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Building Risk Assessment Procedures

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Abstract: This work describes the results of the experience acquired by the authors during their participation to some among the European research programs with the aim to develop a probabilistic risk assessment procedure to analyse the spreading of fatigue-induced damage in typical aeronautical components. The several steps of the procedure are pointed out, and especially the modelling of the damage evolutionary process; the initiation and the transitional probabilities, which characterize the passage from one damage level to a higher one, are fully characterized and their dependence from time and from the damage state of surrounding zones illustrated by various example results.

keyword: Reliability, Damage Tolerance

1 Introduction

The need for a sound risk assessment procedure is nowadays widely felt, at most when dealing with large lightweight structures whose damage can involve the loss of life; it has been accepted, indeed, that in such cases no deterministic design methodology can be found such as to give the certainty of safety for whatever time interval. The random character of damage, in presence of all uncertainties due to the material, manufacturing procedures, exploitation conditions and loads requires probabilistic design methodologies which, coupled with maintenance policies and procedures – also probabilistically defined – have given birth to damage tolerance techniques [Swift (1994)].

Such problems were put to the attention of EC in recent years, when large Consortia were being formed among well known firms to build new aircrafts of unusual size, as a counterpart to analogous activities which were being carried forward in USA; therefore several research projects were activated with the general aim to study the propagation of damage in aircrafts subjected to widespreading fatigue because of the progressive deterioration following a normal exploitation.

The present paper concerns the experience acquired by the authors thanks to their participation to some of the said research programs and mainly to SMAAC (Structural Maintenance of Ageing Aircraft) and ADMIRE (Advanced Design concepts and Maintenance by Integrated Risk Evaluation), where the probabilistic perspective was more deeply investigated; several renewed firms, research institutes and universities were involved in those programs and a very useful cooperation followed, which led to relevant achievements and developments, but in this paper the attention will be focused only on the activity of the authors [Soprano and Caputo et alii, (2003)], leaving to the other researchers to report their own work.

All above stated, we can proceed by observing that in general a large and complex structure isn't subjected to an uniform state of stress, but it consists of differently loaded regions which, because of their particular geometries, are affected by higher or lower stress levels and gradients; unfortunately, the nowadays available probabilistic procedures involve time-expensive analyses which are too heavy to deal with such a complex structure in a single computer step; therefore, tree-procedures have been developed, as for example in Darwin code [Gonnet and Hallet (1997)], where a general FEM step identifies the most stressed regions to which a reliability analysis is subsequently applied.

In our case, as the main scope was to define a rather general procedure [Soprano, Caputo and Lamanna (2004)], a reference structure was defined, which consisted of a simple sheet (Fig. 1) subjected to a normal load – neglecting secondary bending – because of the loads transmitted by three rows of rivets; that example component not only can constitute an useful reference structure, but it is also periodic for both geometry and loading, so that it can decomposed rather easily in simpler details, as it will be shown in the following sections; one of the explored topics, indeed, was to define how to extract and investigate limited details and how to extend the obtained results to the whole structure, for which such a component is quite adapt.

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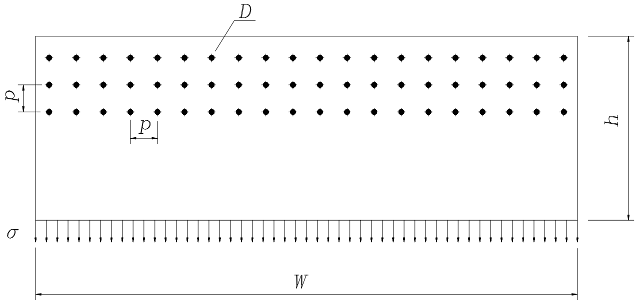


Figure 1 : Reference Structure

2 General probabilistic aspects

A risk assessment procedure is meant to be a tool such as to give a rather complete picture of the behaviour of a given structure with reference to evolutive damage, or, equivalently, its probability of failure in time as well as the statistics of damage levels for assigned instants.

It is well known [e.g. Madsen, Krenk and Lind (1986); Ditlevsen and Madsen(1996); Melchers (1999); Rackwitz (2001)] that to obtain the probability of failure of a structure whose random character is defined by a set of design variables $\{\mathbf{X}\}$ with a joint probability density function (pdf) given by f($\{\mathbf{X}\}$), one has to evaluate the following integral:

$$P_f(t) = \int_{\Omega_f} f_{\mathbf{X}}(\{\mathbf{X}\};t)\,dt = \int_{\Omega_f} f_{x_1x_2\cdots}(x_1(t),x_2(t),\ldots;t)\,dt, \quad (1)$$

where the variables are shown to be time-dependent and where Ω_f represents the failure region; the integral above would let us follow the behaviour of probability of failure in both time and space, but unfortunately it cannot be solved exactly but in a few simple cases and therefore one must refer to one of the available numerical procedures, which we assumed to be of Monte-Carlo (M-C) type for our case.

It is also well known that simpler and less time-consuming techniques exist for the evaluation of spatial probability, as FORM or even SORM, but their most effective advantage is limited to those cases where an explicit limit state function can be found, or at least, using for example the Response Surface Method, when a single design point is to be expected.

In any case, at first we adopted FORM technique for an intermediate step of our procedure, as we will show later with the reasons why we discarded its use in a subsequent version.

On the contrary, the structure we have assumed as a reference is such that a rather uniform state of stress is induced by external loads and where the most probable locations to be assumed as crack origins are all probabilistically equivalent to each other; in fact, we limited our analysis to the third (most internal) row of loaded holes where the sum of the by-pass load and of that transferred by rivets is the highest, and at the same time we considered as negligible the reduction of stress which is induced in the outermost holes because of the side effects, as they can be easily counterbalanced by other (manufacturing) effects.

Therefore, under the said hypotheses such questions are meaningless as the probability of one or another assigned hole to be cracked, and the only answers one can obtain are of more general kind, i.e. what is the most probable time when a such and such scenario will take place, for example when the largest crack attains a given length, wherever it is located.

In those conditions, the simulation type of techniques are therefore specifically suitable, at most when dealing with large structures or complex MSD (multi-site damage) cases, where a general result is required or when an analytical function cannot be used to define the limit state surface, or even when many equivalent design point exist.

On the other hand, a generally accepted result is that the number of M-C trials to be carried out to evaluate a p probability with a maximum error e_{max} and with an α confidence level is given by

$$N = \left(\frac{2\cdot\alpha}{e_{\max}}\right)^2 \frac{1-p}{p} \quad , \quad (2)$$

which reveals to be a very high number, if acceptable valued are give to α and e_{max} and when p is very small, as it has to be when dealing with such cases as air transportation, where many lives are at the stake.

In order to limit the number of trials to more acceptable figures, a rather wide spectra of variance reduction techniques have been insofar developed, as importance sampling, directional simulation and others, but their use has been discarded in our case as they need at some point the identification of the reliability index or of the limit state surface; furthermore, it must to be pointed out that all those techniques were basically introduced in order to evaluate only spatial probabilities and their application to time analyses would result to be rather cumbersome. In fact, if M-C technique can solve the problem of spatial probability, the need to define the evolution of damage in time requires the introduction of a hypothesis regarding the kind of stochastic process to be considered; for that aspect it is immediate to observe that the simplest process one can use is of Markow type, for which it is assumed that the state of damage which can appear at time "t+Δt" is only dependent on the state at time "t", with no memory of previous times. It is to be noted that this assumption doesn't imply any independence from load history, but only that the situation at a given time, in terms of state of stress, net sections, and so on – as build from previous instants – is sufficient to define the scenario which will take place in the immediately following time.

It has to be further noted that the introduction of a Markow process requires in turn the definition of an initial scenario and of the transitional probabilities which affect the passage from one state of damage to the following.

All the previous characters stated, we had therefore to follow a step-by-step procedure to build the code, covering different topics, as the definition of the design variables, that of the structural behaviour of the component adopted and the probabilistic environment to be used when following the evolution of damage in order to obtain a well organized procedure, as it will be shown in the next sections.

3 The identification of the design variables

If we consider the particular component we adopted as a reference, we can observe that its damage behaviour is affected by many parameters, as geometry (pitch and diameter of holes, thickness of the sheet), distribution of load level, size of the initial defect (length and depth) and parameters of the propagation law. As the global analysis of the component can become more or less time expensive according to the number of design variables we consider, a first analysis was carried out in order to define which ones were the relevant parameters we had to account for.

Therefore, as a first step we resolved to carry out an extensive number of trials to investigate the influence exerted by each one of those parameters on the statistics of the number of cycles to failure; in any case, as pitch and diameter of the holes are rather standardized in size, we disregarded their analysis, while the sheet thickness was assumed as a deterministic parameter, varying between 1.2 and 4.8 mm; it follows that the only parameters we investigated were the stress level distribution, the size of the initial defect and the parameters of the propagation law, which we assumed to be of Paris' type.

