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Rotational Flexibility for Detecting Low Level Damage in Beam-Like Structures

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Abstract: This paper proposes a methodology for damage detection in beam like structures using vibration characteristics obtained from transfer matrix technique. At first, vibration characteristics of beam-like structure have been determined with the help of a computer program developed based on the formulations presented in this paper. Then, a detailed study has been carried out to categorise the influence of damage on frequency and mode shape (both displacement and rotational) information. For a structure with known magnitude and location of damage(s), frequencies and mode shape information are obtained and the same has been used in determining the damage in the structure. It is observed that the change in frequency with damage is very small but it is easy to measure. Mode shape information is very sensitive to damage but it is prone to experimental errors. Hence, a methodology for damage detection, specifically for low level of multi-damage cases, has been proposed using both frequency and rotational mode shape information obtained from a damaged structure. Curvature of rotational flexibility has been proposed and demonstrated to be an improved indicator of damage than the established ones. Further, uniform moment surface has been developed by incorporating number of lower mode information to reduce the error from any particular mode. The proposed damage indicator is enhanced by Chebyshev polynomials for better performance. It is observed that the proposed damage indicator is capable of identifying locations of damage in structure even for case of very low damage. Here, no prior knowledge or information on the undamaged (intact) stage of the structure is required for this proposed methodology. Hence, the methodology presented here provides a simple and efficient tool for detection and localisation of damage(s) in distressed structure.

Keywords: structural damage; detection; frequency; curvature; rotational mode

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shape; moment surface

1 Introduction

Since damage produces changes in physical properties (i.e., stiffness, mass, and damping) of structures, these changes are accompanied by changes in the modal characteristics of the structure (i.e., natural frequencies, mode shapes, and modal damping). Specifically, damage in a structure would cause a reduction in natural frequencies, a change to the mode shapes and an increase (may not be true always) in modal damping. This phenomenon has widely been used for detecting damage or health monitoring of structures. There has been a significant effort to detect the location of damages using one or more of these characteristics.

Perhaps, first research article to address a method for damage detection using vibration measurements was proposed by Lifshitz and Rotem (1969) where the change in the dynamic moduli was related to the frequency shift. Cawley and Adams (1979) are the first researchers to give a formulation for damage detection from changes in frequency obtained from undamaged- and damaged- structure. Yuen (1985) examined changes in the mode shape and mode-shape-slope parameters to simulate the reduction in stiffness in each structural element, then the predicted changes were compared to the measured changes to determine the damage location. Rizos et al. (1990) presented a simple and easy method for damage detection in one-dimensional structures. The natural frequencies were established as functions of the depth and location of crack. A study reported by Pandey et al (1991) has investigated the change in curvature of mode shape for a damaged beam. It was showed that the difference between curvature mode shapes for an intact and damaged beam could find a localized change in elastic modulus of about 30%.

A method based on perturbing the stiffness matrix of a finite element model and comparing the resulting changes in natural frequencies with observations was presented by Hearn and Testa (1991). An alternative to the stiffness matrix, proposed by Pandey and Biswas (1994), is to consider the flexibility matrix, which converges better on increasing frequency. Baruh and Ratan (1993) developed a method for the detection of the existence and location of structural damage where the identified eigen solution was used together with properties of the eigenvalue problem to detect the damaged components. Slater and Shelley (1993) presented a method by using frequency-shift measurements to detect damage in a smart structure. Theory of modal filters was used to track the frequency changes over time. A transfer matrix technique was used by Choy et al. (1995) to detect damage for beam like structures. Yuen (1995) investigated the systematic change in the fundamental mode shape of a cantilever with respect to the location of the damage. Ratcliffe (1997) developed a technique for identifying the location of structural damage in a beam using modified Laplacian Operator on mode shape data. Doebling et al. (1998) brought out an excellent review of vibration-based damage identification methods. Zhao and Dewolf (1999) presented a theoretical study comparing the use of natural frequencies, mode shapes, and modal flexibility for structural damage detection. Results demonstrated that the modal flexibility is more likely to indicate damage than frequency or mode shape. A numerical study of the relationship between damage characteristics and the changes in the dynamic properties was presented by Abdo and Hori (2002). It was found by the authors that the rotation of mode shape is a sensitive indicator of damage localisation. Lu et al. (2002) pointed out that Pandey's (1991) method is difficult to locate multiple damages, and they recommended the modal flexibility curvature for multiple damage localization due to its high sensitivity to closely distributed structural damages. A damage localisation method was proposed by Parloo et al. (2003) based on the use of mode shape sensitivities. Differences in the dynamic behaviour of the structure in its undamaged and damaged conditions were used for damage localisation of beam like structures.

Another damage localisation method based on changes in uniform load surface curvature is developed by Wu and Law (2004). Escobar et al. (2005) presented a method for detecting damage by calculating the changes in element stiffness. An iterative scheme was employed to derive the geometrical transformation matrix which was used to obtain the stiffness matrix of a damaged structure from its undamaged state. Kim et al. (2006) proposed a vibration-based damage evaluation method for detection of damage by utilizing only a few of the lower mode shapes. The proposed method was particularly advantageous for beam-like structures. Ge and Lui (2005) proposed a method for identification and quantification of damage by using finite element model of the structure in its undamaged state as well as damaged state of the structure to obtain the frequencies and mode shapes. Damage is quantified by comparing the change in stiffness and mass properties of the structural elements in its both states. Alvandi and Cremona (2006) reviewed usual vibration-based damage identification techniques for structural damage evaluation. With the help of a simple supported beam with different damage levels, the reliability of these techniques were investigated by using only few mode shapes and/or modal frequencies of the structure that can be easily obtained by dynamic tests. A crack detection methodology was presented in (Zhong and Oyadiji, 2007) by using the stationary wavelet transform (SWT) of two sets of mode shape data obtained from simply supported beam-like structure where the modal responses of the damaged simply supported beams used are computed using the finite element method. Galvanetto and Violaris (2007) proposed that orthogonal decomposition is an efficient damage indicator and the efficacy of the proposed indicator was examined using numerical investigations. Brasiliano et al. (2008) used Residual Error

Method to identify and quantify damages in concrete beam. This method is based on changes in the dynamic properties of structures due to damage. It was shown that the proposed method is able to locate and to quantify damages in concrete beam. Giridhar and Gopalakrishanan (2009) proposed a damage detection method using frequency domain strain energy. The author of the present paper along with the co-researchers [Srinivas et al. (2009)] presented the model and non-model based methodologies using frequency match, change in mode shape, mode shape curvature and artificial neural network (ANN) for detection and localisation of damage in distressed structure using the vibration characteristics obtained from transfer matrix formulation.

From the established research, it is observed that change in natural frequency is the single most effective dynamic indicator of structural damage but multi-damage cases can not be identified by using frequency information alone. Most of the published work requires reference to an undamaged dynamic model in some form. Requirement of information about the undamaged state of the structure can limit the application of these techniques. It is also noticed that the established methodologies for detecting single damage is much more advanced than that for multi-damage cases, specifically cases with low-level of multi-damage. In view of this, in the present study curvature of rotational flexibility has been identified as an improved indicator of damage than the established ones. Curvature of rotational flexibility has been formed by considering both the frequency and mode shape information from a damaged structure. Further, uniform moment surface has been developed from modal rotational flexibility matrix to incorporate the contribution from number of lower modes to reduce the influence of experimental error in any particular mode. Curvature of rotational flexibility and the final uniform moment surface have been proposed as new and improved local damage indicator. The proposed damage indicator is enhanced by Chebyshev polynomials for better performance. It is observed that the proposed damage indicator is capable of identifying locations of damage in structure even for case of very low damage. It is worth mentioning that no prior knowledge or information about the undamaged (intact) stage of the structure is required for identifying locations of distress in a damaged structure.

