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Output-only System Identification and Damage Assessment through Iterative Model Updating Techniques

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Abstract: Model updating may be defined as an adjustment on the FE model through modal parameters experimentally obtained, in order to better represent its dynamic behavior. From this definition, structural health monitoring (SHM) methods can be considered closely related with these procedures, because it refers to the implementation of in situ non-destructive sensing and analysis of the dynamic system characteristics, which aims to detect changes that could indicate damage. Within this context, the present paper evaluates an iterative model updating approach when it is subjected to experimental vibration data. In addition, after getting the experimental adjusted model, a numerical damage detection procedure is also proposed. Since in ambient vibrations situations, it is only available the structural response, particular emphasis is given on output-only system identification. Two damage cases are created and dynamic tests are numerically simulated with a varying added measurement noise. In order to localize and quantify the damage, an index (damage detection index - DDI) is also proposed in this paper. It can be noticed that the obtained results were very accurate, and the proposed index DDI was able to correctly localize and quantify the damage in all situations, even considering an output only system identification procedure, noise presence and multiple damage case.

Keywords: Model Updating, Indirect Methods, System Identification.

1 Introduction

Reliable mathematical models are essential to assess the integrity or to control vibrations of structures subjected to dynamic loads. This requires the comparison of experimental results with predictions of the model under consideration. Since this process typically leads to differences between the theoretically predicted and the experimentally determined structural responses, a number of schemes known as

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model updating techniques were proposed to modify the properties of the numerical model in order to achieve an optimum fit with the experimental data (Mottershead and Friswell, 1993).

Model updating techniques in Structural Dynamics may be divided into two main groups, namely the direct and the iterative methods. In the first group, the model is expected to match some reference data, usually consisting of an incomplete set of eigenvalues and eigenvectors derived from field or laboratory measurements. These direct methods are also known as representation models because they are able to replicate the measured data, but their main drawbacks are the need for high quality measurements as well as very accurate modal analysis to allow the expansion of the experimentally obtained mode shapes to include all the components of the finite element model. The iterative updating methods aims at improving the correlation between the experimental and analytical models via a penalty function. Because of the general nature of penalty functions, the problem has to be linearized and thus optimized iteratively. Since the penalty function is usually non-linear, the iterations may not converge. In any case, iterative methods have two main advantages: first, a wide range of parameters may be updated simultaneously and second, both measured and analytical data can be weighted, a feature that allows the introduction of subjective judgment in the numerical procedure.

Within this context, structural health monitoring (SHM) procedures, which have undergone significant progress in the last decade, are closely related to model updating methods. In SHM the modal parameters extracted from dynamic tests during the lifetime of the structure, under different operating conditions, are compared with reference modal parameters corresponding to a model of the structure in an undamaged condition [e.g. Kaminski Jr. and Riera (1996); Doebling, Farrar, Prime and Shevitz (1996); Riera and Rios (2000); Sohn, Farrar, Hemez, Shunk, Stinemates, Nadler and Johnson (2003); Riera (2004); Amani, Riera and Curadelli (2006); Fadel Miguel, Miguel, Riera and Ramos de Menezes (2007); Brasiliano, Souza, Doz and Brito (2008)]. Then, if differences in the identified modal parameters are found, they may lead to the identification of structural damage.

Hence, the process of locating and quantifying damage may be thought as part of a model updating procedure, in which the main goal is to establish a mathematical model that matches the measured structural response. In certain structural systems in which the influence of damping may be neglected in the identification process, the updating approach would lead to the identification, in the original stiffness matrix, of the components corresponding to the generalized coordinates (which define the configuration of the damaged elements) that were affected by damage. Nevertheless, it is germane to call attention of the reader to the fact that determination of changes in the stiffness matrix alone may in certain structural systems be insufficient to quantify damage [Riera (2004); Curadelli, Riera, Ambrosini and Amani (2008)].

In this context, the present paper describes an iterative model updating technique based on a penalty function applicable to situations in which experimental vibration data is available. After experimentally adjusting the model, a numerical damage detection study is carried out and a damage detection index - DDI is determined. The approach is illustrated with a simple reduced scale one-bay, three-stories high plane frame. First, the results of a finite element model are compared with experimental values, which are used to conduct the updating procedure proposed herein. In a second stage, employing the experimentally adjusted model for the undamaged condition, two cases of damage are introduced by reducing selected stiffness coefficients and dynamic tests are numerically simulated considering the presence of simulated measurement noise. An output-only system identification procedure, namely the stochastic subspace identification (SSI) method is resorted to for this purpose [Van Overschee and de Moor (1993); Peeters and de Roeck (1999); Fadel Miguel, Miguel, Ramos de Menezes, Kaminski Jr. (2006)]. After the spectral properties are determined, the structure is updated again and the damage detection index - DDI evaluated, thus completing the process of localization and quantification of damage in the example.

The paper is organized as follows: first, the so-called stochastic subspace system identification (SSI) method is introduced, describing its methodology. Next, a comprehensive survey of the literature related to iterative updating techniques based on penalty functions is presented. Finally, in order to assess the updating procedure as well as the proposed damage detection approach, a simple illustrative example is presented. It is shown that, at least in the case considered, the parameters and frequencies converge with few iterations and the proposed damage index is able to correctly localize and quantify damage.

