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Sample Size Determination for Development of S-N Curve of A356.2-T6 Aluminum Alloy

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Abstract: This paper presents the details of method of sample size determination to estimate the characteristic fatigue life of aluminum alloy. The characteristic fatigue life of Aluminum alloy A356.2-T6 has been estimated by assuming a two parameter Weibull distribution model. A stepwise procedure is outlined to determine the number of specimens required at a predetermined stress amplitude to estimate the fatigue life within an acceptable error at a given probability and confidence level. The percentage of error is calculated at various probabilities and confidence levels (C.L). The probabilities considered are 50%, 90% and 95% whereas C.Ls considered are 90%, 95% and 97.5%. Maximum percentage of errors has also been calculated for the above probabilities and C.Ls. Details of development of S-N curve for aluminum alloy A356.2-T6 have been explained. Weibull slope have been plotted to represent the variation in the sample data at a particular stress level. From this study, it is concluded that the estimated fatigue life is reliable and the sample size considered for fatigue life estimation is adequate.

Keywords: Aluminum alloy; fatigue; Weibull distribution; fatigue life; S-N curve

1 Introduction

In recent years aluminum alloy materials have developed rapidly for structural applications. Applications include automobile industry particularly for the manufacturing of wheels. The wheels are one of the most critical components of automobiles, which must perform their intended function to human safety. Most of the structural components/structures made up of these aluminum alloys are subjected to fatigue loading during their service conditions.

The loading may be of either constant amplitude loading or variable amplitude loading. In the present scenario, the interest to researchers/designers is to understand the fatigue/fracture behaviour of these alloys under fatigue loading. Statistical evaluations are important because of different distributions of the test results in aluminum samples. For safe and reliable applications of the materials in industry, their fatigue data as statistically must be known well. The statistical properties used, in general related to distribution in mean strength. Weibull distribution is widely used statistical model than other distributions in fatigue data evaluations from the point of variables in endurance life and strength parameters. Applications of Weibull distribution includes aero- space, electronics, materials and automotive industries. Recent advances in Weibull theory have also created numerous specialized Weibull applications. Modern computing technology has made many of these techniques accessible across the engineering spectrum. Luko (1999) reviewed generally used Weibull distribution models including discussion and illustrations. Monto Carlo method was also discussed in brief.

It is generally known that life testing of components during the period of useful life is generally based on the exponential model. The failure rate of a component may not be constant throughout the period under investigation. In some instances, the period of initial failure may be so long that the component's main use is during this period.

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In other instances the main purpose of life testing may be that of determining the time to wear out failure rather than chance failure. In such cases, the Weibull distribution adequately describes the failure times of components when their failure rate either increases or decreases with time. Further, the exponential distribution, because of its constant hazard function, is not very useful for modeling life data in components that wear out with age (i.e. mechanical components).

It is generally known that fatigue life of any structure/structural component is the number of stress or strain cycles to cause failure in anticipated stress amplitude. Fatigue life is a function of many variables, including environmental and metallurgical conditions of the material, and exhibits scattered results, even where the specimens are taken from the same lot and the applied stress or strain cycles are equal. Due to this scattered nature, it is difficult to evaluate the fatigue life of the component or to establish the prediction interval from a limited number of sample data.