In the formulation proposed in this study for detection of damage, dynamic analysis for damaged structure has been performed using transfer matrix method because of its versatility and ease with which it can be applied to a structure of either uniform or non-uniform cross section and under a variety of boundary conditions. Analysts of beam-like structures are perennially challenged by complex geometries which are not readily translated into beam element models (Vest and Darlow, 1991). Further, for performing iterative studies and determination of required vibrational characteristics such as rotation of mode shape information can be easily obtained from transfer matrix formulation compared to FE analysis. As the transfer matrix method for structural analysis is well reported, only a brief on transfer matrix formulation for dynamic analysis has been presented to understand its implications and limitations towards judicious applications, and for ready reference as well. Then, theoretical developments of the methodology for detection of damage are presented followed by detailed numerical studies to demonstrate the efficacy of the proposed method.

2 Transfer matrix approach for calculating dynamic characteristics of beam-like

For computing plane flexural vibrations of a straight beam using transfer matrix method, the beam section is modelled by discrete uniform structural elements interconnected at the nodal points. Using the conventional assumption of a mass-less beam, the inertia effects of the beam element are dynamically represented by two lumped masses at both ends of the element (as shown in Figure 1). Each individual beam is considered to be of individual homogenous material property and geometry which can be represented by area moment of inertia and Young's modulus of that particular element. Two displacements, viz., vertical deflection (η) and rotation (φ) and the corresponding forces viz., shear force (V) and bending moment (M) are considered for describing the state array variables at each section and the sign convention of the state array variables is shown in Figure 2.

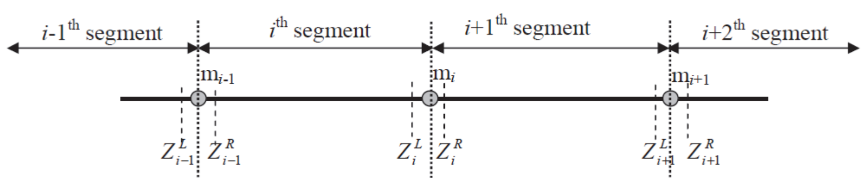

Figure 1: Beam with concentrated masses

The equilibrium between sections i and $i-1$ of an element will be maintained by Pestel and Leckie (1963)

$$V_i^L - V_{i-1}^R = 0 \tag{1}$$

$$M_i^L - M_{i-1}^R - V_i^L l_i = 0 \tag{2}$$

where the superscript L and R stands for left and right side of a section respectively.

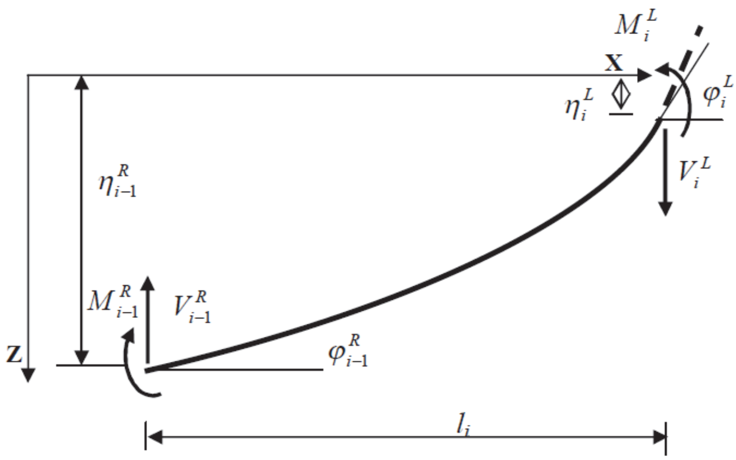

Figure 2: End forces and deflections for massless beam

Two more equations those are required for solving the problem can be obtained from compatibility conditions and the final equations can be expressed as

$$\eta_i^L = \eta_{i-1}^R + l_i\varphi_{i-1}^R + \frac{l_i^2}{2\,(EI)_i}M_{i-1}^R + \frac{l_i^3}{6\,(EI)_i}V_{i-1}^R \tag{3}$$

$$\varphi_i^L = \varphi_{i-1}^R + \frac{l_i}{(EI)_i}M_{i-1}^R + \frac{l_i^2}{2\,(EI)_i}V_{i-1}^R \tag{4}$$

$$M_i^L = M_{i-1}^R + l_iV_{i-1}^R \tag{5}$$

$$V_i^L = V_{i-1}^R \tag{6}$$

and can be expressed in matrix form as,

$$\begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_i^L = \begin{bmatrix} 1 & l & \frac{l^2}{2EI} & \frac{l^3}{6EI} \\ 0 & 1 & \frac{l}{EI} & \frac{l^2}{2EI} \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_{i-1}^R \tag{7}$$

So, from Eq. (7), the field matrix ($\mathbf{F}_i$) connecting Z_i^L with Z_{i-1}^R can be expressed as

$$Z_i^L = F_iZ_{i-1}^R \tag{8}$$

The point matrix ($\mathbf{P}_i$) connecting Z_i^R with Z_i^L is found by using continuity of deflection, slope and moment across the concentrated mass m_i, $\eta_i^R = \eta_i^L$; $\varphi_i^R = \varphi_i^L$ and

$$M_i^R = M_i \tag{9}$$

The vibrating mass, however, introduces the inertial force which causes discontinuity in shear. The free-body diagram shown in Figure 3 yields a relation from simple equilibrium considerations as:

$$V_i^R = V_i^L \pm m_i \omega^2 \eta_i \tag{10}$$

(in formulation, a particular sign convention has been followed)

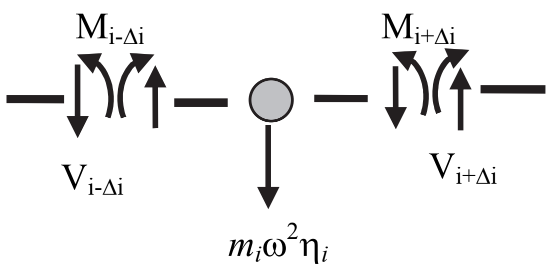

Figure 3: Free-body diagram of mass m_i

Eqs. (9) and (10) can be expressed in matrix form as:

$$\begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m_i\omega^2 & 0 & 0 & 1 \end{bmatrix}_i \begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_i^L \tag{11}$$

$$Z_i^R = P_i Z_i^L \tag{12}$$

By combining both field and point matrices, relation between the state vectors of adjacent ends ($i$and i-1) can be obtained as

$$Z_i^R = P_i F_i Z_{i-1}^R \tag{13}$$

2.1 Transfer matrix for Frequency determinant

The transfer matrix method can be applied to solve more complicated problems by considering a beam that is made up of piecewise uniform mass-less elements, with masses concentrated at discrete points. If a structural element is made up of n segments (between the ends 0 to n), relationship between the state vectors at the extreme ends (0 and n) of the beam can be obtained as

$$Z_n = F_n P_{n-1} F_{n-1} \ldots\ldots\ldots. P_4 F_4 P_3 F_3 P_2 F_2 P_1 F_1 Z_0$$

$$Z_n = U Z_0 \tag{14}$$

Eq. (14) can be written in full, as

$$\begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_n = \begin{bmatrix} u^n_{11} & u^n_{12} & u^n_{13} & u^n_{14} \\ u^n_{21} & u^n_{22} & u^n_{23} & u^n_{24} \\ u^n_{31} & u^n_{32} & u^n_{33} & u^n_{34} \\ u^n_{41} & u^n_{42} & u^n_{43} & u^n_{44} \end{bmatrix}_i \begin{bmatrix} -\eta \\ \varphi \\ M \\ V \end{bmatrix}_0 \tag{15}$$

where the coefficients u^n_{11} to u^n_{44} are functions of circular frequency ω. Boundary conditions can be applied to the equations formulated from Eq. (15) to arrive at the frequency determinant. For example, a beam (consists of n segments) with simply supported ends can be solved as follows:

The boundary conditions at simply supported ends are $\eta_n = 0$, $M_n = 0$, $\eta_0 = 0$, and $M_0 = 0$; By substituting these boundary conditions in Eq. (15), the following relation can be obtained

$$u^n_{12}\varphi_0 + u^n_{14}V_0 = 0 \tag{16a}$$

and

$$u^n_{32}\varphi_0 + u^n_{34}V_0 = 0 \tag{16b}$$

where u^k_{ij} is element of i^{th} row and j^{th} column of the transfer matrix which can be obtained by using Eq. (15) and superscript k denotes the number of segments. The normal modes can be found for the system using the following procedure.