2 Subspace System Identification Method (SSI)

The identification method considers a discrete-time state-space model, discussed by Peeters and de Roeck (1999), described by eqs. (1):

$$\begin{aligned} \vec{x}(k+1) &= \mathbf{A}\vec{x}(k) + \vec{w}(k) \\ \vec{y}(k) &= \mathbf{C}\vec{x}(k) + \vec{v}(k) \end{aligned} \tag{1}$$

in which k denotes the discrete time instant such that $t = k\Delta t$, $\vec{x}(k)$ is the state vector, $\vec{y}(k)$ the output vector, A is known as the state matrix and C is the output matrix. The vectors $\vec{w}(k)$ and $\vec{v}(k)$ represent the noise due to disturbances and modeling inaccuracies and measurement noise, respectively.. They are both not measurable vector signals assumed to be zero mean, white noise type vectors with covariances matrices defined in eqs. (2):

$$E\left[\begin{pmatrix}\vec{w}_p \\ \vec{v}_p\end{pmatrix}\begin{pmatrix}\vec{w}_q^T & \vec{v}_q^T\end{pmatrix}\right] = \begin{pmatrix}\mathbf{Q} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R}\end{pmatrix}\delta_{pq} \tag{2}$$

in which E[ ] denotes the expected value operator, δ_{pq} is the Kronecker delta and p,q are two arbitrary time instants. In that sense, in the ensuing stochastic representation the input is implicitly modeled by the noise terms assumed stationary white noise with zero mean.

The main property of those systems indicates that the output covariances can be considered as Markov parameters of the deterministic linear time-invariant system, constituting the solution to the stochastic identification problem: the output covariance sequence can be estimated from the measurement data; so, if the estimated output covariance sequence can be decomposed in a similar way, the state-space matrices are found. Starting from this idea, some identification methods were proposed.

However, due its formulation, the stochastic subspace identification method (SSI) avoids this previous computation of covariances between the outputs. It is replaced by projecting the row space of future outputs into the row space of past outputs. The idea behind this projection, which apply robust numerical techniques such as QR factorization, is to retain from the past all the information that is useful to predict the future.

It is useful in the development of the SSI method to gather the output measurements in a block Hankel matrix with $2i$ block rows and N columns, in which N is the number of time samples. The first i blocks have r rows, the last i have l rows. The Hankel matrix can be divided into a past reference and a future part, given in eq. (3):

$$\mathbf{H}^{ref} = \frac{1}{\sqrt{N}}\left[\frac{\begin{bmatrix} y_0^{ref} & y_1^{ref} & \cdots & y_{N-1}^{ref} \\ y_1^{ref} & y_2^{ref} & \cdots & y_N^{ref} \\ \vdots & \vdots & \ddots & \vdots \\ y_{i-1}^{ref} & y_i^{ref} & \cdots & y_{i+N-2}^{ref}\end{bmatrix}}{\begin{bmatrix} y_i & y_{i+1} & \cdots & y_{i+N-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+N-2}\end{bmatrix}}\right] = \left[\frac{\mathbf{Y}_{0|i-1}^{ref}}{\mathbf{Y}_{i|2i-1}}\right]$$

$$= \begin{bmatrix} \mathbf{Y}_p^{ref} \\ \hline \mathbf{Y}_f \end{bmatrix} \begin{bmatrix} \updownarrow \\ \updownarrow \end{bmatrix} \begin{bmatrix} ri \\ li \end{bmatrix} \frac{\text{"past"}}{\text{"future"}} \tag{3}$$

Another division is obtained by adding one block row to the past references and omitting the first block row of the future outputs:

$$\mathbf{H}^{ref} = \begin{bmatrix} \mathbf{Y}_{0|i}^{ref} \\ \hline \mathbf{Y}_{i|i} \\ \hline \mathbf{Y}_{i+1|2i-1} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_p^{ref+} \\ \hline \mathbf{Y}_{i|i}^{\sim ref} \\ \hline \mathbf{Y}_f^- \end{bmatrix} \begin{bmatrix} \updownarrow \\ \updownarrow \\ \updownarrow \end{bmatrix} \begin{bmatrix} r(i+1) \\ l-r \\ l(i-1) \end{bmatrix} \tag{4}$$

As previously mentioned, the projections play an important role in the stochastic subspace system identification. The notation and definition of this projection is (in which $()^\dagger$ denotes the Moore-Penrose pseudo-inverse of a matrix):

$$\mathbf{P}_i^{ref} \equiv \mathbf{Y}_f/\mathbf{Y}_p^{ref} \equiv \mathbf{Y}_f(\mathbf{Y}_p^{ref})^T(\mathbf{Y}_p^{ref}(\mathbf{Y}_p^{ref})^T)^\dagger \mathbf{Y}_p^{ref} \tag{5}$$

Introducing the QR factorization in the Hankel matrix eq. (3) on eq. (5) yields a very simple expression for the projections $\mathbf{P}_i^{ref}$:

$$\mathbf{P}_i^{ref} = \begin{bmatrix} \mathbf{R}_{21} \\ \mathbf{R}_{31} \\ \mathbf{R}_{41} \end{bmatrix} \mathbf{Q}_1^T \in R^{lixN} \tag{6}$$

The main theorem of stochastic subspace identification states that the projection $\mathbf{P}_i^{ref}$ can be factorized as the product of the extended observability matrix $\mathbf{O}_i$ and the Kalman filter state sequence $\vec{X}_i$.

$$\mathbf{P}_i^{ref} = \mathbf{O}_i\vec{X}_i \equiv \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdots \\ \mathbf{CA}^{i-l} \end{bmatrix} \begin{bmatrix} \vec{x}_i & \vec{x}_{i+1} & \ldots & \vec{x}_{i+N-1} \end{bmatrix} \updownarrow n \tag{7}$$

The projection matrix has rank n because it is the product of a matrix with n columns and a matrix with n rows shown by Eq. (7). A reliable tool to numerically evaluate the rank of a matrix is the singular value decomposition (SVD). After omitting the zero singular values and corresponding singular vectors, the application of the SVD to the projection matrix can be carried out in which the extended observability matrix and the Kalman filter state sequence are obtained by splitting this decomposition in two parts:

$$\begin{aligned} &\mathbf{P}_i^{ref} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^T \\ &\mathbf{O}_i = \mathbf{U}_1\mathbf{S}_1^{1/2} \\ &\vec{X}_i = \mathbf{O}_\mathbf{i}^\dagger \mathbf{P}_i^{ref} \end{aligned} \tag{8}$$

in which $\mathbf{U}_1 \in R^{lixn}$ and $\mathbf{V}_1 \in R^{Nxn}$ are othonormal matrices and $\mathbf{S}_1 \in \left(R_0^+\right)^{nxn}$ is a diagonal matrix containing the positive singular values in descending order. In order to obtain the system matrices, two different algorithms can be used:

