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Cohesive Strength and Separation Energy as Characteristic Parameters of Fracture Toughness and Their Relation to Micromechanics

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Abstract: A review on phenomenological fracture criteria is given, based on the energy balance for cracked bodies, and the respective toughness parameters are related to micromechanical processes. Griffith's idea of introducing a "surface energy" and Barenblatt's concept of a "process zone" ahead of the crack tip build the foundation of modern cohesive models, which have become versatile tools for numerical simulations of crack extension. The cohesive strength and the separation energy used as phenomenological material parameters in these models appear to represent a physically significant characterisation of "fracture toughness". Micromechanical interpretations of these parameters can be derived, depending on the specific damage phenomenon.

keyword: Cohesive model, Fracture toughness, Cohesive strength, Separation energy.

1 Introduction

Two essential questions arise with respect to the description of fracture. The first one is the issue of providing a correct and physically meaningful picture of the processes occurring at the crack tip. The second question is on how the fracture toughness of a material in a specific structural geometry can be described. With the focus on a modelling approach both issues have to be addressed. One of the main requirements of models for nonlinear constitutive behaviour is that they must allow for a separation of the energy dissipation connected to the local material separation process from the remote plastic strain energy.

The history of analytical approaches to studying fracture started less than a hundred years ago, which is relatively short compared to other issues of strength of materials. The work of men like Griffith and Irwin has helped to make recent advances in fracture modelling possible. Griffith's theory of brittle fracture [Griffith (1920, 1924)] helps us to understand why fracture occurs in an elastic material. His basic ideas are not restricted to brittle fracture, however, as Irwin (1957, 1964) showed, who took Griffith's work and applied it to moderately ductile materials. After a long period, during which the understanding of fracture toughness was mainly reduced to the question of how to measure a J_R-curve correctly, followed by discussions on "constraint effects" on J_R-curves, people became aware again of Griffith's strain energy concept resulting in proposals of an "energy dissipation rate" [Turner (1990)] and in numerical models for the "cohesive zone" at the crack tip [Needleman (1990), Yuan and Cornec (1996), Tvergaard (2001)].

2 Griffith's Energy Concept of Fracture

2.1 Fracture of Brittle Materials

Griffith (1920) started his fundamental paper with discussing the "effect of surface treatment on the strength of machine parts subjected to alternating or repeated loads", which could not be explained with the help of the hypothesis of maximum tensile stress. The stresses due to typical scratches, which he had calculated using Inglis' equations [Inglis (1913)], "could increase the maximum stresses from two to six times, ... and these maximum stresses were to all intents and purposes independent of the absolute size of the scratches". The resulting conclusions were "in direct conflict with the results of alternating stress tests", and Griffith concluded that "the ordinary hypotheses of rupture, as usually interpreted, are inapplicable to the present phenomena". Thus, he attacked the problem of rupture of elastic solids from a new standpoint, based on the theorem of minimum energy. The relevant passage is quoted below, as its arguments appear as quite recent with respect to both continuum mechanics and micromechanics [Griffith (1920)]:

"Rupture of the solid has occurred, if the system can pass from the unbroken to the broken condition by a process involving a continuous decrease in potential energy. In order, however, to apply this extended theorem to the problem of finding the breaking loads of real solids, it

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is necessary to take account of the increase in potential energy, which occurs in the formation of new surfaces in the interior of such solids. It is known that, in the formation of a crack in a body composed of molecules which attract one another, work must be done against the cohesive forces of the molecules on either side of the crack. This work appears as potential energy, and if the width of the crack is greater than the very small distance called the 'radius of molecular action', the energy per unit area is a constant of the material, namely its surface tension".

Considerations of the molecular theory of strength phenomena were also addressed by Smekal (1922), around the same time as Griffith established his concept. But it took about 60 more years and the increasing capacities of modern computers that atomistic models and the molecular dynamics technique were applied to study the micromechanics of brittle fracture, in particular microcrack initiation and crack growth, e.g. Machova (1992).

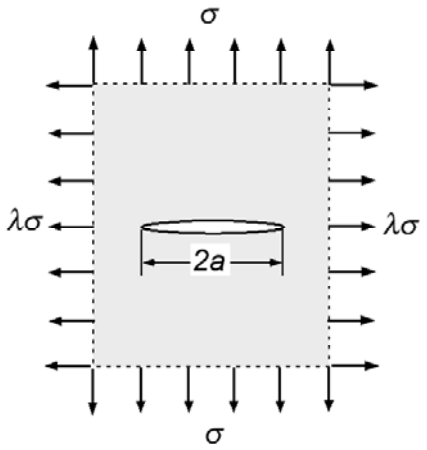

Figure 1 : Center-cracked panel under biaxial loading (biaxiality factor, λ) used to derive Griffith's fracture concept for mode I

Applying his theory to an "infinite" cracked panel, the subsequently renowned "Griffith crack" configuration for mode I fracture, Fig. 1, he came up with the condition that the crack may extend as

$$\frac{\partial}{\partial a}\left(U_{rel}^{el}-U_{sep}\right)\geq 0 \tag{1}$$

where U_{rel}^{el} is the change of elastic strain energy due to the crack.

