Copyright © 2012 Tech Science Press SDHM, vol.8, no.4, pp.295-305, 2012

Relaxation of Residual Stress under Fatigue Load Described in Terms of Cyclic-Plastic Deformation Model

S. Kwofie[1]

Abstract: Fatigue life of components may be enhanced by mechanical surface treatments, such as shot penning, which induce compressive residual stresses in the component's surface. Under cyclic/fatigue loads, however, relaxation of the residual stress may occur thus, reducing the optimum benefit of the surface treatment. For health monitoring / life prediction under such conditions it is important to be able to assess stress relaxation in real-time. However, the phenomenon of cyclic relaxation of residual stress is not well understood and its tracking during component operation is a technical challenge. By means of cyclic plastic deformation model and the Matlab/Simulink Program cyclic relaxation of residual stress is simulated. Simulated results show that residual stress relaxation under cyclic loading is associated with mean-strain shift, which is indicative of stress redistribution. The significance of this mean strain shift is briefly discussed in terms of tracking and quantitatively determining residual stress relaxation in real-time. Reduction in residual stress may be estimated as the product of *mean-strain shift* and the *elastic modulus* of the material.

Keywords: Residual stress; stress amplitude; hysteresis loops; mean-strain shift; residual stress relaxation;

1 Introduction

Fatigue is a surface phenomenon occurring in the following sequence: (i) localized cyclic plastic deformation (ii) crack initiation (iii) crack propagation to critical size and (iv) final fracture. In general, tensile residual stresses promote while compressive residual stresses retard fatigue damage. Therefore, it is an engineering practice to induce compressive residual stresses onto surfaces of components for the purpose of retarding fatigue damage and/or increasing fatigue life. Such induced stresses exist and remain within components prior to the application of ex-

---

[1] Department of Materials Engineering, College of Engineering, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

ternal loads hence, known as residual stresses, Sigwart (1957). Mechanical surface treatments used to induce compressive residual stresses include shot-peening, laser shock peening, low-plasticity burnishing, autofretage, and hole-expansion, Zhuang and Halford (2001). The full benefit of the mechanical treatment would be realized if the induced compressive stresses were to remain stable during component operation. Thus, all things being equal, higher induced compressive stress is expected to yield higher life and vice versa. However, during component operation under cyclic/fatigue loads, relaxation of the induced compressive stress may occur and this would reduce the benefit of the surface treatment and hence the fatigue life would be lower than expected or predicted, e.g., Mattson and Coleman (1954). For purposes of component design, and health monitoring/life management, it is important to be able to assess the effect of induced residual stresses and/or their relaxation on the fatigue crack initiation and propagation.

In practice, residual stresses may be evaluated using hole-drilling techniques, e.g. ASTM Standard E837, and X-Ray diffraction methods, e.g. Prevey (1986), Prevey (1990) and Prevey(1991). For components operating under cyclic/dynamic loads, evaluation of residual stress relaxation by these techniques involves measuring residual stresses *before* and *after* load application; the difference between these two measured stresses is the reduction in residual stress. Obviously, the hole-drilling and the X-ray diffraction techniques/methods are not suited for monitoring residual stress/relaxation in real-time, i.e., during component operation under dynamic or fatigue loads. The difficulty in measuring cyclic relaxation of residual stress during component operation is a technical challenge that impedes the consideration of tracking and assessing its effect on remaining life. This could be due, partly, to the lack of complete understanding of the phenomenon of residual stress relaxation under cyclic/dynamic loads. Therefore an understanding of the phenomenon is required and necessary for the development of techniques/methods suitable for monitoring and evaluating cyclic relaxation of residual stress in real-time.

In this paper, a cyclic-plastic deformation model of Kwofie (2003) is modified and / or extended to simulate residual stress relaxation under cyclic/fatigue loads. The objectives are (a) to enhance understanding of the phenomenon of cyclic relaxation of residual stresses and (b) to propose/suggest a method of tracking and evaluating residual stress relaxation in real-time.