Algorithm 1:

Using the Hankel matrix, another projection can be defined:

$$\mathbf{P}_{i-1}^{ref} \equiv \mathbf{Y}_f^- / \mathbf{Y}_p^{ref+} = \begin{bmatrix}\mathbf{R}_{41} & \mathbf{R}_{42}\end{bmatrix} \begin{bmatrix}\mathbf{Q}_1^T \\ \mathbf{Q}_2^T\end{bmatrix} = \mathbf{O}_{i-1}\hat{\mathbf{X}}_{i+1} \tag{9}$$

The extended observability matrix $\mathbf{O}_{i-1}$ is obtained after rejecting the last l rows of $\mathbf{O}_i$, and the state sequence $\hat{\mathbf{X}}_{i-1}$ can be computed as:

$$\vec{X}_{i+1} = \mathbf{O}_{\mathbf{i}-1}^{\dagger}\mathbf{P}_{i-1}^{ref} \tag{10}$$

The system matrices can now be evaluated from following set of linear equations:

$$\begin{bmatrix}\mathbf{A} \\ \mathbf{C}\end{bmatrix} = \begin{bmatrix}\vec{X}_{i+1} \\ \mathbf{Y}_{i|i}\end{bmatrix} \vec{X}_i^{\dagger} \tag{11}$$

Algorithm 2:

The system matrices can be directly determined from the extended observability matrix given in Eq. (8). Using MATLAB notation it is possible to determine the state matrix (in which *pinv* denotes the Moore-Penrose pseudo-inverse of a matrix):

$$\mathbf{A} = pinv(\mathbf{O}_i(1:l(i-1),:))\mathbf{O}_i(l+1:li,:) \tag{12}$$

And the output matrix:

$$\mathbf{C} = \mathbf{O}_i(1:l,:) \tag{13}$$

After obtained the system matrices they should be used for a modal analysis of the structure. The dynamic behavior is characterized by their eigenvalues and eigenvectors through its transformation for the continuous time:

$$\mathbf{A} = \mathbf{\Psi}\mathbf{\Lambda}\mathbf{\Psi}^{-1}, \quad \mathbf{\Lambda} = diag(\lambda_q), \quad \mathbf{\Psi}_c = \mathbf{\Psi}, \quad \lambda = \frac{\ln(\lambda_q)}{\Delta t} \tag{14}$$

in which $\mathbf{\Lambda} = diag(\lambda_q)$ is a diagonal matrix containing the discrete-time eigenvalues. The eigenvalues occur in complex conjugated pairs and they can be written as:

$$\lambda_{c_q}, \lambda_{c_q}^* = -\xi_q\omega_q \pm j\omega_q\sqrt{1-\xi_q^2} \tag{15}$$

The modal shapes at sensor locations are the observed parts of the eigenvectors of the system $\boldsymbol{\Psi}$ and they are obtained in the following way:

$$\boldsymbol{\phi} = \mathbf{C}\boldsymbol{\Psi} \tag{16}$$

In this way, the modal parameters ω_q, ξ_q and Φ_q are obtained from the identified system matrices, $\boldsymbol{A}$ and $\boldsymbol{C}$. There are several variants of stochastic subspace identification, differing in the multiplication of a weight function in the projection matrix $\mathbf{P}_i^{ref}$, before the singular value decomposition, determining the state-space basis in which the model will be identified:

$$\mathbf{W}_1\mathbf{P}_i^{ref}\mathbf{W}_2 = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{17}$$

Three versions of the stochastic subspace system identification method (SSI) are presented: PC (principal component), UPC (unweighted principal component) and CVA (canonical variate algorithm) [Van Overschee and de Moor (1993)]. Table 1 shows the weight functions of these variants, in which $\boldsymbol{I}$ is the identity matrix.

Table 1: Weight functions.

	$\boldsymbol{W}_1$	$\boldsymbol{W}_2$
PC	$\boldsymbol{I}$	$\mathbf{Y}_p^{ref}\left[\mathbf{Y}_p^{ref}\mathbf{Y}_p^{ref^T}\right]^{-1/2}\mathbf{Y}_p^{ref}$
UPC	$\boldsymbol{I}$	$\boldsymbol{I}$
CVA	$\left[\mathbf{Y}_f\mathbf{Y}_f^T\right]^{-1/2}$	$\boldsymbol{I}$

3 Iterative Updating Techniques Based on Penalty Functions

Penalty function methods express the modal data as a function of the unknown parameters using a truncated Taylor series expansion. The series is truncated to yield the linear approximation:

$$\delta\mathbf{z} = \mathbf{S}\delta\boldsymbol{\theta} \tag{18}$$

in which $\delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}_j$, $\boldsymbol{\theta}_j$ is the current value of the parameter vector, $\boldsymbol{\theta}$ is the estimated vector, $\delta\mathbf{z} = \mathbf{z}_e - \mathbf{z}_j$, $\mathbf{z}_e$ is the measured output, $\mathbf{z}_j$ is the current estimate of the output, $\mathbf{S}$ is the sensitivity matrix containing the first derivative of the eigenvalues and mode shapes with respect to the parameters, evaluated at the current parameter estimate $\boldsymbol{\theta}_j$.

