 SDHM, vol.4, no.1, pp.19-27, 2008

Modeling of a Surface Contact Stress for Spur Gear Mechanism using Static and Transient Finite Element Method

F. R. M. Romlay[1]

Abstract: This paper presents a surface contact static stress of a spur gear system combined with dynamic characteristic using transient Finite Element Method (FEM). Traditionally, the static stress analysis is done separately with dynamic properties due to limitation of complex equation and avoiding of error occurred. However, in this paper, static stress information is combined with the dynamic mechanism due to the time consuming during the design and analysis stage. A transient FEM analysis is carried out to formulate and solve large systems of algebraic equations in order to obtain a relationship between the contact parameter and the kinematics function. The methodology of the research is started with static stress analysis on tooth surface contact of a pair of the spur gear. Finite element modeling is run by choosing a certain static condition. The loading conditions are applied suitable with the gear mechanism. Degree of freedom controlled is based on the transmission system. The process is repeated until diagnosing work is satisfied. The result of the surface contact stress is visualized at each condition. Modeling of spur gear system is continued by combining stress analysis with dynamic characteristic via transient finite element method. Analysis of gear mechanism is obtained by investigate the stress distribution on real time application. Time range is set at the beginning of the analysis. Duration of the analysis is depended on a time frame chosen. By the transient FEM analysis, the stress occur at each step of the work cycle is performed. Results of the kinematics functions are derived and qualitative kinematics variations due to contact changes in time-step domain is identified. The simulation results from static and transient FEM are compared due to the validating procedure. The finite element results are in good agreement compared with the theory calculation.

Keyword: Gear mechanism, transient, Finite Elements Method

1 Introduction

Now days, there are so many mechanisms those involve with load and requirement to understand the stress in component is increased. The mechanisms and the stress always come together and they have a strong relation between each other.

In real application, displacement and stress are dependent with the dynamic criteria. The loads that apply to the model that got motion consider as transient load. This will involve kinetic and kinematic criteria of the model.

With the current trend now, modeling and simulation using computer is growing very fast and the demand is increasing dramatically by a year. Mathematical model through FEM is most suitable application in engineering design with high capability in analysis option. The performance of the FEM solver is influenced by the computer performance too. With the mature of the computer technology expansion, benefit for the computational mechanics is increased.

Presently, the model is analyzed by finite element method to identify the stress and displacement. The same model need to be design again or is possible, is exported to the dynamic analyzer for a purposed of dynamic study. This needs a long time to finish a simulation work. The CAD design also can be damage during the file transfer process. This factor can affect the accuracy of the result.

[1] Faculty of Mechanical Engineering, Universiti Malaysia Pahang, 25000 Kuantan, Pahang, Malaysia. Email: fadhlur@ump.edu.my

Using of finite element result for the dynamic analysis input is very difficult because of the involvement of a complex mathematical equation and high possibility of error to be occurred. That is why both of the analysis needs to be combined in a single analyzer.

2 Gear system

Gear system is very common component in mechanical system and world wide usage in machine driving mechanism. One of the functions of the gear system is to increase or decrease a load transfer in machine component. More than that, it is also use to change the angular velocity of the mechanism, direction of the rotation, location of the load transfer and some more application mostly in machine mechanism. As a critical part for load transfer and transmission system, gear structure must be robust enough. The reliability of the load transfer system depends on the gear performance during the operation.

For gear system, modeling stage is very important to determine the increasing or decreasing of the load ratio. In design stage, size, type of gear and material properties play an important rule in giving the specific output for the gear system as a torque. Modeling will help the designer to fabricate a suitable characteristic for a specific application.

In gear system, a contact of a gear tooth is very complex problem to solve. The complexity is increase with the influence of manufacturing and assembly technique. The basic errors that usually happen are lead crowning and shaft misalignment. Over contact stress is happened by these errors. A partial mating between the teeth is caused by improper alignment [Bensely, Jayakumar, Lal, Nagarajan and Rajadurai (2006)]. It develops high stress between the teeth in contact, leading to larger load acting on a very small area during sliding. This resulted in teeth chipping all around the edges of the crown wheel.

