A superimposed single overload or spike load during CAL is the simplest case of VAL. The application of single overload was observed to cause significant decrease in the crack growth rate for a large number of cycles subsequent to the overload as shown in Figure 1. This phenomenon is referred to as the crack retardation. Further, it is observed that higher OLRs can arrest the crack growth. Increase in OLR value results in increase in number of delay cycles, N_D. Application of fatigue underloads [negative overloads] has the detrimental effect on the fatigue crack initiation and crack growth. The crack growth rate is augmented and fatigue life will be reduced as reported by Stephens et al. (1976), Marissen et al. (1984), Carlson and Kardomateas (1994), Rama Chandra Murthy et al. (2004). The combination of overloads and underloads, singly or in blocks, produces much more complex situation. The application of an underload immediately following overloading diminishes the effect of the latter depending on their relative values, reported by Carlson and Kardomateas (1994), Jaime (1994), Dawicke (1997), Rushton and Taheri (2003). It was observed

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that the retardation effect is lesser due to a tensile overload followed by a subsequent higher compressive overload, which can even lead to an acceleration of the crack growth. Also, application of an underload prior to overloading may have no influence or even decrease the retardation effect of the overload, depending on the particular loading conditions. In practice, however, the effects of compressive underloading are often neglected for remaining life prediction. Jaime (1994) explained some of the transient effects on crack growth and is illustrated in Figure 2.

Several theories are available in the literature, reported by Elber (1971), Wheeler (1972), Newman (1997), Ramos et al. (2003), Kim and Shim (2003), Rama Chandra Murthy et al. (2004) to explain the crack retardation, including crack tip blunting, crack closure effects and compressive residual stresses at the crack tip. Some investigators such as Rama Chandra Murthy et al. (2004) reported that plasticity induced closure is the major cause for the retardation. Many others such as Ramos et al. (2003), Kim and Shim (2003) and Rama Chandra Murthy et al. (2004) reported that compressive residual stresses are the primary cause for retardation. However, the retardation models based on compressive residual stress concept are found to be more effective and perform better as reported by Rama Chandra Murthy et al. (2004). In the present studies, it is proposed to use the concept of residual stress to represent the retardation effect due to overloads-underloads. The retardation due to an overload is a complex phenomenon. Two primary influences on crack growth behaviour are retardation following overloads and acceleration following underloads. There are number of empirical models for representation of retardation/acceleration effects, which contain one or more curve fitting parameters that are to be obtained experimentally. The widely used models are crack tip plasticity models (yield zone models), crack closure models and statistical models.

The crack closure models such as Elber (1971) and Newman (1997) are based on the assumption that crack growth is controlled not only by the behaviour of the plastic zone but also by residual deformations left in the wake of the crack as it grows through previously deformed material. Since analytical modeling of crack closure is very difficult, models based on yield zone concept such as Wheeler (1972), Gallagher and Hughes (1974) and Gray and Gallagher (1976) are generally employed in the analytical investigations. Barsom (1976) and Hudson (1981) models are based on the root mean square approach for evaluating the average fatigue crack growth rate without accounting for the load interaction effects. If the influence of load interaction is small, these models predict reasonable result.

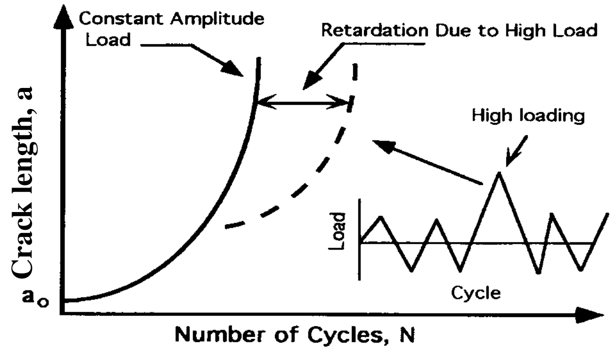


Figure 1 : Decrease in the rate of crack growth due to the overload followed by the CAL

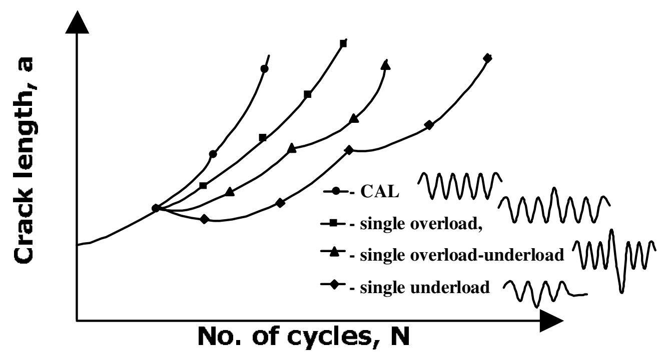


Figure 2 : Crack growth produced by (a) CAL, (b) single overload, (c) single overload-underload and (d) single underload