2 Cyclic relaxation of residual stress: Model and simulation

As mentioned above fatigue damage occurs as a result of localized cyclic plastic deformation that exhibits cyclic hysteresis behaviour. For materials that exhibit nonlinear/strain-hardening monotonic behaviour, e.g. copper, the cyclic elastic-plastic deformation behaviour has been described by Kwofie (2003) as:

$$\frac{d\sigma}{d\varepsilon} = E, \quad for\ \sigma\left(\frac{d\sigma}{dt}\right) < 0 \tag{1a}$$

$$\frac{d\sigma}{d\varepsilon} = E\left(1 - \frac{\sigma}{\sigma_{pt}}\right),\ for\ \sigma\left(\frac{d\sigma}{dt}\right) \geq 0\ and\ \sigma > 0, \tag{1b}$$

$$\frac{d\sigma}{d\varepsilon} = E\left(1 - \frac{\sigma}{\sigma_{pc}}\right),\ for\ \sigma\left(\frac{d\sigma}{dt}\right) \geq 0\ and\ \sigma < 0\ , \tag{1c}$$

$$\frac{d\sigma_{pt}}{d\varepsilon_p} = \theta_0\left(1 - \frac{\sigma_{pt}}{\sigma_{pts}}\right) \tag{1d}$$

$$\frac{d\sigma_{pc}}{d\varepsilon_p} = \theta_0\left(1 - \frac{\sigma_{pc}}{\sigma_{pcs}}\right) \tag{1e}$$

$$\varepsilon_p = \varepsilon_a - \frac{\sigma_{pt}}{E} \tag{1f}$$

where, σ is the applied cyclic stress, ε is the corresponding strain, E is the elastic modulus of the material, $\theta_0$is the initial strain-hardening parameter of the material, σ_{pt} and σ_{pc} are the tensile and compressive peak stresses within a cycle, respectively, σ_{pts} and σ_{pcs} are the respective peak stresses at steady state conditions, ε_p is the cumulative plastic strain, ε_a is the strain amplitude, and t is time.

Eqn.(1a) describes the elastic portions of the hysteresis loop while eqns.(1b) and (1c) describe the plastic (inelastic) portions of the loop during tensile and compressive loading, respectively. For elastic-plastic cyclic loops the peak stresses occur in the plastic regime and vary monotonically with cumulative plastic strain. Eqn. (1d) and (1e) describe the slope of peak stress versus cumulative plastic strain curve as a function of the tensile peak stress, and compressive peak stress, respectively. With appropriate boundary conditions, eqn.(1) may be used to simulate cyclic phenomena such as cyclic hardening or softening under strain-control cycling, cyclic creep/ratcheting under stress-control cycling, and cycle-dependent stress-relaxation due to biased strain-control cycling. In particular, *cyclic hardening* of copper under strain-control loading and *cyclic creep* of copper in stress-control loading have been described and simulated by Kwofie (2003) and Kwofie (2005), respectively.

In this paper the model of eqn. (1) is extended to simulate and describe relaxation of compressive residual stress under constant-amplitude cyclic/fatigue loading condition. For this purpose it is considered that the residual stress is a *virtual mean stress* and that under applied constant amplitude cyclic stress the material would experience internal bias stress cycling such that:

$$\sigma_{pc} = -(\sigma_a + \sigma_R) \tag{1g}$$

$$\sigma_{pt} = \sigma_a - \sigma_R \tag{1h}$$

$$\sigma_{pt} - \sigma_{pc} = 2\sigma_a \tag{1i}$$

where σ_a is the applied stress amplitude, and σ_R is the residual stress. Note that compressive stresses are negative while tensile stresses are positive. Under conditions when the sum of *stress amplitude* and *residual stress* exceeds the local *yield stress* of the material, cyclic plastic deformation would occur, which condition would result in residual stress relaxation. For the case of induced compressive residual stress relaxation occurring under constant stress amplitude σ_a the magnitude of the peak compressive stress σ_{pc}, would, in the context of eqn.(1g), decrease, whilst the magnitude of the peak tensile stress σ_{pt} would increase according to eqn.(1h). The opposite is expected to be the case if the surface residual stress were tensile.

Residual stress is associated with residual strain which also acts as *virtual mean strain* under constant amplitude cyclic loading. This would also result in biased-strain loading condition and, for the case of compressive residual strain, would aid deformation in the compressive direction but oppose deformation in the tensile loading direction. Thus, plastic deformation in the compressive direction within a cycle would not be completely recovered during tensile or forward loading. The net effect would be that the cyclic stress-strain curve and, hence, the mean strain would shift with cycle towards the compressive strain direction. The cyclic relaxation of residual stress as described by eqn. (1) may be simulated using the Matlab/Simulink Software Program. Figure.1 shows the Matlab/Simulink Block Model of eqn. (1) for simulating cyclic relaxation of residual stress. A simulated stress-strain curve illustrating stress relaxation is shown in Figure.2. The simulated cyclic stress-strain response of Figure.2 is characterized by hysteresis loops that shift upwards in the stress-axis direction and leftwards in the strain-axis direction, with cycle. Note that the cyclic stress-strain response of Figure.2 is exclusive of the initial monotonic loading, but commences from the initial compressive peak stress. Since the stress amplitude remains constant the vertical (upward) shift is attributed to *reduction in residual stress*, whereas the horizontal (leftward) shift is due to *changes in mean strain* position and is indicative of residual stress redistribution towards achieving equilibrium.

