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Fatigue Crack Growth Modelling of Aluminium Alloy under Constant and Variable Amplitude Loadings

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Abstract: This paper presents a study on fatigue crack growth modelling of thin walled tube aluminium alloy with circumferential crack under constant and variable amplitude loadings. Three fatigue crack growth models were investigated for this purpose, i.e. Walker, Forman and NASGRO. The results showed the differences in the fatigue crack growth simulation under both loading types, although, they have the same stress ratio for the maximum value. Subsequently, it was found that, different models gave different fatigue crack growth behaviour. In another case, many factors were identified towards the fatigue crack growth affection in structures. Hence, an initial crack length and the stress ratio are then studied and they showed a great influence in the life of the thin walled tube. From the detail analysis of this paper, the NASGRO model was found to be the most appropriate model for the variable amplitude loading. Therefore, this model can be suggested for the use in critical applications with respect to the fatigue crack growth studies for different structures under variable amplitude loadings.

Keywords: Constant amplitude; variable amplitude; fatigue crack growth; initial crack; stress ratio; thin wall tube.

Nomenclature

$K_{\max}$	Minimum stress intensity factor
$K_{\min}$	Minimum stress intensity factor
ΔK	Stress intensity factor range
K_C	Fracture toughness
C_F	Material constant (Forman model)
M_y	Material constant
a_n	Current crack length
K_c	Fracture toughness

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a_{OL}	Overload crack length
Q	Elliptical integral of second kind
Φ	Semi elliptical angle
ΔK_{eff}	Effective stress intensity factor
da/dN	Fatigue crack growth rate
K_{OL}	Stress intensity factor during the overload cycle
R	Stress ratio
γ_{POL}	Plastic zone created by overload
γ_w	Curve fitting parameter
C_p	Paris law coefficient
m_w	Paris law exponent
t	Thickness

1 Introduction

Crack growth in structures from the load point of view depends on the amplitude, stress ratio, and frequency of the load. Due to the random nature of variable loading by means of the presence of overload and under load, it is difficult to model all these influential parameters correctly. Overloads are known as an occurrence to retard crack growth, while under loads are the events to accelerate crack growth relative to the background rate (Huang, Moan and Cui, 2008). These interactions, which are highly dependent upon the loading sequence, make the prediction of fatigue life under variable amplitude loading (VAL) is more complex compared to the ones under constant amplitude loading (CAL). The accurate estimation of fatigue life of metals in service environments is still a challenge for the designer and engineer. These prediction models for crack growth under variable amplitude loading vary from simple modifications on the constant amplitude baseline up to complex models with detailed description of the relevant fracture mechanisms. Some models to calculate the crack growth by averaging over the applied load spectrum, while many other models tend to calculate the crack growth with cycle-by-cycle analysis.

Despite the ongoing development of prediction models towards more accurate description of phenomena, there seems to be no general agreement about which mathematical description is the most useful. Even the simple prediction models are still used by many engineers. The fracture mechanics based crack growth models have been developed by previous researchers (Khan, Alderliesten, Schijve and Benedictus, 2007; Schijve, 2009) to support the economical fail-safe and damage tolerance concepts. In general, these models can be divided in global analysis and cycle-by-cycle analysis, as presented in Fig. 1.

The global analysis concept predicts the fatigue crack growth considering the aver-

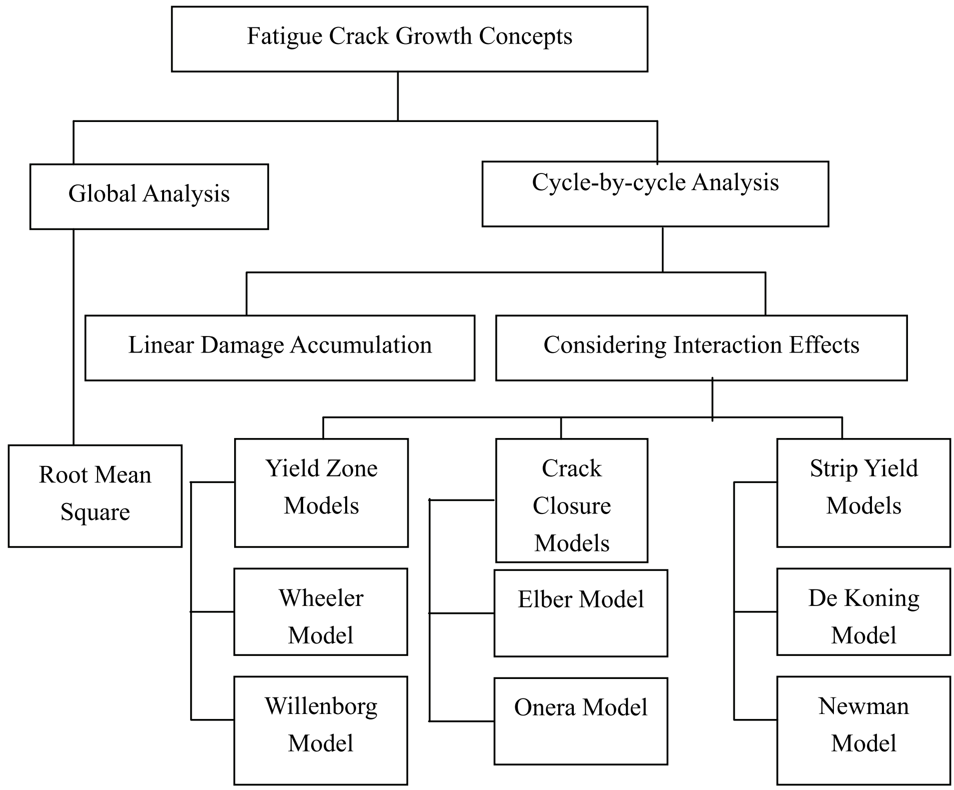


Figure 1: Classification of fatigue crack growth concepts

age of the applied loading cycles. The cycle-by-cycle analysis evaluates the crack growth for each load cycle and determines the crack growth life by accumulation. The root mean square model (Basrsom, 1976) is a global analysis model which, averages the stress intensity factor ranges following from the load spectrum using the root mean square technique to obtain one value for stress intensity factor range (ΔK) that can be used in the Paris relation for crack growth.