For a nontrivial solution of Eqs. (16a) and (16b), the determinant of the coefficients must be zero, that is

$$\begin{Vmatrix} u^n_{12} & u^n_{14} \\ u^n_{32} & u^n_{34} \end{Vmatrix} = 0 \tag{17}$$

The same procedure can be followed for other boundary conditions also. Since, the elements u_{ij} are functions of the circular frequencyω, this determinant serves to compute the natural circular frequencies. In view of the fact that a beam which possesses *n* segments will have *n*-1 discrete masses, the expansion of the frequency determinant leads to an equation of *n*-1 degree in ω^2.

2.2 Determination of the normal modes

Once the natural frequencies of an elastic system are found by means of transfer matrices, it is easy to compute the normal modes. As explained earlier, the state vector at i^{th} section Z_i can be expressed in terms of the unknowns at point 0, namely, the slope φ_0 and the shear V_0. When the boundary conditions are applied, φ_0 and V_0 (using Eqs. (16a) and (16b)) as, $V_0 = -\left(\frac{u_{12}^n}{u_{14}^n}\right)\varphi_0$ where *n* is number of segments considered.

The column vector $\left\{\varphi_0 \quad V_0\right\}^T$ can be rewritten in terms of φ_0 alone as $\left\{1 \quad \frac{-u_{12}^n}{u_{14}^n}\right\}^T \varphi_0$. All the state vectors may then be expressed in terms of φ_0, which is undetermined, but can be arbitrarily chosen as unity. When this is done, the expressions for the state vectors of a structure with '*n*' segments can be obtained by

$$Z_n = \begin{bmatrix} u_{12}^n & u_{14}^n \\ u_{22}^n & u_{24}^n \\ u_{32}^n & u_{34}^n \\ u_{42}^n & u_{44}^n \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{u_{12}^n}{u_{14}^n} \end{bmatrix} \tag{18}$$

Using Eq. (18), normal modes can be found for the system.

3 Influence of location and magnitude of damage on vibration characteristics

In this study, a beam like structure is considered for calculating the natural frequencies and corresponding mode shapes. The material and sectional properties of the beam are: E= 25×10^6 kN/m^2, I=0.001333 m^4, A = 0.1 m^2. For transfer matrix formulation, 31 elements (as shown in Figure 4) are considered for representing the whole beam. As discussed in the preceding section, the determinant for the whole beam incorporating the boundary condition is found out for an assumed (initial) natural frequency. Then, an iterative study has been carried out for different natural frequencies to get the determinant of the transfer matrix and the locations where the determinant value is near zero, is assigned as the natural frequency of the beam.

Though changes in the natural frequencies have been used in many studies as a parameter for detection of damage, question may arise that how far the frequencies of a structure is influenced by the damage in its particular element(s). From the review of literature on detection of damage using frequency or mode shape, it is found that the number of research works carried out in this area is quite considerable. But, there is no confirmation on the superiority of any method over the others. Further, how the location and magnitude of damages influence the vibration characteristics need to be understood. In view of this, a detailed study has been carried out to evaluate the influence of level of damage and its location on frequency or mode shapes of a structure. A beam is simulated (consists of 31 segments) with damage of different magnitude (percentage) is assumed to be located at 2^{nd}, 7^{th} and 15^{th} element (refer to Figure 4). In this study, presented results are restricted to first 4 modes because it may have practical difficulty to extract information in higher modes. Figures 5-7 show the percentage change in natural frequency for first 4 modes with different degrees of damage in 2^{nd}, $7^{th}$and 15^{th} element, respectively. Similarly, first two displacement mode shapes due to different degrees of damage located at 2^{nd}, $7^{th}$and 15^{th} element of the beam are shown in Figures 8-10 and 11-13, respectively. For brevity, the study is restricted to first two mode shapes only.

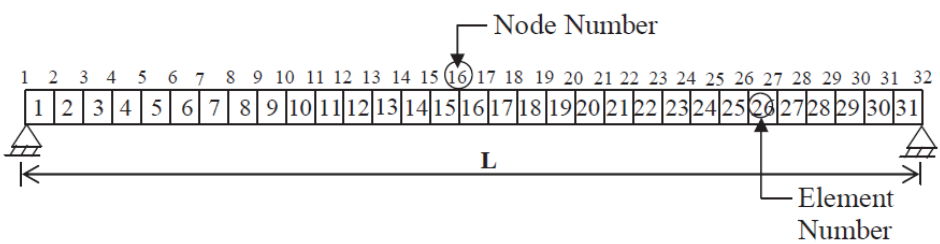

Figure 4: A typical beam like structure with elements and node numbers

It is experienced that many dynamics problems demand the response of a structure at all possible degrees of freedom, nevertheless standard traditional experimental determination of responses are effective only for translational and a very limited subset of rotational DoF. So, most of the researchers have considered only the displacement mode shape as a reference to analyse the dynamic behaviour of the structure for detecting damage. However, in recent years, major advances have been taken place in the field of mechanical vibration measurements. The introduction of Scanning Laser Doppler Vibrometer (SLDV) has revolutionized dynamic testing and analysis. A laser system has been developed by Ratcliff and Lieven (2000) which enables rotations to be extracted by a simple plane fitting technique. To extend the study, it is also considered (Abdo and Hori, 2002) that rotational mode shape information may also provide significant information for detecting damage

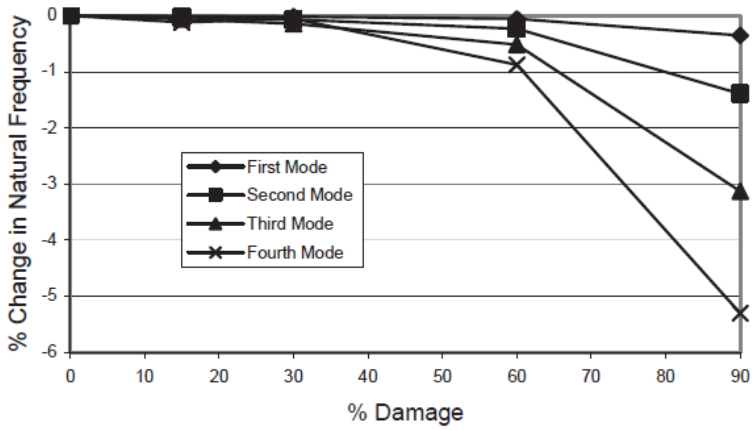

Figure 5: Percentage change in natural frequency due to damage in 2^{nd} element

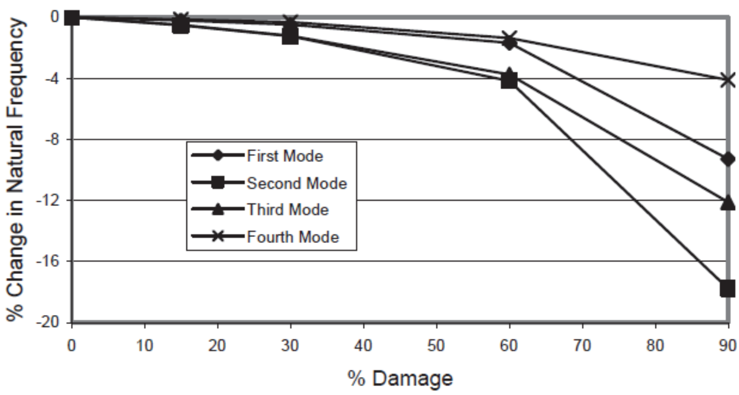

Figure 6: Percentage change in natural frequency due to damage in 7th element

in structures. First two rotational mode shapes of the beam with different degrees of damage located at 2^{nd}, $7^{th}$and 15^{th} element are shown in Figures 14-16 and 17-19 respectively.