The widely used Wheeler and Generalised Willenborg residual stress models are based on the assumption that crack growth retardation is caused by compressive residual stresses acting at the crack tip. Wheeler residual stress model is the simplest model among the various models available for VAL employing the yield zone concept. Wheeler model uses the retardation parameter (C_{pi}) to represent retardation due to overload. C_{pi} is the power function of the ratio of current plastic zone size and the distance from the crack tip to the border of the overload plastic zone size. The power coefficient used in the function for retardation parameter is generally called as the

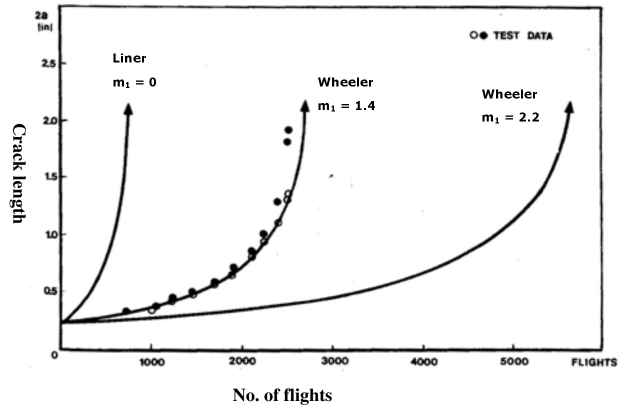


Figure 3 : Influence of shaping exponent on remaining life

shaping exponent (m_1). The shaping exponent is an adjustable calibration parameter that depends on the level of applied overload, crack length and plate width. Figure 3 presents a graph of crack length against no. of flights for typical values of the shaping exponent, given by Broek (1989). From Figure 3, it can be observed that the remaining life significantly increases as m_1 increases.

In general, the shaping exponent can be obtained by using one of the following procedures.

1. Calibrating ΔK and da/dN through number of experiments

2. Analytically by trial and error procedure

Sheu et al. (1995) conducted experiments on CT specimen to evaluate the shaping exponent considering different OLRs and initial crack lengths. After regression analysis, the following equations for m_1 in terms of OLR and a/w (where a = crack length and w = plate width) were proposed.

$$m_1 = 2.522(OLR)^{0.733} - 3.092\left(1+\frac{a}{w}\right)^{1.273} \quad \text{for 5083-O Al alloy} \tag{1}$$

$$m_1 = 0.089(OLR)^{2.982} + 0.856\left(1+\frac{a}{w}\right)^{1.212} \quad \text{for 6061-T651 Al alloy} \tag{2}$$

Rushton and Taheri (2003) conducted experiments on centre cracked plates made up of 350WT Steel to evaluate the shaping exponent considering different OLRs and stress ratios (SR). After regression analysis, they proposed the following equations for m_1 in terms OLR.

$$m_1 = 4.00(OLR)^2 - 13.04(OLR) + 12.62 \quad \text{for SR = 0.1} \tag{3}$$

$$m_1 = 3.32(OLR)^2 - 10.84(OLR) + 10.43 \quad \text{for SR = 0.3} \tag{4}$$

From close examination of eqns. 1 to 4, it can be seen that the indices and multiplication factors are different for different materials and it is difficult to predict the physical significance of these. Further, the above equations are valid for the specific materials. For any other material, the above equations will not be applicable straightaway to compute the retardation parameter and one would need to conduct number of experimental studies. Finney (1989) studied the sensitivity of fatigue crack growth prediction using the Wheeler residual stress model. He expressed that the calibrated m_1 value is not a material constant but depends on the maximum stress in the spectrum and the crack shape. This contradicts the results reported by Sheu et al. (1995). Gray and Gallagher (1976) and Arone (1990) concluded from their experimental studies that the shaping exponent is a function of OLR and the ratio of crack length to width of plate panel. Taheri et al. (2003) conducted number of experiments on 350WT Steel under VAL for remaining life prediction for center cracked plate panels and compared the experimental values with the Wheeler residual stress model. They assumed the shaping exponent value by trial and error procedure, which provides the best fit of the experimental data. This resulted in predicting different shaping exponent for each OLR. This means that one need to conduct number of experiments for each specific loading scenario, requiring a huge database of test data.