Some time, tooth modification is needed in order to fix the tooth assembly. A precise theoretical method to be able to calculate surface contact stress and root bending stress of a pair of spur gears with considering manufacturing error, assembly error and tooth modification is provided.

With the demand of modeling using computational mechanics is increased, the researchers were developed a gear mechanism simulation using computer aided design (CAD) software [Brauer, (2000) and Lunin, (2001)]. How ever, in term of time consume, is not very efficient. It is because a lot of mesh involved in a 3D modeling.

In order to expand the fundamental of gear modeling, Brauer was provided a general finite element model of involute gear. The model present was complete with the mathematical description include the root surface. The capability of addendum modification is also provided for the flexibility of gear modeling [Brauer, (2004)].

Studying a contact of a pair of gear must be considering static stress. Static stress exists because of the load applied and by the same load, the part move simultaneously. That means the equation of the dynamic mechanism is dependent with the static stress result. Consider this factors, modeling the part by finite element method will make simulation become easier. The stress and displacement results will come together with the kinetic and kinematic information.

The critical part that needs a particular observation in modeling is at contact area or the surface. Some advance interpolation technique is needed to model that critical area. Than, the stress will be well distribute among the gear teeth and body of the gear. This will generate more accurate FEA results for further research at any interest point or area.

Prior to this section, this research focus on understanding of stress distribution in gear system. This paper will presents a modeling of surface contact stress between a pair of spur gear using finite element method. The results is validate with deterministic calculation.

3 Spur gear

Spur gear as shown in Fig.1 has a straight tooth and it is hold by a shaft as shown in Fig. 2. Of all types, the spur gear is the simplest and for this reason, spur gear model will be used to develop the primary kinematics relationship of the tooth form [Shigley and Mischke, (2003)].

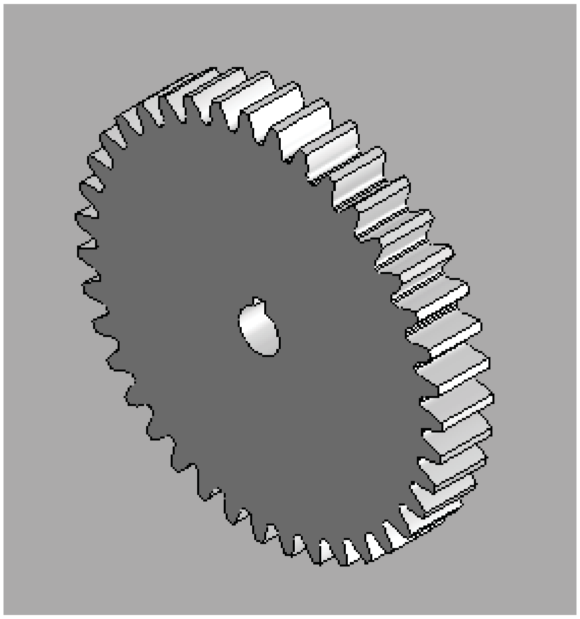

Figure 1: Spur gear with a shaft hole.

Combination more than one gear is called as gear system. Gear ratios express a mathematical relationship of one gear to another. Gear ratio depends on the geometry properties of the gear such as pitch diameter and gear tooth quantity. The force transmitted form one gear to another is calculated base on gear ratio given.

A gear ratio also expresses the amount of torque multiplication or reduction between gears. The ratio is obtained by dividing the diameter, d or number of teeth, N of the driven gear by the diameter, d or number of teeth, N of the drive gear as shown in Eq. 1. The rotation frequency of driving and driven gear is told by gear ratio too.

$$GearRatio = \frac{N_{Driven\ gear}}{N_{Drive\ gear}} = \frac{d_{Driven\ gear}}{d_{drive\ gear}} \tag{1}$$

The system that developed by more than two gears is categorized as planetary gear. Calculating the gear ratio of the planetary gear system need a different formula which is involve with parameter submission. The planetary gear ratio is given in Eq. 2 which is number of teeth, N is utilized and