From the above study, a number of important observations have been made which has paved the way for further development. These observations are ***i***) Due to a damage in a structure, both frequency and mode shape changes with respect to an undamaged one, ***ii***) For a particular degree of damage, reduction in frequency of higher modes is more predominant than the lower ones, ***iii***) Location of damage has a considerable role on degree of reduction in natural frequency. For example,

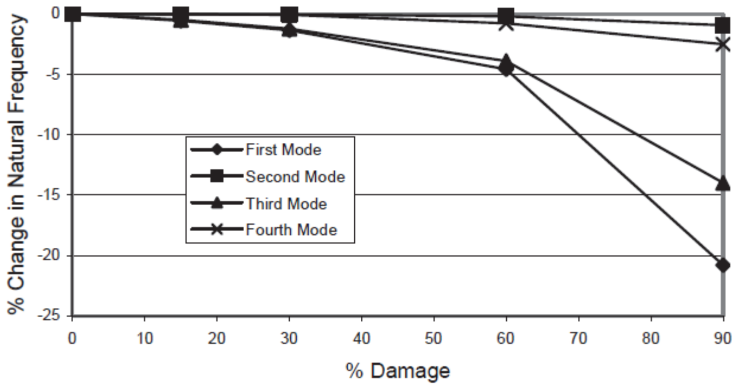

Figure 7: Percentage change in natural frequency due to damage in 7th element

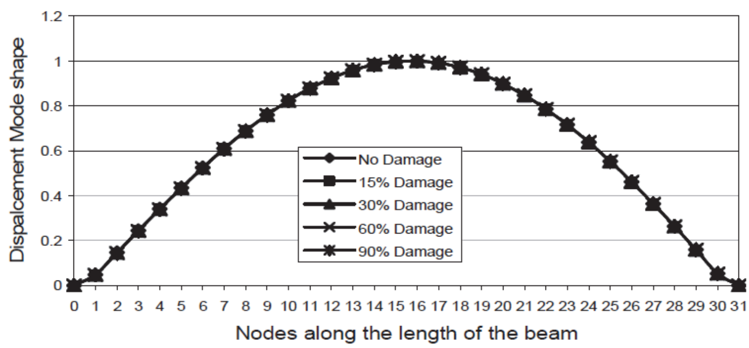

Figure 8: First Mode Shape (displacement) of the beam with damage of different magnitude in 2nd element

a beam with 90% damage will produce reduction of 0.35%, 9.28% and 20.82% in first mode frequency when damage is located at 2^{nd}, 7^{th} and 15^{th} element respectively (as shown in Figures 5-7), ***iv***) Location of damage towards centre of the beam will have more reduction in frequency than a location near support. However, it is significant to note that the location of damage, mode of vibration and reduction in frequency are inter-related, ***v***) It is observed from Figs. 8-13 that displacement mode shapes are more sensitive to damage than the shift in frequencies, ***vi***) Higher modes are more predominant in showing the shift in mode shape displacements due to damage in the structure, ***vii***) Shift in mode shape largely depends on the location of damage and the mode considered. In other words, higher mode will magnify the shift in mode shape provided that the damage location does not fall near the zero-

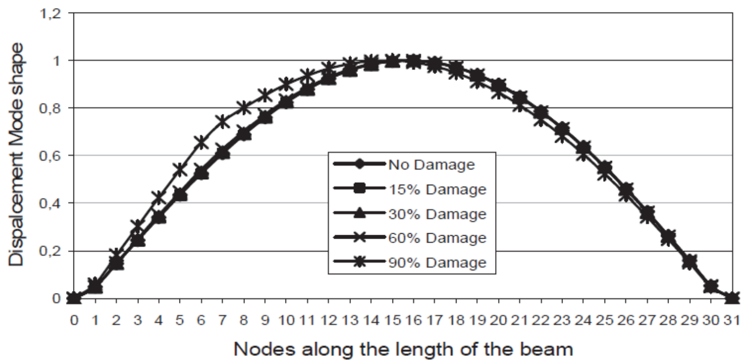

Figure 9: First Mode Shape (displacement) of the beam with damage of different magnitude in 7th element

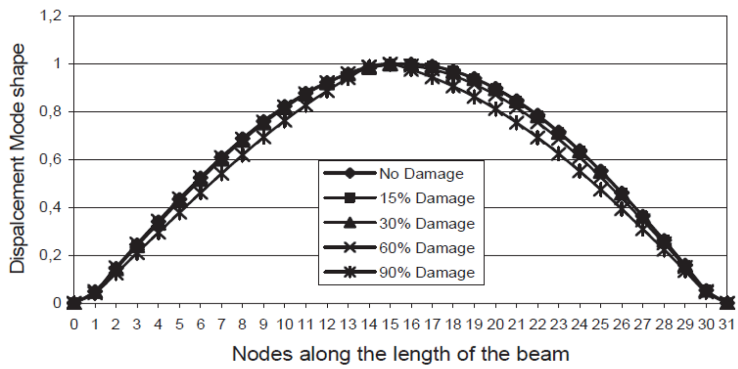

Figure 10: First Mode Shape (displacement) of the beam with damage of different magnitude in 15th element

displacement points, ***viii***) It is therefore significant to note that shift in mode shape can reflect the degree as well as the location of damage whereas frequency change can (if possible) only reflect the degree of damage, not location, unless treated otherwise, ***ix***) It is important to mention in this context that any shift in mode shape of a damaged structure with respect to the mode shape of undamaged mode shape may lead to an interpretation of a damage in that location but in many cases it may be wrong. For example, Fig. 10 shows the first mode shape (displacement) of the beam with damage of different magnitude in 15^{th} element (at middle of the beam) where shift in mode shape occurs towards supports, not at centre. Further, if the damage is located at a location where zero displacement occurs in that particular mode, shift in mode shape will be reflected in place other than the actual place

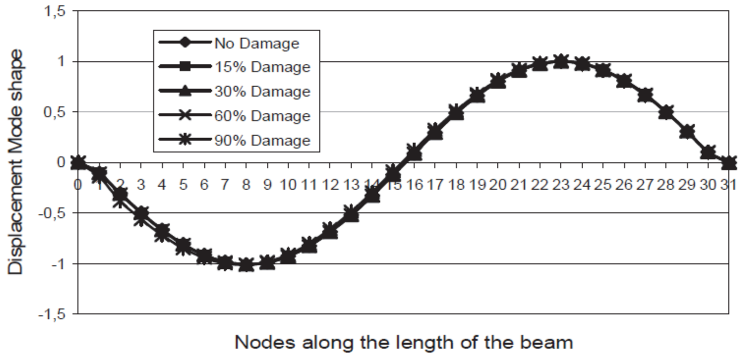

Figure 11: Second Mode Shape (displacement) of the beam with damage of different magnitude in 2nd element

