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On the Structural Response of Elasto/Viscoplastic Materials Subject to Time-Dependent Loadings

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Abstract: The influence of different loading rates on the structural response of elasto/viscoplastic materials is illustrated with specific numerical examples. An associated formulation of the evolutive laws in elasto/viscoplasticity is presented within the framework of the generalized standard material model with internal variables. An appropriate solution scheme is applied which is capable to be adopted for different constitutive models. Different loading programs are analyzed by considering different values of the loading rate and of the intrinsic properties of the material. Computational applications and examples are illustrated which describe the rate-dependency of the elasto/viscoplastic material behavior. The significance of the loading program and the loading rate is therefore emphasized with respect to the nonlinear structural response of elasto/viscoplastic materials.

Keywords: Elasto/viscoplasticity, Structural response, Time-dependent loadings.

1 Introduction

Rate-independent plasticity has achieved in the last two decades a significant progress not only in the definition of an appropriate theoretical framework of the phenomenon but also in the computational treatment of the model, see among others Crisfield (1997), Simo and Hughes (1998), and Zienkiewicz and Taylor (2005).

On the other hand the problem of the numerical integration in viscoplasticity may not be considered as trivial, see e.g. Simo (1991) and Simo and Govindjee (1991). Considerations on the stability of the generalized midpoint rule integration algorithms are reported for elastoplasticity by Ortiz and Popov (1985), while for viscoplasticity by Hughes and Taylor (1978), Simo (1991) and Simo and Govindjee (1991).

In viscoplasticity a complete variational formulation of the structural evolutive problem with internal variables is provided, e.g., by DeAngelis (2000). The deriva-

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tion of a class of return mapping algorithms associated with the system of variational inequalities is discussed, e.g., by Crisfield (1997), Simo and Hughes (1998), and Zienkiewicz and Taylor (2005). More specifically, integration procedures were developed by Zienkiewicz and Cormeau (1974) who considered time step restrictions for the Euler forward difference method in quasi-static elasto/viscoplasticity. Hughes and Taylor (1978) reconsidered the application of implicit methods by the use of an algorithmic procedure which requires the inversion of a compliance matrix. Szabo (1990) compared different time integration schemes in a review article. Integration algorithms for viscoplastic models involving non-smooth yield surfaces are reported by Simo, Kennedy, and Govindjee (1988), while stability properties of the algorithms are investigated by Simo (1991) and Simo and Govindjee (1991). Integration procedures for viscoplastic models are also presented by Ju (1990) and by Peric (1993). In the latter a perturbation method is also proposed for the solution of stiff equations arising in low-rate-sensitive materials. Alternative integrator formulations have been investigated by Freed and Walker (1992) and Freed and Walker (1993). Chaboche and Cailletaud (1996) showed integration schemes specifically developed for complex material models and analyzed integration methods for plastic and viscoplastic models encompassing nonlinear kinematic hardening behavior. General solution procedures suitable to be applied to different constitutive models have been proposed by Alfano, DeAngelis, and Rosati (2001). Extensions to include rate plasticity have been investigated by DeAngelis, Cancellara, Modano, and Pasquino (2011), DeAngelis and Cancellara (2012a) and DeAngelis and Cancellara (2012b). For a comprehensive account on the computational modeling of elastoplasticity and elasto/viscoplasticity see, among others, Crisfield (1997), Simo and Hughes (1998) and Zienkiewicz and Taylor (2005).

In the present paper the influence of different loading rates on the nonlinear structural response of elasto/viscoplastic materials is illustrated and specific numerical examples are detailed. A solution procedure is adopted which can be applied to different viscoplastic constitutive models. The treatment is developed within the framework of the generalized standard material model with internal variables and an associated formulation of the evolutive laws in elasto/viscoplasticity is presented. Different loading programs are evaluated by considering different values of the loading rates and of the intrinsic properties of the material. The loading is enforced by increasing the prescribed boundary displacements and, accordingly, a non-dimensional loading program parameter is introduced which is appropriate to such cases. Numerical computations and results for both rate-independent and rate-dependent loadings are reported. Computational applications are illustrated for evaluating the effects of the loading rates on the structural response of elasto/viscoplastic materials. The significance of the type of loading program and loading rate on the structural response of elasto/viscoplastic materials is therefore illustrated in detail with specific numerical examples.