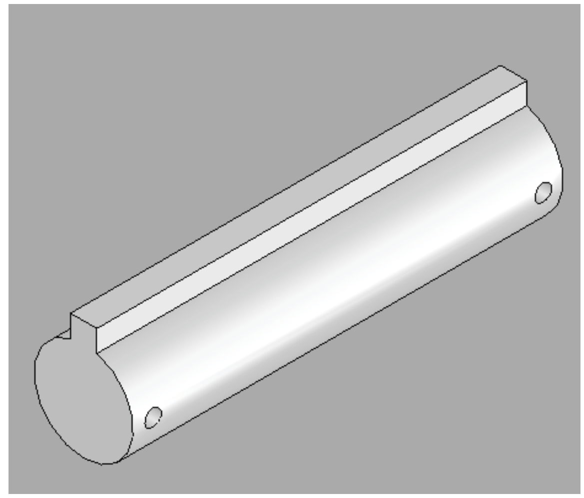

Figure 2: Shaft used for gear rotating and force transmitting.

Eq. 3 for diameter, d is used [Waldron (2003)].

$$GearRatio = \frac{N_{Drive\ gear} + N_{Driven\ gear}}{N_{Drive\ gear}} \tag{2}$$

$$GearRatio = \frac{d_{Drive\ gear} + d_{Driven\ gear}}{d_{Drive\ gear}} \tag{3}$$

4 Stress calculation

Stress distribution around the gear teeth contact is investigated by numerical and validated by deterministic method. The numerical is run by FEM. The investigation of static and transient conditions is considered as a major part of the stress analysis.

4.1 Surface contact static stress of the spur gear

Hertz formula is often used to calculate surface contact stress of gears when tooth load is applied. However, the formula is very complex to use if assembly error, machining error and tooth modification are considered. So this paper calculates the surface contact stress of gears with a "Unit Force" method. It means, calculation is generated by define the tooth load distributed on unit contact area of a tooth surface. When the tooth load distributions are obtained, the result is derived a stress and compare with FEM results [Shuting (2007)].

K is an arbitrary reference point on driving gear andK' is a responsive contact point on tooth surface of driven gear. K and K' are used as a pair of contact points in 2D view as shown in Fig.3. Distance between any two points touched is considered as contact line and contact area is defined when contact line is time to the gear thickness.

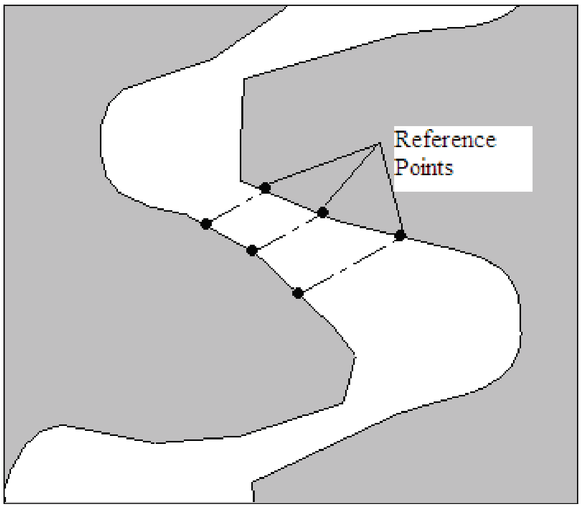

Figure 3: Contact point of spur gear system.

The force is transmitted through the contact line along the gear profile and directly become a surface contact force when the gear thickness is considered. The contact between the gears surfaces are considered as elastic contact bodies. In gear study, an external force, P is assumed to be equal to the sum of all the contact force, F_j, (j=1 to n). The relationship between the external force, P and contact force, F_j, are given by the Eq. 4 below:

$$P = \sum_{i=1}^{n} F_i \tag{4}$$

The contact surface area on specific time for spur gear is very small. By this condition, it is assumed that all the common normal lines of the contact point pairs are approximately parallel with the external load [Shuting, (2007)].