Let U_0^{el} be the elastic strain energy per unit thickness stored in a panel under biaxial tension, which does not contain a crack. It depends on the panel dimension and becomes infinite as the panel size does. If a hole is cut into the panel, its stress and strain state and, hence, its strain energy change. Depending on the boundary conditions, the strain energy may increase or decrease. Assuming "fixed-grip" conditions, strain energy is released, $U^{el}=U_0^{el}-U_{rel}^{el}$. The decrease of strain energy due to a crack of length $2a$ is

$$U_{rel}^{el}=\frac{\pi a^2\sigma^2}{8G}(1+\kappa), \tag{2}$$

where $G=E/2(1+\nu)$ is the shear modulus, and κ is a parameter depending on Poisson's ratio, ν, and the stress state, namely $\kappa=3-4\nu$ for plane strain and $\kappa=(3-\nu)/(1+\nu)$ for plane stress. Stresses parallel to the crack do not affect the strain energy released in the case of an ideal crack.

The second term in Eq. 1, U_{sep}, is the potential energy of the two crack surfaces per unit thickness of the panel,

$$U_{sep}=4a\gamma, \tag{3}$$

where γ is the "surface tension", having the dimension of energy per area.

An existing crack will extend unstably, if the equals sign holds in Eq. 1, i.e. if the "*energy-release rate*",

$$\mathcal{G}^{el}=-\frac{\partial U^{el}}{B\,\partial(2a)}=\frac{\partial U_{rel}^{el}}{B\,\partial(2a)}=\frac{\pi a\sigma^2}{2G}(1+\kappa), \tag{4}$$

which is the elastic energy being released by a unit crack extension under "fixed-grip" conditions, equals the "*separation energy*" (SE),

$$\frac{\partial U_{sep}}{B\,\partial(2a)}=2\gamma=\Gamma_c, \tag{5}$$

which is necessary to create new fracture surfaces and which is supposed to be a material constant in fracture mechanics,

$$\mathcal{G}^{el}(a)=\Gamma_c. \tag{6}$$

From this criterion, the (global) fracture stress of the panel is derived as

$$\sigma_f=\sqrt{\frac{2E'\gamma}{\pi a}}, \tag{7}$$

with $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain.

Note that σ_f depends on the crack length, a, and tends to approach infinity if a approaches zero, which means that no fracture can occur in an ideally flawless structure. If a material contains microcracks of a certain size, a, however, the assumption of a critical SE implies a critical fracture stress or "*cohesive strength*" (CS), $\sigma_c = \sigma_f(a)$, as a material parameter under plane strain conditions. Invariably, the size of microcracks in a brittle material is not constant, but follows some density distribution. Together with the weakest-link assumption, that a structure will fail if a representative volume element (RVE) fails, Weibull's statistical theory of the strength of materials [Weibull (1939)] can be derived from Eq. 7. This concept has found application in the "local approach" to cleavage fracture of ferritic steels [Mudry (1987)].

2.2 Extension to Small-Scale Yielding

Griffith's work was significant, however it did not include ductile materials in its consideration. Almost four decades later, Irwin investigated how the theory would apply to "somewhat brittle fracture", meaning "that a region of large plastic deformations may exist closer to the crack but does not extend away from the crack by more than a small fraction of the crack length" [Irwin (1957)], which we would call "small-scale yielding conditions", today. Remarkably, he already came up with a two-parameter approach of fracture long before similar concepts were discussed intensively within the fracture-mechanics community: "the influence of the test configuration, loads and crack length upon the stresses near an end of the crack may be expressed in terms of two parameters. One of these is an adjustable uniform stress parallel to the direction of a crack extension. ... The other parameter, called the stress intensity factor, is proportional to the square root of the force tending to cause crack extension" [Irwin (1957)]. Actually, the first parameter is Rice 's T-stress [Rice (1968)], and the crack driving force is Griffith's strain-energy release rate,

$$\mathcal{G}^{el} = \frac{K^2}{E'}, \tag{8}$$

which is proportional to the square of the mode I stress intensity factor, K. In order to account for crack-tip plasticity, Irwin (1964) assumed that "the stress relaxation from local yielding is equivalent to a small additional crack extension called r_Y. The value of r_Y was fixed partly from experiment and partly from theoretical reasoning at"

$$r_Y = \frac{1}{2\pi}\left(\frac{K}{\sigma_Y}\right)^2. \tag{9}$$

The energy-release rate of a centre-cracked panel, Fig 1, accounting for crack-tip plasticity then becomes

$$\mathcal{G} = \frac{\sigma^2\pi\,(a+r_Y)}{E'} = \mathcal{G}^{el} + \mathcal{G}^{pl}, \tag{10}$$