For what refers to the load, we supposed to be in presence of traction load cycles with R = 0 and with a mean value which followed a Gaussian probability density function (pdf) around 60, 90 and 120 MPa, with a coefficient of variation varying according assigned steps; initial crack sizes were considered as normally distributed from 0.2 mm up to limits depending on the examined case, while for what concerns the two parameters of Paris' law, they were considered as characterized by a normal joint pdf between the exponent n and the logarithm of the other one.

Initially, an extensive exploration was carried out, considering each variable in turn as random, while keeping the others as constant and using the code NASGRO 3.0 [Nasgro (2001)] to evaluate the number of cycles to failure; an external routine was written in order to insert the crack code in a M-C procedure. CC04 and TC03 models of NASGRO library were adopted in order to take into account corner- as well as through-cracks.

For all analyses we carried out 1,000 trials/point, which was assumed as a convenient figure to be accepted to obtain rather stabilized results, while preventing the total runtimes from growing unacceptably long; by performing the said M-C procedure for an assigned statistics of one input variable at the time, we studied its effect on the statistics of the number of cycles before failure.

The results obtained can be illustrated by means of the following pictures and first of all of the Fig. 2, where we report the dependence of the mean value of life from the mean amplitude of remote stress for different cases where the CV (coefficient of variation) of stress pdf was considered as being constant.

The figure assesses the increase of the said mean life to failure in presence of higher CV of stress, as in this case rather low stresses are possible with a relatively high probability and they influence the rate of propagation in

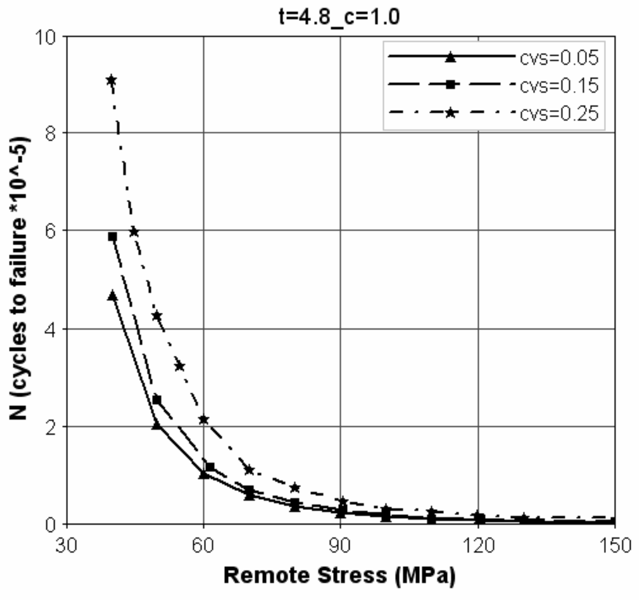


Figure 2 : Life as a function of remote stress

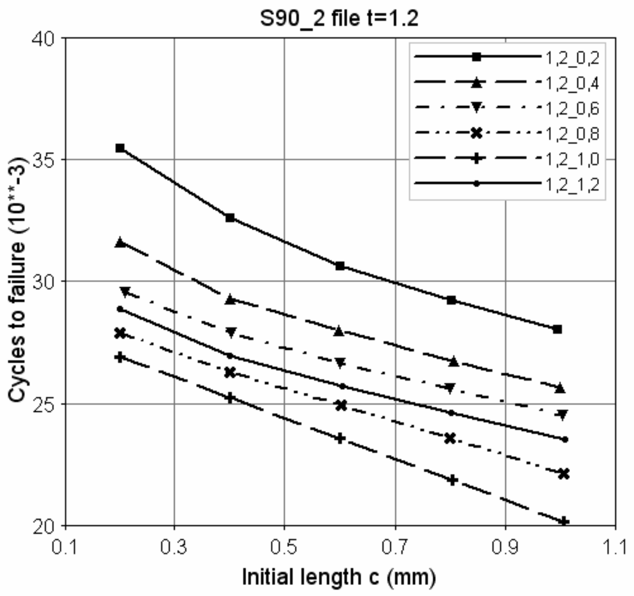


Figure 3 : Life as function of initial crack length

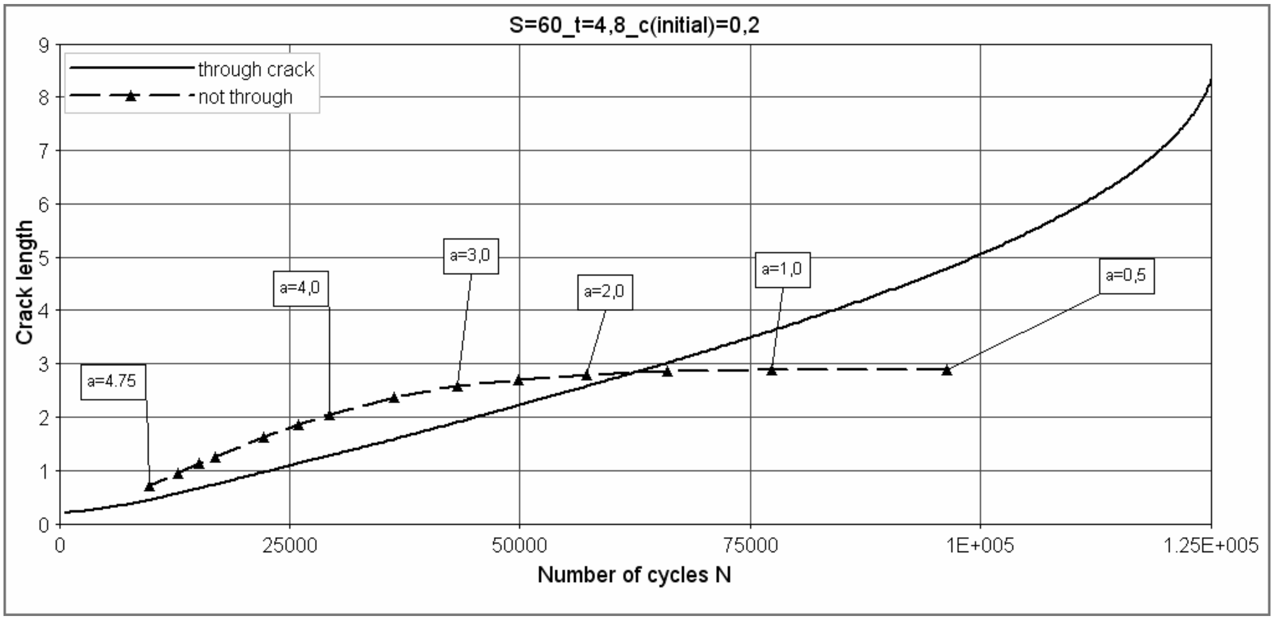


Figure 4 : Comparison between growth times of through and corner cracks

a higher measure than large ones. In Fig. 3 we represent the influence of the initial geometry of a corner crack, considered to be elliptical in shape, with length c and depth a; a very interesting aspect of the consequences of a given shape is that for some cases the life for a through crack is longer than the one recorded for some deep-corner ones; that case can be explained with the help of the plot of Fig. 4, where the growth of a through crack is compared with those of quarter corner cracks, recording times when a corner crack becomes a through one: as it is clarified in the boxes in the same picture, each point of the dashed curve references to a particular value of the initial depth.

It can be observed that beyond a certain value of the initial crack depth, depending on the sheet thickness, the length reached when the corner crack becomes a through one is larger that the one obtained after the same num-

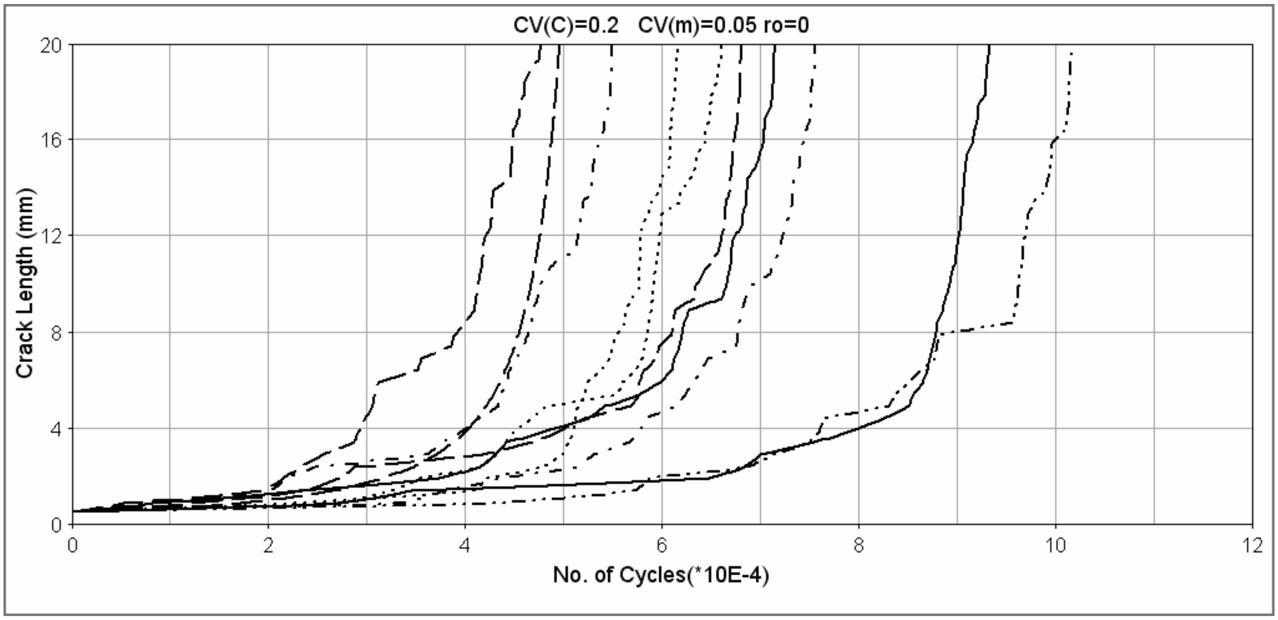


Figure 5 : Scatter of propagation curves because of spatial randomness of growth parameters