Ramamurtyraju et al. (2007) generated S-N curve for aluminum alloy A356.2-T6 and estimated fatigue life under radial fatigue load. Safety factor has been suggested for reliable fatigue life estimation by conducting a parametric finite element studies. Many researchers have developed graphical and analytical methods to evaluate the fatigue life or strength and S-N curve from a limited amount of data (Gope 1994, Parida et al. 1990, ASTM STP 9 1963, Gao 1984, Nakazawa 1987). Most of the analytical methods are based on either normal or log normal distribution. As the fatigue testing is time consuming and costly, setting of the minimum sample size required to extract the statistical information is of great importance (Parida et al. 1990, ASTM STP 9 1963, Gao 1984, Nakazawa 1987, Wilks 1942, Lawless 1973, Gope 1999,2002). Gope (1999, 2002) presented a method of sample size determination to estimate the fatigue life, confidence level and maximum acceptable error. Log normal or Weibull distributions were used for statistical analysis. Castillo et al. (2006) deals with the problem of estimating the S-N field based on samples with different lengths and testing the hypothesis of length independence of fatigue lifetimes. A weibull model developed is used to discuss the problem and analyze two data samples of prestressing wires and prestressing strands. Zhao et al. (1998) carried out a statistical investigation of 23 groups of fatigue life data on Q235 steel – welded joints in terms of linear regression analysis. By comparing the effects of fits to the six assumed distributions (Three parameter weibull, Two parameter weibull, Extreme minimum value, Extreme maximum value, Normal and Lognormal distributions). Gao et al. (1999) describe applications of the recent developments in modeling cleavage fracture to predict the behavior for various crack configurations of an A515- 70 pressure vessel steel, including surface crack specimens loaded by different combinations of tension and bending. They briefly reviewed the conventional, two-parameter weibull stress model and the modified, three-parameter weibull stress model, and outlined the strategy to calibrate weibull stress parameters. Belmonte et al. (2008) proposed a Weibull based methodology for assessing the condition of pipes based on strength characteristics obtained from small samples. Khandaker et al. (2008) applied a modified Weibull failure theory to biomaterial specimen under thermal loading.

From the wide literature, it has been observed that the research on sample size determination to estimate the characteristic fatigue life of aluminum alloy A356.2-T6 is limited. The present work focuses on sample size determination to estimate the characteristic fatigue life of aluminum alloy A356.2-T6 at the desired probability of survival confidence level. Details of development of S-N curve for aluminum alloy A356.2-T6 have been explained.

2 Test set-up and experimentation

To find the fatigue properties of aluminum alloy A356.2-T6 for actual manufacturing conditions, a test is carried out on specimens taken from the wheels. For all 43 identical specimens which are machined from the spokes of alloy wheels (David Gerkin 1999), rotary bending fatigue test is conducted according to Standards (IS: 5075 1985). The schematic diagram of specimen geometry and dimensions are shown in Figure 1. Test set-up is shown in Figure 2. The wheels, from which specimens are machined, are manufactured at low pressure die casting followed by T6 heat treatment process.

The results obtained from the rotary bending fatigue test are given in Table 1. Results include applied stress, fatigue life and observation on each sample. The scattered points obtained from the rotary bending fatigue test are shown in Figure 3.

One of the premier aluminum alloys for wheels in use today is AlSi7Mg. The chemical composition of AlSi7Mg alloy is shown in Table 2.

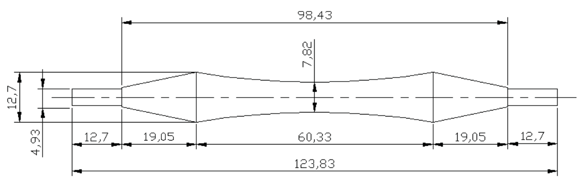

Figure 1: Rotating bending fatigue test specimen (All dimensions are in mm)

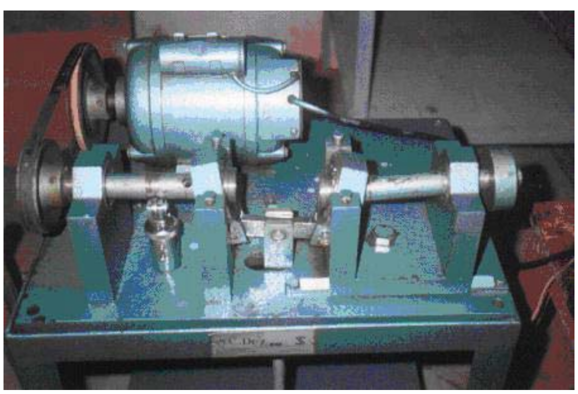

Figure 2: Typical test set-up

The following are monotonic material data for the specimens taken from finished wheels.