4.2 *Transient stress for gear mechanism*

Usually, stress analysis for the dynamic component is done by stand alone analyzer using finite element method. Using a traditional method, the stress of the dynamic mechanical part is estimated by separated time step using quasi-static stress analysis approach [Haiba, Barton, Brooks and Leveslay (2002)]. The equation is given by Eq. 5, 6 and 7.

$$\sigma_x(t) = \sum_{i=1}^{n} \sigma_{xi} F_i(t) \tag{5}$$

$$\sigma_y(t) = \sum_{i=1}^{n} \sigma_{yi} F_i(t) \tag{6}$$

$$\tau_{xy}(t) = \sum_{i=1}^{n} \tau_{xyi} F_i(t) \tag{7}$$

where n is the number of applied load histories and $\sigma_{xi}(t)$ $\sigma_{yi}(t)$ $\tau_{xyi}(t)$are the stress due to a unit load in a function of time. The stress is applied at a specific nodal and a same direction with the load history $F_i(t)$.

In this condition, the analysis is chosen just for a certain moment from the whole mechanism. In most of the time, engineer need to predict and choose the critical time of the failure that will be happen which is some is not accurate. That is why results for the whole process or mechanism are very important.

5 Modeling strategy

Prior to this section, this research focus on understanding of stress distribution in gear mechanism. The spur gear model was developed as illustrates in Fig. 4, Fig. 5 and Fig. 6.

5.1 *Computer aided design of gear system*

The modeling is started with a design in computer aided design (CAD) software. The design of the gear must properly construct in order to avoid a modeling error. The error occur at design stage will dramatically increase in analysis stage especially by the involvement of multiple function [Ariffin & Romlay, 2004)].

The gear system is developed to decrease the force. It is using nine type of gear which is variable in size. The gear ratios are decreasing the torque to 21.125 times and there have three levels of gears.

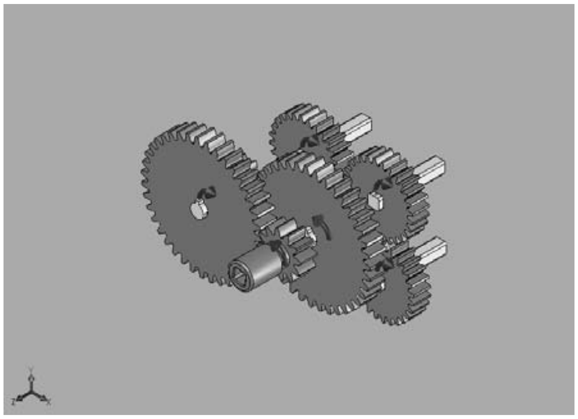

Figure 4: Isometric design view and direction of the motion.

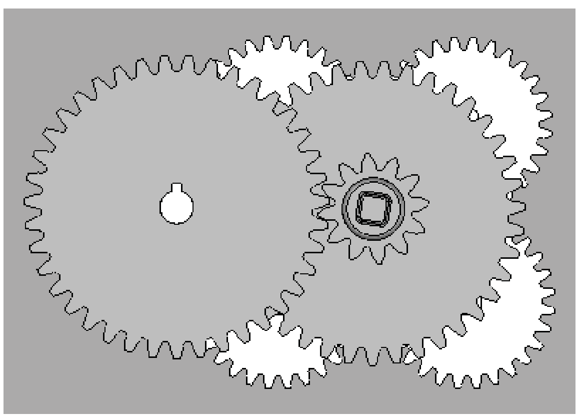

Figure 5: Front view of spur gear system.

For the first level, it has two type of gear; the spur gear with 12 teeth as a drive gear and another one is gear with 39 teeth as a driven gear. Ratio for the load transmission is 3.25. For the second level, it contains two types of gear which are the spur gear with 12 teeth and a gear with 39 teeth. The ratio reduction for this level is also 3.25. For the third level, it is attached by five types of gears. A single spur gear at the middle of the level and four gears connect this gear in rectangle. The spur gear is fabricated with 14 teeth and the rest four gears are fabricated with 28 teeth. Ratio for this gear system is two. Therefore, the overall ratio is 21.125.

An equation control is used for the purpose of geometry correction and updating. A complete design file is exported to computer aided engineering (CAE), as a first step for analyze the reliability of the model.