Calculate those first derivatives of the eigenvalues and mode shapes with respect to the parameters is computationally intensive and efficient methods for their computation are required. Fox and Kapoor (1968) calculated the derivative of the *ith* eigenvalue, λ_i, with respect to the *jth* parameter, θ_j, by taking the derivative of the eigenvector equation, to give:

$$\left(\frac{\delta \mathbf{K}}{\delta \theta_j} - \lambda_i \frac{\delta \mathbf{M}}{\delta \theta_j}\right) \boldsymbol{\phi}_i - \frac{\delta \lambda_i}{\delta \theta_j} \mathbf{M} \phi_i + (\mathbf{K} - \lambda_i \mathbf{M}) \frac{\delta \boldsymbol{\phi}_i}{\delta \theta_j} = 0 \tag{19}$$

Pre-multiplying by the transpose of the eigenvector, φ_i, and using mass orthogonality and the original definition of the eigensystem produces:

$$\frac{\delta \lambda_i}{\delta \theta_j} = \boldsymbol{\phi}_i^T \left(\frac{\delta \mathbf{K}}{\delta \theta_j} - \lambda_i \frac{\delta \mathbf{M}}{\delta \theta_j}\right) \boldsymbol{\phi}_i \tag{20}$$

These authors have also suggested two methods for calculating the first derivative of the eigenvectors. Lim (1987) suggested an approximate method for calculating the first derivative of the eigenvectors which is only valid for the low frequency modes. Other methods for calculating mode shapes derivatives have been suggested by Chu and Rudisill (1975), Ojalvo (1987) and Tan and Andrew (1989).

The penalty functions methods differ in the choice of design parameters and the definition of optimization constraints. Design parameters such as individual elements of the mass and stiffness matrices, sub-matrices, geometric or material properties can be defined. Constraints are usually imposed on natural frequencies and mode shapes.

Usually, the number of design parameters and measurements is not equal and hence the matrix **S** in (18) is not square. The case in which there are more design parameters than measurements was considered by Chen and Garba (1980). The parameter vector closest to the original analytical parameters was sought which reproduce the required measurement change. They found the solution to the problem by seeking a set of design parameters by minimizing the norm as an additional constraint equation:

$$Q = \sum_j \Delta \boldsymbol{\theta}_j^2 \tag{21}$$

Similarly, the SVD technique was used by Hart and Yao (1977) and Ojalvo, Ting, Pilon, Twomey (1989) for a case with less design parameters than measurements. The solution of equation (18) can be calculated by minimizing the penalty function:

$$J(\delta \boldsymbol{\theta}) = (\delta \mathbf{z} - \mathbf{S} \delta \boldsymbol{\theta})^T (\delta \mathbf{z} - \mathbf{S} \delta \boldsymbol{\theta}) \tag{22}$$

in which $\varepsilon = \delta\mathbf{z} - \mathbf{S}\delta\boldsymbol{\theta}$ is the error in the predicted measurements based on the updated parameters. Differentiating J with respect to $\delta\boldsymbol{\theta}$ and setting the result equal to zero, it can be shown that the solution is given by:

$$\delta\boldsymbol{\theta} = \left[\mathbf{S}^T\mathbf{S}\right]^{-1}\mathbf{S}^T\delta\mathbf{z} \tag{23}$$

and an updated estimate of the unknown design parameter vector is obtained by:

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \delta\boldsymbol{\theta}_j \tag{24}$$

In practical situations, all measured data does not have the same accuracy. Usually, mode shape data are less accurate than natural frequency data. Also the higher natural frequencies are not measured as accurately as the lower ones. The relative accuracy of measured data can be incorporated into the updating process by including a diagonal positive definite weighting matrix $\mathbf{W}_{\varepsilon\varepsilon}$, whose elements are given by the reciprocals of the variance of the corresponding measurements. Equation (22) becomes:

$$J(\delta\boldsymbol{\theta}) = (\delta\mathbf{z} - \mathbf{S}\delta\boldsymbol{\theta})^T\mathbf{W}_{\varepsilon\varepsilon}(\delta\mathbf{z} - \mathbf{S}\delta\boldsymbol{\theta}) \tag{25}$$

The minimization of equation (25) yields:

$$\delta\boldsymbol{\theta} = \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S}\right]^{-1}\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\delta\mathbf{z} \tag{26}$$

or in full,

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S}\right]^{-1}\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}(\mathbf{z}_m - \mathbf{z}_j) \tag{27}$$

In either solution - equation (23) or equation (27) - the number of measurements was assumed to be larger than the number of parameters. Under this assumption the matrix is square being full rank, so the equations may be solved. However, in almost all practical cases this situation will not occur, i.e., the number of unknown parameters will exceed the number of measured data points. Due to this problem $\mathbf{S^TS}$ will be rank deficient because the number of equations in equation (1) is less than the number of unknowns. An alternative approach (Natke,1988) is to add an extra term to minimize the change of the design parameters. The extended weighted penalty function can be expressed as:

$$J(\delta\boldsymbol{\theta}) = \varepsilon^T\mathbf{W}_{\varepsilon\varepsilon}\varepsilon + \delta\boldsymbol{\theta}^T\mathbf{W}_{\theta\theta}\delta\boldsymbol{\theta} \tag{28}$$

in which again $\varepsilon = \delta\mathbf{z} - \mathbf{S}\delta\boldsymbol{\theta}$ is the error in the predicted measurements based on the updated parameters. Next, it is added a positive definite weighting matrix $\mathbf{W}_{\theta\theta}$

chosen to be a diagonal matrix with the reciprocals of the estimated variances of the corresponding parameters as the elements. These variances are not an easy task and some engineering insight is required. The solution of $\delta\boldsymbol{\theta}_j$ is given by:

$$\delta\boldsymbol{\theta} = \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S} + \mathbf{W}_{\theta\theta}\right]^{-1}\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\delta\mathbf{z} \tag{29}$$

or in full,

$$\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S} + \mathbf{W}_{\theta\theta}\right]^{-1}\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\left(\mathbf{z}_m - \mathbf{z}_j\right) \tag{30}$$