and in order to satisfy the fracture criterion of Eq. 9, an additional plastic contribution of the SE has to be included on the right-hand side,

$$\mathcal{G}(a) = \Gamma_c = \Gamma^{el} + \Gamma^{pl}. \tag{11}$$

This plastic SE, i.e. the work of stresses on plastic deformations per unit crack area, has the same dimension as Γ^{el}, namely energy per area. However, it may not be understood as a "surface energy" like in Griffith's concept, any more, because the plastic strain energy, ΔU^{pl}, dissipated during a time increment refers to a volume,

$$\Delta U^{pl} = \int\limits_{V_{pl}} \left(\int_t^{t+\Delta t} \overline{\sigma} d\overline{\varepsilon}^{pl} \right) dV = \Delta \overline{u}^{pl} V_{pl}, \tag{12}$$

where $\overline{\sigma}$ and $\overline{\varepsilon}^{pl}$ are von Mises effective stress and plastic strain, respectively, $\overline{u}^{pl}$ is the average plastic strain energy density in the plastically deformed volume, V_{pl}.

As long as plastic deformations are restricted to a small vicinity of the crack tip, assumed as a "process zone", this phenomenological enhancement of Griffith's theory will not cause severe problems despite the fact that the elastic and plastic contributions to Γ_c cannot be separated. The conceptual difference between the two has severe implications for fracture under large-scale yielding, however, for the following reasons.

As long as local and remote plastic strain energy cannot be separated, there is no chance of obtaining physically meaningful and geometry-independent values of SE and, hence, for fracture toughness.

The CS, σ_c, retains some significance as a stochastical material parameter in Weibull's theory and in the local approach to cleavage fracture, but would require a general re-definition for ductile rupture processes, provided

that the concept of a ductile rupture CS has any meaning at all.

Calculating SE as mechanical work dissipated in a process zone requires the introduction of a length scale, h_0, namely the height of the process zone. This problem shows up in various guises, in the "paradoxon of elastic plastic fracture" that the "near-tip" J-integral approaches zero for vanishing contour height [Rice (1979), Brocks and Yuan (1989)] as well as in a "pathological mesh-dependence" of numerical results obtained from local damage-mechanics models [Sun and Hoenig (1994), Bernauer and Brocks (2002)].

3 Ductile Rupture

3.1 J-Integral and Dissipation Rate

The 1970s and 80s were the decades of the J-integral, as introduced by Cherepanov (1967) and Rice (1968) in elasto-plastic fracture mechanics. Everything looked quite consistent at the beginning. In elasticity, J equals G^{el}, and in deformation plasticity, J is an energy-release rate, again, $J = -\partial U \big/ B\partial a$, and was found to be the amplitude of the singular stress and strain fields at the tip of a stationary crack, the so-called HRR field. Though local and remote contributions of the plastic strain energy could not be separated, any geometry dependence of critical J-values for initiation of crack extension under monotonic loading was found not to be significant but within the scatter of material properties. This changed with the application of J for characterizing crack extension in the form of J_R-curves, $J(\Delta a)$. Several problems and inconsistencies came up, from the path dependence of J [Brocks and Yuan (1989)] to significant geometry dependencies of R-curves, and numerous attempts to cope with these problems were undertaken, from defining limits for J-controlled crack growth [Shih and German (1985)] to various investigations on "constraint effects" and two-parameter approaches [Betegon and Hancock (1991), O'Dowd and Shih (1991), Wang (1993), Brocks and Schmitt (1993)].

From today's point of view, the fracture-mechanics community missed two essential points:

The cumulative quantity J, which increases with crack length, ceases to be an energy-release rate, as soon as the crack starts extending, and an incremental quantity is required in plasticity instead, as Turner (1990) has pointed out in a basic discussion on the necessity of defining an alternative measure of tearing toughness. What has been understood to be an extension of Griffith's theory was not, in the end.

Although physically meaningful models of the failure processes occurring at the crack tip were available [Rice and Tracey (1969), Thomason (1985)], the purely phenomenological J concept did not consider them. Analogous local models appeared separately as the "local approach", "micromechanical models" and "damage mechanics" [Rousselier (1987), Needleman and Tvergaard (1984), Sun et al. (1992), Brocks et al. (1995)] and were in the beginning eyed with distrust by the J-community.