2 Continuum problem

2.1 General remarks

Let $\Omega \subset \Re^n$, $1 \leq n \leq 3$ be the reference configuration of the body $\mathscr{B}$ and particles labelled by their position vector $\mathbf{x} \in \Omega$ relative to a Cartesian coordinate system. Let $\mathscr{T} \subset \Re_+$ be the time interval of interest. We denote with V the space of displacements, D the strain space and S the dual stress space. We also denote by

$$\mathbf{u} : \Omega \times \mathscr{T} \to V \tag{1}$$

the displacement field and by

$$\sigma : \Omega \times \mathscr{T} \to S \tag{2}$$

the stress field. The compatible strain field is defined as

$$\varepsilon(\mathbf{u}) = \nabla^s(\mathbf{u}) : \Omega \times \mathscr{T} \to D, \tag{3}$$

where ∇^s is the symmetric part of the gradient.

The hypothesis of small strains is adopted and viscous effects are assumed to show beyond the elastic range. We therefore consider the class of material behavior often referred to in the literature as rate-sensitive materials, see e.g. Naghdi and Murch (1963) and Skrzypek and Hetnarski (1993). For a survey account see also Duvaut and Lions (1972) and Lemaitre and Chaboche (1990). Accordingly, we denote with ε^e the elastic strain. The strain difference

$$\varepsilon^{vp} = \varepsilon - \varepsilon^e \tag{4}$$

is denoted as the viscoplastic strain, in which combined viscous and plastic effects are represented.

The elastic energy $\mathscr{W} : D \to \Re$ and the complementary elastic energy $\mathscr{W}^* : S \to \Re$ in case of linear elasticity are expressed in the quadratic forms

$$\mathscr{W}(\varepsilon^e) = \frac{1}{2}\langle \mathbf{C}\varepsilon^e, \varepsilon^e \rangle, \qquad \mathscr{W}^*(\sigma) = \frac{1}{2}\langle \sigma, \mathbf{C}^{-1}\sigma \rangle, \tag{5}$$

where $\mathbf{C}$ is the elastic stiffness and the symbol $\langle \cdot , \cdot \rangle$ denotes a non-degenerate bilinear form acting on dual spaces.

In order to represent hardening behavior we consider a dual pair of kinematic $\alpha = (\alpha_{kin}, \alpha_{iso}) \in X \times \Re$ and static $\chi = (\chi_{kin}, \chi_{iso}) \in X' \times \Re$ internal variables,

where $\alpha_{iso} \in \Re$ and $\chi_{iso} \in \Re$ model isotropic hardening and $\alpha_{kin} \in X$ and $\chi_{kin} \in X'$ model kinematic hardening, X and X' being dual spaces. The hardening matrix $\mathbf{H} = \mathrm{diag}[\mathbf{H}_{kin}, H_{iso}]$ is introduced, so that static and kinematic internal variables are linked by the relation $\chi = \mathbf{H}\alpha$.
A convex yield function $f(\sigma, \chi_{kin}, \chi_{iso})$ defines the convex elastic domain $\mathscr{E}$ as

$$\mathscr{E} \stackrel{\text{def}}{=} \{(\sigma, \chi_{kin}, \chi_{iso}) \in S \times X' \times \Re : f(\sigma, \chi_{kin}, \chi_{iso}) \leq 0\}. \tag{6}$$