Ultimate Tensile Strength (S_u): 250 MPa

Yield Strength (S_y) : 230 MPa

Elongation (e) : 5%

Hardness (HB) : 90

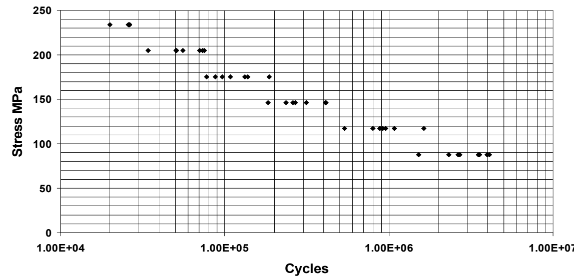

Figure 3: Scattered points at different stress levels

The Manufacturing of aluminum passenger car wheels involves low pressure die casting process. The molten aluminum kept in a gas tight heat insulated container flows under a mild pressure of approximately 70-100 KPa via a standpipe to escape through vent-holes and enters the die without turbulence. After solidification of the material in the die, the container is depressurized and the molten contents of the standpipe flow back into the container. The wheel is then machined.

3 Determination of minimum sample size

A stepwise procedure is outlined to determine the number of specimens required at a predetermined stress amplitude to estimate the fatigue life within an acceptable error at a given probability and confidence level (Gope 1999). Figure 4 shows the sequential steps to be followed for determination of minimum sample size.

i) Mean, $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n-1}$

ii) Standard deviation, $S = \sqrt{\frac{\sum_{i=1}^{n} (x_i-\overline{x})^2}{n-1}}$

iii) Characteristic fatigue life, $\overline{\theta} \approx$ also known as scale factor

iv) Weibull slope, $\overline{\beta} \approx$ shape parameter

v) $\overline{\xi} = ln(\overline{\theta})$

vi) $\overline{\delta} = \frac{1}{\beta}$

vii) $d = \left\{ \frac{-a_{12}.u^2 - n.\lambda + u.\in}{n - a_{22}.u^2} \right\}$

Table 1: Rotating bending fatigue tests results

Sample No.	Stress, MPa	Fatigue life, Cycles N	Remarks	Sample No.	Stress, MPa	Fatigue life, Cycles N	Remarks
1	88	3960264	Specimen failed	23	146	413472	Specimen failed
2	88	1518389	Specimen failed	24	146	77300	Specimen failed
3	88	2704850	Specimen failed	25	176	96453	Specimen failed
4	88	2317776	Specimen failed	26	176	185900	Specimen failed
5	88	3556811	Specimen failed	27	176	108392	Specimen failed
6	88	2627425	Specimen failed	28	176	87413	Specimen failed
7	88	3493790	Specimen failed	29	176	132868	Specimen failed
8	88	4083373	Specimen failed	30	176	137530	Specimen failed
9	117	537516	Specimen failed	31	176	34033	Specimen failed
10	117	953019	Specimen failed	32	205	50967	Specimen failed
11	117	797731	Specimen failed	33	205	70386	Specimen failed
12	117	878631	Specimen failed	34	205	74845	Specimen failed
13	117	1636424	Specimen failed	35	205	72999	Specimen failed
14	117	877734	Specimen failed	36	205	55365	Specimen failed
15	117	917730	Specimen failed	37	205	50326	Specimen failed
16	117	1081674	Specimen failed	38	205	25974	Specimen failed
17	146	236452	Specimen failed	39	234	19932	Specimen failed
18	146	409950	Specimen failed	40	234	26212	Specimen failed
19	146	269709	Specimen failed	41	234	25974	Specimen failed
20	146	183567	Specimen failed	42	234	26212	Specimen failed
21	146	259141	Specimen failed	43	234	26402	Specimen failed
22	146	312707	Specimen failed				

Table 2: Chemical composition of AlSi7Mg (in %)

Si	Fe	Cu	Mn	Mg	Zn	Ti
6.5-7.5	0.15	0.03	0.10	0.3-0.45	0.07	0.10-0.18

viii) $\in = [(a_{12}^2 - a_{11}a_{12}).u^2 + n.a_{11} + 2.n.a_{12}.\lambda + n.a_{22}.\lambda^2]^{1/2}$, where, u is the normal deviate corresponding to α probability of failure and a_{ij} are the co-efficient for the asymptotic variances and covariance of $\frac{\overline{\delta}}{\delta}$ and $\frac{\overline{\xi}}{\xi}$.

The value of a_{11}=1.10876; a_{22}=0.6079 and a_{12}=-0.25702.