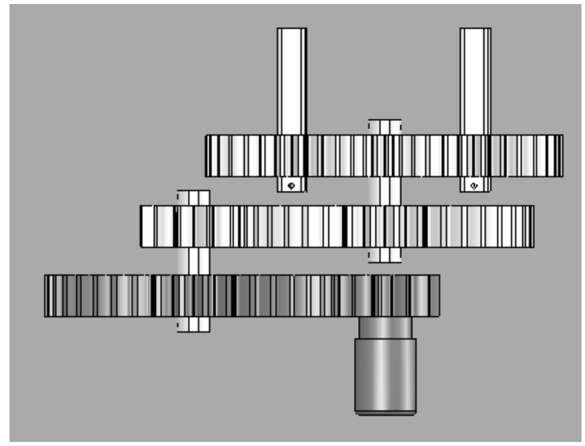

Figure 6: Top view of spur gear system which divide by 3 level.

5.2 Computer aided engineering by FEM

The gear assembly was defined by finite element method. The CAD model was meshed and defined using brick and triangular elements as shown in Fig. 7. The constraint was declared by a pin at the centre of the gear. A torque is applied at the pin to drive the gear mechanism and at the same time applied the load to the part.

Consider assembly of the main and pinion gear, the affect of the contact surface will result the stress around the gear part. To make it happen, surface contact need to be identify and it is driven by actuator element. It should be noted that, actuator elements are used to specify the relative motion of two points of a structure or mechanism.

Consider the dynamic affect direct to the model, the angular velocity is applied and the time-step needs to be defined. All these kinetic and kinematic parameters were contributed the mechanism of gear motion.

5.3 Transient modeling

Transient dynamic analysis is used to predict the motion behavior of the component. The dynamic

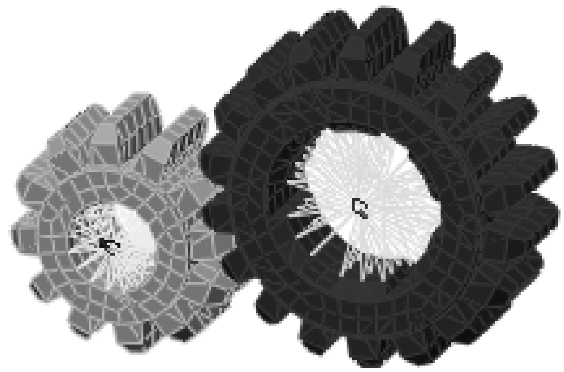

Figure 7: Modeling of main and pinion gear with respect to the torque at a pin for each gear.

modeling includes a path and trajectory. To be clear, the path only considers the coordinate and vector of the line, while the trajectory integrated a time domain of the path.

Modeling the mechanism of the part by transient analysis parallel with the finite element method will make simulation become easier. The stress and displacement results will come together with the kinetic and kinematics information. The model is verified by repeating the program in an iteration form which is defined by time-step function.

Static stress analysis exist because of the load applied and by the same load, the part got a motion. That means the equation of the dynamic mechanism is dependent with the static stress result. Consider this factor, the error occur from static stress analysis will dramatically increase the error of the dynamic modeling especially by the involvement of multiple function. That is why the performance of error control is very important in combining static stress analysis and transient dynamic modeling.

6 Deterministic and FEA results of surface contact static stress of the spur gear system

This calculation is to determine surface contact stress between a pair of spur gear. In this analysis, the result from the deterministic method is compared with the finite element analysis (FEA).

6.1 Level 3 of the gear system analysis

Denoting by F, the magnitude of the tangential force between gear teeth, F = 3.71 kN. The area of the gear contact surface is 8.27×10^{-5} m^2. The stress is determined as 44.99 MNm^{-2}. Force at gear with 28 teeth is calculated as 220.47 kN while force at gear with 14 teeth is equal to 93.33 kN.

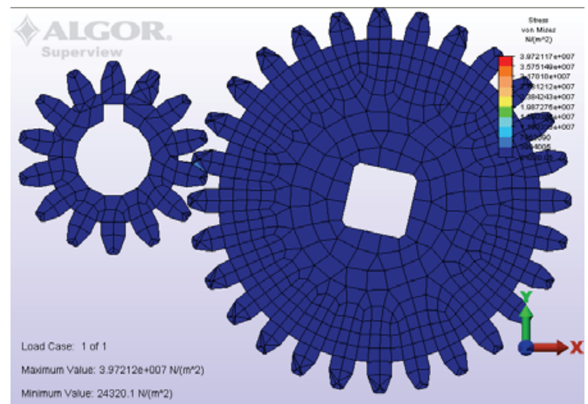

Figure 8: Stress analysis of level 3.