An important class of viscoplastic hardening materials arises when the yield function is expressed by

$$f(\sigma, \chi_{kin}, \chi_{iso}) = F(\sigma - \chi_{kin}) - \chi_{iso} - y_o, \tag{7}$$

where y_o is a material parameter.
The hardening potential $\mathscr{H}(\alpha) : X \times \Re \to \Re$ and the complementary hardening potential $\mathscr{H}^*(\chi) : X' \times \Re \to \Re$ are introduced for modeling hardening phenomena. For linear hardening they are expressed in the quadratic forms

$$\begin{aligned} \mathscr{H}(\alpha) &= \frac{1}{2}\mathbf{H}_{kin}\alpha_{kin} \cdot \alpha_{kin} + \frac{1}{2}\mathrm{H}_{iso}\alpha_{iso}^2, \\ \mathscr{H}^*(\chi) &= \frac{1}{2}\chi_{kin} \cdot \mathbf{H}_{kin}^{-1}\chi_{kin} + \frac{1}{2}\mathrm{H}_{iso}^{-1}\chi_{iso}^2. \end{aligned} \tag{8}$$

The Helmholtz free energy is thus expressed as

$$\Psi(\varepsilon^e, \alpha) = \mathscr{W}(\varepsilon^e) + \mathscr{H}(\alpha), \tag{9}$$

and the complementary free energy as

$$\Psi^*(\sigma, \chi) = \mathscr{W}^*(\sigma) + \mathscr{H}^*(\chi). \tag{10}$$

2.2 The generalized standard material model

In the generalized standard material model, introduced by Halphen and Nguyen (1975), strains and kinematic internal variables, as well as the corresponding dual ones, are collected in suitably defined generalized variables

$$\mathbf{E} = (\varepsilon, \mathbf{o}), \quad \mathbf{E}^e = (\varepsilon^e, \alpha), \quad \mathbf{E}^{vp} = (\varepsilon^{vp}, -\alpha), \quad \Sigma = (\sigma, \chi), \tag{11}$$

for which the same formal rules of the basic variables apply. Accordingly the generalized kinematic and static variables are defined respectively in the product spaces

$\tilde{D} = D \times X \times \Re$ and $\tilde{S} = S \times X' \times \Re$. We remark that the additive decomposition of the strains is carried over to the generalized variables, so that

$$\mathbf{E} = \mathbf{E}^e + \mathbf{E}^{vp}. \tag{12}$$

The duality product between the generalized variables is induced by the corresponding ones between D and S as well as the ones between X and X', so that

$$\langle \Sigma, \mathbf{E} \rangle = \langle \sigma, \varepsilon \rangle, \qquad \langle \Sigma, \mathbf{E}^e \rangle = \langle \sigma, \varepsilon^e \rangle + \langle \chi, \alpha \rangle, \qquad \langle \Sigma, \mathbf{E}^{vp} \rangle = \langle \sigma, \varepsilon^{vp} \rangle - \langle \chi, \alpha \rangle, \tag{13}$$

where, for simplicity, the same symbol has been used to denote duality products defined on different pairs of dual linear spaces.

The consistency condition is now enforced on the generalized stress Σ and states that Σ must belong to the closed generalized convex elastic domain $\tilde{\mathscr{E}} \subseteq \tilde{S}$ defined as

$$\tilde{\mathscr{E}} \stackrel{\text{def}}{=} \{\Sigma \in \tilde{S} : \tilde{f}(\Sigma) \leq 0 \iff (\sigma, \chi_{kin}, \chi_{iso}) \in S \times X' \times \Re : f(\sigma, \chi_{kin}, \chi_{iso}) \leq 0\}, \tag{14}$$

where $\tilde{f} : \tilde{S} \to \Re$ is the yield function expressed in terms of generalized stresses.

2.3 Evolutive equations

The principle of maximum plastic dissipation (see, e.g., Hill (1950)) plays a crucial role in the formulation of elasto/viscoplasticity by supplying the associative form of the flow rule.