The linear damage accumulation model is simply a summation of calculated crack growth increments. As a result, it is the simplest model to predict the crack growth rate under VAL. The majority of the previous studies (Wheeler, 1970; Willenborg, Engle and Wood, 1971; Arone, 1990; Schijve, 1997; Meggiolaro and Casto, 2001) reported the presence of interaction effects under VAL and hence mentioned that the linear damage accumulation rule is physically unrealistic. The presence of interaction effects is the evident from experiments and fracture mechanics, that is dealing with the fatigue failure under VAL. These effects always alter the crack growth rate under the application of VAL. For correctly predicting the crack growth under VAL, it is necessary to involve the interaction effects while developing the prediction models.

According to Gallagher (1974) and Schijve (2001), the models that try to explain the interaction effect by considering the condition in front of crack tip are known as Yield Zone Models. Wheeler (1970), Willenborg (1971), and Yuen and Taheri (2006) started this generation of prediction models involving interaction effects in the crack growth prediction. The occurrence of crack closure of a fatigue at a positive tensile stress level after removing the load on the specimen is a physical reality (Elber, 1970). During crack growth, the plastic zone is moving with the tip of the crack as well as increasing in size; this phenomenon in literature is referred to as "Crack Closure". It was firstly observed by Elber (1970) and it is sometimes referred to as the Elber mechanism. The presence of this phenomenon can be justified either by stiffness measurement (Schijve, 1981), which is not an accurate way of measurement, or by the effect on fatigue crack growth. Many models have been used to model crack growth rates under variable amplitude loads (Schijve, 1981; Newman, 1984; Voorwald and Torres, 1991; Ray and Patanker, 2001(part I); Ray and Patanker, 2001(part II); Newman, Phillips and Everett, 2001).

Strip Yield Models are based on the Dugdale model (Dugdale, 1960; Jones, Pitt and Peng, 2008; Molent, McDonald, Barter and Jones, 2008). The Dugdale model was used to estimate the size of the plastic zone at the tip of the crack. Dugdale assumes that yielding occurs in a narrow strip ahead of the crack tip. The material response to plastic deformation is rigid-perfectly plastic, which leads to a constant stress (yield stress) in the plastic zone. To solve the elastic-plastic crack problems approximately for the plane stress state, a strip yield models were proposed by Budiansky and Hutchinson (1978) and Ibrahim, Thompson and Topper, (1986). The most famous strip yield models are the ones developed by Newman (1974) and De Koning and Liefting (1988). These models were later used by Zapatero, Moreno and Gonza (2005) for representing the retardation effects produced during the growth process. The main difference between these two models is the definition of the constraint factor. Newman assumes that the state of stress depends on crack growth rate, with low crack velocities being under plane strain conditions and high rates being under plane stress. The effects of the constraint parameter on life prediction in crack closure model are also examined by Christopher and Stephens (2006). Kassim, Emerson, and Leonardo (2008) used NASGRO and Walker models in order to show the effect of some factor on fatigue crack growth. The Prediction of fatigue crack growth might appear to be a simple procedure. For each cycle the crack extension Δa is equal to the crack rate in that cycle which can be obtained from a calibration curve, $da/dN = f(\Delta K)$, as obtained in a crack growth test on a simple specimen. However, there are some pitfalls (Schijve, 2009), i.e. firstly, the crack growth is depending on the stress ratio R which implies $da/dN = f(\Delta K, R)$. Secondly, the crack growth is also depending on the material thickness which is associated with the significant plane-strain/plane-stress conditions. Moreover, a through crack might still have one simple dimension, which is the crack length a. It had to be admitted that highly accurate predictions can not be guaranteed.

Due to the number and complexity of the mechanisms involved in this problem, it has been found from the literature review that no universal model has been developed to analyse the crack growth condition under VAL. Kujawski (2001) clearly indicated that there is no general agreement among researchers regarding the significance of closure concept on fatigue crack behaviour. Therefore this study discuss the fatigue crack growth (FCG) behaviour of thin walled tube aluminium alloy with circumferential crack. The analysis are performed with different models, i.e. Walker, Forman and NASGRO, under different load histories of CAL and VAL. For the analysis, the effects of initial crack length and stress ratio exhibited greater influence to the fatigue crack growth. Finally, this study indicates, higher value of initial crack length tend to lower number of cycles for crack growth, and it also showed that the increment of the stress ratio has lead to the tendency of the increment in the crack growth rates.

2 Theoretical Background

Although a general understanding of many aspects of fatigue crack growth behaviour was established in the early 1960s, a specific accumulation of damage model for computation of growth under a wide variety of service loads was lacking. The reason for building models is to link theoretical ideas with the observed data to provide a good prediction of future observations. Modelling of fatigue crack growth rate (FCGR) data has enhanced the ability to create damage tolerant design philosophies.