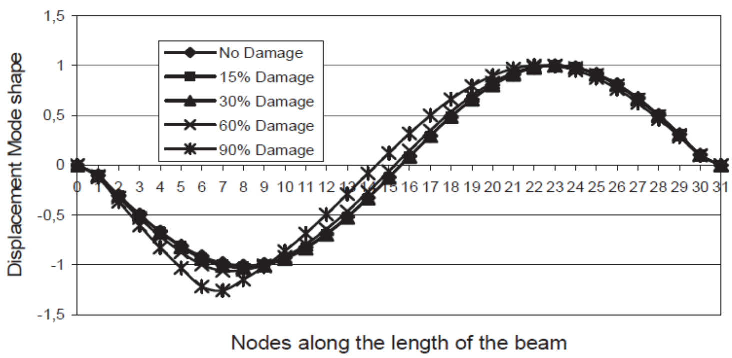

Figure 12: Second Mode Shape (displacement) of the beam with damage of different magnitude in 7th element

where damage has really taken place (Figure 13), ***x***) Though in higher modes, shift in mode shape is predominant than the lower modes, it may show a number of locations with shift in mode shape with respect to a undamaged mode shape which prompts to interpret them as the location of damages, but in reality it may be totally different, ***xi***) It is worthy to note that the rotation of mode shapes is highly sensitive to the damage and its location, as can be seen from Figures 14-19, and ***xii***) Considering a case with significant damage, the shift in rotation of mode shape for any mode is much more than that of a displacement mode shape, ***xiii***) Unlike displacement mode shape, rotational mode shape would not have a shift (with respect to a mode shape corresponding to an undamaged case) in many places in case of a single damaged condition. In other words, rotation of mode shape has the capability

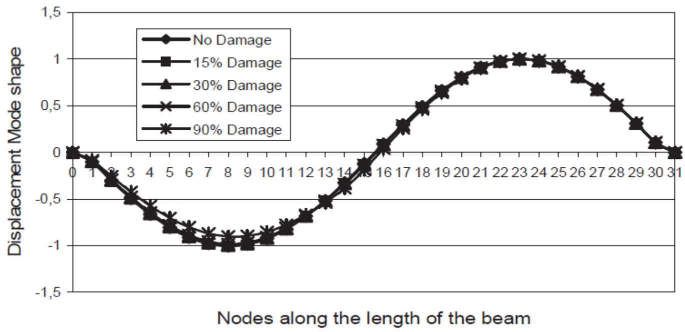

Figure 13: Second Mode Shape (displacement) of the beam with damage of different magnitude in 15th element

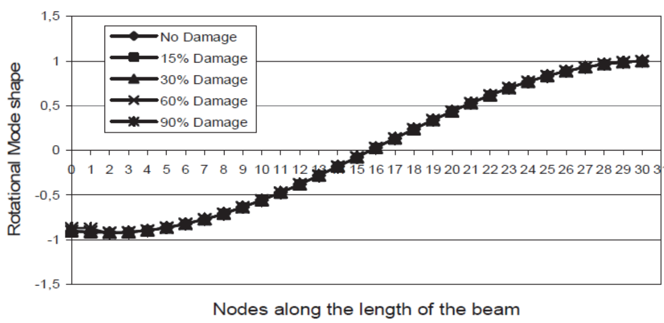

Figure 14: First Mode Shape (rotation) of the beam with damage of different magnitude in 2nd element

of detecting location of damage with better accuracy than the displacement mode shape.

The observations mentioned above have indicated that the rotational mode shape with a proper treatment can be used as an improved damage indicator. It is also understood that the consideration of few lower modes would reduce the influence of experimental error induced in a particular mode. Further, it is identified that usage of both frequency and rotation of mode shape information would bring more stability and generality to the proposed indicator for damage detection. Based on the above notes, the proposed method is formulated and presented in steps.

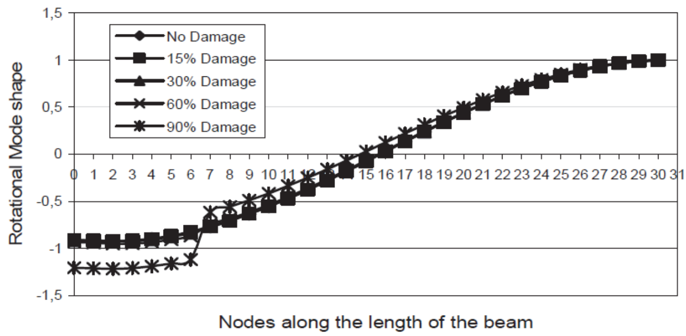

Figure 15: First Mode Shape (rotation) of the beam with damage of different magnitude in 7th element

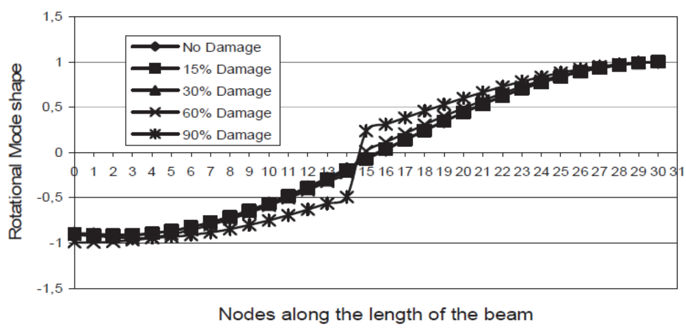

Figure 16: First Mode Shape (rotation) of the beam with damage of different magnitude in 15th element

4 Detection of multi-damage cases using rotational mode shape information

From the study discussed in the earlier section, it is understood that the information obtained from rotation of mode shape is very sensitive and more suitable for detection of damage in the structure. On the other hand, despite of its disadvantages, it is straight forward to appreciate that natural frequencies are easy to measure. Hence, in this study, a methodology has been proposed utilising both frequency (easy to measure and less prone to error) and rotational mode shape information (very sensitive to damage and its location), obtained from damaged structures for detecting its damage. It is also important to mention that in practical use, it is very difficult to presume the number of damages prior to analysis and multi-damage problem may arise in many cases. Hence, in this study, multi-damage cases with different

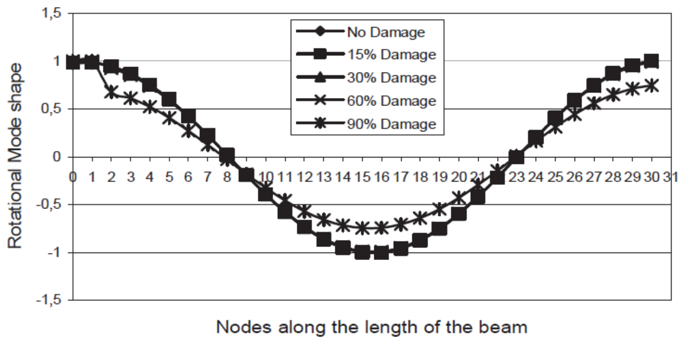

Figure 17: Second Mode Shape (rotation) of the beam with damage of different magnitude in 2nd element

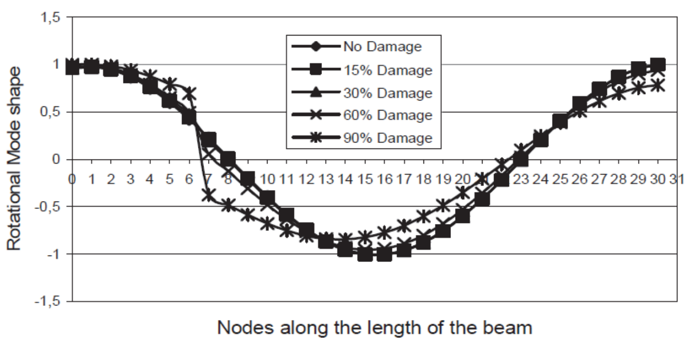

Figure 18: Second Mode Shape (rotation) of the beam with damage of different magnitude in 7th element

degrees of damage have been considered. To detect the damages, in this paper two damage indicators are proposed viz., curvature of rotational flexibility and curvature of uniform moment surface. The proposed methodologies and their efficacies are presented in the following sections.