Initially, along with other distributional parameters (such as mean and standard deviation), Characteristic fatigue life $(\overline{\theta})$ and Weibull Slope $(\overline{\beta})$ is estimated through probability plot. Parameters ξ and d are calculated to compute the coefficient of variation of Weibull distribution. Along with probability and confidence level (C.L), an acceptable error level is chosen and the corresponding error factor is selected from tables (Gope 1999).

The Percentage of error is then calculated by multiplying the error factor with coefficient of variation. The above procedure is repeated until the computed percentage of error is less than the acceptable error level. Once that stage is reached, the experimentation for that stress level can be stopped and the fatigue life is calculated based on the data obtained by the present sample size.

4 Method applied to present study

Initially, a minimum of three tests were conducted and calculated the error of estimation using a stepwise statistical procedure. Likewise, tests were further conducted at each of the stress levels until the error of estimation converges to an acceptable level.

If fatigue life (N_f) follows Weibull distribution with cumulative distribution function (CDF) then,

$$F_{Nf}(N_f) = 1 - exp\left\{\frac{N_f}{\overline{\theta}}\right\}^{\beta} \tag{1}$$

$X = \ln(N_f)$ will then follow extreme value distribution with CDF as

$$F_x[\ln(N_f)] = 1 - exp\left\{\left(\frac{x-\xi}{\delta}\right)\right\} \tag{2}$$

The safe fatigue life at α percent of probability and γ percent of confidence level can be determined from the relation as

$$\mu_x = \overline{x} + (d+\lambda)\overline{\delta} \tag{3}$$

where $\overline{x} = \overline{\xi} + \lambda.\overline{\delta}$ and $\lambda = \ln[1 - \ln(1-\alpha)]$.

The coefficient of variation is given by,

$$\phi_\omega = \frac{\overline{\delta}}{\left[\mu_x + (d+\lambda).\overline{\delta}\right]} \tag{4}$$

The percent of error involved in the fatigue life experiment can be defined as the error made when the sample life is accepted to be the population life.

Let the expected population life be μ_x and expected sample life be E (μ_x), then the percentage of error (R_w) can be obtained as,

$$R_w = \frac{E(\mu_x) - \mu_x}{\mu_x} \times 100 \tag{5}$$

Now the error factor $K_{w,\alpha,\gamma}$ for Weibull distribution, at α percent of probability and γ percent of confidence level is defined to be,

$$K_{w,\alpha,\gamma} = \frac{R_w}{\phi_w} \tag{6}$$

The distributional parameters for the life data at various stress levels has been calculated and are shown in Tables 3 and 4. One method of calculating the parameters of the Weibull distribution is by using probability plotting and the method described in the following paragraph.

Identical specimens are reliability-tested under identical test conditions at each of the stress levels. It can be observed from Table 3 that, all the specimens are failed during the test after certain number of cycles. The procedure for determining the parameters of the Weibull probability distribution function (pdf) representing the data using probability plotting are outlined below.