The reason for building models is to link theoretical ideas with the observed data. Modelling of FCG rate data has enhanced the ability to create damage tolerant design philosophies (Kassim, Emerson, Leonardo, 2008). The first paper proposed the well-known crack growth law was published by Paris, Gomez and Anderson (1961) and it turned out to be a milestone publication. The major limitation of the Paris law is its inability to account for the stress ratio. This drawback notified Walker (1970) to improve the Paris model by estimating the effect of stress ratio. Walker proposed the parameter of K, which is an equivalent zero to maximum (R = 0) stress intensity factor that causes the same growth rate as the actual $K_{\max}$ and

Rcombination, as expressed by

$$\overline{\Delta K} = K_{\max}\left(1-R\right)^{\gamma_W} \tag{1}$$

where $K_{\max} = K/(1-R)$, and is the parameter which, controls the extent of modification. Then Eq. 1 becomes:

$$\overline{\Delta K} = \frac{\Delta K}{\left(1-R\right)^{1-\gamma_W}} \tag{2}$$

The Walker law of da/dN is represented by the following expression:

$$\frac{da}{dN} = C_W\left(\overline{\Delta K}\right)^{m_W}$$

or

$$\frac{da}{dN} = C_W\left[\frac{\Delta K}{\left(1-R\right)^{1-\gamma_W}}\right]^{m_W} \tag{3}$$

For R = 0, Eq. 3 is formulated to form

$$\frac{da}{dN} = C_W\left(\Delta K\right)^{m_W} \tag{4}$$

and Eq. 4 is equivalent to the Paris law with $Cp = C_w$ and $m_p = m_w$.

The significance of this equation is that the log-log plot of da/dN versus K should result a single straight line that regardless the stress ratio for which the data was obtained. The ability to account for the above log-log results in the introduction of a third curve fitting parameter γ_ω, γ_ω is determined by trial and error. However, its value is the one that best consolidates the data along a single straight line on the log-log plot of da/dN versus $\overline{\Delta K}$. If there is no value of γ_ω would be found, thus, the Walker equation cannot be used. If the γ_ω value is equal to one, $\overline{\Delta K}$ equal to K that indicates the stress ratio has no effect on the data.

Although Walker improved the Paris model by taking account of the stress ratio, neither models could account for the instability of the crack growth when the stress intensity factor approaches its critical value. Forman (1972) improved the Walker model by suggesting a new model, which is capable of describing region III of the fatigue rate curve and includes the stress ratio effect. The Forman law is given by the mathematical relationship

$$\frac{da}{dN} = \frac{C_F\left(\Delta K\right)^{m_y}}{\left(1-R\right)K_C-\Delta K} = \frac{C_F\left(\Delta K\right)^{m_y}}{\left(1-R\right)\left(K_C-K_{\max}\right)} \tag{5}$$

where K_c is the fracture toughness for the material and thickness of interest. Eq. 5 indicates when $K_{\max}$ approaches K_c , therefore da/dN tends to have the infinity values. Based on this situation, the Forman equation is capable to represent the stable intermediate growth (region II) and the accelerated growth rates (region III). The Forman equation is also capable to represent data for various stress ratios by computing the following quantity for each data point, i.e.

$$Q = \frac{da}{dN}\left[(1-R)K_C - \Delta K\right] \tag{6}$$

If the various K and R combinations fall together on a straight line on the log-log plot of Q versus K, the Forman equation is applicable and it can then be used. Comparing Eqs. 5 and 6, the Forman equation can be represented as:

$$Q = C_F\,(\Delta K)^{m_y} \tag{7}$$

Further modifications of the Forman's expression to represent region I, II and III have been accomplished by including the threshold stress intensity parameter K.

Another related development has lead NASGRO to extends the generalized Willenborg model (Willenborg, Egle and Wood, 1971; Kassim, Emerson and Leonardo, 2006) by taking into account the reduction of retardation due to underloads. The NASGRO equation represent the most comprehensive growth law formulation comprising mean stress (R-ratio) effect, threshold, the honest of fast fracture and crack closure (nCode, 2003). The NASGRO formula is expressed as:

$$\frac{da}{dN} = C\left(\frac{1-f}{1-R}\Delta K\right)\frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{K_{\max}}{K_C}\right)^q} \tag{8}$$

Where C, n, p and q are the empirically derived coefficients from the measured data, and the other parameters are determined from the following formula.

The crack tip opening function f is determined from the following formulation:

$$f = \begin{cases} \max\left\{(R), (A_0 + A_1R + A_2R^2 + A_3R^3)\right\} & \text{if } (R \geq 0) \\ A_0 + A_1R & \text{if } (-2 \leq R < 0) \\ A_0 - 2A_1 & \text{if } (R < -2) \end{cases} \tag{9}$$

where

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)\left[\cos\left(\frac{\pi}{2}SR\right)\right]^{1/\alpha}$$

$$A_1 = (0.415 - 0.071\alpha)SR \tag{10}$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$A_3 = 2A_0 + A_1 - 1$$

SR is the ratio of the maximum applied stress to the flow stress. These values are all empirically derived. The threshold stress intensity is obtained from this equation:

$$\Delta K_{th} = \Delta K_o \frac{\sqrt{\frac{a}{a+a_o}}}{\left[\frac{1-f}{(1-A_0)(1-R)}\right]^{1+C_{th}R}} \tag{11}$$

where ΔK_o is the threshold stress intensity range at R= 0, obtained from test results, a is the crack length, a_o is the intrinsic crack length given as the constant, a_o = 0.0381 mm and C_{th} is the threshold coefficient obtained from test results.

The critical stress intensity K_C is expressed as a function of thickness, i.e.

$$K_C = K_{1C}\left(1 + B_K e^{(A_K t/t_o)^2}\right) \tag{12}$$

where K_{1C} is the plain strain fracture toughness, A_K and B_K are fitting parameters obtained from test and t is the thickness. The t_o value is the reference thickness for plain strain conditions, and it is given as:

$$t_o = 2.5\left(\frac{K_{1C}}{\sigma_y}\right)^2 \tag{13}$$

where σ_y is the yield stress.