4.1 Damage indicator using rotational flexibility

In this study, a methodology has been formulated based on flexibility matrix (Yan and Golinval, 2005) and uniform moment surface concept where both the mode shape and natural frequency at few lower modes are utilised to minimise any experimental error from a particular mode and application of both frequency and mode shape would claim better performance.

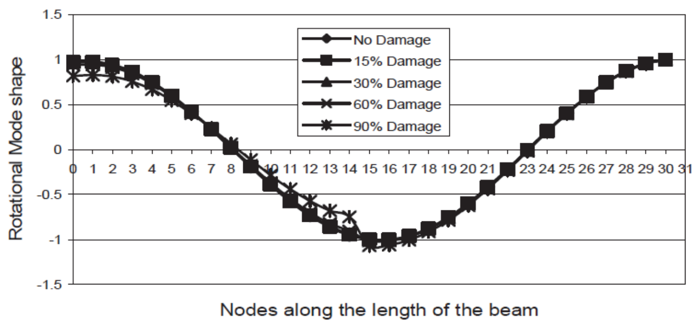

Figure 19: Second Mode Shape (rotation) of the beam with damage of different magnitude in 15th element

For a structural system with n degrees of freedom, the flexibility matrix can be expressed by superposition of the mass normalised rotational modes φ_r as

$$F = \sum_{r=1}^{n} \frac{\varphi_r \varphi_r^T}{\omega_r^2} \tag{19}$$

where ω_r is the r^{th} natural frequency. It is evident from Eq. (19) that the modal contribution to the flexibility matrix decreases rapidly as the frequency (ω_r) increases and ensures rapid convergence. So, the requirement of number of modes can be reduced that provides a practical implication as well. When n numbers of lower modes are available, the modal flexibility matrix of the structure can be approximated as

$$f_{i,j} = \sum_{r=1}^{n} \frac{\varphi_r(i)\varphi_r(j)}{\omega_r^2} \tag{20}$$

In which the modal flexibility, $f_{i,j}$, at the i^{th} point under the unit moment at point j is the summation of the products of two related modal coefficients for each available mode.

Further, the uniform moment surface (UMS) is defined as the rotation vector of the structure under uniform moment. For a linear system, the modal rotation at point i under uniform unit moment all over the structure can be approximated (for total y number of segment) as

$$u(i) = \sum_{seg=1}^{y} f_{i,j} = \sum_{r=1}^{n} \frac{\varphi_r(i) \sum\limits_{seg=1}^{y} \varphi_r(j)}{\omega_r^2} \tag{21}$$

As it is described in Eq. (21), the contribution from all the modal coefficients of the corresponding mode, makes UMS is less sensitive to measurement noise and rapid in convergence. These important properties make the UMS a potentially stable and sensitive indictor for structural damage detection and proposed in this study as a damage indicator. Further, the damage indicator has been enhanced by introducing Chebyshev polynomials (Ismail et al., 2006) for better efficiency and wide applicability.

4.2 Application of Chebyshev polynomial on damage indicator

To enhance the damage indicator (curvature of flexibility), the used Chebyshev polynomial is with r degrees, as

$$T_r(x) = \cos(r\cos^{-1} x) \tag{22}$$

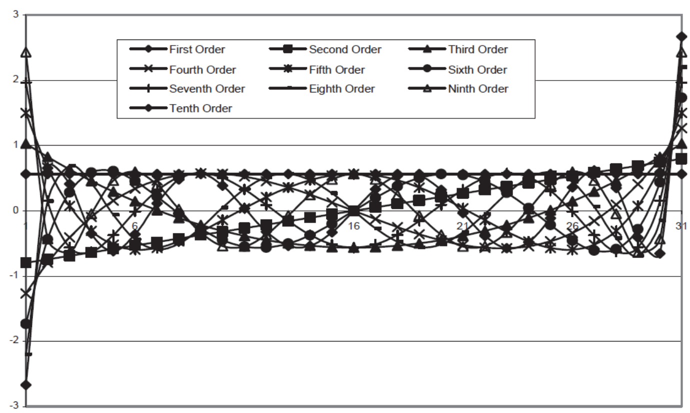

Figure 20: Chebyshev polynomials (shown upto 10th order only)

The uniform moment surface (UMS) estimated using Chebyshev polynomial is

$$u(x) = \sum_{p=1}^{order(p)} C_p T_p(x) \tag{23}$$

where $T_p(x)$ is the Chebyshev polynomial and p is its order. Chebyshev polynomials (for clarity upto 10^{th} order) are shown in Figure 20. C_i is the coefficient vector.

By making use of the orthogonal property of Chebyshev polynomial, the curvature of the UMS can be approximated as

$$\text{Curvature} = u''(x) = \sum_{p=1}^{p} c_p \frac{\partial T_p^2(x)}{\partial x^2} \tag{24}$$

The curvature of the uniform moment surface obtained from the formulation mentioned above can be used for detection of damage. Before proceeding further, it is to mention that modal displacement- and rotational- flexibilities are calculated from displacement and rotational mode shapes, respectively. First and second derivatives of the flexibility matrix provide the rotation and curvature of respective flexibility matrix.

4.3 Detection of damage using the proposed methodology

An attempt has been made to study the suitability of this methodology which uses the information of rotational mode shape and frequency, to detect multi-damages cases. In view of this, a simply supported beam structure is considered with the geometric and mechanical properties same as mentioned for single damage case, as discussed earlier (section 3.0). The simulated model is analysed by using transfer matrix method as formulated in this study. Three damages located at 0.2L, 0.5L and 0.8L is considered where L is the total unsupported length of the beam. Here, three locations are chosen in such a way that the influence of support zone, point of zero-rotation and the problems from symmetric damage locations (which can not be distinguished from frequency information alone) can be studied. From the study, as discussed in the preceding section, it is clear that rotation of mode shape information is very sensitive to damage and it is also noticed that for major damage cases (more than 50% damage), both displacement and rotational mode shape information is capable in identifying the damage location. But, in real practice, when large damages are already included in the structure, a sophisticated methodology for damage detection is not required; rather it can be located either by visual observation or simple inspection techniques. So, in this study, low levels of damages are considered to check the acceptability the methodology. Hence, in this study, low to very low level of damages, i.e. 30%, 10% and 1% is simulated at different locations (0.2L, 0.5L and 0.8L) of structure to study and verify the suitability and applicability of the methodology proposed in this study. It is to mention that a (considerably large) number of case studies have been presented here to ensure the efficiency and applicability and to find out the limitations (if any) of 'curvature of rotational flexibility' as a better damage indicator proposed in this study. It is important to note here that the very low level of damage such as 1%, as considered in this study, has no such physical significance for detection of damage because such a minor reduction in flexural rigidity (causing damage) might neither be a concern for a structural engineer nor practically possible to measure. But, in this study, such a low level of damage has been considered symbolically, just to check the performance of the proposed method in case of a very low level of damage, which is a challenging issue in this regard. Considering the possible availability of mode shape information from field experiments, first 3 modes are considered for the study. To check the superiority of the proposed methodology over the existing ones, damage cases with different location and magnitude, as mentioned earlier, has been compared with the established method based on curvature of modal flexibility using displacement information which is treated as one of the most efficient methods in identifying damage.

Curvature of modal flexibility using displacement mode shapes for first three modes of a beam with different degrees of damage (30% damage, 10% damage and 1% damage) has been shown in Figures 21-23, respectively. Then, curvature of modal flexibility using rotational mode shapes (which has been identified in this study as a better indicator of damage) for first three modes and curvature of UMS as proposed in this study using those three modes of a beam with different degrees of damage (30% damage, 10% damage and 1% damage) has been shown in Figures 24-26 and Figure 27, respectively.