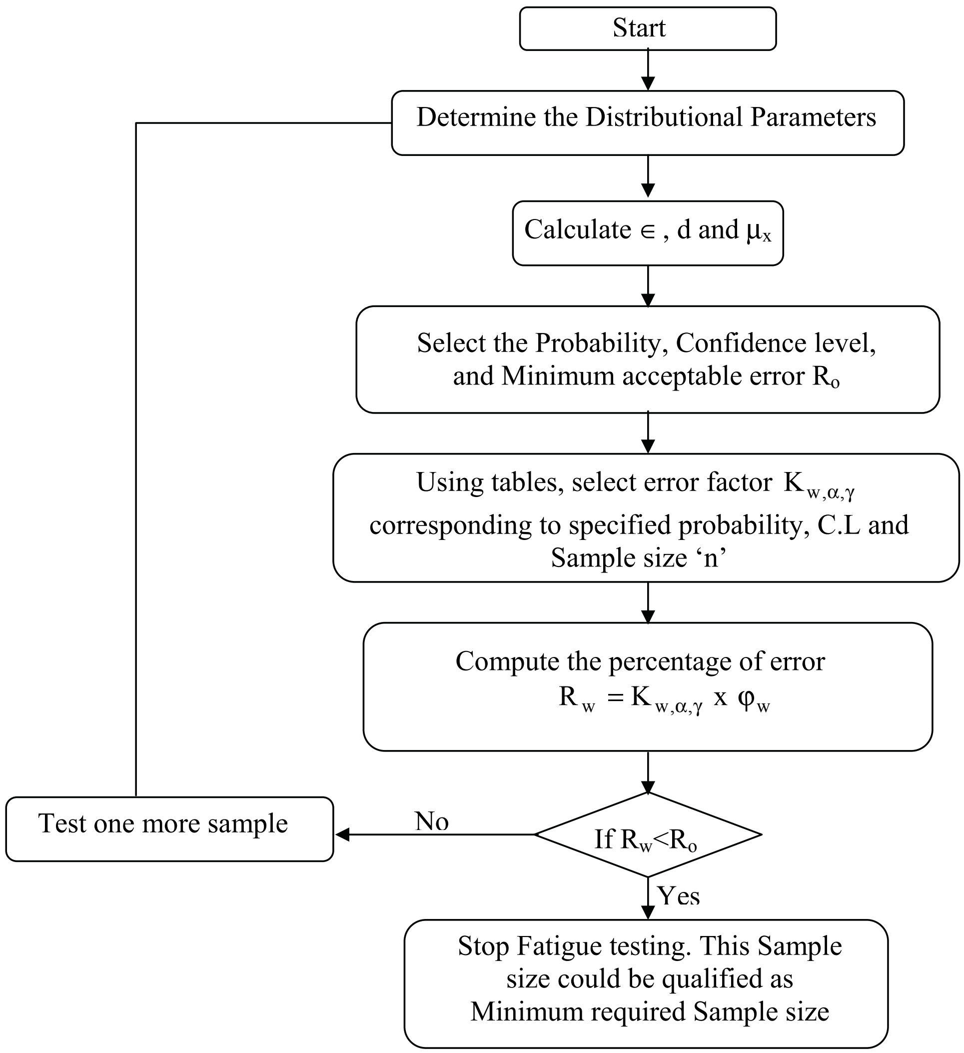

Figure 4: Flow chart for determination of minimum sample size

Table 3: Distributional parameters for the life data at various stress levels

Parameter	88 Mpa	117 Mpa	146 Mpa	176 Mpa	205 Mpa	234 Mpa
$\bar{x}$	14.88	13.73	12.39	11.51	10.89	10.1
S	0.33	0.31	0.53	0.54	0.36	0.12
$\bar{\theta}$	3342140	1067077	303097	125450	63055	25891
$\bar{\beta}$	4.36	3.32	2.86	2.79	4.4	18.24
$\bar{\xi}$	15	13.88	12.62	11.74	11.05	10.16
$\bar{\delta}$	0.23	0.3	0.35	0.36	0.23	0.05

Table 4: Other parameters at various probabilities

Stress level (s)	Sample size (n)	$\%\alpha$	u	λ	$\in$	d	μ_x
88 Mpa	8	50	0	-0.36	3.29	0.36	14.88
		90	1.28	0.83	3.05	-0.32	14.99
		95	1.64	1.1	3.33	-0.41	15.03
117 Mpa	8	50	0	-0.36	3.29	0.36	13.73
		90	1.28	0.83	3.05	-0.32	13.88
		95	1.64	1.1	3.33	-0.41	13.94
146 Mpa	8	50	0	-0.36	3.29	0.36	12.39
		90	1.28	0.83	3.05	-0.32	12.57
		95	1.64	1.1	3.33	-0.41	12.63
176 Mpa	7	50	0	-0.36	3.08	0.36	11.51
		90	1.28	0.83	2.87	-0.28	11.71
		95	1.64	1.1	3.14	-0.34	11.78
205 Mpa	7	50	0	-0.36	3.08	0.36	10.89
		90	1.28	0.83	2.87	-0.28	11.02
		95	1.64	1.1	3.14	-0.34	11.06
234 Mpa	5	50	0	-0.36	2.6	0.36	10.1
		90	1.28	0.83	2.45	-0.15	10.13
		95	1.64	1.1	2.7	-0.11	10.15

Table 5: Median ranks for the sample data

Cycles-to-failure	Failure Order Number out of a Sample Size of 8	Median Ranks
3960264	1	8.33
1518389	2	20.2
2704850	3	32.1
2317776	4	44.0
3556811	5	55.9
2627425	6	67.8
3493790	7	79.7
4083373	8	91.66

First, rank the cycles-to-failure in ascending order as shown in Table 5. Obtain their median rank plotting positions. Median ranks can be found either from the standard text books or by using the following equation (Luko 1999).

$$MR\% \approx \frac{i-0.3}{n+0.4} * 100, \tag{7}$$

where i is the failure order number and N is the total sample size.