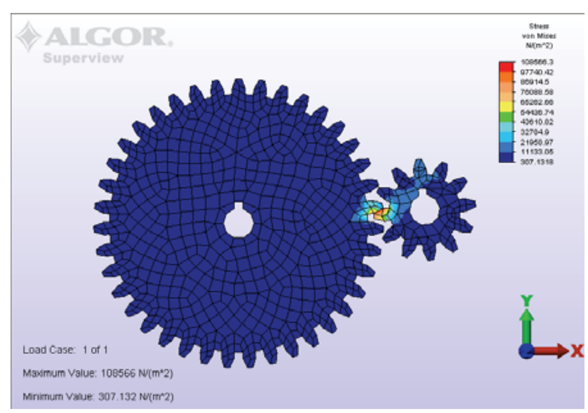

Figure 9: Stress analysis of level 2.

Fig. 8 shows the stress analysis of spur gear (Level 3) which is the maximum stress is 44.99 MNm^{-2}. The value stress from calculation is 39.72 MNm^{-2} as shown in Table 1.

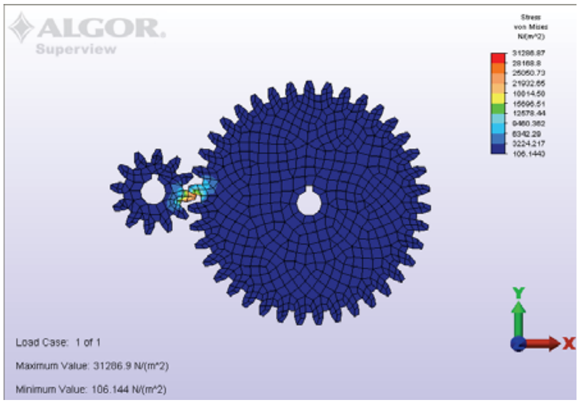

Figure 10: Stress analysis of level 1.

6.2 *Level 2 of the gear system analysis*

Denoting by F, the magnitude of the tangential force between gear teeth, F = 4.24 kN and the surface contact stress equal to 131 MNm^{-2}. Force at gear with 39 teeth is determined as 373.33 KN while force at gear 12 tooth is equal to 114.86 KN.

Fig. 9 shows the stress analysis of spur gear (Level 2) which is the maximum stress is 131 kNm^{-2}. The value stress from calculation is 109 kNm^{-2} as shown in Tab. 1.

6.3 *Level 1 of the gear system analysis*

Finally, for level one, the magnitude of the tangential force between gear teeth, F = 1.30 kN and the surface contact stress, σ = 398.47 kNm^{-2}. Force at gear 39 teeth is calculated as 114.86 kN and force at gear 12 teeth, is determined as 35.34 kN.

Fig. 10 shows the stress analysis of spur gear (level 1) which is the maximum stress is 398 kNm^{-2}. The value stress from calculation is 312 kNm^{-2}as shown in Table 1. The results comparison of maximum stress between deterministic and FEM show that the minimum error is 11.49% and the maximum error is 21.68%.

The error occur because of the area calculation is not very precise. The touching area cannot specify accurately base on the mesh shape is not symmetry.

The result of the maximum stress is happened at the contact surface of a pair of spur gear. The maximum force is transmitted by the maximum stress along the area contact. The calculation of the contact area needs to specify very well. Overload is directly effected the contact surface between the gear and probability of failure is among this region.

Table 1: Comparison results of the deterministic and FE maximum stress at the contact surface of the spur gear.

Gear	$\sigma_{Deterministic}$ (kNm^{-2})	σ_{FEM} (kNm^{-2})	**Error** %
Level 3	44.99×10^3	39.72×10^3	11.49
Level 2	131	109	17.06
Level 1	398	312	21.68

However, the failure time is not provides by static FEA. There is no hint or information available to predict the failure of the components. So, transient FEA is done in order to give some sort of the failure time for the components.