The stress-strain response of Figure.2 describes what pertains *internally* within the material hence, may not be observed experimentally or in practice with the use of strain-sensors such as strain-gauges. This is because strain-sensors cannot detect or measure residual (internal) stresses directly, but only detects changes in strains associated with the stresses.

Thus, stress-strain curves from strain-gauges would rather be similar to that of Fig-

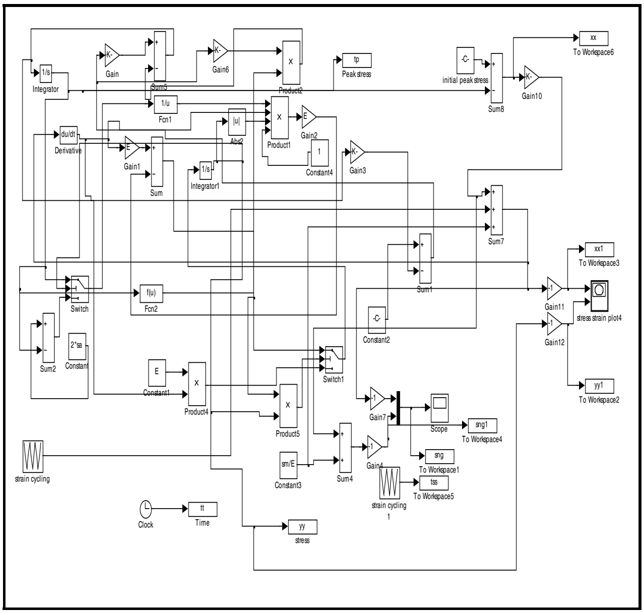

Figure 1: Matlab/Simulink Block Model for simulating residual stress relaxation under cyclic loads

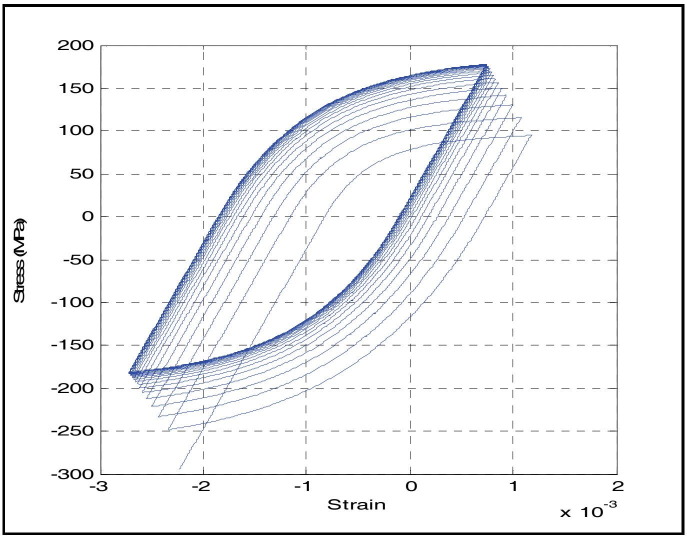

Figure 2: Simulated cyclic stress-strain response of material showing stress relaxation of compressive residual stress subjected to constant amplitude cyclic loading. Initial monotonic compressive loading curve is excluded.