The effective stress intensity ΔK is obtained as follows for values of $R < R_{\min}$:

$$\Delta K = \begin{cases} K_{\max}(1-R) & \text{if } (R < R_{\min}) \\ K_{\max} - K_{\min} & \text{otherwise} \end{cases} \tag{14}$$

$K_{\max}$ is also adjusted for values of $R > R_{\min}$ as follows:

$$K_{\max} = \begin{cases} \frac{\Delta K}{1-R_{\max}} & \text{if } (R > R_{\max}) \\ K_{\max} & \text{otherwise} \end{cases} \tag{15}$$

Methodology

Pipes or tubes contain defects from the manufacturing, installation and servicing processes. The defects can affect the safety of the structures, and even depress their service life that may lead to enormous economic costs and jeopardise the surrounding ecological environments. Aluminium alloys are widely used in the design of many engineering application, due to their good mechanical properties and low densities. In this application a thin walled tube of aluminium alloy (500 mm in radius and 50 mm in thickness) with circumferential crack under tension was analysed. The chemical composition and mechanical and fatigue properties of this material are shown in Tab. 1 and Tab. 2, respectively (ASM, 1985,1990,1993; Structural Alloys Handbook, 1996).

Table 1: Chemical composition of aluminium alloy 2024 T3

Component	Wt%	Component	Wt%
Al	90.7-94.7	Cr	Max. 0.1
Cu	3.8-4.9	Fe	Max. 0.5
Mg	1.2-1.8	Mn	0.3-0.9
Si	Max. 0.5	Ti	Max. 0.15
Zn	Max. 0.25	Other, each	Max. 0.05
Other, total	Max. 0.15		

Table 2: Mechanical and fatigue properties of aluminium alloy 2024 T3

Yield Stress (MPa) *YS*	345
Ultimate Tensile Strength (MPa) *UTS*	483
Plane Strain Fracture Toughness (MPa.$\sqrt{}$m) K_1C	36.262
Plane Stress Fracture Toughness (MPa $\sqrt{}$m) K_1D	72.524
Part Through Fracture Toughness (MPa $\sqrt{}$m) K_1E	50.547
Forman Exponent m_y Forman Co-efficient (m/MPa(m^1/2)^(n-1) C	3.284 1.5451e-10
Walker Exponent m_w	0.3
NASGRO Exponent p NASGRO Exponent q Modulus of Elasticity (GPa) E Fatigue Strength coefficient (MPa) f Elongation at Break (%)	0.5 1 73.1 130 18

Components and structures that are subjected to quite diverse load histories, their histories may be rather simple and repetitive and at the other extreme, they may be completely random. The cycle-by-cycle analysis can be performed with or without involving the interaction effects, i.e. the effect of a load cycle on crack growth in later cycles. The selected variable and constant load histories are shown in Figs. 2

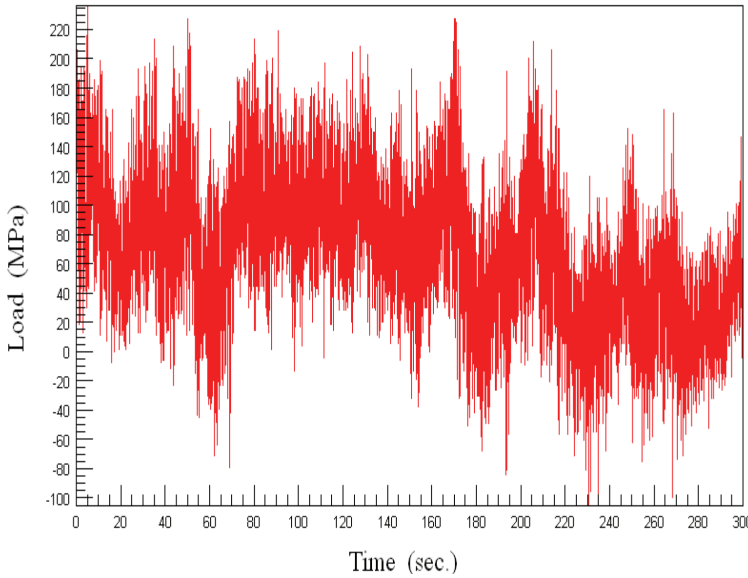


Figure 2: Display of variable amplitude loading used in the simulation

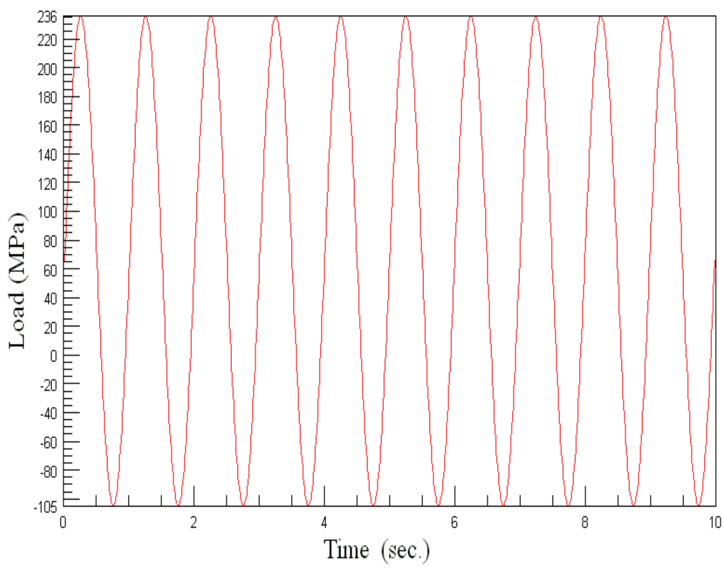


Figure 3: Display of constant amplitude loading used in the simulation

and 3, respectively. Many engineering structures are subjected to random loading in service. The fatigue growth life will be affected by load sequence. Neglecting the effect of cycle interaction in fatigue calculations under variable amplitude loading can lead to completely invalid life predictions (Huang, Zhang, Cui and Leng, 2005). However, for design purposes it is particularly difficult to generate an algorithm to quantify these sequence effects on fatigue crack propagation, due to the number and to the complexity of the mechanisms involved in this problem (Meggiolaro and Castro, 2003).The presence of interaction effects is always alter the crack growth rate under the application of VAL. For correctly predicting the crack growth under VAL, it is necessary to involve the interaction effects while developing the prediction models as a part of cycle by cycle analysis using different models. The CAL have been developed to correlate fatigue crack growth rates for different values. One of the purposes of this paper is to address how to characterize the effect of variable amplitude loading in fatigue crack propagation.To account load ranges and mean of the used load history, the rainflow counting method was then used. The modelling and simulation of the analysis were performed using the commercial software package.