Given a generalized viscoplastic strain $\dot{\mathbf{E}}^{vp}$, among all possible generalized stresses $\Gamma = (\tau, \mathbf{q}) \in \tilde{S}$, the actual generalized stress $\Sigma = (\sigma, \chi)$ satisfies the condition of maximum dissipation

$$\mathscr{D}^{vp}(\dot{\mathbf{E}}^{vp}) = \sup_{\Gamma \in \tilde{S}} \{\langle \Gamma, \dot{\mathbf{E}}^{vp} \rangle - \Pi^*(\Gamma)\}, \tag{15}$$

where $\Pi^*(\Gamma)$ indicates a viscoplastic convex potential.

Problem (15) can be expressed by enforcing the stationarity condition for the Lagrangian

$$\mathscr{L}^{vp}(\Gamma) \stackrel{\text{def}}{=} -\langle \Gamma, \dot{\mathbf{E}}^{vp} \rangle + \Pi^*(\Gamma), \tag{16}$$

which yields

$$0 \in [\partial \mathscr{L}^{vp}(\Gamma)]_{(\Sigma)} \qquad \iff \qquad \dot{\mathbf{E}}^{vp} \in \partial \Pi^*(\Sigma). \tag{17}$$

The flow rule $(17)_2$ is given in components as

$$\dot{\varepsilon}^{vp} \in \partial_{\sigma}\Pi^*(\sigma,\chi),$$

$$-\dot{\alpha}_{kin} \in \partial_{\chi_{kin}}\Pi^*(\sigma,\chi_{kin},\chi_{iso}), \tag{18}$$

$$-\dot{\alpha}_{iso} \in \partial_{\chi_{iso}}\Pi^*(\sigma,\chi_{kin},\chi_{iso}),$$

which express the flow law of the viscoplastic strain and the evolutive equations of the kinematic internal variables.

In the sequel we represent the viscoplastic problem as a penalty regularization of the plastic problem, see e.g. Yosida (1980). A penalty function $g^+(x)$ of the constraint $f(\Gamma)$ is thus introduced and the regularized form of the Lagrangian (16) has the expression

$$\mathscr{L}^{vp}_{\eta}(\Gamma) \overset{\text{def}}{=} -\langle \Gamma, \dot{\mathbf{E}}^{vp} \rangle + \frac{1}{\eta} g^+(f(\Gamma)), \tag{19}$$

where $\eta > 0$ is a penalty parameter which has the meaning of a viscosity coefficient.

Accordingly, the viscoplastic dissipation is expressed in the regularized form

$$\mathscr{D}^{vp}_{\eta}(\dot{\mathbf{E}}^{vp}) = \sup_{\Gamma \in \tilde{S}} \{ \langle \Gamma, \dot{\mathbf{E}}^{vp} \rangle - \frac{1}{\eta} g^+(f(\Gamma)) \}. \tag{20}$$

The penalty function $g^+(x)$ is required to be continuous in $[0,\infty)$, $g^+(x) \geq 0$ and convex in $[0,\infty)$, with $g^+(x) = 0$ if and only if $x \leq 0$. In these assumptions the solution Σ_η of the regularized problem tends to the solution Σ of the constrained plastic problem for $\eta \to 0^+$, see e.g. Luenberger (1973).

For linear viscous effects the penalty function g^+ may be assumed in the form

$$g^+(x) \overset{\text{def}}{=} \begin{cases} \frac{1}{2}x^2 & \text{for} \quad x \geq 0 \\ 0 & \text{for} \quad x < 0, \end{cases} \tag{21}$$

and the derivative is given by $\frac{dg^+(x)}{dx} = < x >$, where the MacAuley brackets $< \cdot >$ are defined as $< x > = (x + |x|)/2$. In order to model nonlinear viscous effects a flow function $\Phi(x)$ is introduced such that $\frac{dg^+(x)}{dx} = < \Phi(x) >$. Accordingly, the stationarity condition for the regularized viscoplastic potential $\mathscr{L}^{vp}_{\eta}(\Gamma)$ yields the viscoplastic flow law of the Perzyna (1963) constitutive model

$$0 \in \left[\partial \mathscr{L}^{vp}_{\eta}(\Gamma)\right]_{(\Sigma)} \qquad \Longleftrightarrow \qquad \dot{\mathbf{E}}^{vp} \in \frac{1}{\eta} < \Phi(f(\Sigma)) > \partial f(\Sigma). \tag{22}$$