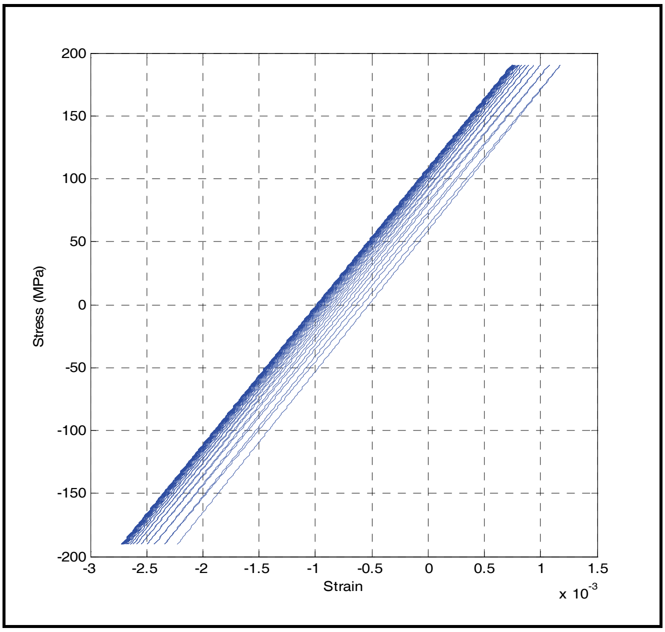

Figure 3: Expected cyclic stress-strain output from strain gauge during stress relaxation of compressive residual stress under constant amplitude loading. The shift of mean strain with cycle is leftwards and is evidence of stress relaxation.

ure.3, which shows linear elastic cyclic stress-strain curves shifted horizontally in the biased-strain direction (leftward) with cycle, but without any shift in the vertical direction. The strain shift is initially high but decreases with cycle (or time) up to the stable stage when further residual stress relaxation does not occur and the cyclic stress-strain path remain constant. Note that the stresses of Figure.3 are without residual stresses, but only the cyclic stress of constant amplitude. The shift of strain with time as simulated from the Block Diagram (Figure.1) of eqn.(1) is shown in Figure.4. It can be seen that although the stress-strain output of Figure.3 suggests elastic material response, the strain-time output of Figure. 4 is nonlinear showing inelastic response typical of creep.

It is interesting to note that strain-time output of Figure.3 is found similar to observations by Rao, Wang, Chen, and Ni (2007), and Dawson and Moffat (1980), while the stress-strain output of Figure.4 is similar to observations by Rao, Wang, Chen and Ni (2007). Note that for the cases of Rao, Wang, Chen and Ni (2007) and Dawson and Moffat (1980), residual stresses are tensile and are relieved by Vibration Stress Relief (VSR) treatment, and strain shifts were in the tensile (biased) direction. Thus, in general, the strain shift occurring during residual stress relaxation under cyclic/dynamic loads would be in the sense of residual stress; that is, strain would shift in the positive or negative strain axis direction depending on whether the residual stress is positive or negative, respectively.

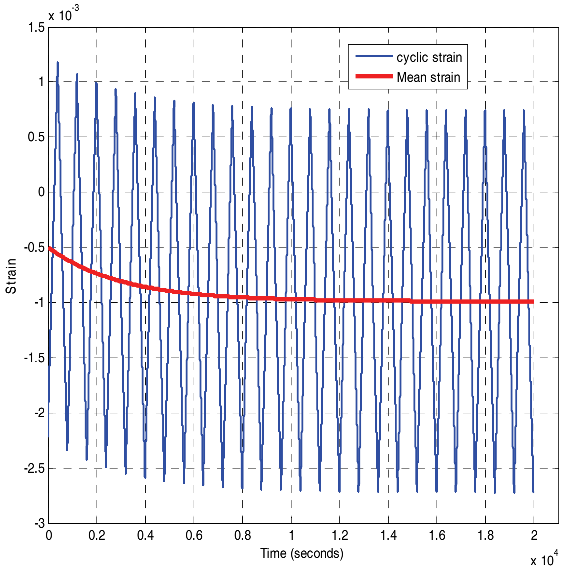

Figure 4: Simulated cyclic and mean strains as a function of time during stress relaxation of compressive residual stress. The shift of cyclic strain and mean strain is apparent.

3 Discussion

Although cyclic relaxation of residual stress is known to occur, the phenomenon is not well understood. To enhance the understanding of the phenomenon, the *internal* cyclic stress-strain behaviour during cyclic relaxation of residual stress was simulated using the cyclic deformation model of eqn. (1) and the Matlab/Simulink Program. Fig. 2 shows simulation of the internal stress-strain characteristics illustrating cyclic relaxation of compressive residual stress. Superposition of cyclic stress on residual (mean) stress results in a condition of biased cyclic loading. Under conditions when cyclic plasticity occurs, the material response would exhibit hysteresis behaviour. Cyclic relaxation of residual stress occurring under constant stress amplitude is characterized by vertical shift of stress-strain hysteresis loop towards the zero mean stress position and a corresponding horizontal shift of the loop in the direction of biased strain. The vertical shift is indicative of reduction in residual stress, while the horizontal shift is indicative of stress redistribution that follows stress relaxation.