Let us consider, like Griffith (1920) did, the energy balance for an incremental crack extension under quasistatic loading:

$$\frac{\partial W_{ex}}{B\partial a} = \frac{\partial}{B\partial a}\left(U^{el} + U^{pl} + U_{sep}\right), \tag{13}$$

where W_{ex} is the work done by external forces, U^{el} and U^{pl} are the elastic and plastic fractions, respectively, of the internal energy of the body, and U_{sep} is the work required for material separation in the process zone. The total energy dissipation rate, R, is then defined as in [Turner (1990)] by

$$\begin{aligned} R &= \frac{\partial U_{dis}}{B\partial a} = \frac{\partial}{B\partial a}\left(W_{ex} - U^{el}\right) \\ &= \frac{\partial U^{pl}}{B\partial a} + \frac{\partial U_{sep}}{B\partial a} = R^{pl} + \Gamma_c. \end{aligned} \tag{14}$$

"Dissipated energy" is understood as "non-recoverable mechanical energy". In an elastic-plastic finite element (FE) analysis of crack extension the plastic strain-energy fraction of the dissipation rate can be calculated directly from Eq. 12. If the simulation reflects plastic processes only, i.e. if it is simply based on the Mises-Prandtl-Reuss equations and crack extension is controlled by an experimental R-curve, the dissipation rate, R, equals $\partial U^{pl} \big/ B\partial a$ as in fracture mechanics testing.

When introducing R, Turner generally doubted that splitting it into local and global contributions would ever be possible, as only external work and elastic energy can be measured. Thus, every measured ductile crack-extension resistance will necessarily contain remote plastic work, which in general is much larger than the (local) SE. The problem of geometry dependence of ductile tearing resistance is therefore inherent and appeared unsolvable.

Recent developments, both in experimental techniques and in micromechanical modeling of fracture phenomena allow a further step forward, however, in analyzing the various contributions to energy dissipation by ductile tearing and its geometry dependence.

3.2 Modeling of the Process Zone

The physical processes which occur within the process zone characterize the material at the onset of fracture. Understanding of the energy dissipation mechanisms in the process zone is hence precisely what is necessary to identify "fracture toughness" as a material property. Generally, there are two types of mechanisms, involving different length scales and resulting in significantly different amounts of dissipated mechanical energy. The first is the emission and motion of dislocations from the crack tip [Rice and Thomson (1974), Rice (1992), Kysar (2003)], the second is the formation, growth and coalescence of voids [Rice and Tracy (1969), Tvergaard (1982), Thomason (1985)]. The following considerations restrict to ductile crack extension by void nucleation and growth.

Stüwe (1980) developed a simple analytical model to estimate the specific plastic energy for the formation of dimples on the fracture surface and Kolednik (1993) applied it for qualitatively explaining the geometry dependence of $J-\Delta a$ curves. By this approach, the energy required for material separation for ductile rupture can be determined experimentally from analyses of the topology of the fracture surfaces [Stampfl et al. (1996a, 1996b, 2000)].

Uncoupling of R^{pl} and Γ_c can also be realized in numerical models, if according to Barenblatt's idea [Barenblatt (1962)] a specific "process zone" ahead of the crack tip is introduced, where material degradation and separation occur [Siegmund and Brocks (1998, 1999, 2000)]. The simulations require a constitutive description of the material behavior in the process zone, which can mirror the local loss of stress carrying capacity. In general, two alternatives are used:

Damage models based on the micromechanisms of ductile rupture [Thomason (1985), Tvergaard (1982)], namely the nucleation, growth and coalescence of voids, as e.g. the most commonly used model of Gurson, Tvergaard and Needleman (GTN-model) [Gurson (1977), Needleman and Tvergaard (1984)]. The respective constitutive equations of porous metal plasticity are relations between stresses and strains in solid elements, representing the microstructure of the material in an average sense. In order to obtain physically meaningful results for the dissipation rate in the process zone, a length parameter, h_0, has to be introduced, however. This parameter depends on the average spacing of void nucleating particles and the hardening behavior of the metallic matrix. The respective relation is established by an energy equivalence for a micromechanically representative volume element (RVE). The work of separation, ΔU_{sep}, per incremental crack extension, Δa, is

$$\begin{aligned}\Gamma_c &= \frac{\Delta U_{sep}}{B\Delta a} \\ &= \frac{1}{B\Delta a}\int\limits_{V_0}\left(\int_t^{t+\Delta t}(1-f)\overline{\sigma}\,d\overline{\varepsilon}^{pl}\right)dV \\ &= \overline{u}_{sep}\,h_0 \end{aligned} \tag{15}$$

with f and $\overline{u}_{sep}$ being the average void volume fraction and the average SE density in the RVE of volume $V_0 = h_0 B\Delta a$, respectively. If "local" constitutive equations are applied, which do not contain an intrinsic length scale, the height of the finite elements in the ligament has to be considered as a characteristic material parameter [Sun and Hoenig (1994), Bernauer and Brocks (2002)].