From the foregoing discussions, it is observed that this limits the usefulness of the Wheeler residual stress model. Further, it is noted that the remaining life predictions are dependent upon the empirically determined shaping exponent. To the best of authors' knowledge, there are no general expressions available in the literature to evaluate the shaping exponent. As large amount of time and effort is required to calibrate the model through experiments to evaluate the shaping exponent and also to overcome trial and error procedure, there is a need to develop simplified and general expression to evaluate the shaping exponent for any material with reasonable accuracy. In order to predict the remaining life of cracked plate panels under VAL, an improved Wheeler residual stress model consisting of simple, reliable and general expressions for the shaping exponent taking into account of tensile overloads have been proposed in the earlier paper authored by Rama Chandra Murthy et al. (2004).

The limitation of the Wheeler model is that it does not consider the acceleration effects due to compressive underload. Calculation of the plastic zone size under tensile-compressive overload is one of the important parameters to evaluate the retardation coefficient. In the present study, this aspect is addressed in depth.

This paper presents an improved Wheeler residual stress model for remaining life prediction of cracked plate panels under VAL. The improvement to the Wheeler residual stress model is in two folds. One is expressions for the shaping exponent, which are generally obtained through experiments. Another is calculation of the effective plastic zone size to incorporate the sequent effects under tensile-compressive overloading. The remaining life prediction has been carried out by employing the linear elastic fracture mechanics (LEFM) principles. Studies on remaining life prediction of cracked plate panels subjected to tensile and tensile-compressive overloading have been carried out for validating the improved Wheeler model. It is observed from the studies that the predicted remaining life using the improved Wheeler model for these cracked plates are found to be in close agreement with the corresponding experimental values reported in the literature.

2 Wheeler residual stress model

Wheeler (1972) employed the residual stress retardation model to account for the crack growth retardation due to tensile overload (Figure 4). The development of Wheeler model begins with the basic crack growth equation

$$\frac{da}{dN} = f(\Delta K) \tag{5}$$

Since the load is discontinuous variable, the crack growth can be computed using cycle-by- cycle approach

$$a_n = a_o + \sum_{i=1}^{N} f(\Delta K_i) \tag{6}$$

where, a_n = final crack length after N cycles, a_0 = initial crack length, and ΔK_i= stress intensity factor (SIF) range for cycle i.To account for crack growth retardation, Wheeler introduced a retardation parameter, C_{pi}, eqn. (6) then reduces to

$$a_n = a_o + \sum C_{pi} f(\Delta K_i) \tag{7}$$

The retardation parameter can be calculated using

$$C_{pi} = \left(\frac{r_p}{(a_p - a)}\right)^{m_1} \quad \text{for } (a + r_p) < a_p$$
$$= 1.0 \quad \text{for } (a + r_p) > a_p \tag{8}$$

where, r_p = extent of current plastic zone size (a_p - a) = distance from crack tip to elastic - plastic interface (refer Figure 4)m_1= shaping exponent, which is generally obtained through experiments. The value of m_1 depends on the applied overload, crack size and width of the plate. It is observed that C_{pi} is minimum immediately after the application of overload, when (a_p - a) has its maximum value. As ‘a’ approaches a_p, C_{pi} increases.

From the Wheeler’s model, it can be observed that

1. the retardation decreases proportionately to the penetration of the crack into the overload plastic zone.
2. the retardation occurs as long as the current plastic zone is within the overload plastic zone and ceases as soon as it touches the boundary of the overload plastic zone.

3 Improved wheeler residual stress model

As mentioned earlier the shaping exponent is primarily dependent on OLR and the ratio of crack length to width of plate panel. In the present study, the degree of influence of OLR and shape factor (β) on the shaping exponent have been investigated for remaining life assessment of cracked plate panels under tensile overload. Extensive analytical investigations including regression analysis were conducted by Rama Chandra Murthy et al., (2003) that contain different expressions for the shaping exponent in terms of OLR and β to study the varying degree of influence on the remaining life prediction. Number of example problems were solved by Rama Chandra Murthy et al. (2003) using these expressions for different materials such as steel and aluminium alloys. Extensive studies were carried out that considered