Under completely-reversed constant amplitude loading, residual stress σ_R and its corresponding strain ε_m form a mean-strain-mean-stress coordinate pair (ε_m, σ_R), within the material. Now let the initial coordinates with respect to the loading cycle be (ε_{m0}, σ_{R0}) and those after stress relaxation at cycle N, be (ε_{mN}, σ_{RN}). Then the shift in the vertical (stress) axis is equal to $(\sigma_{R0}-\sigma_{RN})$ while that of the horizontal (strain) axis is $(\varepsilon_{m0}-\varepsilon_{mN})$. Since residual stresses and strains are elastic the ratio of *change in residual stress* to *change in mean strain* must be equal to the elastic modulus of the material. That is

$$\frac{(\sigma_{R0}-\sigma_{RN})}{(\varepsilon_{m0}-\varepsilon_{mN})}=-\frac{\Delta\sigma_R}{\Delta\varepsilon_m}=E \tag{2a}$$

or

$$\triangle\sigma_R=-E\left(\triangle\varepsilon_m\right) \tag{2b}$$

$$\varepsilon_{mi}=\frac{\varepsilon_{max}+\varepsilon_{min}}{2} \tag{2c}$$

where ε_{mi} is the mean strain at the ith cycle, ε_{max} is the maximum strain within a cycle, ε_{min} is the corresponding minimum strain, ε_{m0} is the initial mean (residual) strain, ε_{mN} is the mean (residual) strain at cycle N, σ_{R0} is the initial residual stress, σ_{RN} is the residual stress at cycle N, ?σ_R is the reduction in residual stress after N cycles, and $\Delta\varepsilon_m$ is the mean-strain shift after N cycles. From eqn. (2b) the reduction in residual stress ($\Delta\sigma_R$) after N cycles of loading can be evaluated as the product of the mean strain shift ($\Delta\varepsilon_m$) and the elastic modulus (E) of the material. The negative sign serves to indicate that whereas the shift of *residual (mean) stress* is towards the zero mean stress position that of *residual (mean) strain* is away from the zero mean strain position.

Since cyclic relaxation is expected to occur when the sum of residual stress and the cyclic stress exceeds the local yield stress of the material, the implication is that

$$\sigma_R+\sigma_a>\sigma_y \tag{3}$$

where σ_y is the yield stress of the material. Note that $\sigma_R+\sigma_a$ defines peak stress, σ_{pk}, within a cycle. As stress relaxation occurs, the magnitude of σ_R would decrease until eqn. (2) is no longer satisfied when stress relaxation ceases. Let σ_{Rs} be the stable residual stress at which further relaxation does not occur. Then mathematically, stress relaxation is expected to stop when

$$\sigma_{Rs} + \sigma_a < \sigma_y \tag{4}$$

Since σ_{R0} is the initial residual stress prior to cyclic loading, the maximum possible residual stress relaxation $\Delta\sigma_{Rmax}$ that may be achieved is given as

$$\Delta\sigma_{Rmax} = \sigma_{R0} + \sigma_a - \sigma_y \tag{5a}$$

But the maximum residual stress relaxation is also numerically equal to the difference between the initial and the stable values. Hence we have

$$\Delta\sigma_{Rmax} = \sigma_{R0} - \sigma_{Rs} \tag{5b}$$

Therefore from eqns. (5a) and (5b) we have

$$\sigma_{Rs} = \sigma_y - \sigma_a \tag{5c}$$

For design and life management purposes, the stable residual stress is desired, for which eqn.(5c) is useful. Thus, the stable residual stress sustainable after cyclic relaxation depends on the *yield stress* of the material and the applied *stress amplitude* of loading. For compressive residual stress induced to enhance fatigue life of components, high value of σ_{Rs} is desirable thus, component material of high surface yield stress is beneficial. However, for cases where residual stresses are undesirable, e.g. tensile residual stresses due to welding, and which may be reduced by Vibration Stress Relief (VSR) techniques e.g. Rao Wang, Chen and Ni (2007), and Dawson and Moffat (1980), the remaining/stable residual stress σ_{Rs} needs be as small as possible, and this, according to eqn. (5c) requires higher stress amplitude. Theoretically, complete stress relaxation would occur if the cyclic stress amplitude is at least equal to the yield stress of the material (eqn.(5c)).