Phenomenological "cohesive models" (CM) describing various kinds of decohesion processes, see Fig. 2, by a relation between surface tractions, σ_n, and material separation, δ_n. For this, cohesive surface elements are introduced at the boundaries of solid elements along a predefined crack path. The constitutive relation of the interface elements represents the effective mechanical behavior due to the physical processes of micro-void nucleation, growth and coalescence in a ductile material. Commonly, the cohesive law is defined by two parameters, a cohesive strength (CS), σ_c, and a critical separation, δ_c, or, alternatively, a separation energy (SE), Γ_c, which simply represents the area under the traction-separation law.

$$\Gamma_c = \int\limits_0^{\delta_c}\sigma_n(\delta_n)\,d\delta_n \tag{16}$$

The cohesive elements, in particular, can be understood as a renascence of Griffith's concept of a surface energy. The significant differences, however, are that

Though Γ_c is supposed to be a "surface" energy, the respective physical separation process occurs in a volume

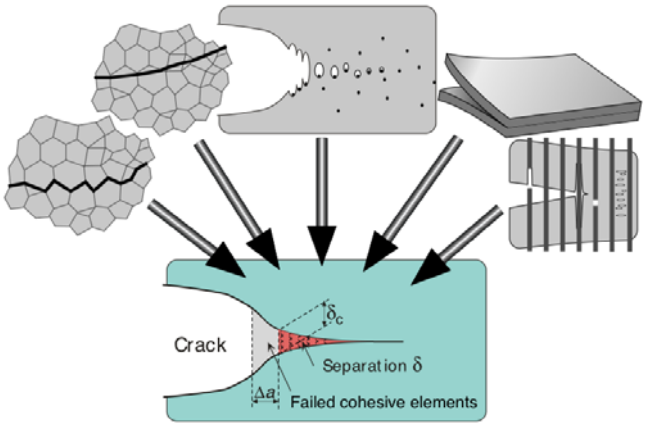

Figure 2 : Schematic of cohesive model for various failure phenomena: damage is localized in an interface

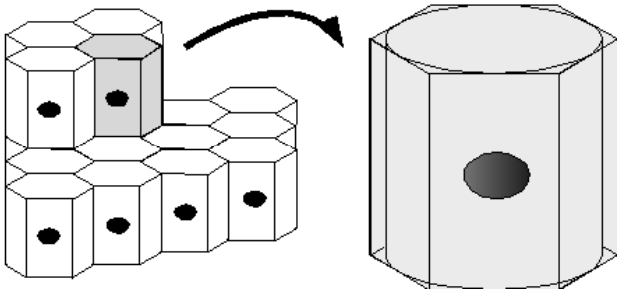

Figure 3 : Unit cell as representative volume element (RVE) of a periodic microstructure containing voids

of finite, though commonly small thickness, in reality, and

The CS, σ_c, is an additional independent, phenomenological parameter, representing the maximum tensile stress acting on an RVE.

The advantage of the CM in comparison to the local continuum models of damage is, that they do not show pathological mesh dependence and do not require the introduction of a length parameter via the FE mesh, as they include an intrinsic length parameter, δ_c, already. A major disadvantage is their restriction to pre-defined crack paths within the FE mesh.

The question of how to obtain a physically meaningful cohesive law required for the cohesive elements has also to be addressed. As it is supposed to represent micromechanical processes of material degradation and damage in a phenomenological way, micromechanically based models of void growth may help to identify it. These processes have been investigated by several authors, based on numerical simulations of RVEs or "unit cells" containing a void [Koplik and Needleman (1988), Brocks et al. (1995)], which are supposed to represent a typical periodic microstructure of ductile materials, see Fig. 3.

The mechanical behavior of an RVE is described in terms of averaged "mesoscopic" stresses and strains,

$$\Sigma_{ij} = \frac{1}{V_0}\int_{V_0} \sigma_{ij}(x_k)\, dv = \frac{1}{V_0}\int_{\partial V_0} x_i t_j\, da \tag{17}$$

$$E_{ij} = \frac{1}{V_0}\int_{V_0} \tfrac{1}{2}\left(u_{i,j}+u_{j,i}\right) dv = \frac{1}{V_0}\int_{\partial V_0} \tfrac{1}{2}\left(n_i u_j + n_j u_i\right) da \tag{18}$$

The lower case letters, σ_{ij}, u_i, t_j represent the non-uniform "microscopic" field quantities of stresses, displacements and surface tractions, respectively.

The mesoscopic stress-strain curves, Σ_{33} vs E_{33}, for a cylindrical elastic-plastic unit cell of diameter D_0 and height $h_0 = D_0$, i.e. volume $V_0 = {}^1\!/_4\pi D_0^3$, containing a spherical void of 0.5% volume fraction under triaxial tension, $\Sigma_{33} > \Sigma_{22} = \Sigma_{11}$ are displayed in Fig. 4, The curves depend on the applied mesoscopic stress triaxiality, T, defined as the ratio of hydrostatic to equivalent stress,

$$T = \frac{\overline{\Sigma}}{\Sigma_h} = \frac{\sqrt{3/2\Sigma'_{ij}\Sigma'_{ij}}}{1/3\Sigma_{kk}} = \frac{3\left(\Sigma_{33}-\Sigma_{11}\right)}{\Sigma_{33}+2\Sigma_{11}} \tag{19}$$

but have a typical shape: Tensile stresses increase up to a maximum, representing the CS, σ_c, of the volume element, remain at an approximately constant level for a while and decrease again, when plastic collapse of the cell, indicating the start of coalescence with neighboring voids, is reached. The final coalescence of voids, which would lead to a total loss of stress-carrying capacity of the volume element, cannot be modeled with single cells, of course. Qualitatively similar results would be obtained from single-element calculations applying the GTN model [Siegmund and Brocks (1999].