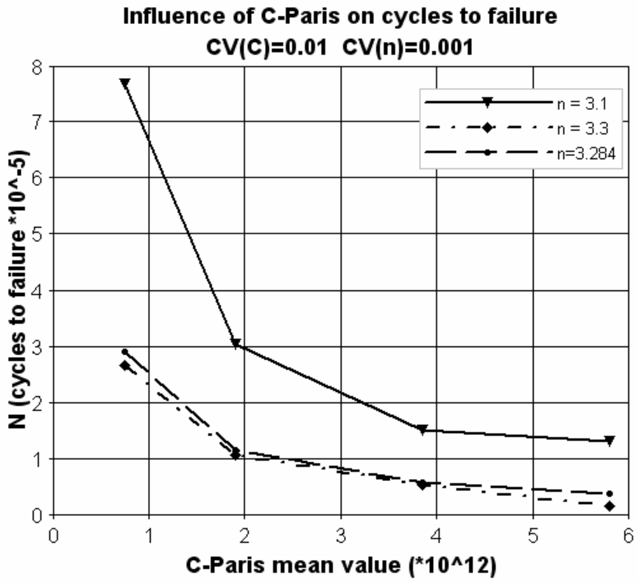


Figure 6 : Influence of the parameters of Paris' law

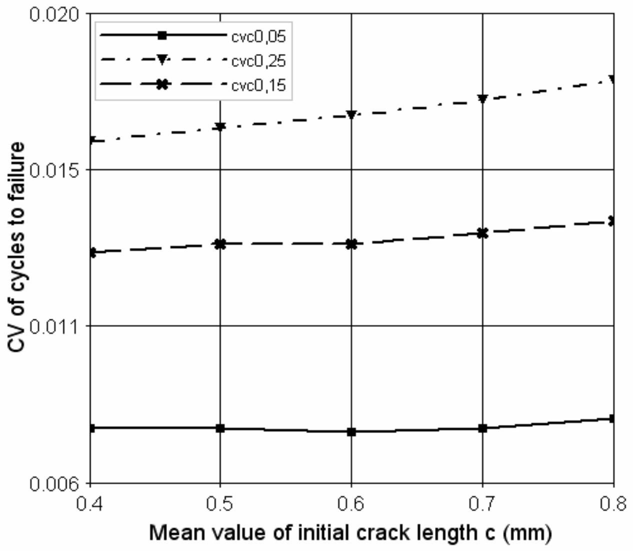


Figure 7 : Influence of the statistics of initial length

ber of cycles when starting with a through crack, and this effect is presumably connected to the bending effect of corner cracks.

For what concerns the influence exerted by the growth parameters, *C* and *n* according to the well known Paris' law, a first analysis was carried out in order to evaluate the influence of spatial randomness of propagation parameters; therefore the analysis was carried out considering that for each stage of propagation the current values of *C* and *n* were randomly extracted on the basis of a joint normal pdf between ln*C* and *n*. The results, illustrated in Fig. 5, show a strong resemblance with the well known experimental results [Wirkler, Hillberry and Goel (1976)].

At last, the influence of the mean value of *C* parameter on the mean value of number of cycles to failure is represented in Fig. 6. Using the results of the same analysis, an investigation was carried out about the influence of the

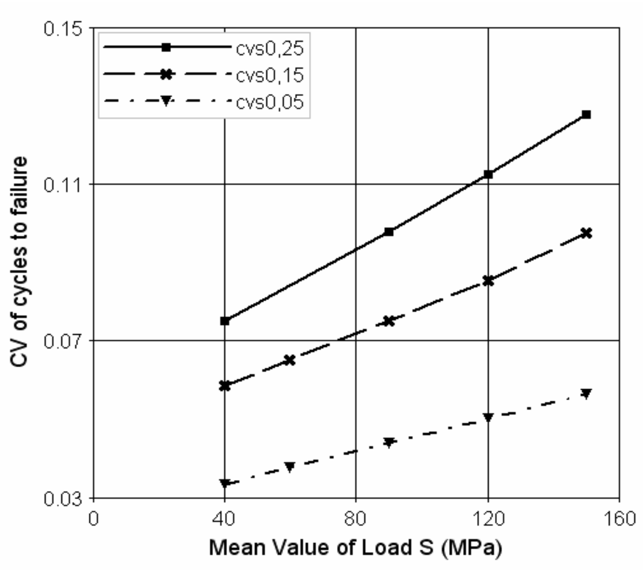
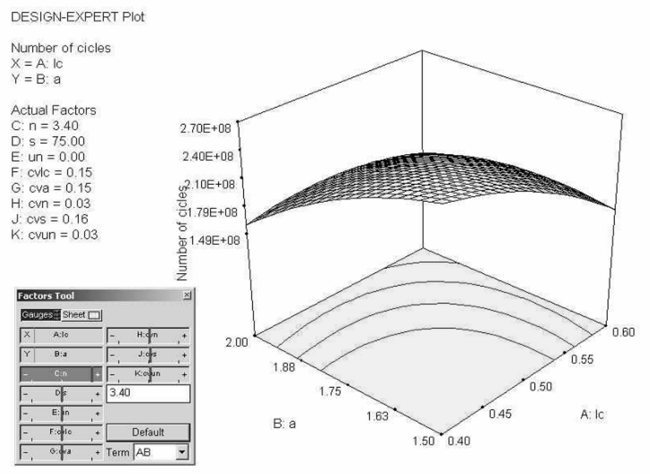


Figure 8 : Influence of the statistics of remote stress


DESIGN-EXPERT Plot

Number of cicles
X = A: lc
Y = B: a

Actual Factors
C: n = 3.40
D: s = 75.00
E: un = 0.00
F: cvlc = 0.15
G: cva = 0.15
H: cvn = 0.03
J: cvs = 0.16
K: cvun = 0.03

Figure 9 : Life as a funct. of both crack length and depth

same ruling parameters on the variance of cycles to failure. The results are recorded in Fig. 7 and 8; the former shows that the mean value of the initial length has a little influence on the CV of cycles to failure, while on the contrary the same results are largely affected by the CV of the said geometry.

On the other hand, both statistical parameters of the distribution of remote stress have a deep influence on the CV of fatigue life, as it is shown in Fig. 8.

As the influence exerted by one of the parameters on the number of cycles to failure can be opposed or strengthened by the presence of one of the others, a second part of this first phase of our analysis was devoted to the study of interactions between parameters. A powerful tool which was used with that scope was found in DOE (Design of Experiment), as implemented in the statistical software package JMP by SAS Institute [Jmp (2000)]; using 8 variables (the mean value of all parameters, plus the CV of load, initial length and n_{Paris}) we applied Plackett-Burnam method to reduce the number of analyses required and for each of them we performed 1,000 M-C trials, recording the mean value of results, which were subsequently examined by the profiler method or by 3D boxes; an example is shown in Fig. 9.

The conclusions reached at the end of the previous analyses were that the mean value of the number of cycles to failure is largely dependent on the mean value of all the examined parameters and only in a lesser degree on their CV; at the same time the CV of life is affected by the CV's of all variables, but in a lesser degree and in some cases that effect can be disregarded; at last, the correlation between the two parameters of Paris' law, if existent, doesn't show a relevant influence on lifetime and therefore the said variables could also be considered as uncorrelated, at least for our application, without an appreciable loss of confidence of the results.

4 Deterministic structural behaviour of the examined component

Once the design variables were identified, we had to focus our attention to the type of structure we wanted to use as a reference; as we already stated, a simple riveted lap joint for aeronautical application was chosen (Fig. 1); it is composed by two 2024-T3 aluminium sheets, each 1 mm thick, with 3 rows of 10 columns of 5 mm rivets and a pitch of 25 mm.