The times-to-failure, with their corresponding median ranks, are shown in Table 5.

On a Weibull probability paper, plot the times and their corresponding ranks. Draw the best possible straight line through these points, as shown below, then obtain the slope of this line by drawing a line, parallel to the one just obtained, through the slope indicator (refer Figure 5). This value is known as shape parameter $\overline{\beta}$, in this case $\overline{\beta}$ is obtained as 1.4.

At the $Q(t) = 63.2\%$ ordinate point, draw a straight horizontal line until this line intersects the fitted straight line. Draw a vertical line through this intersection until it crosses the abscissa. The value at the intersection of the abscissa is the $\overline{\eta}$. This is always at 63.2% since

$$Q(T) = 1 - e^{-1} = 0.632 = 63.2\%$$

From Normal distribution tables, u value corresponding to 50, 90 and 95 percent probability would be 0, 1.28 and 1.64 respectively. Similarly, λ value can be given as -0.36, 0.83 and 1.1 at 50, 90 and 95 percent of probability respectively. Now the values of error factor $K_{\omega,\alpha,\gamma}$ corresponding to α percent of probability and γ percent of confidence level are taken from tables (Gope 1999). The percentage of error is then calculated by multiplying coefficient of variation with the error factor, and the results are tabulated in Table 6. From Table 6, it can generally be observed that the percentage of error is increasing with increase of percent of probability and C.L for a particular stress level.

5 Development of S-N curve

From the study, it has been observed that a minimum Weibull slope of 2.79 at 176 MPa and a maximum Weibull slope of 18.24 at 234 MPa. This can easily be attributed to the variation in the corresponding sample data. Consequently, more number of experiments were conducted at the stress levels, which showed relatively smaller Weibull slope. This is done in order to keep the error factor within the acceptable limit. The fatigue life is estimated from these minimum samples, represents the population life at 50 percent probability and 90 percent confidence level with a maximum of 2.4 percent error. Similarly, the maximum error percentages in estimating the fatigue life at typical probabilities and confidence levels are shown in Table 6. The S-N curve obtained from the curve fitting of the experimental data for the A356.2-T6 is presented in Figure 6.

6 Summary and concluding remarks

The details of method of sample size determination to estimate the characteristic fatigue life at the desired probability of survival confidence level and maximum acceptable error have been presented. The characteristic fatigue life of alloy A356.2-T6 has been estimated by assuming a two parameter weibull distribution model. Fatigue life for all the samples has been obtained by conducting rotary bending fatigue test. Detailed procedure for Weibull distribution including estimation of error has been given. Weibull slopes have been plotted to represent the variation in the sample data at a particular stress level. Initially, a minimum of three tests were conducted and calculated the error of estimation using a stepwise statistical procedure. Likewise tests were further conducted at each of the stress levels until the error of estimation converges to an acceptable level. The percentage of error is increasing with increase of percent of probability and confidence level for a particular stress level. Details of development of S-N curve for aluminum alloy A356.2-T6 have been explained. From the study, it is concluded that the estimated fatigue life is reliable and the sample size considered is adequate.