The mechanical behavior of this RVE may be idealized

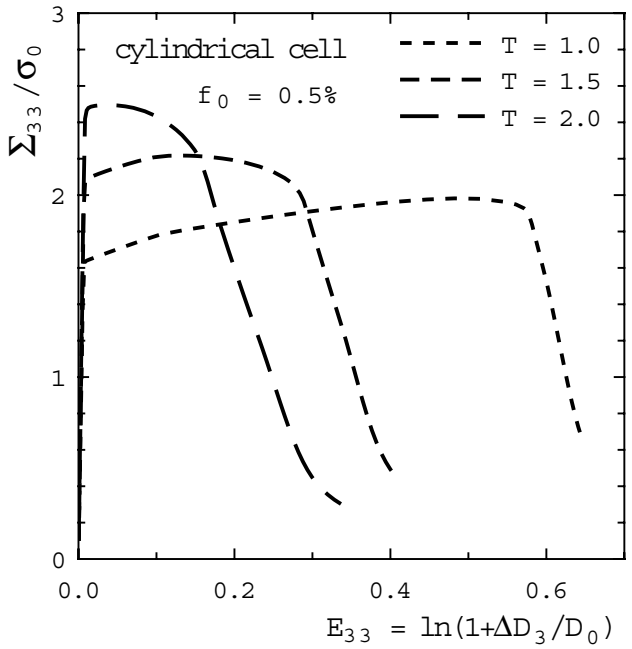

Figure 4 : Macroscopic stress-strain curves of a cylindrical elastic-plastic unit cell containing a spherical void of 0.5% volume fraction under varying triaxiality; σ_0 is the yield stress, and D_0 denotes the cell size.

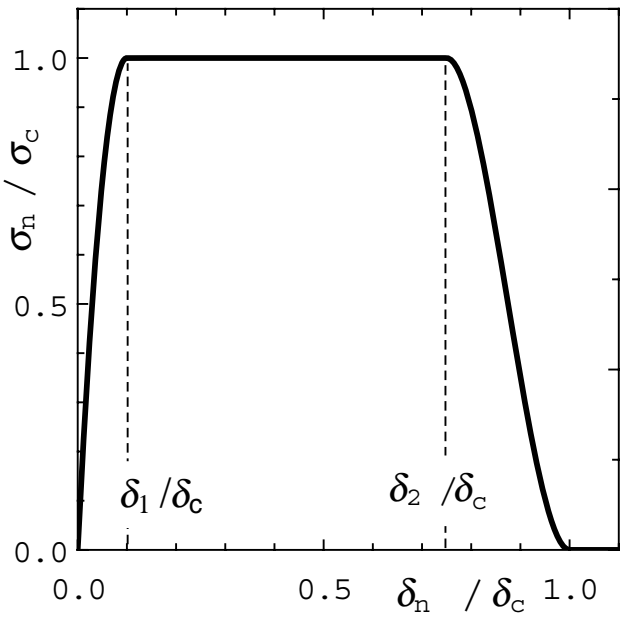

Figure 5 : Cohesive law for ductile tearing with two shape parameters, δ_1, δ_2.

as in Fig. 5, namely

$$\sigma_n = \sigma_c \begin{cases} 2\left(\frac{\delta_n}{\delta_1}\right) - \left(\frac{\delta_n}{\delta_1}\right)^2 & \text{for } \delta_n \le \delta_1 \\ 1 & \text{for } \delta_2 \le \delta_n \le \delta_c \\ 2\left(\frac{\delta_n-\delta_2}{\delta_c-\delta_2}\right)^3 - 3\left(\frac{\delta_n-\delta_2}{\delta_c-\delta_2}\right)^2 + 1 & \text{for } \delta_2 \le \delta_n \le \delta_c \end{cases} \tag{20}$$

with

$$\Gamma_c = \sigma_c \left(\frac{1}{2} - \frac{1}{3}\frac{\delta_1}{\delta_c} + \frac{1}{2}\frac{\delta_2}{\delta_c}\right), \tag{21}$$

where δ_1 and δ_2 represent additional shape parameters [Scheider and Brocks (2003)]. But various other relations between normal tractions and separation, based on different micromechanical substantiations, have been used successfully in the literature, e.g. [Needleman (1990), Yuan and Cornec (1996), Tvergaard (2001), Wnuk (2002, 2003)]. Cohesive laws may be established for mixed mode separation processes, too. They will require an additional assumption on the interaction of the three modes [Scheider and Brocks (2003)].