Many reasons induced us to analyze such a structure before beginning the probabilistic study; first of all, our interest was directed to evaluate the interactions between existing singularities when a MSD (multi-site damage) or even a WFD (widespread fatigue damage) takes place.

Several studies have been carried out, in fact [for example, Horst 2005], considering a probabilistic initiation of cracks followed by a deterministic propagation, on the basis that such a procedure can use very simple techniques, such as compounding [Rooke 1986]. Even if such a possibility is a very appealing one, as it is very fast, once the appropriate fundamental solutions have been found and recorded, some doubts arise when one comes to its feasibility.

The fundamental equation of compounding method is indeed as follows:

$$K = K' + \sum \left(K_i - K'\right) + K_e \quad , \tag{3}$$

where the SIF at the crack tip of the crack we want to investigate is expressed by means of the SIF at the same location for the fundamental solution, K', plus the increase, with respect to the same 'fundamental' SIF, $(K_i - K')$, induced by each other singularity, taken one at a time, plus the effect of interactions between existing singularities, still expressed as a SIF, K_e.

As the largest part of literature is related to the case of a few cracks, generally the K_e term is neglected, but that assumption appears to be too weak when dealing with WFD studies, where singularities approach each other; therefore one of the main reasons to carry out such deterministic analysis was to verify the extent of this approximation.

We have also to stress that no widely known result is available for the case of rivet-loaded holes, at least for cases matching with the object of the present analysis; even the well known papers by Tweed and Rookes (1979-1980) deal with the evaluation of SIF for cracks which initiate on the edge of a loaded hole, but in our case we are interested on the consequence of rivet load on cracks which arise elsewhere.

Another aspect, related to the previous one, was the analysis of the load carried by each pitch as damage propagates; as compliance of partially cracked pitches increases with damage, one is inclined to guess that the mean load carried by those zones decreases, but the nonlinearity of stresses induced by geometrical singularities makes the quantitative measure of such variation difficult to forecast; what's more, the usual expression adopted for SIF's comes from fundamental cases where just one singularity is present and it is given as a linear function of remote stress. One has to guess if such a reference variable as the stress at infinity is still meaningful in WFD cases.

Furthermore, when we started to study the reference structure, an appealing idea to get a fast solution was to decompose the structure in simple and similar details, each including one pitch, to be analyzed separately and then added together, considering each of them as a finite element or better as a finite strip; that idea induced us to consider the problem of the interactions between adjacent details.

In fact, even if the structure is considered to be a two-dimensional one, the propagation of damage in different places brings the consequence of varying interactions, for both normal and shearing stresses.

For all reasons above, an extensive analysis of the reference structure was carried out in presence of different MSD scenarios; in order to get fast solutions, use was made of the well known BEASY commercial code, but different cases were verified by means of more complex Ls-Dyna models (Fig. 10).

Of course, as Ls-Dyna is not a crack-oriented code, we didn't get any hint about SIF's, but it was a valuable tool to verify the general stress-strain field in the structure, letting us to use about 216,000 elements to model sheets and rivets; in fact, the explicit technique it uses let us obtain the desired solution in a relatively short time. In any case, also some FEM analyses were carried out, using ANSYS, in order to get a further insight in the structure.

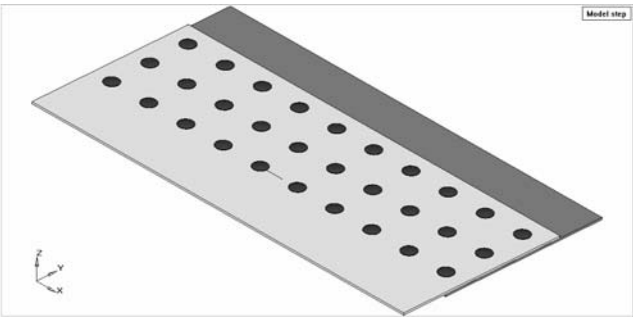


Figure 10 : Ls-Dyna model of the joint

Ls-Dyna, for example, let us analyze the behaviour of rivets, as it is shown in Fig. 11, where three adjacent pins are illustrated, with the central one mounted in a hole which has a crack at its right side; is it very clear, through the contour map, that because of the singularity the rivets are more stressed on their right side, but that the central one carries a lower load.

On the basis of the said controls, we explored a wide set of scenarios, with two, three and also four cracks existing at a time, using a two-dimensional DBEM model, considering a 100 MPa remote stress which was transferred to the sheet through the rivets according to a 37%, 26% and 37% distribution of load, as it is usually accepted in

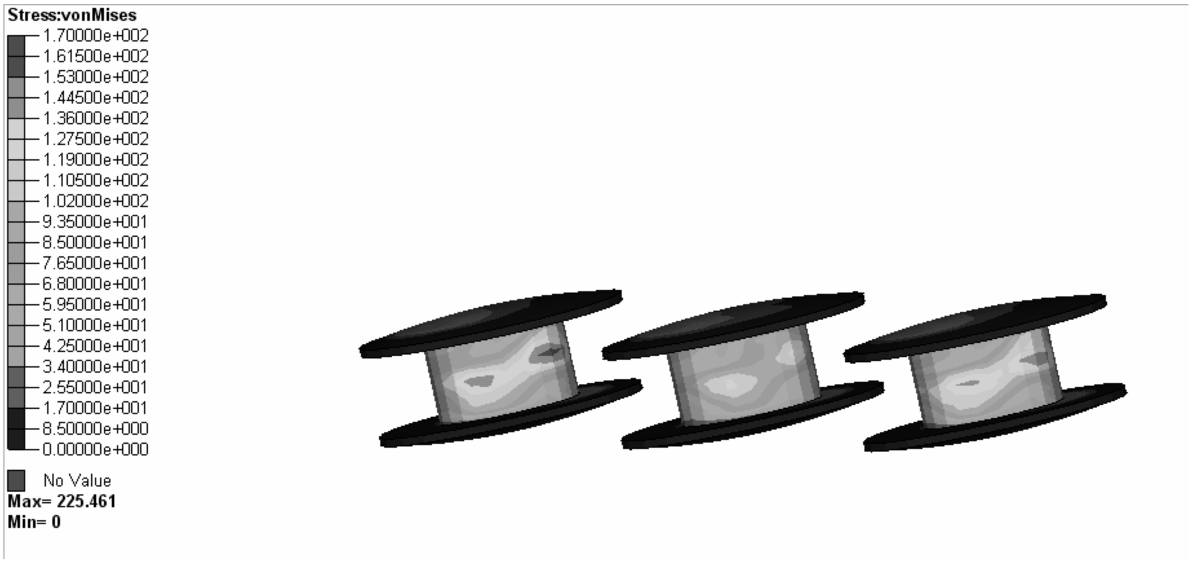


Figure 11 : Comparison among loads carried by three rivets, with the central one mounted in a cracked hole

literature; that load is applied considering an opportune pressure distribution on the edge of each hole.

This model, therefore, cannot take into account two effects, i.e. the limited compliance of holes, due to the presence of rivets and the variations of the load carried by rivets mounted in cracked holes; both those aspects, however, were considered as not very relevant, following the control runs carried out by FEM.

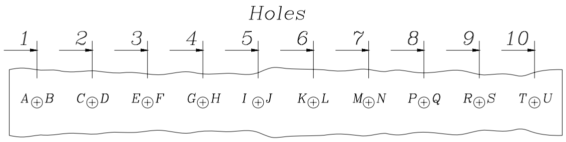


Figure 12 : Coding adopted to illustrate the results

For a better understanding of the following illustrations, one has to refer to Fig. 12, where we show the code adopted to identify the cracks; each hole is numbered and each hole side is indicated by a capital letter, followed, if it is the case, by the crack length in mm; therefore, for example, E5J7P3 identifies the case when three cracks are present, the first, 5 mm long, being at the left side of the third hole (third pitch, considering sheet edges), another, 7 mm long, at the right side of the fifth hole (sixth pitch), and the last, 3 mm long, at the left side of the eight hole (eighth pitch).

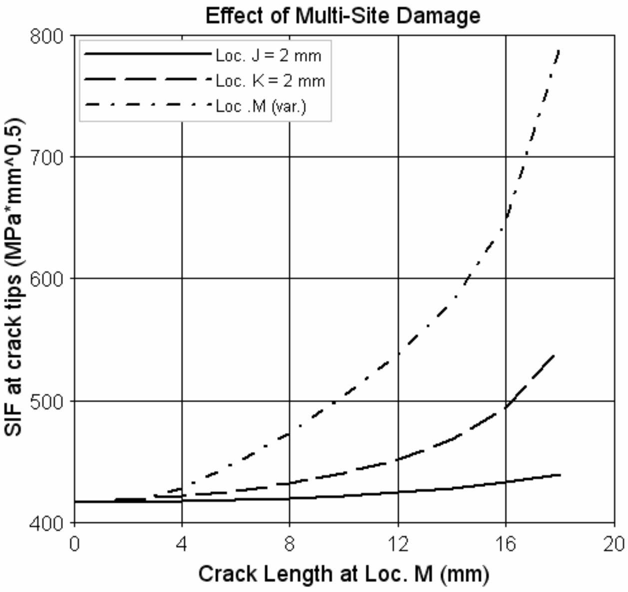


Figure 13 : SIF's for three-crack cases

In Fig. 13 a three cracks scenario is represented, where in pitch 6 there are two cracks, each 2 mm long and another crack is growing at the right edge of the seventh hole, i.e. in the adjacent seventh pitch; if we consider only LEFM, we can observe that the leftmost crack (at location J) is not much influenced by the presence of the propagating crack at location M, while the central one exhibits an increase in SIF which can reach about 20%.