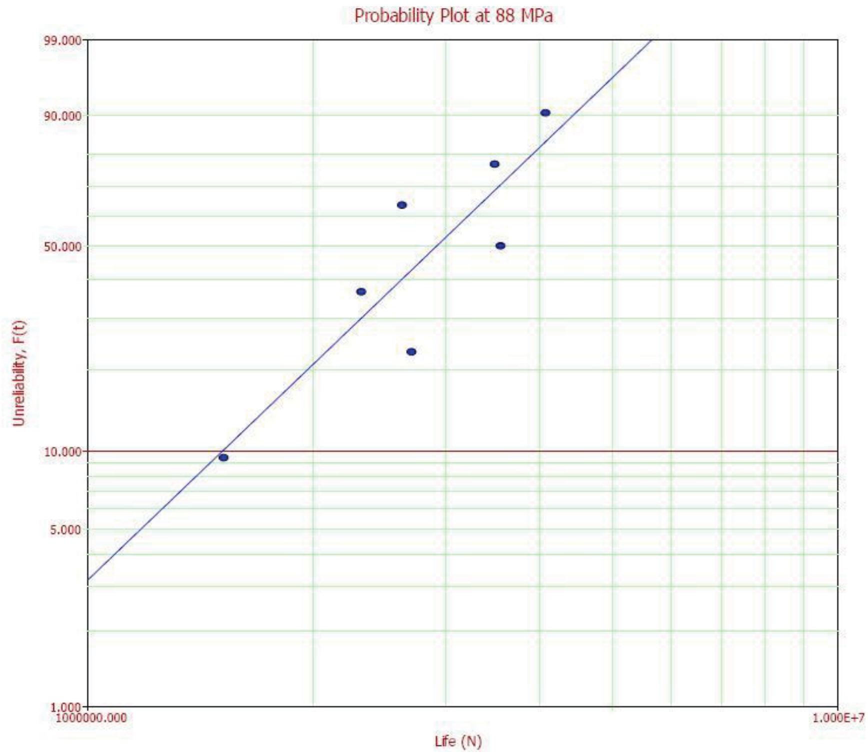

Figure 5: Fatigue life vs unreliability

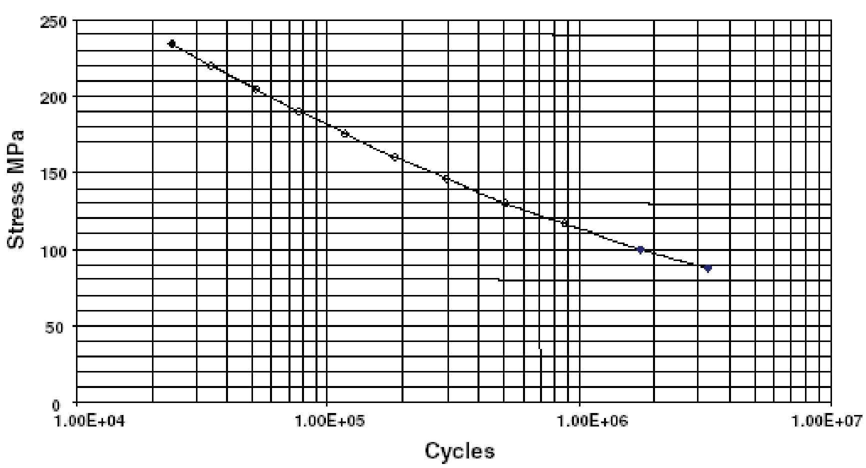

Figure 6: S-N curve of A356.2 material

Table 6: Percentage of error at various Probability and C.L

			C.L=90%		C.L=95%		C.L=97.5%	
Stress level	% p	ϕ_w	K_w	R_w	K_w	R_w	K_w	R_w
88 Mpa (n=8)	50	0.015	0.672	0.01	0.885	0.013	1.093	0.016
	90	0.015	1.171	0.017	1.189	0.018	1.721	0.026
	95	0.015	1.489	0.022	2.103	0.031	1.992	0.03
117 Mpa (n=8)	50	0.022	0.672	0.015	0.885	0.019	1.093	0.024
	90	0.021	1.171	0.024	1.189	0.025	1.721	0.036
	95	0.021	1.489	0.031	2.103	0.044	1.992	0.042
146 Mpa (n=8)	50	0.028	0.672	0.019	0.885	0.025	1.093	0.031
	90	0.027	1.171	0.032	1.189	0.032	1.721	0.046
	95	0.027	1.489	0.04	2.103	0.057	1.992	0.054
176 Mpa (n=7)	50	0.031	0.798	0.024	0.989	0.031	1.231	0.038
	90	0.030	1.434	0.043	1.333	0.04	1.886	0.056
	95	0.030	1.672	0.05	2.491	0.075	2.109	0.063
205 Mpa (n=7)	50	0.021	0.798	0.017	0.989	0.021	1.231	0.026
	90	0.02	1.434	0.029	1.333	0.027	1.886	0.038
	95	0.02	1.672	0.033	2.491	0.05	2.109	0.042
234 Mpa (n=5)	50	0.005	1.015	0.005	1.38	0.007	1.767	0.009
	90	0.005	1.928	0.01	1.869	0.009	2.615	0.013
	95	0.005	2.64	0.013	3.242	0.016	2.852	0.014

References

ASTM STP 9. (1963): A guide for fatigue testing and statistical analysis of fatigue data.