Whereas cohesive models of ductile fracture are mainly based on the assumption, that CS and SE are material constants, numerical studies on cell models have demonstrated, however, that material separation based on void growth and coalescence strongly depends on the stress triaxiality, T, see Fig. 4. The observed "constraint" effect is quite obvious: higher triaxiality causes an increase of the "fracture stress", i.e. a higher CS, and a decrease of ductility, i.e. a lower SE, see Fig. 6. The particular micromechanical process of void growth and coalescence governing ductile rupture thus shows a local constraint effect, which adds to global constraint effects on the overall plastification of the structure.

Respective modifications have been proposed and applied [Siegmund and Brocks (1998, 1999, 2000), Wnuk (2002, 2003)]. As the (local) SE is very small compared to the (global) plastic work per crack extension, however, the effect of triaxiality on the cohesive parameters is commonly negligible from an engineering point of view [Siegmund and Brocks (1998, 2000)].

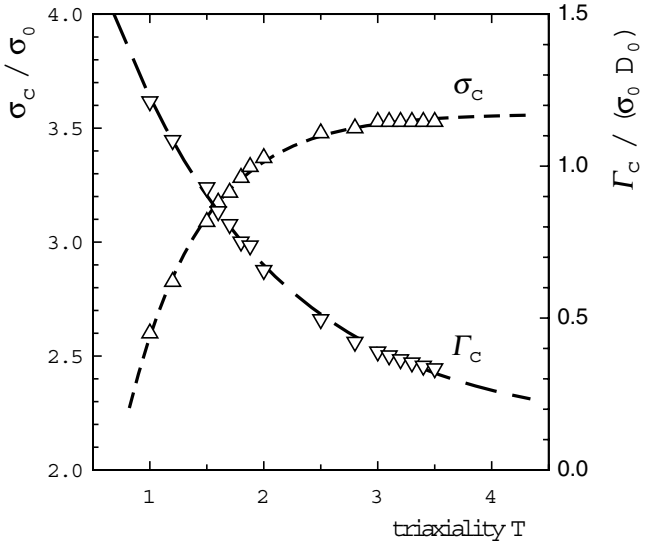

Figure 6 : Triaxiality dependence of cohesive parameters for a ferritic steel [Siegmund and Brocks (1998)].

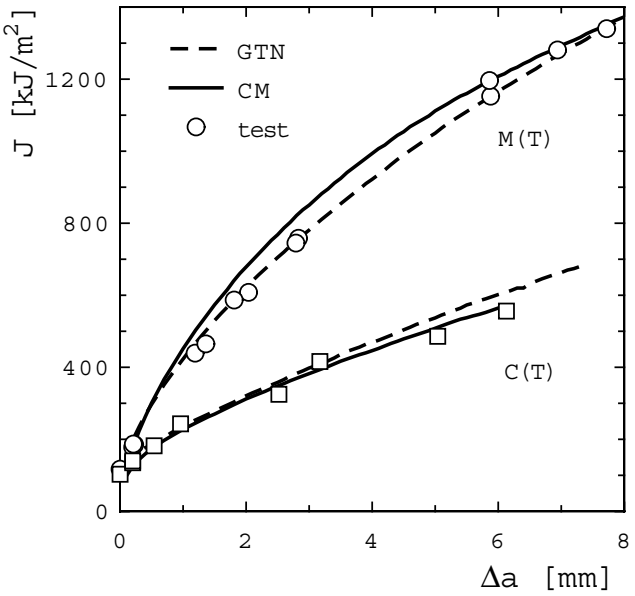

Figure 7 : J_R-curves of C(T) and M(T) specimens as predicted from FE simulations applying GTN and CM.

3.3 Prediction of R-Curves

The CM has become a versatile, simple and numerically stable tool for simulating ductile tearing in specimens and structures [Cornec et al. (2003)]. The following example of numerical simulations of fracture tests on side-grooved standard compact specimens, C(T), and centre-cracked panels, M(T), with the CM and, for comparison, with the GTN model, shows that the geometry dependence of J_R-curves is captured well by both models, one of which is a phenomenological, the other a micromechanically based model [Siegmund and Brocks (2000)]. The investigated material is a ferritic steel DIN StE 460.

The predicted J_R-curves are plotted in Fig 7 in comparison to the test results, which are well met. For the simulations with the CM, a triaxiality independent value of the SE, Γ_c = 53.3 kJ/m^2, was chosen. The CS was determined from the cell model calculations as $\sigma_c = 3.36\ \sigma_0 = 1580$ MPa for the relevant range of triaxiality. The comparably simple CM, which gets by with only two material parameters, is well capable of predicting the large difference in the "crack-extension resistance" of the two specimen types, as conventionally measured in terms of J_R-curves.