During service loading, sub-critical cracks nucleate from these sites and grow till catastrophic failure (unstable crack growth) takes place when the crack length reaches a critical dimension. Most of the previous studies used the aspect ratio as 0.05 to 0.5 (ASTM E399; ASTM E647; Skorupa, Machniewicz, Schijve, Skorupa, 2007; Huang and Moan, 2007; Mohanty, Verma and Ray, 2009). In this paper, the ratio was taken at the values between 0.08 to 0.24, which means the initial crack length is ranging from 4 to 12 mm.

In this overview the three main input parameters are geometry, material and loading. The process proceed by selection of the fatigue crack growth model to show the behaviour of the geometry. The next step is to show the effects of different factors such as; initial crack length and stress ratio. The results of the previous process predict the fatigue life and fatigue crack growth. At each cycle, to get a new result it is possible to change any of the factors (fatigue crack growth model, geometry, material, loading, initial crack length and stress ratio), which means the ability to make a new prediction. The detail flow of such process is shown in Fig. 4.

3 Results and Discussion

It is evident from the work reported in the literature, that the global analysis model and linear damage accumulation models cannot be used for the prediction of fatigue crack growth under VAL (Khan, Alderliesten, Schijve and Benedictus, 2007). The cycle-by-cycle analysis can be performed with or with out involving the interaction effect, i.e. the effect of load cycle on crack growth in later cycles. It is necessary