In Fig. 14 the same scenario is represented, with the dif-

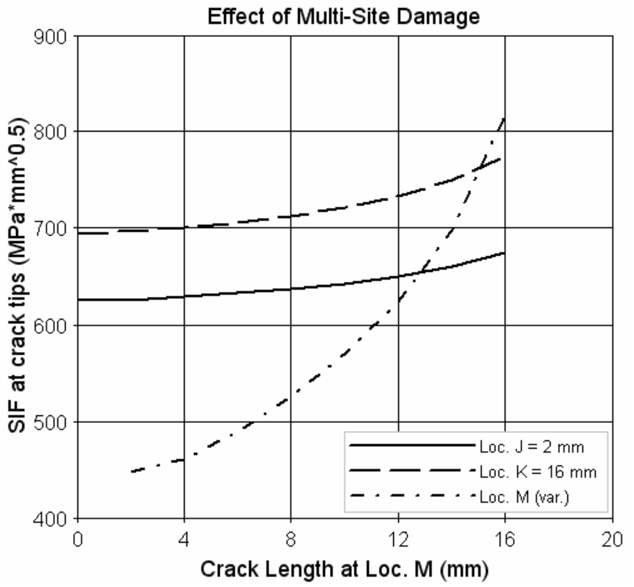


Figure 14 : SIF's for three-crack cases

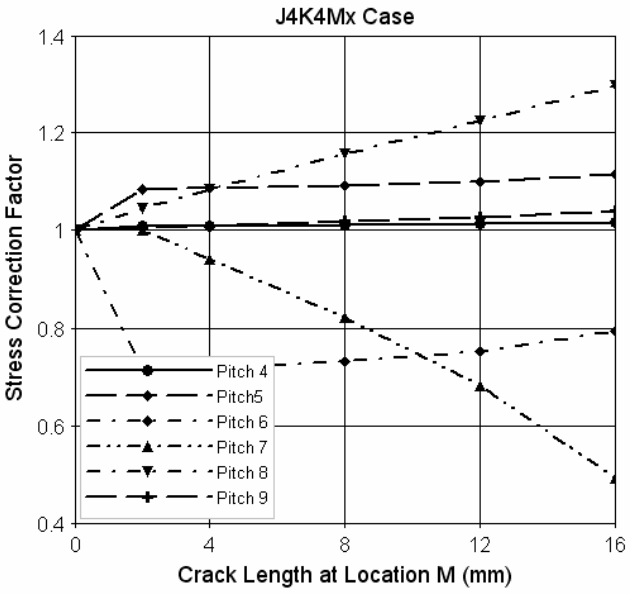


Figure 15 : Stress Redistribution – Stress concentration factor as a function of a crack growing in pitch 7

ference that the crack at location K is now 16 mm long; in the same LEFM sense, we can observe that also in this case SIF at location J is not influenced by the growth of crack at location M, even if the SIF level is now much higher, because of larger length of the crack at location K.

Considering now the effects of complex scenarios on the load carried by each ligament between holes, we first introduce a stress correction factor as:

$$SFC = \frac{\sigma_{eff}}{\sigma_{net}} \quad , \tag{4}$$

where σ_{eff} is the mean longitudinal stress recorded in the runs, obtained as the ratio between the transmitted load and the net section and σ_{net} is the mean longitudinal stress evaluated by compliances, simply dividing the load transmitted in a safe joint by the net section.

In Fig. 15 the results obtained are shown for the case of two cracks in the sixth pitch, each 2 mm long, in presence of a growing crack in pitch 7, at location M. We can observe that, on the right side of the propagating crack, the effect on SFC is negligible for pitch no. 9, with a maximum increase of about 5%, while pitch no. 8 is largely affected, as it now carries up to about 30% more than expected.

On the left side, pitch no. 4 isn't really influenced by what happens at location M, while pitch no. 5, because of the near crack at locations J and K, is subjected to an increase of the transmitted load which attains about 10%; in the sixth pitch, because of the simultaneous presence of two cracks (even if rather short ones) and the proximity of the larger one, suffers a sudden decrease in transferred load, even if for longer cracks at location M the said load increases because of the severe reduction of net section.

At the same time, as expected, the SFC for pitch no. 7 is progressively reduced in respect to what comes from compliances, as the stress flux lines are largely displaced and distributed among safe pitches.

Those effects can be observed as absolute values considering the values of mean longitudinal stress for different scenarios, as illustrated in Fig. 16, 17 and 18 for the same cases already illustrated; in the first one, we can observe a progressive increase in the mean longitudinal stress around pitch no. 6, which is the most severely reduced and the influence of the small crack at location M is not very high.

As the length of crack in pitch 7 increases, however, the mean longitudinal stresses in both pitches 6 and 7 becomes quite similar and much higher of what is recorded in safe zones, where the same longitudinal stresses are not much increased in respect to what is recorded for a safe structure, because the transfer of load is distributed

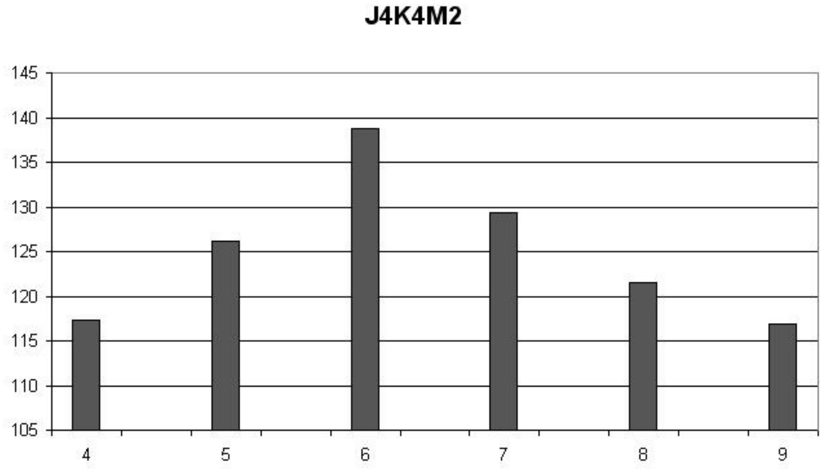


Figure 16 : Mean longitudinal stress loading different pitches for a 2 mm crack in pitch 7

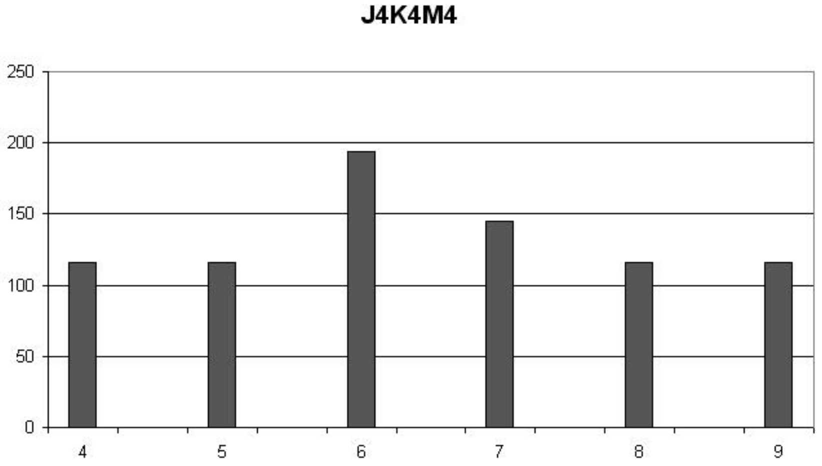


Figure 17 : Mean longitudinal stress loading different pitches for a 4 mm crack in pitch 7

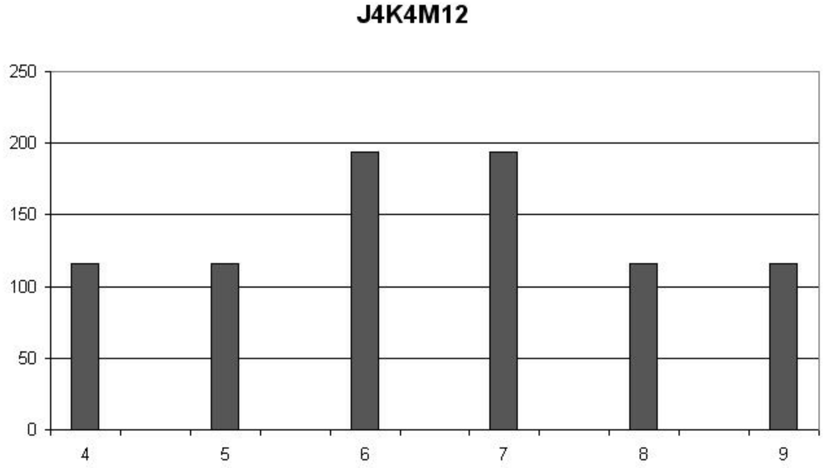


Figure 18 : Mean longitudinal stress loading different pitches for a 12 mm crack in pitch 7

among many pitches.