7 Simulation of transient finite element

The transient FEM is started by simulate the mechanism of the gear system. The time step is set up to 100 steps which is 0.0002 seconds per step.

Fig. 11 illustrated the result of the surface contact stress distribution for the forth time-step. The maximum Von Mises stress is 1.61 GPa. The maximum Von Mises stress is decrease to 1.17 GPa at eighth time-step as shown in Fig. 12. The maximum Von Mises stress value continues decreases to 975.91 MPa at twelfth time-step as shown in Fig. 13. However the value increases to 7.12 GPa at sixteenth time-step as shown in Fig. 14. At this moment, there is a high stress concentration at certain surface which is a possibility of failure tip start to be growth.

Fig. 15 shown the result of transient finite element method validated with deterministic approach by quasi-static equation. The quasi-static equation is applied by consider$n$ is the number of applied load histories and $\sigma_{xi}(t)$ $\sigma_{yi}(t)$ $\tau_{xyi}(t)$are the stress due to a unit load in a function of time. The stress

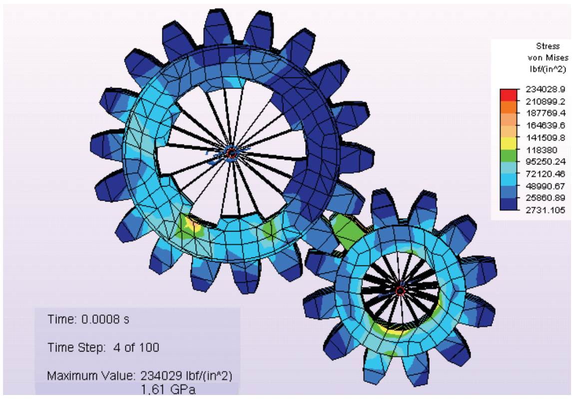

Figure 11: Stress distribution for fourth time-step.

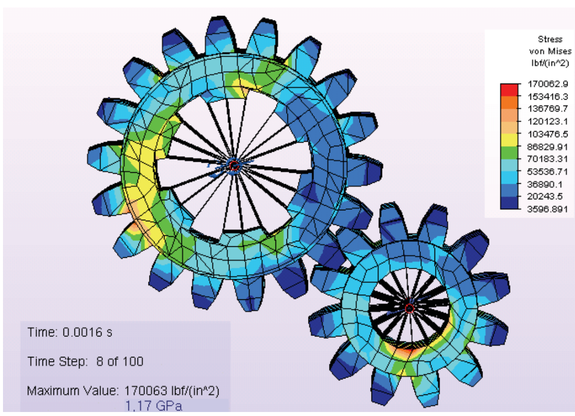

Figure 12: Stress distribution for eighth time-step.

is applied at a specific area and a same direction with the load history $F_i(t)$.

Both of the curves are not in uniform shape and the maximum Von Mises stress is variable along the time step of the event. This is happen due to the variable mate surface along the gear rotation period. Different location of the contact will provide difference surface stress. By investigate more detail of the gear tooth, it is understand that the gear tooth profile with certain pressure angle is the main influence factor that support the result in Fig. 15.

To sum up, it is clear that the geometry properties of the gear tooth are the main criteria that influence the surface contact stress in gear mechanism system.

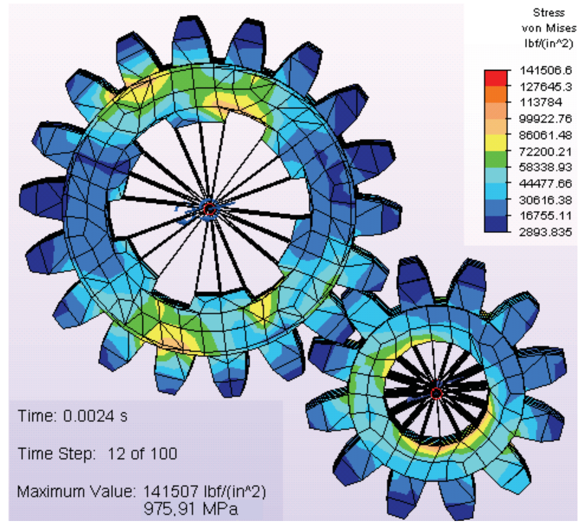

Figure 13: Stress distribution for twelfth time-step.