A similar approach to obtaining a well conditioned set of equations is to weight the initial estimates of the unknown parameters. This more accurately reflects the engineer's desire to weight the change in parameter from the initial estimated values, rather than the parameter change at every iteration. Thus, the new penalty function is given by:

$$J\left(\delta\boldsymbol{\theta}\right) = \varepsilon^T\mathbf{W}_{\varepsilon\varepsilon}\varepsilon + \left(\boldsymbol{\theta} - \boldsymbol{\theta}_0\right)^T\mathbf{W}_{\theta\theta}\left(\boldsymbol{\theta} - \boldsymbol{\theta}_0\right) \tag{31}$$

in which the solution is:

$$\delta\boldsymbol{\theta} = \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S} + \mathbf{W}_{\theta\theta}\right]^{-1}\left(\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\delta\mathbf{z} - \mathbf{W}_{\theta\theta}\left(\boldsymbol{\theta}_j - \boldsymbol{\theta}_0\right)\right) \tag{32}$$

or in full,

$$\begin{array}{l}\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \left[\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\mathbf{S} + \mathbf{W}_{\theta\theta}\right]^{-1} \\ \left(\mathbf{S}^T\mathbf{W}_{\varepsilon\varepsilon}\left(\mathbf{z}_m - \mathbf{z}_j\right) - \mathbf{W}_{\theta\theta}\left(\boldsymbol{\theta}_j - \boldsymbol{\theta}_0\right)\right)\end{array} \tag{33}$$

A comprehensive literature survey concerning iterative model updating may be found in Mottershead and Friswell (1993).

After the model updating, a finite element model adjusted for the healthy condition of the structure has been determined. The damage assessment may be thought as an extension of this procedure, in which the modal parameters for the current state (and possible a damage state) of the structure that are extracted from dynamic tests during its lifetime and under different operating conditions, can be used as the starting point for the model updating approach.

Within this context, if a structure is damaged the updating procedure will establish a mathematical model that matches the measured structural parameters for this condition. Thus, the damage detection approach can be carried out through a direct comparison between the stiffness matrices on healthy and damaged states, i.e., the main goal is to identify on the original stiffness matrix the degrees of freedom (related to the damaged elements) that were changed due to damage.

While the damaged element (or elements) may be located observing the relation between changes on the stiffness matrix and its correspondent connectivity, the damage quantification can be carried out evaluating the amount of rigidity loss through the two structural conditions.

4 Application: Shear Building Plane Frame

The iterative model updating and the subsequent damage detection approach are verified using the typical reduced scale three-story plane frame shown in Figure 1. This model was built and tested at the Laboratory of Structural Dynamics and Reliability (LDEC) of the Federal University of Rio Grande do Sul (UFRGS), Brazil, for validating several identification procedures.

Figure 1: Shear Building Model.

The model has three stories, which can be considered as rigid plates, and two elastic columns. This assumption is valid because the stiffness of the girders is much higher than the stiffness of the columns, which allows neglecting the flexibility of the former. Each of the two steel columns has cross section dimensions b = 19mm $\times$ t = 0.62mm and Young's modulus equal to 2×10^{11}N/m^2. The two highest stories have a floor-to-ceiling height, *h*, of 93mm while the ground story has a

100mm floor-to-ceiling height. The structural columns are tightly clamped at each floor. The mass assigned to each degree of freedom includes the mass of the floor and the contributions of the columns, accelerometers and accelerometers' supports. Geometrical and physical details are shown in Figure 2.

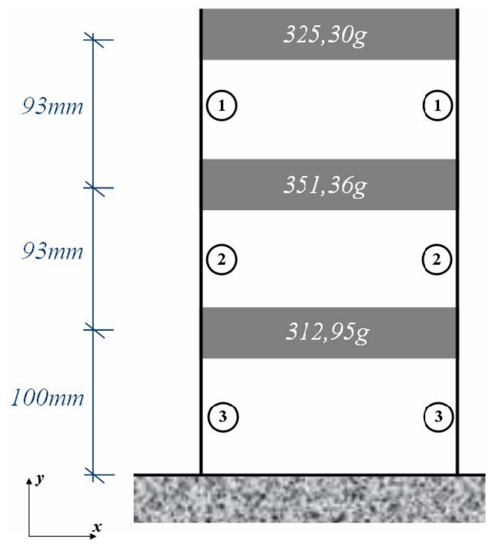

Figure 2: Geometrical and physical details.

Once the geometrical and physical properties of the model were defined, a finite element analysis was performed employing the initial stiffness and mass matrices. Bruel & Kjaer piezoelectric accelerometers and signal amplifiers and the software HP VEE 5.0 (Hewlett Packard) were used to measure the response of the structure. Three accelerometers were located above each girder, allowing the measurement of the three DOF. The sampling frequency was 1024Hz. The experimental frequencies were selected as the peaks of the response spectrum when the reduced scale model was subjected to free vibration tests. Table 2 presents a comparison of the finite element model predictions and the experimental results.

Table 2: Compared frequency results.

Modes	Theoretical Analysis (Hz)	Experimental Results (Hz)
1^{st}	5.47	5.2
2^{nd}	16.04	16.3
3^{rd}	23.27	23.0

As shown in this table, both sets of results are very close but not identical. Hence, a model updating process must be carried out. Three parameters may be updated, namely the stiffness of each degree of freedom for which purpose also three experimental values are available, namely the three natural frequencies. Thus, the number of unknown parameters is equal to the number of measurements, which allows the solution of the updating problem via equation (27). The sensitivity matrix is a square S_{3x3} matrix determined herein through Fox and Kapoor [19] procedure defined in equation 20, while the weighting matrix $\mathbf{W}_{\varepsilon\varepsilon}$ was formed by the reciprocals of the variance of the corresponding measurements, as pointed out in Section 3.

After the model updating was completed, it was observed (Table 3) that the frequencies of the FE model converge rapidly to the measured values, reproducing them with sufficient accuracy after just six steps. The updated stiffness parameters are indicated in Table 4, which also shows a fast convergence.

Table 3: Convergence of the natural frequencies.