The main results obtained through the previously discussed analysis can be summarized by observing that in complex scenarios high interactions exist between singularities and damaged zones, which can prevent the use of simple techniques such as compounding, but that the specific zone to be examined gets up to a single pitch beyond the cracked ones, of course on both sides.

At the same time, as expected, we can observe that for WFD conditions, in presence of large cracks, the stress levels become so high that the use of LEFM can be made only from a qualitative standpoint.

5 Probabilistic structural behaviour of the examined component

Even if the main object of this step was discarded in following steps of our procedure, as we report later, it can be in any case noteworthy to recall it, as a significant effort to obtain a fast integration method and also because it gave us some further insight of the connected problems.

The main scope of this step was to investigate the application of fast probabilistic methods to the crack propagation problem in a multi-site damage condition, as opposed to those deterministic techniques often met in applications as we noted in the previous sections; after several trials, FORM (First Order Reliability Model) was chosen thanks to its intrinsic simplicity, even if it required the use of small integration steps because of the linearization of LSS (Limit State Surface) it adopts.

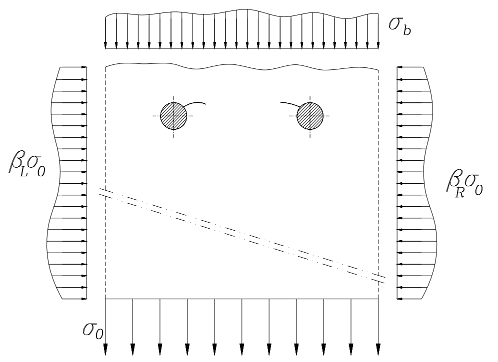


Figure 19 : Structural detail analyzed with FORM

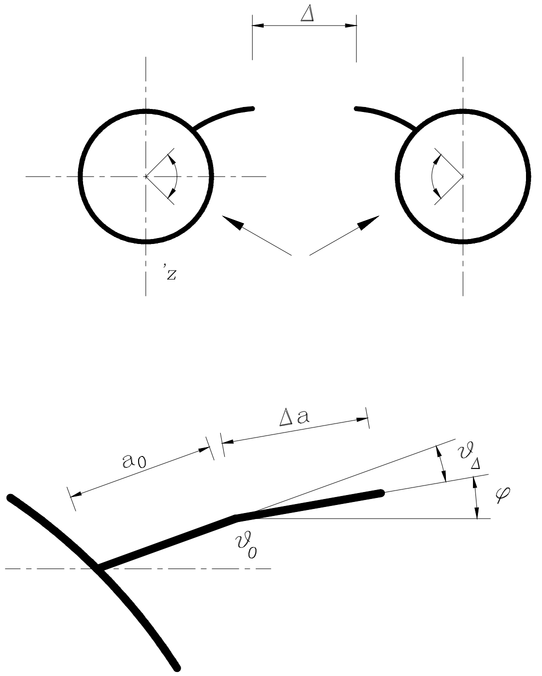


Figure 20 : The two-crack case analyzed through FORM

Considering the periodic character of the reference structure, we tried to obtain a simple method to deal with a limited zone and then to extend the results to the whole joint; thus, we extracted a limited detail, which included two columns of holes (Fig. 19), the innermost ones being the origins of corresponding cracks.

In order to analyze such a detail, we considered the studies by Belytschko and Liu (1993-1996) who successfully applied FORM in connection with BEM to evaluate the curvilinear propagation path of a crack. The method we applied was built as a generalization of the said technique, considering the simultaneous presence of two cracks (fig. 20) which would grow together, what let us take into account the existing interactions.

The set of design variables included the initial crack length and direction, the remote stress and the parameters of the propagation law and one LSS was written for each set of one hole and crack, as follows:

$$\begin{aligned} g_L &= \left(\int_{a_i}^{a_f} \frac{da}{C\,(\Delta K_{eff})^n} \right)_L - N \\ g_R &= \left(\int_{a_i}^{a_f} \frac{da}{C\,(\Delta K_{eff})^n} \right)_R - N \quad . \end{aligned} \tag{5}$$

Those two functions were included in an iterative procedure where one crack was considered first to find the time

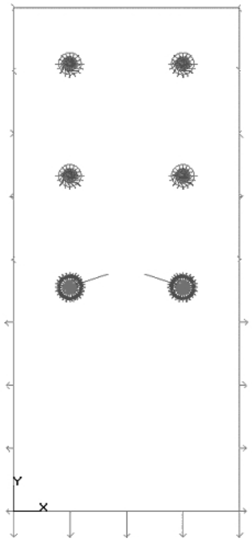


Figure 21 : BEASY model of the detail

required to obtain, with an assigned probability, a given growth step, while the second one was subsequently analysed to find the length increment obtained in the found time with the same probability; the reason why such a path was followed is to be found in the inherent difficulties related to other treatments; for example, if we would look for the most probable length increment to be developed for the first crack in a given time step, the same procedure for the second hole would give a different probability and therefore the two results would not be congruent; this problem, which can be solved with an opportune application of system reliability, was left to further research.

The code developed was written in Fortran for the probabilistic part, while for the structural aspects use was made of Beasy, as in the previous steps (Fig. 21). The obtained results included time histories of damage for a given propagation probability and they were stored in a database for reference or for further use in the following steps of the procedure; one example of the said results is shown in Fig. 22, where the abscissas of each crack tip is recorded as a function of time (or number of cycles).

This technique, even if it brought the previous interesting results, as we referred in Soprano and Caputo (2004), was discarded in later steps as it showed several drawbacks for the present application, namely the need to build a large database where the detail histories were stored for all combinations of loads, geometry, probability and growth parameters; therefore, even if the following risk assessment procedure resulted to be a fast one, it required a very long preliminary work.

Another aspect which induced us to leave that technique is related to the behaviour of interactions between details, which proved to be a function of the state of damage existing in adjacent zones, therefore requiring a variations of the lateral loads as damage spread in the structure, and therefore producing another serious increase of the results to be obtained and stored in the database before using them in the risk assessment procedure.

6 Assembling the risk assessment procedure

In a first phase a wide range of solutions were examined, with the aim to use one of the available commercial codes in our procedure; that possibility was then discarded, as those codes are not such as to deal with evolutionary processes and therefore major external routines had to be written and coupled with them in a very intricate way; therefore, the final decision was to write a complete procedure, which could also use the results obtained in the previous steps of our research, as pointed out above.

As already discussed, the adopted integration method was the direct Monte-Carlo, considering the structure entirely safe at the beginning of each trial and then following a damage process which was considered as to be of Markow type. For the sake of brevity we shall not recall here the characters of such a process, which we consider to be widely known today; we simply mention that we have to define the initial scenario, the damage initiation criterion and the transitional probabilities for damage steps.

In our case we considered the reference structure as entirely safe at the beginning of each trial, even if the inclusion of other hypothesis, and first that of an initial damage state as related to EIFS (Equivalent Initial Flow Size) or to the case of a rogue flaw, for example, don't imply any particular difficulty and could be introduced in a later time.