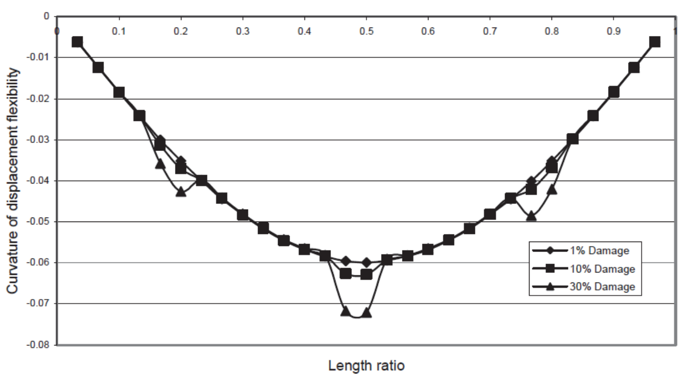

Figure 21: Curvature of first modal flexibility (displacement) with different magnitude of damage

From Figures 21 to 26, it is observed that curvature of rotation flexibility is considerably more sensitive than the curvature of displacement flexibility to detect the locations of damage. It is noted that the curvature of displacement flexibility can identify the case with 30% damage case. It is also to note that curvature of displacement flexibility is capable of identifying a low damage case (say, 10%) provided

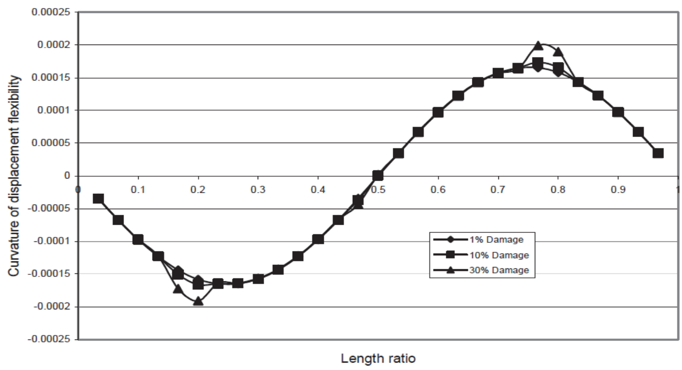

Figure 22: Curvature of second modal flexibility (displacement) with different magnitude of damage

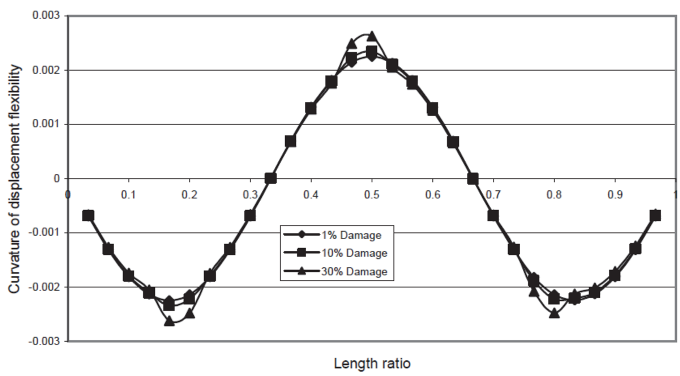

Figure 23: Curvature of third modal flexibility (displacement) with different magnitude of damage

that the damage occurs near the mid of the beam (as seen from Figures 21-23), whereas the curvature of rotation flexibility can even identify and locate very small damage (1%) cases as shown in Figures 24-26. It is significant to mention that curvature of rotational mode shape for individual modes is more sensitive and superior than displacement modes. On the other hand, use of number of available lower modes in UMS assures its stability. But, it is important to note that UMS may not provide sufficient indication of damage when any damage falls near mid zones (as shown in Figure 27). But, it can be solved easily because before calculating UMS, curvature of first rotational mode itself is sufficiently sensitive and capable in indicating any damage (Figure 24). Since it is not clear from the earlier figures

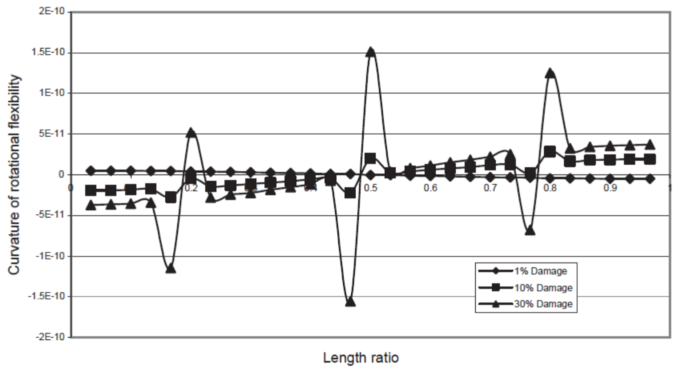

Figure 24: Curvature of first modal flexibility (rotational) with different magnitude of damage

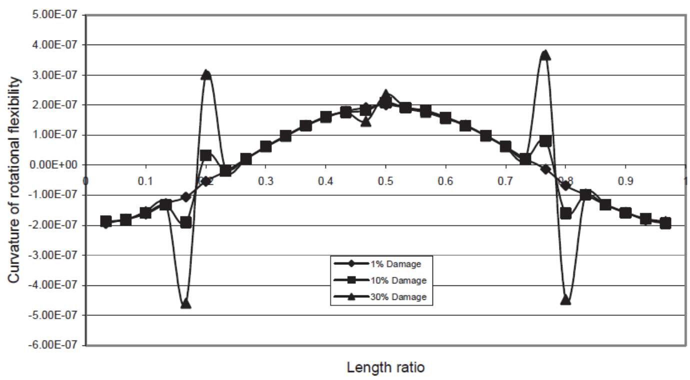

Figure 25: Curvature of second modal flexibility (rotational) with different magnitude of damage

(Figures 21-23 and Figures 24-26), ability in detecting very low level of damages (i.e. 1% damage) by using curvature of displacement flexibility and curvature of rotational flexibility for different modes are shown in Figures 28-29, respectively. The figures prove the superiority of curvature of rotational flexibility as s damage indicator in comparison with curvature of displacement flexibility. So, by judicious instrumentation and from available experiment data, the proposed methodology using information from a few lower modes (or even first mode information alone) from a damaged structure can identify a very low damage cases. It is also important to mention here that, for identifying damage the methodology proposed in this

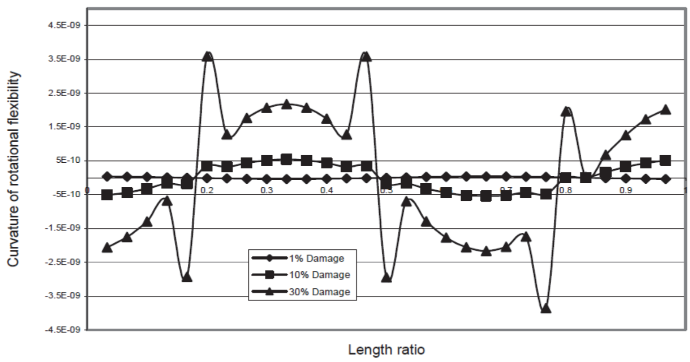

Figure 26: Curvature of third modal flexibility (rotational) with different magnitude of damage

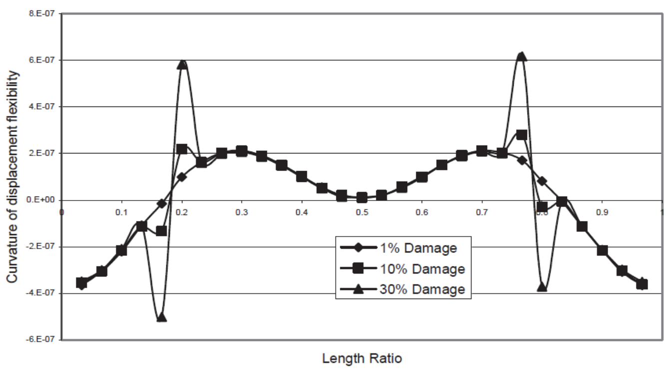

Figure 27: Curvature of uniform moment surface obtained from first three modes

study does not require any information on the intact (undamaged) state of the structure. It is important to mention here that though rotational curvature corresponding to the particular mode is able to identify the damage locations correctly, uniform moment surfaces would also provide an important and significant information by considering number of possible modes.