The flow rule $(22)_2$ is expressed in components as

$$\dot{\varepsilon}^{vp} \in \frac{1}{\eta} < \Phi(f(\sigma, \chi_{kin}, \chi_{iso})) > \partial_{\sigma} f(\sigma, \chi),$$

$$-\dot{\alpha}_{kin} \in \frac{1}{\eta} < \Phi(f(\sigma, \chi_{kin}, \chi_{iso})) > \partial_{\chi_{kin}} f(\sigma, \chi_{kin}, \chi_{iso}), \tag{23}$$

$$-\dot{\alpha}_{iso} \in \frac{1}{\eta} < \Phi(f(\sigma, \chi_{kin}, \chi_{iso})) > \partial_{\chi_{iso}} f(\sigma, \chi_{kin}, \chi_{iso}),$$

which are respectively the flow law of the viscoplastic strain and the evolutive equations of the kinematic internal variables for the Perzyna constitutive model.

For linear viscous effects the flow function of the Perzyna model is often assumed as

$$\Phi(f(\sigma, \chi_{kin}, \chi_{iso})) = f(\sigma, \chi_{kin}, \chi_{iso}). \tag{24}$$

For nonlinear viscous effects other proposed expressions of the flow function are reported e.g. by Perzyna (1963) and Skrzypek and Hetnarski (1993).

In the sequel we assume a Von Mises yield criterion in the form

$$f(\sigma, \chi_{kin}, \chi_{iso}) = \|\mathrm{dev}\sigma - \chi_{kin}\| - \chi_{iso} - y_o = \|\xi\| - R \leq 0, \tag{25}$$

where $\mathrm{dev}\sigma$ is the stress deviator, the relative stress is defined as $\xi \stackrel{\mathrm{def}}{=} \mathrm{dev}\sigma - \chi_{kin}$, the current radius of the yield surface in the deviatoric plane is

$$R = \chi_{iso} + y_o = \sqrt{\frac{2}{3}}(\sigma_{yo} + \mathrm{H}_{iso}\bar{e}^{vp}), \tag{26}$$

σ_{yo} denotes the uniaxial yield stress of the virgin material and $\bar{e}^{vp} = \int_0^t \sqrt{\frac{2}{3}}\|\dot{\varepsilon}^{vp}\|\mathrm{d}t$ represents the equivalent viscoplastic strain. In the following a linear hardening behavior is assumed, with a static internal variable related to isotropic hardening defined by $\chi_{iso} = H_{iso}\alpha_{iso}$ and the dual kinematic internal variable α_{iso} represented by the equivalent viscoplastic strain.

For a prescribed increment of the displacement field $\Delta\mathbf{u}$, the unknown fields are updatet at time $t_{n+1} \in [0, T]$ consistently with the flow rule

$$\dot{\mathbf{E}}^{vp} = \frac{1}{\eta} < \Phi(f(\Sigma)) > \mathrm{d}_{\Sigma} f(\Sigma), \tag{27}$$

where a Perzyna viscoplastic constitutive model has been adopted. Setting $\mathbf{n} = \xi/\|\xi\|$, the flow rule of the viscoplastic strain and the evolutive equations of the internal variables are expressed by

$$\dot{\varepsilon}^{vp} = \frac{1}{\eta} < \Phi(f(\sigma,\chi)) > \frac{\xi}{\|\xi\|} = \frac{1}{\eta} < \Phi(f(\sigma,\chi)) > \mathbf{n},$$