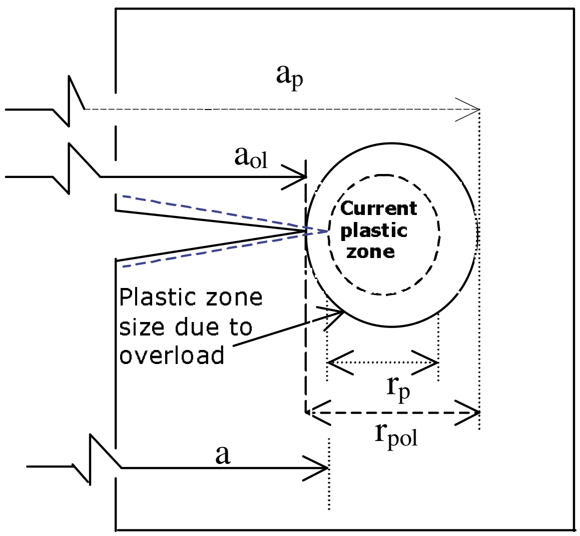


Figure 4 : Wheeler residual stress model

1. variation of parameters β_e and β_c independently and concurrently
2. variation of parameter OLR independently and concurrently
3. various combinations of (i) and (ii)

Based on these studies among the various alternatives, the following shaping exponent expressions for a) the CT specimen and plate panel with b) a center crack and (c) an edge crack under tensile overload were proposed in the improved Wheeler model, reported by Rama Chandra Murthy et al. (2004).

$$m_1 = OLR \quad \text{(for CT specimen)} \tag{9}$$

$$m_1 = OLR + \beta_c^2 \quad \text{(for center crack)} \tag{10}$$

$$m_1 = \frac{OLR}{2}(1+\sqrt{\beta_e}) \quad \text{(for an edge crack)} \tag{11}$$

where OLR= $\frac{\sigma_{\max,OL}}{\sigma_{\max}}$, β_e = shape factor for edge cracked panels and β_c = shape factor for center cracked panels, which can be obtained by using the following equations

For plate with a center crack

$$\beta_c = \left(1-0.025\alpha_c^2+0.06\alpha_c^4\right)\sqrt{\sec\left(\frac{\pi\alpha_c}{2}\right)} \tag{12}$$

where $\alpha_c = \frac{a}{W}$, a = Half crack length, W = Half plate width

For plate with an edge crack

$$\beta_e = 1.12-0.231\alpha_e+10.55\alpha_e^2-21.72\alpha_e^3+30.39\alpha_e^4 \tag{13}$$

where α_e= a/w, a = crack length, w = plate width

4 Wheeler model for tensile-compressive overloads

Figure 5 shows the illustration of various aspects related to overload and underload. With reference to Figure 5, the maximum load during the overload cycle is given by

$$P_{\max,OL} = (P_{\max,CAL})(OLR) \tag{14}$$

$$P_{\min,CAL} = (R)(P_{\max,CAL}) \tag{15}$$

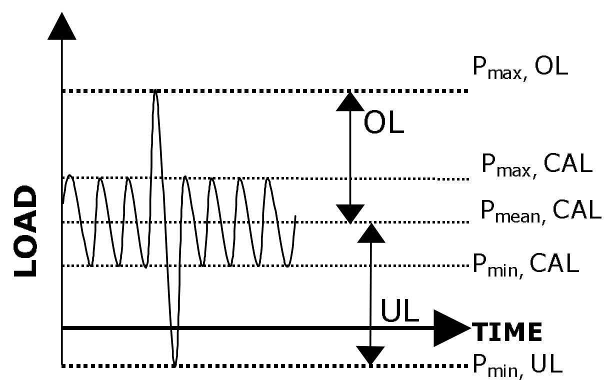


Figure 5 : Schematic diagram of overload and underload

From Figure 5, the following simple expressions can be deduced.

$$\text{Overoad, OL} = P_{\max,OL} - P_{\text{mean},CAL} = P_{\max,CAL}\times\left(OLR-\frac{1+R}{2}\right) \tag{16}$$

where,

$$P_{\text{mean},CAL} = P_{\max,CAL}\times\left(\frac{1+R}{2}\right) \tag{17}$$

Combining the above relations, the following expressions for the minimum load corresponding to the compressive underload, $P_{min,UL}$can be derived.

$$P_{\min,UL} = P_{\max,CAL}\times\left[\left(\frac{1+R}{2}\right)-\frac{1}{OL/UL}\left(\text{OLR - }\left(\frac{1+\text{R}}{2}\right)\right)\right] \tag{18}$$