Belmonte, H.M.S.; Mulheron, M.; Smith, P.A.; Ham, A. (2008): Weibull based methodologies for condition assessment of cast iron water mains and its applications. *Fatigue Fract. Engg. Mater. Struct.*, vol. 31, pp. 370-385.

David Gerken, T. (1999): Rick Neal Squeeze cast (SCPM) light weight aluminum front knuckle case study. *Int Congress and Exposition,* Detroit, Michigan.

Enrique Castillo; Manuel Lopez-Aenlle; Antonio Ramos; Alfonso Fernandez-Canteli; Rolf Kieselbach; Volker Esslinger (2006): Specimen length effect on parameter estimation in modeling fatigue strength by weibull distribution. *Int. Jl. Fatigue,* vol. 28, pp. 1047-1058.

Gao, X.; Dodds, R.H. Jr; Tregoning, R.L.; Joyce, J. A.; Link, R. E. (1999): A Weibull stress model to predict cleavage fracture in plates containing surface cracks. *Fatigue Fract. Engg. Mater. Struct.*, vol. 22, pp. 481-493.

Gao, Z. (1984): The confidence level and determination of minimum specimens and fatigue testing. *Proc 2nd In: Conf fatigue and fatigue threshold EMAS Ltd,* West midlands, pp. 1203-115.

Gope, P.C. (2002): Determination of minimum number of specimens in S-N testing. *Jl. Engg. Mat. Tech.*, vol. 124, pp. 421-427.

Gope, P.C. (1994): A method for the sample size determination to estimate average fatigue life", *Proc IX ISME Conf. Mech. Eng.*, pp.1497-501.

Gope, P.C. (1999): Determination of sample size for estimation of fatigue life by using weibull or log normal distribution. *Int. Jl. Fatigue*, vol. 21, pp. 745-752.

IS: 5075 (1985): Method of rotating bending fatigue testing of metals. *Bureau of Indian Standards*, New Delhi.

Khandaker, M.P.H.; Ekwaro-Osire, S.; Gautam, K. (2008): Applying modified Weibull failure theory to biomaterial specimen under thermal loading. *Fatigue Fract. Engg. Mater. Struct.*, vol. 31, pp. 281-294.

Lawless, J.F. (1973): On the estimation of safe life when the underlying life distribution is weibull. *Technomatrics*, vol. 15, pp. 857-65.

Nakazawa, H.; Kodamas. (1987): Statistical research on fatigue and fracture. *In: Tanaka T, Nishijima S, Ichikawa M. editors current Japanese materials research.* Vol. 2, Elsevier applied science pub ltd, pp. 59-69.

Parida, N.; Das, Gope, S.K.; Mohanty, P.C. (1990): On the probability confidence and sample size in fatigue testing. *Jl. Testing Eval.*, vol. 18 (6), pp. 385-9.

Ramamurthy, raju, P.; Satyanarayana, B.; Ramji, K.; Suresh Babu, K. (2007): Evaluation of fatigue life of aluminum alloy wheels under radial loads, *Engg. Failure. Anal., vol.* 14, pp. 791-800.

Stephen, N Luko. (1999): A review of the weibull distribution and selected engineering applications (the Torrington company). *SAE j* , 1999-01–2859, pp. 398-412.

Wilks, S.S. (1942): Determination of sample size for setting tolerance limits. *Annals of Mathematical Statistics*, 6, pp. 12-91.

Zhao, Y.-X.; Gao, Q.; Sun, X.-F. (1998): A statistical investigation of fatigue lives of Q235 steel -welded joints. *Fatigue Fract. Engg. Mater. Struct.*, vol. 21, pp. 781-790.