Modes		1^{st}	2^{nd}	3^{rd}
Theoretical Analysis (Hz)		5.47	16.04	23.27
Iterations	2	5.1505	16.1450	23.1201
	4	5.1879	16.2865	23.0123
	6	5.2000	16.3000	23.3000
	12	5.2000	16.3000	23.3000
	20	5.2000	16.3000	23.3000
	100	5.2000	16.3000	23.3000
Experimental Results (Hz)		5.2	16.3	23.0

Table 4: Convergence of the updated parameters.

		θ_1	θ_2	θ_3
Initial Value		2251.8509	2251.8509	1811.292
Iterations	2	2589.8896	1990.4474	1590.5425
	4	3019.4311	1362.3138	1982.3421
	8	2909.4511	1514.8507	1897.6986
	12	2909.4511	1514.8507	1897.6986
	20	2909.4511	1514.8507	1897.6986
	100	2909.4511	1514.8507	1897.6986
Units		N/m	N/m	N/m

After the finite element model of the structure was adjusted to predict the response of the reduced scale model in the initial condition, two different damage cases were numerically simulated. In Case 1 the stiffness of element 1 was reduced 20% while in Case 2 elements 2 and 3 present stiffness reductions of 20% and 10% respectively. Table 5 shows the theoretical frequencies for the undamaged model and for both damage cases, being slightly lower for the latter.

Table 5: Compared theoretical frequency results for the damage cases.

Modes	Adjusted Model(Hz)	Damage Case 1 (Hz)	Damage Case 2 (Hz)
1^{st}	5.2	5.1614	4.8178
2^{nd}	16.3	15.7616	15.3259
3^{rd}	23.0	21.4338	22.4033

Two standard excitations were simulated: an impulsive (Figure 3) and an ambient excitation (Figure 4). The latter was modeled by 3 uncorrelated Gaussian white noise signals (generated with MatLab), with zero mean and standard deviation equal to one, applied at all generalized coordinates of the structure. This representation seems adequate to simulate a broad band, ambient excitation of the structure, as suggested in several experimental studies [Brownjohn, Lee, Cheong (1989), Peeters and de Roeck (2001)]. The impulsive loading is represented by the application of an impact at node 2 in the x-direction.

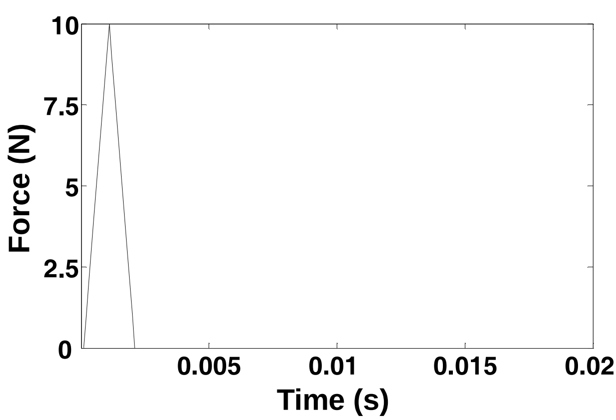

Figure 3: Impulsive excitation.

For these two different damage cases, the structure is numerically modeled using a MatLab finite element code. The dynamic problem is solved by numerical integra-

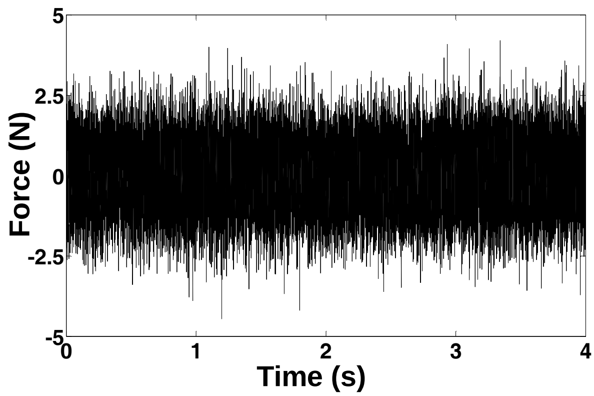

Figure 4: Ambient excitation.

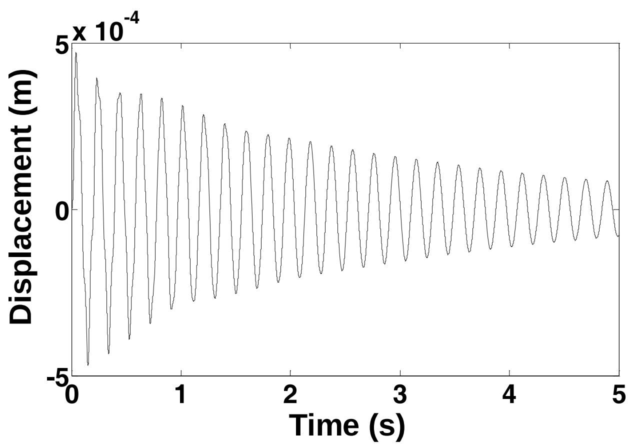

Figure 5: Impulsive excitation.

tion of the equations of motion using Newmark method, with an integration time step equal to 0.0001s for the free vibration case and 0.001s for ambient vibrations. Damping of the structure is assumed proportional to the mass and stiffness matrices. The proportionality constants were determined to yield damping ratios in the 1st mode equal to $\xi = 1\%$.

For the identification procedure, the response is calculated for a time interval of 5s for the transient condition and for a time interval of 400s for ambient vibrations. To reduce the number of data points and to make the identification more accurate in the range of frequency of interest, the output data are filtered with an eight-order

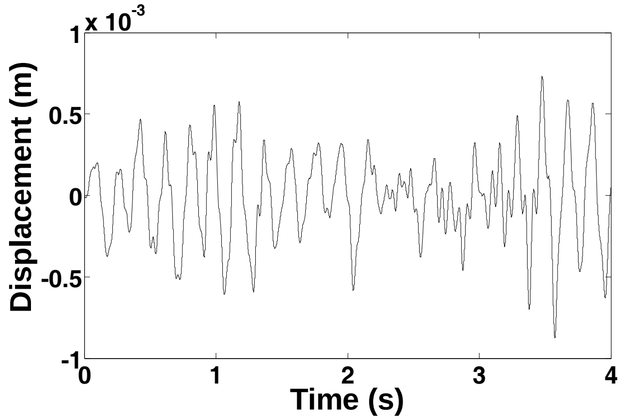

Figure 6: Ambient excitation.