Rearranging the above equations, the following expressions for compressive underload, UL can be obtained

$$UL = \frac{P_{\max,CAL}}{OL/UL}\left(OLR - \left(\frac{1+R}{2}\right)\right) \quad (19)$$

It is observed from the literature that effect of a compressive plastic zone at the crack tip when considering the crack growth acceleration or reduction in crack growth retardation induced by compressive loading is often neglected. This is based on the presumption that upon closure of a fatigue crack, the applied compressive stress becomes distributed evenly across the section. As observed earlier, increasing the magnitude of the applied compressive underload (i.e., decreasing the OL/UL ratio), may significantly reduce crack growth retardation, depending the overload ratio [refer Figure 2]. The reduction in crack growth retardation may be due to the reason that the plastic zone size created by the tensile overload is significantly reduced by the compressive underload.

In order to facilitate the inclusion of compressive loading effects into the Wheeler retardation model, some modifications have been proposed. It is known that the Wheeler's original retardation parameter (C_p) was based on the premise that the crack growth retardation should cease when the boundary of the current plastic zone has extended to or beyond the outermost boundary of any previous tensile overload. Figure 6 summarizes the basic concepts and terms describing the Wheeler model to account for the compressive underloading effects.

Effective plastic zone is given by

$$r_{eff} = r_p - r_{cp} \quad (20)$$

where r_p = plastic zone due to tensile overload, r_{cp} = plastic zone due to compressive underload

Plastic zone size due to tensile overload can be calculated by using Irwin's equation which is given below

$$r_p = \frac{1}{C_1\pi}\left[\frac{K_{\max}}{\sigma_y}\right]^2 \quad (21)$$

where C_1 = a factor accounting for state of stress at crack tip, K_{max} = maximum SIF, σ_y = material yield stress

Plastic zone size due to compressive underload can be calculated by using the following expression

$$r_{cp} = \frac{1}{C_1\pi}\left[\frac{\Delta K_{UL}}{2\sigma_y}\right]^2 \quad (22)$$

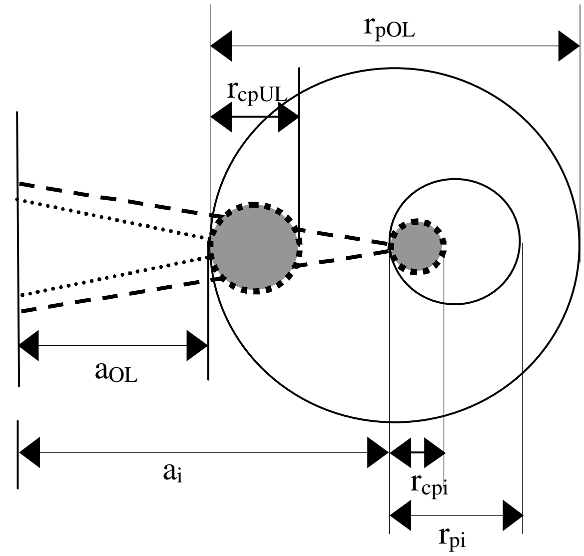


Figure 6 : Wheeler model for tensile-compressive overloads

where ΔK_{UL} = K*-$K_{min,UL}$, K* = minimum of ($K_{min,CAL}$, K_{th})

where $K_{min,CAL}$ = SIF corresponding to min CAL and K_{th} = Threshold SIF

From Figure 6, it can be observed that there is no retardation if

$$a_i + (r_{pi} - r_{cpi}) \geq a_{OL} + (r_{pOL} - r_{cpUL}) \quad (23)$$

where a_i = current crack length corresponding to the i^{th} cycle

r_{pi} = current plastic zone size corresponding to the i^{th} cycle

r_{cpi} = current compressive plastic zone size corresponding to the i^{th} cycle

a_{OL} = crack length at which overload is applied

r_{pOL} = plastic zone size due to tensile overload

r_{cpUL} = compressive plastic zone size due to compressive underload

Rearranging the relation given in eqn. 23 in a manner similar to the Wheeler's original retardation parameter, the following relationship can be arrived at:

$$C_{pi} = \left[\frac{r_{eff,i}}{a_{OL} + r_{eff,OL} - a_i}\right]^{m_1}$$

where $a_i + (r_i - r_{cpi}) < a_{OL} + (r_{pOL} - r_{cpUL})$

$C_{pi} = 1$ where $a_i + (r_{pi} - r_{cpi}) \geq a_{OL} + (r_{pOL} - r_{cpUL})$
where $r_{eff,OL} = r_{pOL} - r_{cpUL}$
m_1 = shaping exponent, same as given in eqns. 9 to 11.