Fig 8 shows that the SE, Γ_c, in the process zone, as calculated from simulations of the fracture tests by the GTN model, does indeed show a dependence on the local stress triaxiality. It differs between the C(T) and the M(T),

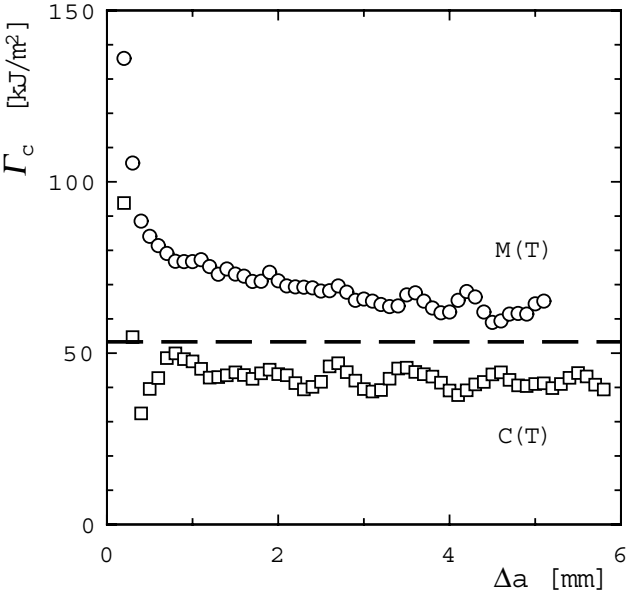

Figure 8 : Separation energy, Γ_c, for a ferritic steel DIN StE 460, calculated from numerical simulations of fracture tests with the GTN model.

where Γ_c of the latter is higher, which is due to the lower triaxiality, see Fig 6, and it depends slightly on the crack extension, Δa.

Both effects are negligible, however, with respect to the global behavior. The reason becomes obvious from the

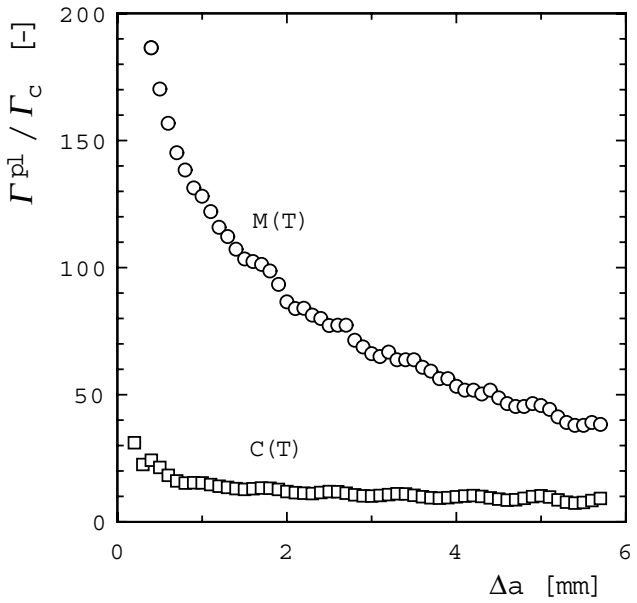

Figure 9 : Ratio of plastic work rate and SE for fracture mechanics specimens, C/T) and M(T), calculated from numerical simulations of tests

evaluation of the global plastic strain energy, which can be calculated according to Eq. 12. The result is plotted in Fig. 9 and reveals that the actual SE, Γ_c, is less than 1 to 10 % of the plastic work per crack increment, R^{pl}. In other words, between 90 and more than 99 % of what is measured in R-curve testing is work of remote plastic deformation and not "fracture energy".

4 Summary and Conclusions

Griffith's approach of establishing the energy balance in a cracked body has been seminal for more than 80 years of fracture-mechanics research and modeling. His idea of introducing a surface energy into this balance has been revitalized in various forms, one of which is the CM, which has become a modern and versatile tool used in numerical simulations.

Griffith's theory of brittle fracture requires just one parameter, the separation energy (SE), Γ_c, as the fracture stress of an RVE containing a micro-crack can be calculated from Γ_c in elasticity. Cohesive models for inelastic failure require (at least) a second parameter, the cohesive strength (CS), σ_c. Both, CS and SE, have a micromechanical interpretation for ductile tearing under mode I. The CS is the ultimate value of the "mesoscopic" maximum principal stress acting on an RVE, and the SE is the mechanical work dissipated by void nucleation, growth and coalescence per unit crack extension in a process zone ahead of the crack tip. Thus, cohesive models provide a veritable "two-parameter approach", which, different from concepts like $J-Q$, has some micromechanical background.

Though SE and CS were found to depend on the triaxiality of the stress state, this dependence may be neglected in numerical predictions of R-curves, as the global plastic strain energy per unit crack extension is much larger than the SE in the process zone

Additional parameters for maximum shear stresses and an interaction law have to be introduced for mixed mode fracture. Other fracture phenomena, which are based on different micromechanical mechanisms, will require respective interpretations of σ_c and Γ_c. In any case, CS and SE appear to be physically significant characterisations of "fracture toughness".

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