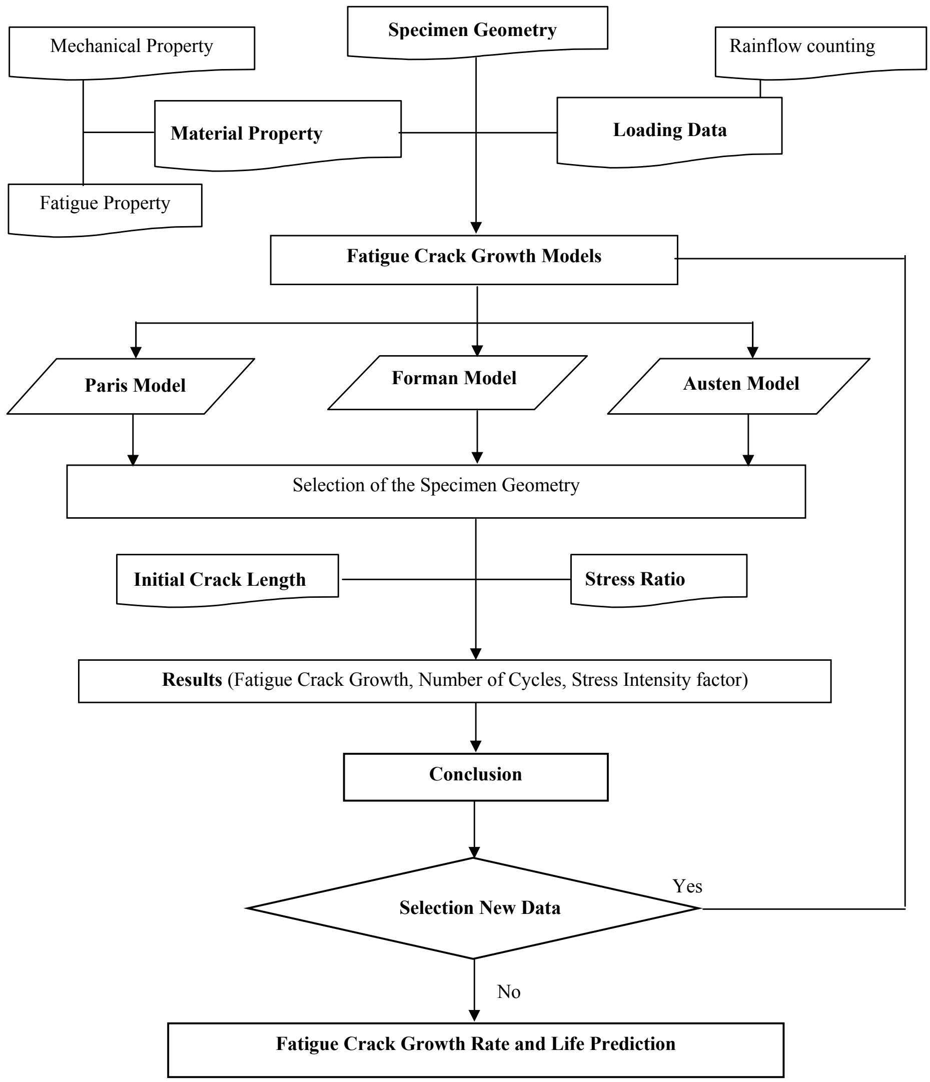


Figure 4: Flow chart of the process

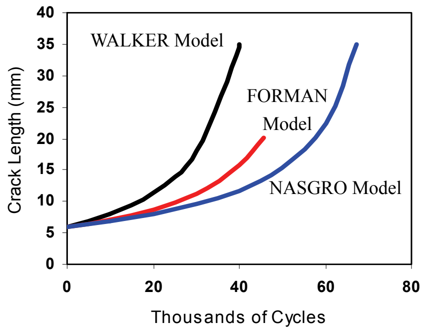


Figure 5: Fatigue crack growth trends using different FCG models under CAL

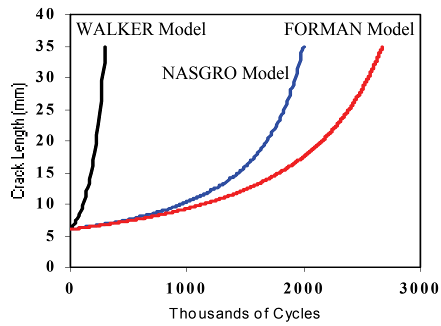


Figure 6: Fatigue crack growth using different FCG models under VAL

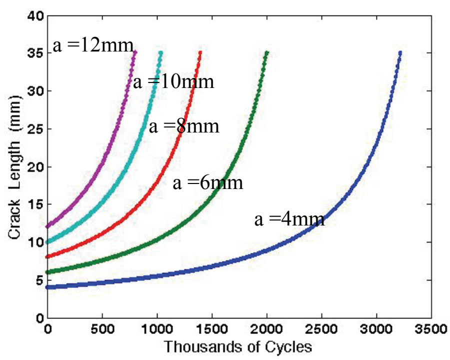


Figure 7: Fatigue crack growth for different initial crack lengths under VAL

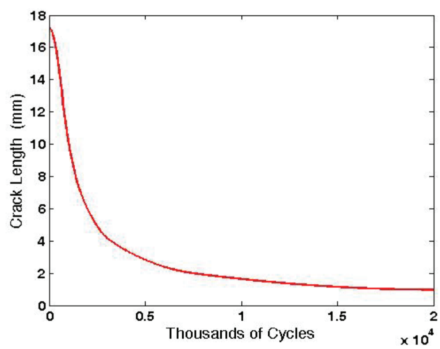


Figure 8: Fatigue life curve with different initial crack lengths

to improve the models by checking and refining the assumptions and extending the predictive capability (i.e. 3D crack geometries). The method of predicting life under VAL becomes very complex and complicated if one aims for an accurate assessment. Introduction of features related with the VAL in the models such as like interaction by means of retardation and acceleration, plastic zone formation and crack closure make the prediction very accurate, but on the expense of complexity and complicated algorithms. In addition, the magnitude of these effects depends on loading variables, specimen geometry, material properties, microstructure and environment. The use of analytical or numerical models to simulate crack growth under random loading is very important to reduce the number of tests required for any fatigue crack growth analysis.

The effects of CAL and VAL loading (using the maximum value of VAL as the value of CAL) on FCG of the tube was studied and they are shown in Figs. 5 and 6. FCG under CAL (Fig. 5) gave less life by 65% to 80% compared to the ones under VAL (Fig. 6) for Forman and NASGRO models. CAL account for all cycles as the maximum value, while VAL account only the peaks. The plastic zone size is relatively small under constant amplitude loading, while the resulting plastic zone becomes larger when a single overload is applied (Huang, Zhang, Cui and Leng, 2005). The fatigue crack growth prediction under VAL was indicated clearly by Ray and Patankar (2001) and Huang, Zhang, Cui and Leng (2005) for aluminium alloy, which is similar to the finding in Fig. 6. Accordingly, Fig. 6 shows the effect of using different FCG models. For the models with VAL, the Forman model gave longer life (2680 kcycles) and it is greater than the NASGRO model by 70%, while the Walker model gave the lowest value (300 kcycles). For CAL, the NASGRO model gave the longer life (68 kcycles), and the Walker model also gave the minimum life (42 kcycles). Forman improved the Walker model by suggesting a new model, which is capable of describing region III of the fatigue rate curve as well as includes the stress ratio effect. The NASGRO model took into account the reduction of retardation due to underloads by extended the generalized Willenborg model. For that reason the Walker model gave the lowest life in two cases (41 and 296 kcycles), while the Forman and the NASGRO models are changeable from case to other by 25 to 32% relating to the load effects and depending on the load sequence effect.

The effect of initial crack length ranging from 4 to 12 mm on FCG using NASGRO model was studied. The results are presented in Fig. 7. The interactions between the initial crack length and the number of cycles indicate that higher value of initial crack length (12 mm) tend to give lower number of cycles (700 kcycles) for the crack growth. The local stress range ahead of the crack tip was found to be higher for a deeper crack compared to shallow crack due to higher stress intensity factor for the case of a deeper crack. In addition, the fatigue life verses the initial crack length shown in Fig. 8, for which it shows maximum life (3220 kcycles) for a shorter initial crack length (4 mm). This explains why the number of cycles required for crack growth in structures having a long crack would be less when comparing to the structures with a small crack.

Many well-known formulations for the effect of R-ratio have been proposed. Can these equations be used to calculate the crack growth life of components subjected to variable amplitude loading? However. There is no definite answer (Huang and Moan, 2007). Most of the mean stress effects on crack growth have been obtained for only positive stress ratios, i.e., $R \geq 0$. In the present work, different stress ratio ranging from negative to positive values is used to show their effect. Fig. 9 shows the variation of the number of fatigue loading cycles versus the corresponding crack