Improved Wheeler residual stress model has been updated to incorporate the sequence effects under tensile-compressive overloading. Remaining life prediction has been carried out by using eqns. 5 and 7.

5 Validation studies

Numerical studies have been conducted to validate the improved Wheeler residual stress model on a plate panel with center crack and a CT specimen subjected to tensile-compressive overloads. Four example problems considering tensile/tensile-compressive overload have been presented herein.

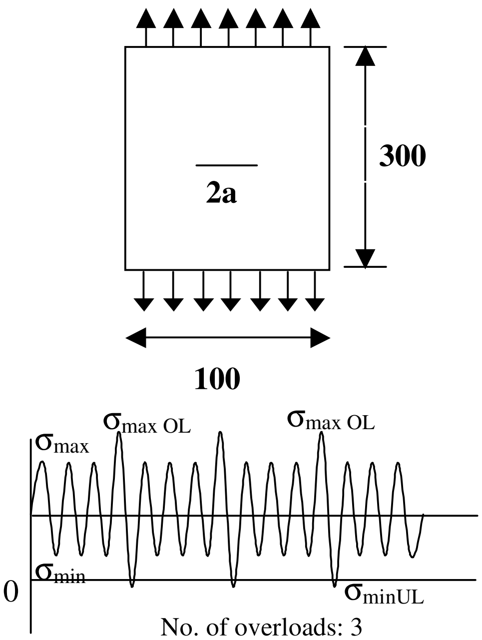
5.1 Plate with a center crack under tensile-compressive overloads

Remaining life prediction for plate with a center crack made up of steel and aluminium subjected to periodic single tensile-compression overload has been carried out for the following data using the improved Wheeler model.

5.1.1 350WT Steel

This example was studied by Rushton and Taheri (2003). The data/information related to this problem is given below (Figure 7).

Material	:	350 WT Steel
Plate dimensions	:	300x100mm
Thickness	:	5 mm
Fracture toughness	:	50 MPa $\sqrt{m}$
Yield Strength	:	350 MPa
Stress ratio	:	0.1
Initial crack length	:	20.0 mm
Overload ratios	:	1.2, 1.5, 1.67, 1.83
OL/UL	:	0.5, 1.0, 2.0
Stress condition	:	Plane stress
Maximum stress (σ_{max})	:	114 MPa
Minimum stress (σ_{min})	:	11.4 MPa
Crack growth equation	:	Paris
C	:	1.02 e-8
m	:	2.94



Crack length at occurrence of overload/underload = 30, 40, 50 mm

Figure 7 : Load Spectrum

No. of overloads and occurrence of overload for each specimen is shown in Figure 7. Using the above data, remaining life computation has been carried out for different OLRs and OL/UL ratios. The computed values are shown in Table 1. From Table 1, it can be observed that the predicted remaining life for different OLRs and OL/UL ratios are in good agreement with the corresponding experimental values available in the literature. The predicted remaining life by considering only OL and for different OLRs is also shown in Table 1. Figure 8 shows the variation of predicted remaining life for different OLRs and OL/UL ratios. Figure 8 also shows the plot of comparison of variation of remaining life for OLR = 1.67 considering various OL/UL ratios with the life predicted under OL alone. From Table 1 and Figure 8, it can be observed that the predicted life decreases when OL/UL ratio decreases for a specific OLR. Further it can also be observed that the decrease in the life is significant for higher OLRs. But compared to the life predicted under CAL, there is considerable gain in the life even with OL/UL ratio is equal to 0.5. In general, it can be observed that the remaining life increases with increase of OLR and OL/UL ratio and is significant for higher OLRs and OL/UL ratios.

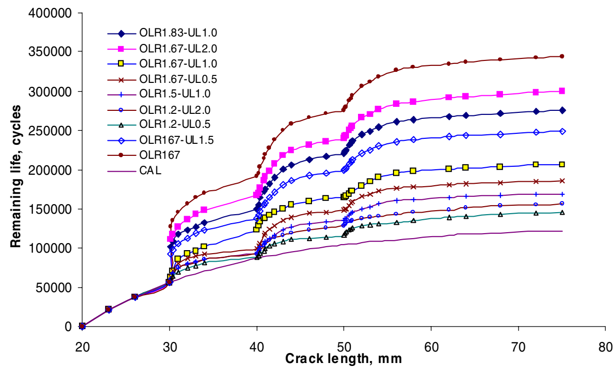


Figure 8 : Crack length vs No. of cycles

5.1.2 2024-T3 Al alloy

Remaining life prediction for plate with a center crack subjected to single tensile overload-underload at every 2500 constant amplitude cycles has been carried out using the following data. This problem was studied by Dawicke (1997).