Two possible crack locations were considered at each hole, corresponding to the direction normal to the remote stress; the initiation of damage was introduced considering, on the basis of experimental tests available in literature, a lognormal pdf of crack appearance in time, given

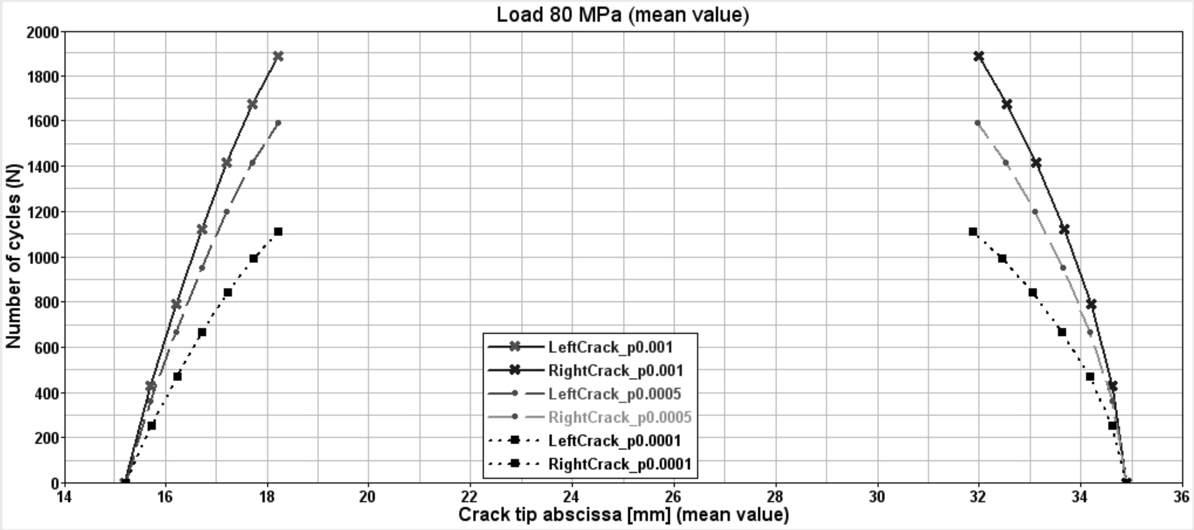


Figure 22 : Propagation histories with different probabilities

by the following function:

$$f\left(N_i\right) = \frac{1}{\sigma_{\ln} N_i \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln\left(N_i\right)-\mu_{\ln}}{\sigma_{\ln}}\right)^2\right] \quad , \tag{6}$$

with an immediate meaning of the different parameters; it has to be noted that in our case we adapted the experimental results available in literature to obtain P-S-N curves, in order to make the statistics dependent on the stress level.

At each time of the analysis the extraction of a random number for each still safe location was carried out to represent the probability of damage cumulated locally and compared with the probability coming from eq. (6) above; in the positive case, a new crack was considered as initiated in the opportune location.

In order to save time, the code doesn't start to perform the search but at a time where the probability to find at least one cracked location is not less than a quantity p chosen by the user; it is well known that, if p_f is the probability of a given outcome, the probability that the same outcome is found at least for one among n cases happening simultaneously is given by:

$$p = 1-\left(1-p_f\right)^n \tag{7}$$

and in our case n is the number of possible locations, thus obtaining, by inverting the probability function corresponding to eq. (6) above, the initial analysis time; in our trials we generally adopted $p = 0.005$, which revealed to be a conservative choice, but of course other values could also be accepted.

A particular choice had also to be made about the kind and the geometry of the initial crack; it is evident that to follow the damage process accurately a defect as small as possible has to be considered, for example a fraction of mm, but in that case two difficulties arise.

The first one is related to the behaviour of such a small crack, which would fall in the range of *short cracks* and which would, therefore, require a different treatment in propagation; even if we could use the idea of the *equivalent crack* as proposed for example by El Haddad, Dowing, Topper, and Smith, (1980) and which could be introduced in the code with very little effort, we didn't adopted it here, as it is quite clear that such a defect would initiate as a corner crack, which would become a through one only in the following stages of propagation and which would require in the intermediate cycle steps a three-dimensional treatment.

In order to limit our analysis to a two-dimensional case, therefore, we had to consider a crack which is born as a through one and therefore we choose it to be characterized by a length equal to the thickness of the sheet, i.e., 1.0 mm in our case.

Our choice was also justified by the fact that generally the experimental tests used to define the statistics represented in eq. (6) above record the appearance of a crack when the defect reaches a given length or, if carried out on drilled specimens, even match the initiation and the failure times, considering that in such cases the propagation times are very short.

Given an opportune integration step, the same random extraction is performed in correspondence of still safe locations, up to the time (cycle) when all holes are cracked, while those already initiated are considered as propagating defects, integrating Paris'-Erdogan law on the basis of SIF values recorded at the previous instant.

Therefore, at each step the code looks for still safe locations, where it performs the random extraction to verify the possible initiation of defect, and at the same time, when it meets a cracked location, it looks for the SIF value recorded in the previous step and, considering it as constant in the step, carries out the integration of the growth law in order to obtain the new defect length.

The core of the analysis is the coupling of the code with a DBEM module, which in our case was the same commercial code BEASY, even if in future steps it will be convenient to write a dedicated module.

A reference input file, representing the safe structure, is prepared by the user and submitted to the code, which analyzes the file, interprets it and defines the possible crack locations; then, after completing the evaluations needed at the particular step, it builds a new file which contains the same structure, but as damaged as it comes from the current analysis and it submits it to BEASY; once the DBEM run is carried out, the code reads the output files, extracts the SIF values pertaining to each location and performs a new evaluation.

For each ligament the analysis ends when the distance between two singularities is smaller than the plastic radius, as given by Irwin

$$r_p = \frac{K_I^2}{\pi\sigma_y^2} \quad , \tag{8}$$

where σ_y is the yield stress and K_I the mode-I SIF; that measure is adopted for cracks approaching a hole or an edge, while for the case of two concurrent cracks the limit distance is considered to be given by the sum of the plastic radiuses pertaining to the two defects.

Once such limit distance is reached, the ligament is considered as broken, in the sense that no larger cracks can be formed; however, to take into account the capability of the ligament to still carry some load, even in the plastic field, the same net section is still considered in the following steps, thus renouncing to take into account the plastic behavior of the material.

Therefore, the generic M-C trial is considered as ended when one of three conditions are verified, the first being the simplest, i.e. when a limit number of cycles given by the user is reached. The second possibility is that the mean longitudinal stress evaluated in the residual net section is equal to the yield stress of the material and the third, obviously, is met when all ligaments are broken.

Several topics are to be further specified and first of all the probabilistic capabilities of the code, which are not limited to the initiation step. The extent of the probabilistic analysis can be defined by the user, but in the general case, it refers to both loading and propagation parameters.

For the latter, user inputs the statistics of the parameters, considering a joint normal density which couples lnC and n, with a normal marginal distribution for the second parameter; at each propagation step the code extracts at each location new values to be used in the integration of the growth law; the observation that variations of the same parameters are to be expected only from 1.5÷2.0 mm upward, as it is mentioned in literature, can be taken into account in future versions of the code.

The variation of remote stress is performed in the same way, but it is of greater consequences; first of all we have to mention that a new value of remote stress is extracted at the beginning of each step from the statistical distribution that, for the time being, we considered as a normal one, and then kept constant during the whole step: therefore, variations which occur for shorter times go unaccounted.

The problem which is met when dealing with a variable load concerns the probability of crack initiation, more than the propagation phase; that's because the variation of stress implies the use of some damage acumulation algorithm, which we used in the linear form of Miner's law, being the most used one.

However, we have to observe that if the number of cycles to crack initiation is a random variable, as we considered above, the simple sum of deterministic ratios which appears in Miner's law cannot be accepted, as pointed out by Hashin (1980 and 1983), the same sum having a probabilistic meaning; therefore, the sum of two random variables, i.e. the damage cumulated and the one

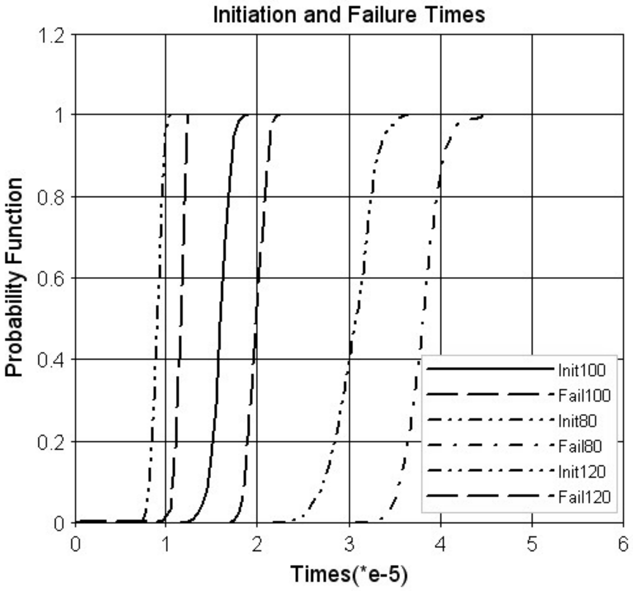


Figure 23 : Cdf's of time for crack initiation and structural failure for different remote stresses

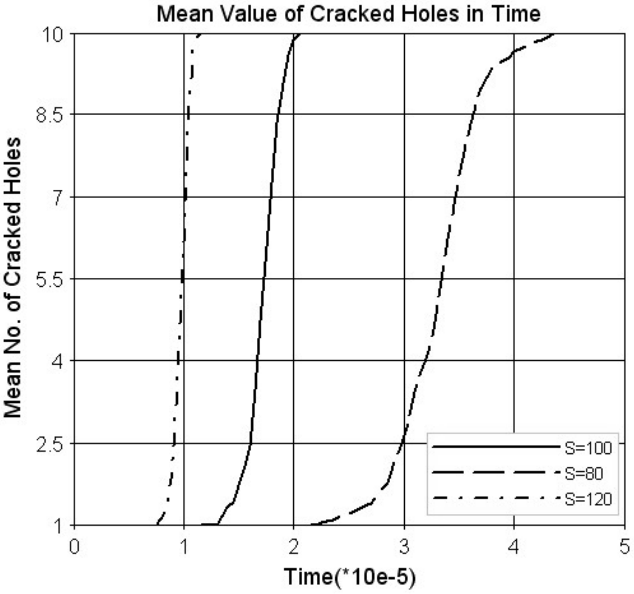


Figure 24 : Cdf's for a given number of cracked holes in time

corresponding to the next step, has to be carried out by performing the convolution of the two pdf's involved.