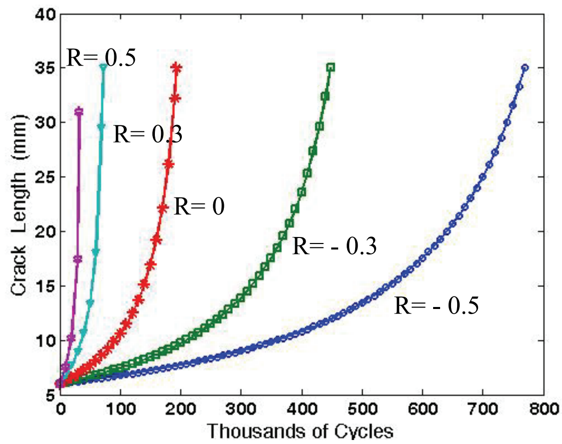
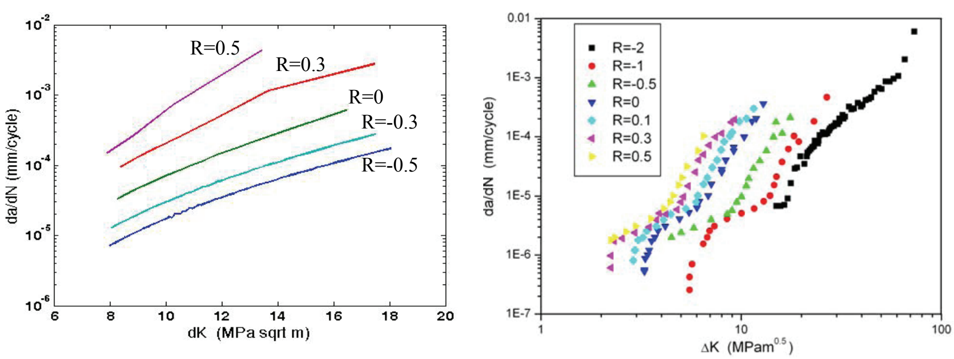


Figure 9: Fatigue crack growths with different stress ratio under VAL



(a) (b)

Figure 10: Fatigue crack growth data for: (a) Al 2024 T3 obtained at stress ratios $-0.5 \leq R \leq 0.5$ (b) Al 2024 T351 obtained at stress ratios $-2 \leq R \leq 0.5$ (Huang and Moan, 2007).

length for stress ratios ranging from –0.5 to 0.5. With an identical stress intensity factor range, higher *R*-ratio tends to give higher crack growth rates. Although the threshold was not experimentally measured, the tendency indicates that the threshold value of the stress intensity factor range increases as the *R*-ratio decreases. It was observed that the FCG affected by different stress ratio, which shown on $a-N$ curve. The increasing of stress ratio (which means increasing the mean stress) has a tendency to increase the crack growth. The results are in a good agreement with the result obtained by Schijve, Skorupa, Skorupa, Machniewicz and Gruszczynski (2004) and Tianwen, Jixi and Yanyao (2008). The maximum difference in positive

R ratios about 6% and 25% for the negative R ratios values.

The R-ratio is an important parameter because it has a significant effect on the crack growth rate and many of engineering structures are subjected to VAL during their service. Various models for crack growth rate, which account for the R-ratio, have been explored. The fatigue crack propagation rate model, which can condense the crack growth rate data under different R-ratios to a single curve of R= 0, is presented in Fig. 10(a). These results are in a good agreement with that presented in Fig. 10(b), which performed by Huang and Moan (2007). The percentage difference in FCGR is 98% for R values ranging from – 0.05 to 0.5 for each of the two figures.

One of the aims of this paper is to show the model capability (NASGRO) to reproduce the effects of the initial crack length and the effect of the overloads and undeloads on the fatigue crack growth rate. It is interesting to compare the growth rate with different initial crack length. Fig. 11 shows the crack growth rates which are different for initial crack lengths, although the curves have the same behaviour. It can also be seen that there is a pattern in the crack growth rate, which is repeated and amplified when the crack grows. It is repeated as a consequence of the repetition of the load sequence. These results are shown a similar behavior to that obtained by the previous finding (Zapatero, Moreno, Herrera and Domingez 2005). The work were carried out on specimens (CT) of 2024-T351 aluminium alloy under initial crack length under different VAL histories.

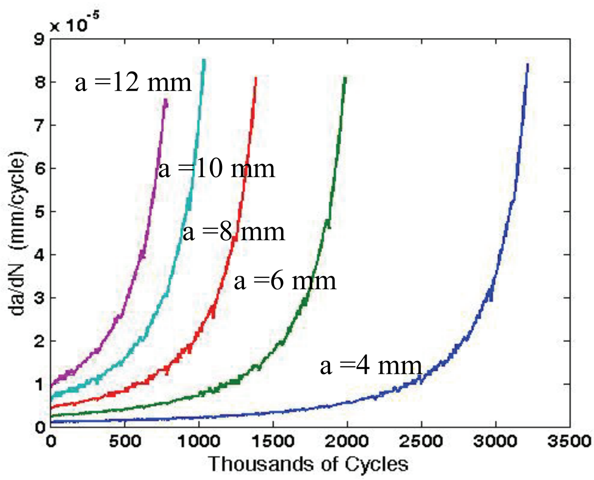
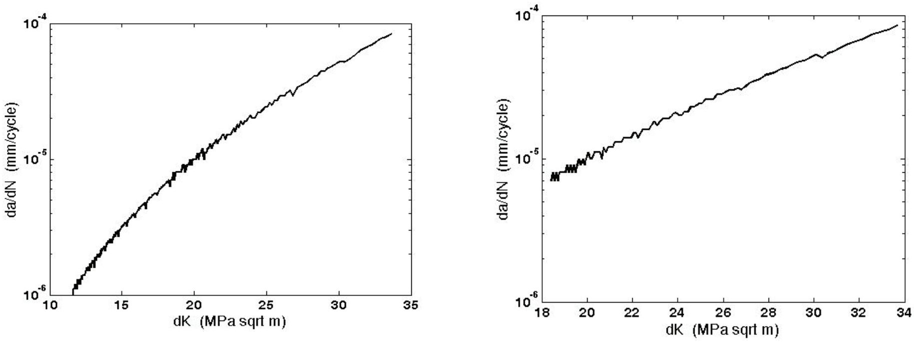
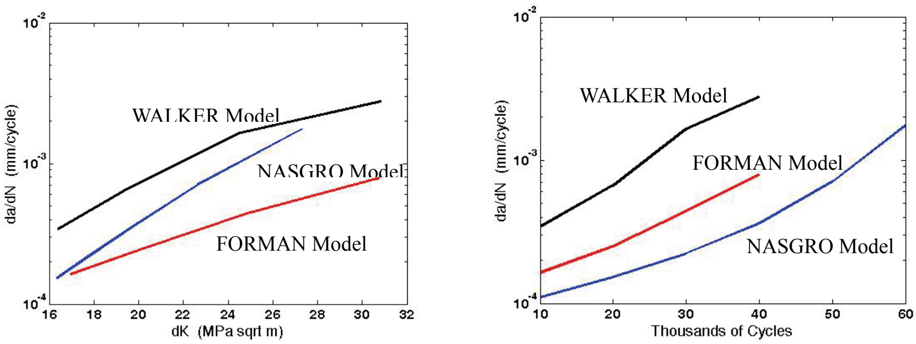