Chebyshev type I lowpass filter with cutoff frequency of 40Hz and the data is re-sampled at a rate of 100Hz. The 3 nodal responses (x-direction) of the frame are considered for the modal parameter estimation, and besides this, since accelerometers are usually used as the measurement transducers, the responses are collected in terms of accelerations. Figures 5 and 6 show the node 1 x-displacement response, both for the impact loading and for the ambient excitation. In order to have a clearer visualization, the ambient excitation signal is partly shown.

In order to determine the variation of structural modal parameters due to noise effects and to evaluate the robustness of the damage detection procedure in situations closer to field conditions, two noise levels were simulated through the addition of white noise signals with RMS amplitude of 5% and 10% of the mean measured response. In real dynamic testing, this is consistent with the assumption of uncorrelation between the primarily electronic noise with the actual measurement signal. In many papers in the technical literature, noise proportional to the signal is assumed, which may grossly misrepresent the effect, by eliminating noise in channels in which the measurement signal is weak, for example, those corresponding to transducers close to modal nodes or to fixed supports.

After getting the output for each noise level, the output-only system identification is carried out using stochastic subspace identification method (SSI), which presents the main advantage of avoiding any preprocessing to obtain spectra or covariances, identifying models directly from time signals. As earlier described by Fadel Miguel, Miguel, Riera and Ramos de Menezes (2007) the performance of the two different algorithms and three different variants is quite similar, thus it a combination of the second algorithm together with the variant CVA was chosen to carry out the identification approach for the two damage cases. The identified frequencies for both damage cases, both excitations and two noise levels are shown in Tables 6 and 7.

Table 6: Identified frequencies for damage case 1.

Modes	Free Vibrations (Hz)			Ambient Vibrations (Hz)		
	Noise 0%	Noise 5%	Noise 10%	Noise 0%	Noise 5%	Noise 10%
1^{st}	5.1611	5.1607	5.1620	5.1641	5.1608	5.1628
2^{nd}	15.7607	15.7603	15.7606	15.759	15.755	15.738
3^{rd}	21.4321	21.4323	21.4319	21.4122	21.4222	21.445

Table 7: Identified frequencies for damage case 2.

Modes	Free Vibrations (Hz)			Ambient Vibrations (Hz)		
	Noise 0%	Noise 5%	Noise 10%	Noise 0%	Noise 5%	Noise 10%
1^{st}	4.8175	4.8179	4.8171	4.816	4.82	4.816
2^{nd}	15.3250	15.328	15.352	15.3224	15.32	15.337
3^{rd}	22.4013	22.4012	22.4015	22.3986	22.3915	22.383

With these identified frequencies the structure is updated again in order to assess the changes caused by damage. Considering that the mass matrix remains unchanged with respect to the undamaged condition, just the stiffness of each degree of freedom is considered in the updating process. Thus, once again, the number of unknown parameters is equal than the number of measurements (the three frequencies), leading the updating solution through equation (27).

Figures 7 and 8 show the difference stiffness matrix for both damage cases, both excitations and for a noise level equals to 10%. This matrix is obtained dividing the coefficients of the stiffness matrix adjusted according to the simulated damage case by the corresponding coefficients of the experimentally adjusted stiffness matrix. It can be noticed that just the DOF related with the damaged members are modified due to damage. For this reason, observing Figure 7, which corresponds to Case 1, it becomes clear that just element 1 presented reduction, i.e., it is the damaged element. Moreover, the terms on the stiffness matrix which presented an absolute uniform change were only those directly related to member 1. Since the term K_{22} presented the same absolute stiffness reduction that the others terms linked to element 1 and the terms K_{23} or K_{32} were not affected by damage, it is evident that member 2 is intact.

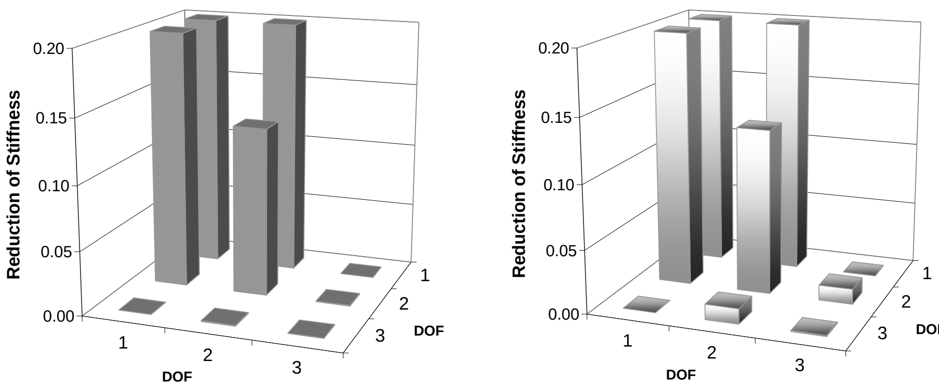

Figure 7: Difference Stiffness Matrix for Damage Case 1 and noise level 10%: (a) Free Vibration, (b) Ambient Vibration.