Material	:	2024-T3 Al alloy
Plate dimensions	:	3.0 x 5.0 in
Thickness	:	0.09 in
Fracture toughness	:	46 ksi $\sqrt{\text{in}}$
Yield Strength	:	53 ksi
Stress ratio	:	0.02
Stress condition at crack tip	:	Plane stress
Maximum stress (σ_{max})	:	10 ksi
Minimum stress (σ_{min})	:	0.2 ksi
Crack growth equation	:	Paris
C	:	0.829e-8
m	:	2.284
Initial crack length	:	1.0 in
Overload ratios		1.125, 1.5
OL/UL		0.375, 0.5, 0.5625, 0.6, 0.75, 1.0, 1.125, 1.5

Using the above data, remaining life prediction has been carried out for different OLRs and OL/UL ratios. The computed values are shown in Table 2. From Table 2, it can be observed that the predicted remaining life for different OLRs and OL/UL ratios are in good agreement with the corresponding experimental values available in the literature. The predicted remaining life by considering only OL and for different OLRs is also shown in Table 2. From Table 2, it can be observed that the predicted life decreases significantly with the decrease in OL/UL ratio. Further, it can be observed that for OLR = 1.5 and OL/UL = 0.5, the predicted remaining life is lesser approximately by 10% compared that of the life predicted under CAL. This may be due to the repetition of overload-underload at every 2500 cycles. It can also be observed that the decrease in the life for different OL/UL ratios compared to the life predicted by considering only OL is significant.

5.2 CT specimen under tensile-compressive overloads

Remaining life prediction for a CT specimen made up of 2024-T3 and 7075-T6 Aluminium alloys subjected to single tensile-compressive overload has been carried out for the following data using improved Wheeler model.

5.2.1 2024-T3 Al alloy

This example was studied by Stephens et al. (1976). The data/information related to this problem is given below.

Material	:	2024-T3 Al alloy
Plate dimensions	:	97.5 x 81.3mm
Thickness	:	9.15 mm
Fracture toughness	:	70.6 MPa $\sqrt{\text{m}}$
Yield Strength	:	355 MPa
Stress ratio	:	0.0
Stress condition at crack tip	:	Plane stress
Maximum stress (σ_{max})	:	103.99 MPa
Crack growth equation	:	Paris
C	:	0.199e-11
m	:	3.282
Overload ratios	:	2.0, 2.5, 3.0
OL/UL	:	1.0
Initial crack length	:	25.4 mm

Overload-underload is applied at initial crack length level. Using the above data, remaining life prediction has been carried out for different OLRs and specific OL/UL ratio which is equal to 1.0. The computed values are shown in Table 3. From Table 3, it can be observed

Table 1 : Remaining life values for plate with centre crack (350WT Steel)

OLR	OL/UL	Remaining life		% diff.	Life predicted by considering only OL	% decrease in life compared to life with only OL
		Improved Wheeler model	Exptl. by Rushton and Taheri (2003)			
1.2	0.5	145775	158000	7.73	174396	16.41
1.2	2.0	156444	168200	6.99		10.29
1.5	1.0	160004	174400	8.25	199271	12.48
1.67	0.5	185920	198000	6.1	351762	47.15
1.67	1.0	206371	224100	7.91		41.33
1.67	2.0	299238	328200	8.82		14.93
1.83	1.0	275299	304700	9.65	386767	28.82

*No. of cycles under CAL = 122171

Table 2 : Remaining life values for plate with centre crack (2024-T3 Al Alloy)

OLR	OL/UL	Remaining life		% diff.	Life predicted by considering only OL	% decrease in life compared to life with only OL
		Improved Wheeler model	Exptl. by Dawicke (1997)			
1.125	1.125	30636	33192	7.70	49871	38.57
1.125	0.5625	27745	29083	4.60	(52840)**	44.37
1.125	0.375	18744	20472	8.44		62.41
1.5	1.5	183565	195075	5.90	365712	49.81
1.5	1.0	122524	135177	9.36	(385332)**	66.49
1.5	0.75	78002	85529	8.80		78.67
1.5	0.6	39309	42042	6.50		89.25
1.5	0.5	24718	25753	4.02		93.24

* No. of cycles under CAL 27339 ** Exptl. by Dawicke (1997)

that the predicted remaining life is in close agreement with the corresponding experimental values reported by Stephens et al. (1976). Further, it can be observed that the predicted life for OL/UL = 1.0 and for various OLRs is significantly less compared to that of the life predicted under OL only.