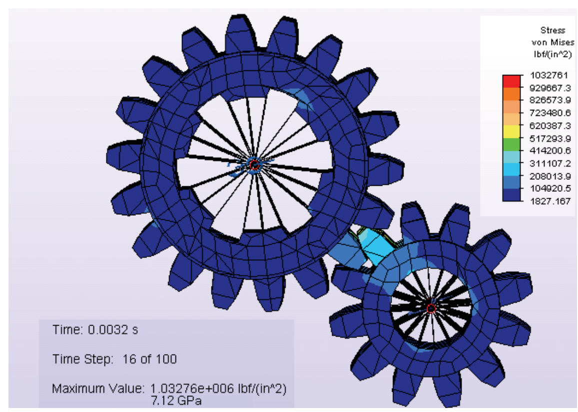

Figure 14: Stress distribution for sixteenth time-step.

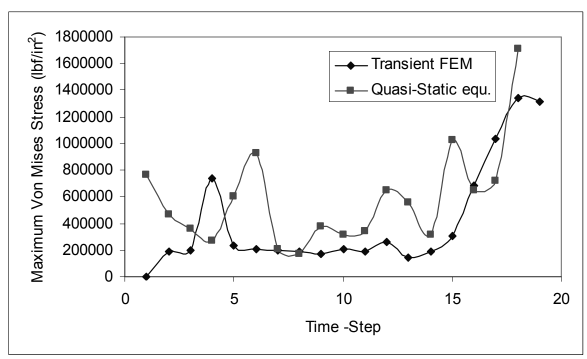

Figure 15: Graph of maximum Von Mises Stress versus time-step.

8 Conclusion

The result of the analysis has shown that the factor of dynamic properties will affect the stress distribution of the component by showing the changing of the stress value in time domain function.

By using transient FEM, not only the stage of the analysis is shorts, it also provides both static and dynamic analysis results simultaneously. This will creates an efficient design and modeling work stage compare to static analysis alone by FEM. This method is useful and practical to be applied for a product development and fabrication of a new product.

To conclude this research work, it is simple to say that, the displacement and stress are dependent with the dynamic criteria. Mechanism which is in function of time will influence the displacement and directly the stress of the components differently along the operation time of any system.

References

Ariffin, A.K.; Romlay, F.R.M. (2004): Deterministic and Probabilistic Approach in Modelling of Fatigue Crack Propagation, *International Conference on Computational Methods*, Singapore.

Bensely, A.; Jayakumar, S.S.; Lal D.M.; Nagarajan, G.; Rajadurai A. (2006): Failure investigation of crown wheel and pinion,*Engineering Failure Analysis,* vol. 13, pp. 1285-1292.

Brauer, J. (2004): A general finite element model of involute gears, *Finite Elements in Analysis and Design,* 40, pp.1857-1872.

Brauer, J. (2000): Towards a dynamic model of a conical involute gear transmission, Licentiate Thesis, KTH, Stockholm, Sweden.

Haiba, M.; Barton, D.C.; Brooks, P.C.; Leveslay, M.C. (2002): Review of life assessment techniques applied to dynamically load automotive components, *Computer and Structures,* vol. 80, pp. 481-494.

Lunin, S. (2001): New method of gear geometry calculation, *JSME International Conference on Motion and Power Transmissions,* Fukuoka, Japan, pp. 472-477.

Shigley, J.E.; Mischke, C.R. (2003): Mechanical engineering design, International Edition, Sixth Metric Edition, pp. 825-856.

Shuting L. (2007): Finite element analyses for contact strength and bending strength of a pair of spur gears with machining errors, assembly errors and tooth modifications, *Mechanism and Machine Theory* vol. 42, pp. 88–114.

Waldron; Kanneth, J.; Kinzel, G.L. (2003): Kinematics, Dynamics, and Design of Machinery, John Wiley & Sons.