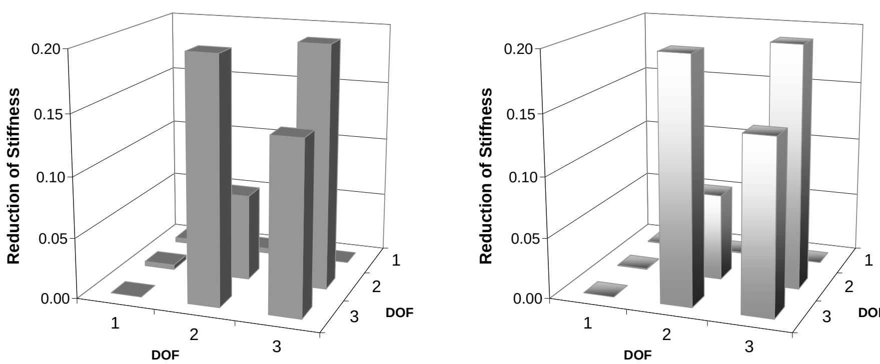

Figure 8: Difference Stiffness Matrix for Damage Case 2 and noise level 10%: (a) Free Vibration, (b) Ambient Vibration.

Figure 8 illustrates the results for damage Case 2. Now the absolute changes in the stiffness matrix are different in terms K_{23} or K_{32} and K_{33}, indicating that more than one element is damaged. Observing the terms on the matrix that presented a stiffness decrease, it is concluded that members 2 and 3 are damaged. This is because the terms K_{23} and K_{32} that receive contributions from member 2 were changed. Moreover, the term K_{33} is also modified, indicating that the two elements are simultaneously damaged.

In order to make the localization clearer and also to quantify damage, an index, herein called DDI - damage detection index, to assess the stiffness reduction is proposed. Its main objective is to evaluate the relative stiffness decrease for all elements. The basic idea is to quantify the relative change in each element of the stiffness matrix resulting from the stiffness reduction of just one element of the system. In case of a multiple damage scenario, a sequential procedure must be followed.

In damage Case 1 just element 1 is damaged. The relative damage in this condition can be evaluated by comparing the stiffness matrix terms K_{11}, K_{12} or K_{21}. Figure 9 shows the damage detection index (DDI) for damage case 1 and both excitations, in which the percentages indicate the noise levels. In Case 2, the quantification must be carried out in sequence. First, the stiffness reduction for element 2 must be determined through terms K_{23} and K_{32}. Next, already knowing the influence of damage of member 2, the damage of element 3 must be determined through term K_{33}. Figure 10 shows the damage detection index (DDI) for Case 2, both excitations and all levels of noise.

It may be seen that the damage detection index (DDI) presented very accurate information, being able to correctly localize damage in all situations, even when employing an output only system identification procedure, presence of noise and multiple damage. Moreover, the quantification errors were always lower than 2%, which indicates the efficiency of the method. Another advantage of the proposed method is that it requires the natural frequency as input, avoiding the use of other modal parameter (such as mode shapes) which are generally more difficult to be experimentally obtained. This is possible due to the iterative model updating formulation, which also requires just natural frequencies as input.

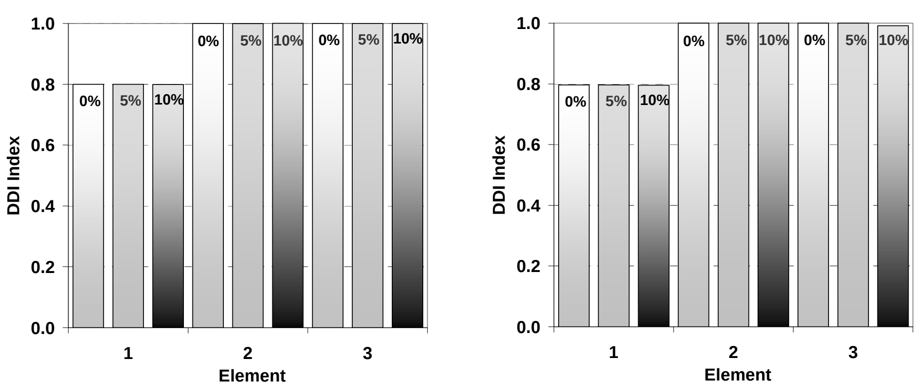

Figure 9: Difference Stiffness Matrix for Damage Case 1 and different noise levels: (a) Free Vibration, (b) Ambient Vibration.

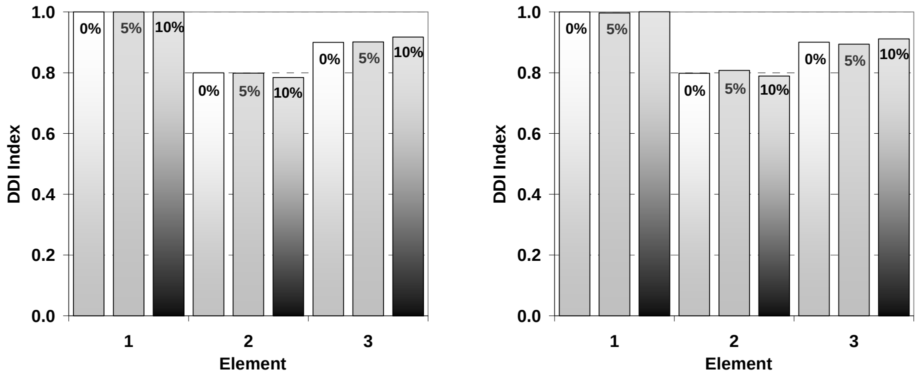

Figure 10: Difference Stiffness Matrix for Damage Case 2 and different noise levels: (a) Free Vibration, (b) Ambient Vibration.

5 Conclusions

The paper describes in detail an iterative model updating procedure, which allows adjusting the coefficients of an initial or prior stiffness matrix of the structure on the basis of its experimentally determined response to an impulsive or broad-band random excitation. When structural damage results primarily in stiffness reduction of damaged components, the method can be used as an effective tool for damage identification. For such purpose, a damage detection index (DDI) is used and assessed in connection with experimental results on a reduced scale laboratory model and data obtained by numerical simulation of the damaged structure. The results confirm the applicability and potential usefulness of the proposed approach.

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