Residual stress/relaxation is difficult to measure directly. Therefore, in practice, changes in strains are rather measured from which reduction in residual stresses are evaluated. Since the algebraic sum of all residual stresses in a component is zero, occurrence of stress relaxation must be accompanied by stress redistribution towards restoring equilibrium. It is this stress redistribution accompanying stress relaxation that causes the shift in strains. Therefore the complex nature of cyclic relaxation of residual stress may be monitored and /or assessed by tracking the strain shifts via the use of strain sensors such as resistance strain-gauges. For this purpose the following procedure is proposed for the evaluation of stress relaxation:

1. *Attach* resistance strain-gauge to critical positions of component prior to the application of cyclic load.
2. *Record* the overall strain-time (or cycle) response during service loads, as in Figure. 4. This may be done by means of X-Y plotter or computer.
3. *Extract* or derive the mean-strain versus time (or cycle) curve or relation from the cyclic strain-time record of (ii)
4. *Evaluate* the reduction in residual stress at given time (or cycle) as the product of the *change in mean strain* and the *elastic modulus of the material*, (Eq. (2b)).

4 Conclusions

The phenomenon of cyclic relaxation of residual stress has been theoretically described, using a cyclic-plastic deformation model of eqn. (1). By means of the Matlab/Simulink Software Program the theoretical cyclic relaxation of compressive residual stress may be simulated as in Figure.2. Cyclic relaxation of residual stress is due to cyclic-plastic deformation occurring within the material, at least locally, and is accompanied by stress redistribution towards attaining equilibrium. The following conclusions could be drawn:

1. Under cyclic/fatigue loads, residual stress can be regarded as virtual mean-stress and causes biased amplitude loading conditions. Thus, the material would be subjected to both cyclic stress and residual (mean) stress.
2. When the sum of cyclic stress and residual stress exceeds the local yield stress of the material, cyclic-plastic deformation would occur.
3. The occurrence of plastic deformation causes reduction in the residual stress which tends to shift the stress-strain curve vertically towards zero mean stress. For the compressive residual stress the shift would be in the upward (tensile) direction, (Figure.2).
4. Plastic deformation in the biased stress/strain direction would not be completely recovered during reversed loading, resulting in the shift of *mean-strain position* on the strain (horizontal) axis, in the direction of bias strain. For the compressive residual stress, the shift is in the compressive direction, (Figure.2)
5. Matlab / Simulink Software Program is useful for simulating cyclic relaxation of residual stresses.

6. Resistance strain-gauges may be used to monitor and evaluation residual stress relaxation during component operation.
7. Reduction in residual stress is directly proportional to the shift in strain, the ratio between them being equal to the Elastic Modulus of the material, (eqn. (2b)).
8. Practical methods and techniques may now be developed for monitoring and evaluating stress relaxation in real-time, for purposes of predicting and managing fatigue life of critical components.

References

ASTM E837: Determining Residual stresses by the Hole-Drilling Strain-Gauge Method, *American Society for Testing and Materials, ASTM Standard E837.*

Dawson R., Moffat D.G. (1980): Vibratory stress relief: A fundamental study of its effectiveness, *J.Eng. Mat.Tech.*, 102, pp 169-176.

Mattson R.L., Coleman W.S. Jr. (1954): Effect of shot peening variables and residual stresses on fatigue life of leaf spring specimens, *Transactions*, *Society of Automotive Engineers* 62, pp. 546–56.

Kwofie, S. (2003): Description of cyclic hysteresis behaviour in terms of one-parameter model, *Materials Science & Engineering A* 357, pp. 86-93.

Kwofie, S. (2005): Cyclic creep behaviour described in terms of a one-parameter kinetic model, *Materials Science and Engineering A* 392, pp. 23–30.

Prevey, P.S. (1986): *Metals Handbook, American Society for Metals*, pp. 380–392.

Prevey, P.S. (1990): Shot Peening Theory and Application, IITT-International, Gournay-Sur-Marne, France, pp. 81–93

Prevey, P.S. (1991): Practical Applications of Residual Stress Technology, *American Society for Metals,* pp. 47–54.

Rao Delin, Wang Dezhong, Chen Ligong, Ni Chunzhen (2007): The effectivness evaluation of 314L stainless steel vibratory stress relief by dynamic stress, *Int. J. Fatigue*, 29, pp. 192-196.

Sigwart H. (1957): Influence of residual stresses on the fatigue limit. *In: Proceedings of the International Conference on Fatigue of Metals, Institution of Mechanical Engineers*, London.

Zhuang W.Z., Halford, G.R. (2001): Investigation of residual stress relaxation under cyclic load, *International Journal of Fatigue* 23, pp. S31–S37.