This task is carried out by the code, in the present version, by a rather crude technique, recording in a file bith the damage cumulated at each location and the new one and then performing the integration by trapezoidal rule, but it is evident that in future version more opportune solutions will be used, and first by means of DFT, as already adopted by Wu and Torng (1990).

At the end of all M-C trials, a final part of our code carries out the statistical analysis of results in such a way as to be dedicated to the kind of problem in hand and to give useful results; for example, we can obtain, as usually, the statistics of initiation and failure times, but also the cumulative density function (cdf) of particular scenarios, as that of cracks longer than a given size, or including an assigned number of holes, as it is illustrated in Fig. 23 and 24.

7 Maintenance problems

One of the main problems that our code wants to deal with refers to the consequences of particular maintenance policies and therefore it has been included in the most recent version of our code.

At the present stage, the user assigns the number and the scheduling of maintenance and at those times the code performs it, considering all locations and looking for a crack. The search is carried out with reference to the POD (probability of detection) which characterizes the particular technique the user wants to adopt, and it is simulated by the extraction of a random number which represent, according to the said POD, the length of the defect which in that particular location the inspection operation is able to detect; if the existing crack is larger than that limit value, it is considered as found and eliminated by the structure, i.e. every detected defect is considered as repaired.

As an example, in Fig. 25 a case study is shown, for a POD which can't detect cracks smaller than 1 mm and has a 99% probability to find 20 mm long defects. Three maintenance operations were assigned, after 200,000, 225,000 and 250,000 cycles and the results on a sample of 200 trials were compared with those obtained in the case of no maintenance.

Considering that "w.m." and "n.m." captions are used in the figure to distinguish between the "with maintenance" and "no maintenance" cases, it is quite obvious that inspection has no influence on initiation times, as it is scheduled at later times and that the small differences between the two curves are connected to the randomness of the samples.

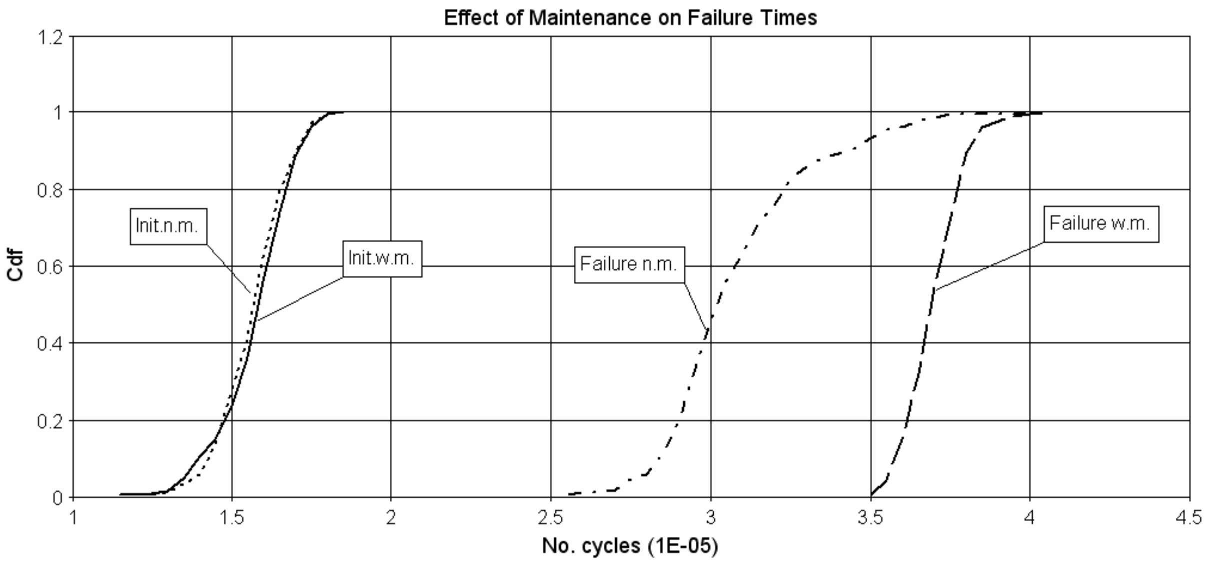


Figure 25 : The effect of maintenance on initiation and failure times

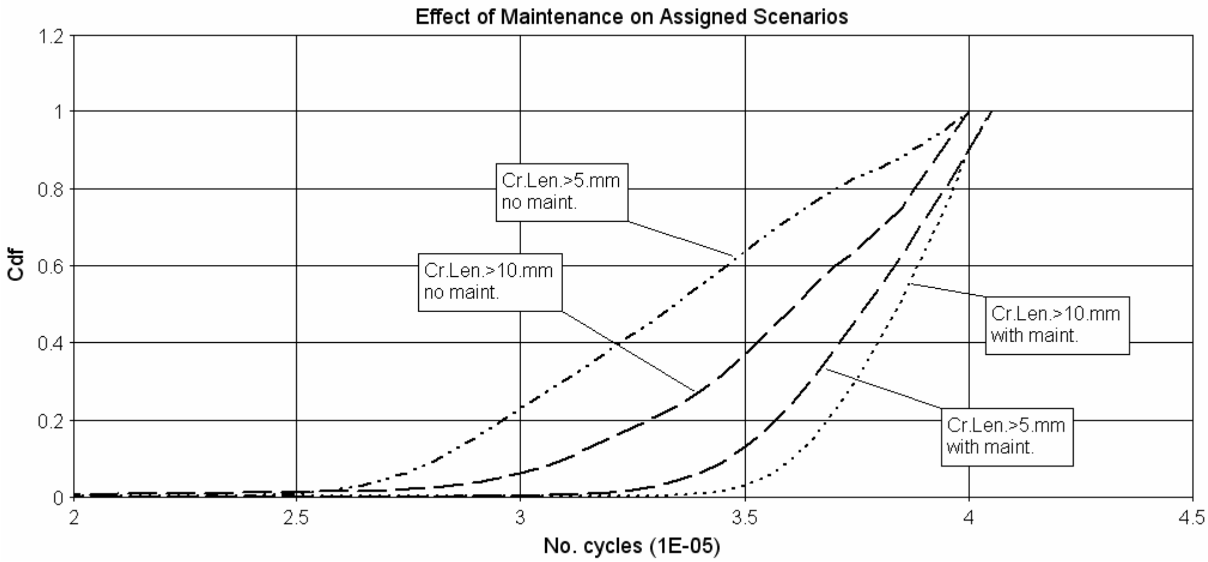


Figure 26 : Probability curves of particular damage scenarios as affected by maintenance

On the other hand, differences between the failure times are evident in the right side of the same figure; it must be pointed out that one single trial was found, for the no maintenance case, where failure time was only a little shorter than the highest results in presence of maintenance.

It is also evident that, even if maintenance was performed in early stages of damage propagation, it heavily influences the following stages of deterioration.

A particular result which can be obtained by such analysis is related to exceedance curves of particular scenarios. In Fig. 26 such results are illustrated, comparing the curves corresponding to the exceedance of 5.0 mm and 10.0 mm long cracks.

It can be observed that the rising of long cracks is much delayed by the maintenance operations; in our case study, the inspections were performed at early stages of damage propagation, and therefore their effect can be appreciated only considering such delay, while no decrease of defects can be graphically observed in the plot.

Therefore, the choice of a particular scheduling of inspections and maintenance operations appears to be a very hard and subtle one, and in any case it is strictly influenced by the NDT adopted; a too early inspection has a small probability to detect the damage, if only small cracks are expected, near to the lower boundary of detection of the adopted technique, which could induce to delay the first inspection, and the same reasoning applies to the choice of the following intervals between inspections.

On the other hand, a "late" inspection and long intervals give a greater probabilities of defect detection, but their choice has to be related to the propagation rate of cracks. It is therefore confirmed that maintenance scheduling can be assessed by means of an optimization procedure, which must take into account all previous considerations.

8 Conclusions

In the previous sections we referred about the analyses carried out by the authors in the development of reliability and risk assessment codes while participating to some of the most known among the research program led by EC. The particular development followed led us to write a completely probabilistic code, which gave good results in the applications examined; in any case, it appears that the application capabilities of the procedure can be still widened, for example with the introduction of different hypothesis about initial scenarios, or with routines devoted to deal with small cracks or, especially, with maintenance policies, what will require, in any case, some changes in various parts of the code.

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