5.2.2 7075-T6 Al alloy

This example was studied by Stephens et al. (1976). The data/information related to this problem is given below.

Overload-underload is applied at initial crack length level. Using the above data, remaining life prediction has been carried out for different OLRs and specific OL/UL ratio which is equal to 1.0. The computed values are shown in Table 4.

From Table 4 it can be observed that the predicted re-

Table 3 : Remaining life values for CT specimen (2024-T3 Al alloy)

OLR	Remaining life		% diff.
	Improved Wheeler model	Exptl. by Stephens, Chen and Hom (1976)	
Only OL			
2.0	79182	85200	7.06
2.5	289920	> 300000	-
OL/UL = 1.0			
2.0	34752	36900	5.82
2.5	31635	34200	7.50
3.0	29400	30900	4.85

Material	:	7075-T6 Al alloy
Plate dimensions	:	97.5 x 81.3mm
Thickness	:	8.9 mm
Fracture toughness	:	51.9 MPa $\sqrt{m}$
Yield Strength	:	558 MPa
Stress ratio	:	0.0
Stress condition at crack tip	:	Plane stress
Maximum stress (σmax)	:	103.55 MPa
Crack growth equation	:	Paris
C	:	0.523e-9
m	:	2.497
Overload ratios	:	2.0, 2.3, 2.5
OL/UL	:	1.0
Initial crack length	:	25.4 mm

Table 4 : Remaining life values for CT specimen (7075-T6 Al alloy)

OLR	Remaining life		% diff.
	Improved Wheeler model	Exptl. by Stephens, Chen and Hom (1976)	
Only OL			
2.0	23812	25900	8.06
2.3	202674	222700	8.99
2.5	52685	57600	5.53
OL/UL = 1.0			
2.3	31594	33900	6.80
2.5	291684	> 300000	-

maining life is in close agreement with the corresponding experimental values reported in the literature. Further, it can be observed that the predicted life for OL/UL = 1.0 and for various OLRs is significantly less compared to that of the life predicted under OL only.

5.3 Observations and Discussion of Results

Number of example problems on remaining life prediction of a CT specimen and plate with a center crack has been solved to validate the improved Wheeler residual stress models under tensile-compressive overloads. From the studies, it is observed that the delay cycles/remaining life values are in good agreement with the corresponding experimental values reported in the literature. But, it is observed from the studies conducted that the differences between the results obtained from the experimental studies and analytical predictions are within 10%. This validates the proposed improved Wheeler model. It is observed that the parameters OLR and β have differing influences on the shaping exponent depending on the crack location for remaining life prediction. Remaining life increases significantly for higher OLRs. Remaining life is also influenced by the number of overloads-underloads, occurrence and their magnitudes. It is observed from the studies that the occurrence of early overloads cause significant increase in the remaining life compared to the occurrence of later overloads. It is also observed that the application of underload following the overload resulted in significant reduction in the remaining life than with those predicted under tensile overload alone. On the application of underload, the crack surfaces yielded in compression, reducing the tensile plastic zone formed due to the overload, thus decreasing the subsequent crack opening stress. For the analysis with compressive underloads, the remaining life decreases with increasing the magnitude of compressive underload.

6 Concluding remarks

An improved Wheeler residual stress model for the remaining life prediction of cracked plate panels such as compact tension (CT) specimen and plate with a centre crack and an edge crack under tensile and tensile-compressive underloads have been presented. The improvement to the Wheeler residual stress model is in two folds. One is expressions for shaping exponent, which is generally obtained through experiments. Another is calculation of effective plastic zone size to incorporate the sequent effects-under tensile-compressive overloading. The remaining life prediction has been carried out by employing LEFM principles. Remaining life prediction has been carried out for cracked plate panels subjected to tensile and tensile-compressive overloading for validating the improved Wheeler model. It is observed from the studies that the predicted remaining life using the improved Wheeler model for these plates are found to be in close agreement with the corresponding experimental values reported in the literature. From the studies, it can be concluded that the effect of compressive underload cannot be neglected for accurate prediction of the remaining life of structural components.

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