4.4 Influence of noise

All the case studies conducted in the earlier section have been carried out with the accurate measured frequencies (free from any noise) from different modes. Though, it is always attempted to obtain noise free experimental data, which would be best suited for the detection of damage, but, in most of the cases, there is a

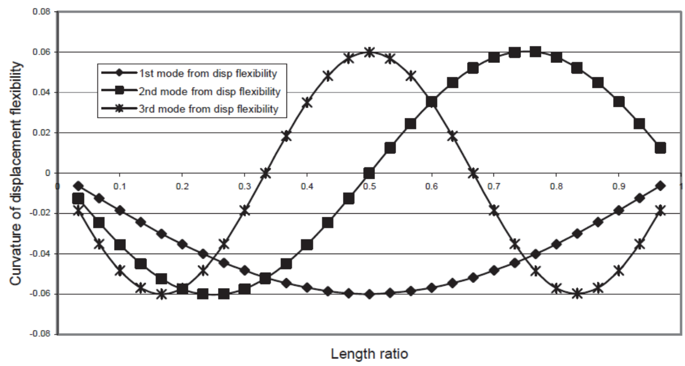

Figure 28: Detection of very small damage (1%) using curvature of displacement flexibility

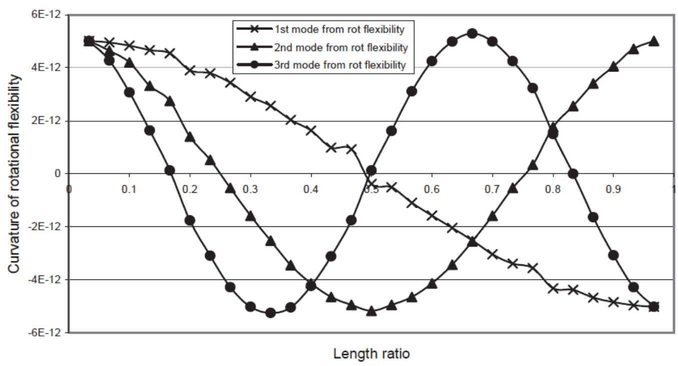

Figure 29: Detection of very small damage (1%) using curvature of rotational flexibility

possibility to contain a certain level of noise in the measured frequencies. Hence, further case studies are conducted to check the feasibility of the methodology by considering measured data with different levels of noise (1%, 2%, 5% and 10%) for different degrees of damage at different locations (same as considered before, i.e. 0.2L, 0.5L and 0.8L). As considered earlier, three representative levels of damage, 30% (medium damage), 10% (low damage) and 1% (very low damage) have been considered to study the efficacy of the proposed methodology. Figures 30-32 show the damage locations for different damage conditions measured from the vibration data which contains certain level of noise. It is noted that the influence of noise is increasingly significant for lower levels of damage. Nevertheless, it is evident from the figures that the proposed methodology is able to correctly identify the locations of damage by using vibration data with noise, which proves that superior character of rotational mode shape, further enhanced by using curvature, in identifying damage.

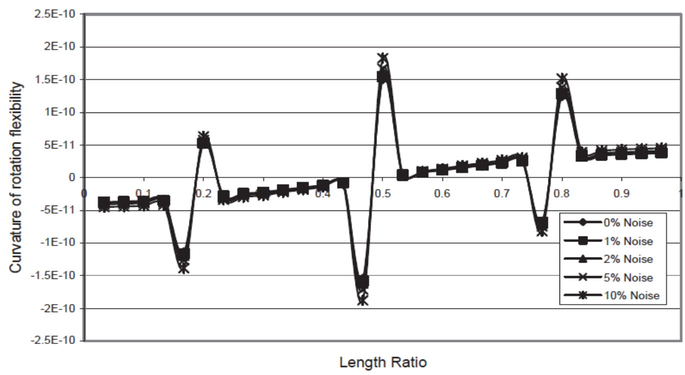

Figure 30: Influence of noise level on detection of damage magnitude 30%

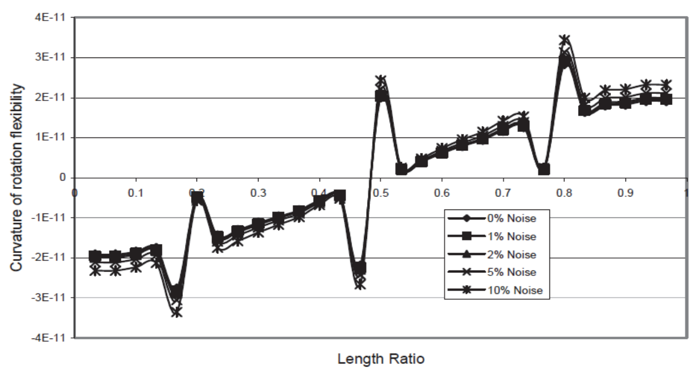

Figure 31: Influence of noise level on detection of damage magnitude 10%

5 Concluding Remarks

As it is noticed from the existing literature that both frequency and mode shape information of a structure can be used for detecting damage, a detailed and systematic study has been carried out to evaluate the efficiency of vibration characteristics,

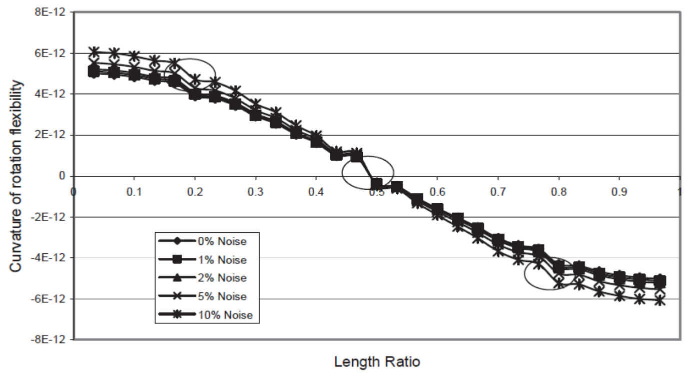

Figure 32: Influence of noise level on detection of damage magnitude 1%

like, frequency, displacement- and rotational- mode shapes in detecting damage in a structure. It is observed from the present study that displacement mode shapes are sensitive to damage and higher modes are more predominant in showing the shift in mode shape displacements due to damage in the structure, whereas a large damage produces a very small shift in frequency. Shift in mode shape largely depends on the location of damage. It is also noticed that the rotational mode shape information is much more sensitive than displacement mode shape information. It is further observed that the curvature of rotational mode shape is capable in detecting damage with greater accuracy than that obtained from displacement mode shape. Hence, a methodology for detection of multi-damage in structures using transfer matrix technique has been proposed based on natural frequencies and rotational mode shape information of few lower modes. Uniform moment surface (UMS) has been considered as the damage indicator calculating using curvature of rotational mode shapes. In this study, instead of finite difference method, to arrive at the curvature, Chebyshev polynomials are used for improving the sensitivity. A detailed study has been carried out to compare the sensitivity of curvature of displacement flexibility (an established method) to that of UMS from curvature from rotational mode shape (as proposed in this study) in detecting damage. It is found that though for a large damage case, both existing and proposed methods are capable in detecting damage, but for small and multi-damage cases, the proposed method using curvature of rotational mode shape and the UMS is better capable in indicating damage and its location(s). It is important to mention that by judicious instrumentation and from available experiment data, the proposed methodology using information from a few lower modes (or even first mode information alone) from a damaged structure can identify a very low damage(s) accurately without any requirement of information on the undamaged state of the structure.

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