Figure 11: da/dN Curves of different initial crack length using NASGRO model under VAL



(a) (b)

Figure 12: FCGR curves of initial crack length using NASGRO model under VAL (a) Initial crack length 4 mm (b) Initial crack length 10 mm



(a) (b)

Figure 13: FCGR curves of different FCG models under CAL (a) with ΔK and (b) with number of cycles

The fatigue crack growth retardation caused by a tensile overload exhibits an initial crack growth acceleration immediately following the overload, and a subsequent delay in reaching the minimum crack growth rate. When multiple overloads are applied, they can interact with each other, and they could either accelerate or decelerate the overall crack growth retardation, which is depending on the frequency of the overload. Furthermore, the crack growth retardation is reduced during the later stage of the fatigue life of a structure when the net section stress approaches the yield strength of the material. Figs. 12 (a and b) show the FCGR with stress intensity factor range (ΔK) for the two selected initial crack length, i.e. 4 and 10 mm. The two curves have the same behaviour of FCGR with stress intensity range.

If we draw them in one, they will be overlap with the a shifting value according to the initial crack length. The effect of retardation and acceleration of the load was shown clearly in that curves. It can be seen from the experimental data (Yuen and Taheri, 2006; Tianwen, Jixi and Yanyao, 2008) that the fatigue life increased with the increment in overload level, as expected, since a higher overload created more crack growth retardation. In addition, up to a point, the fatigue life generally increased as the distance between overload application decreased. However, when the underloads exist it works in opposite and decrease the fatigue life (Figs. 12).

The simplest approach to predicate fatigue crack growth for VAL is to neglect all sequence effects and determine the crack growth on a cycle by cycle basis in conjunction with a constant amplitude fatigue crack growth rule. Figs. 13 (a and b) show the fatigue crack growth rate under CAL with stress intensity range and number of cycles respectively, using different models. These curves show a behaviour similar to the $a-N$ curves. The NASGRO model gave a lower FCGR and the highest given by Walker, while the Forman model in between with a short life. Although Walker improved the Paris model by taking account of the stress ratio, neither model could account for the instability of the crack growth when the stress intensity factor approaches its critical value. The Forman model is capable of describing region III of the fatigue rate curve as well as includes the stress ratio effect.

To ensure design safety, engineers need to be able to model or predict fatigue crack growth behaviour accurately. Under constant amplitude loading, this is relatively simple. When variable amplitude loading is introduced, however, fatigue crack growth modelling becomes more difficult (christopher and Stephens, 2006; Hamam, Pommier and Bumbieler, 2007). A great deal of scientific and academic investigation has been focused in the literature on fatigue crack growth retardation resulting from single and multiple overloads (Huang, Zhang, Cui and Leng, 2005; Mohanty, Verma and Ray, 2009). Studying of fatigue crack growth rate under variable loading is very important for the reliable life prediction of engineering structures.

4 Conclusions

The study of fatigue crack propagation examines how a fatigue crack grows under cyclic load. This topic is the subject of considerable research, mainly dealing with the development of various models to better explain the crack propagation phenomenon. For the modern high performance structures designed for finite service life, fatigue crack growth occurs over a significant portion of the useful life of the structures. Therefore, accurate simulation of crack propagation paths in engineering materials is what designers and engineers are always looking for.

A fatigue crack growth of thin walled aluminium tube with circumferential crack under constant and variable amplitude loading was studied and showed the behaviour of FCG with three different models Walker, Forman and NASGRO based Glyphwork codes. These models applied in both constant and variable amplitude loading histories, which gave different behaviour under the two types of loading. From the simulation, the crack growth rates are different for different initial crack length, although the curves have the same behaviour. It can also be seen that there is a pattern in the crack growth rate repeated and amplified when the crack growths. It's repetition as a consequence of the load sequence.

Fatigue life predicted under CAL is less than that predicted under VAL due to, CAL account for all cycles as the maximum value, while VAL only the peaks. The fatigue crack growth rates under CAL using different models, show a behaviour similar to the $a-N$ curves. The Walker model including the effect of stress ratio while, the Forman model is capable of describing region III of the fatigue rate curve and includes the stress ratio effect. The NASGRO model taking into account the reduction of retardation due to underloads. The NASGRO model was found to be the most appropriate model for the variable amplitude loading. Therefore, this model may be suggested for use in critical applications in studying fatigue crack growth for different structures under variable amplitude loading.

Among many factors affecting the FCG, the effect of initial crack length and stress ratio are shown. This study indicate that the higher value of initial crack length tend to the lower number of cycles for crack growth, and also showed that increasing the stress ratio (which means increasing the mean stress) has tendency to increase the crack growth rates.

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