$$\dot{\chi}_{kin} = \frac{2}{3} H_{kin} \dot{\varepsilon}^{vp} = \frac{2}{3} \frac{< \Phi(f(\sigma,\chi)) >}{\eta} H_{kin} \frac{\xi}{\|\xi\|} = \frac{2}{3} \frac{< \Phi(f(\sigma,\chi)) >}{\eta} H_{kin} \mathbf{n}, \tag{28}$$

$$\dot{\bar{e}}^{vp} = \sqrt{\frac{2}{3}} \frac{< \Phi(f(\sigma,\chi)) >}{\eta}.$$

In the solution procedure an algorithmic scheme has been adopted which is suitable to be applied to different viscoplastic constitutive models (see for details DeAngelis (1998) and Alfano, DeAngelis, and Rosati (2001)). An extension to include nonlinear kinematic hardening rules is illustrated in DeAngelis (2012a). The adopted approach shows to be useful also in the development of variational formulations in rate plasticity, see e.g. DeAngelis (2007a), and in nonlocal plasticity and viscoplasticity, see e.g. DeAngelis (2007b) and DeAngelis (2012b).

3 Validation of the elastoplastic numerical model

In order to describe the behavior of the model and the effective reliability of the numerical results we analyze the problem addressed by Theocaris and Marketos (1964). In their paper Theocaris and Marketos (1964) analyzed the problem of a rectangular strip with a circular hole. The analysis is aimed at determining the elastic-plastic strain and stress distribution that occurs in perforated strips of a strain-hardening material when the applied stress is increased monotonically from the elastic region of loading to values producing an impending plastic flow. In their analysis the values of elastic and plastic components of strains were experimentally determined by using the birefringent coating method complemented with the electrical analogy method which yields the elastic and plastic strain distribution. The complete solution of the strain distribution at the elastoplastic domain of straining was illustrated and reported. The elastoplastic strain distribution that occurs in perforated strips is a good and experimentally verified example and it can be properly considered for the validation of the proposed numerical analysis and for evaluating the soundness and the effective reliability of the adopted computational model.

Accordingly, the problem of an infinitely long rectangular strip with a circular hole is analyzed. The geometry of the problem and the loading conditions are illustrated

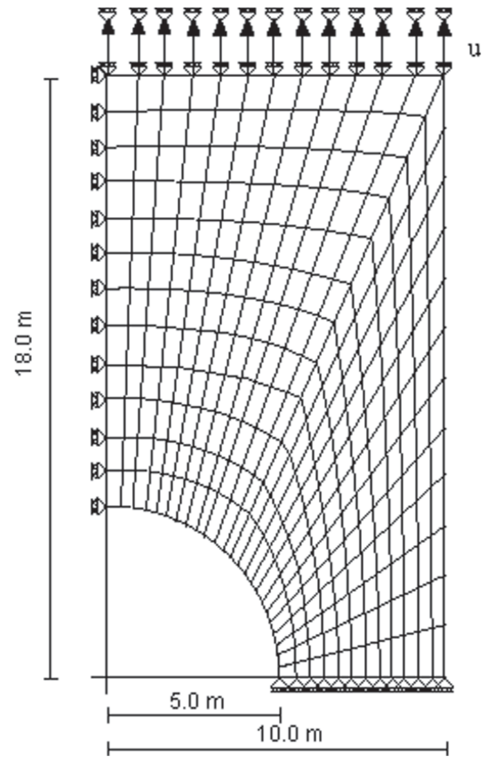

Figure 1: Perforated strip: geometry of the problem and finite element mesh.

in Fig. 1. The loading is performed by controlling the vertical displacement of the top and bottom boundaries of the strip. An increasing displacement of the boundary upper edge is prescribed in the direction perpendicular to the axis of the strip and parallel to one of its sides. For symmetry reasons only one quarter of the section is analyzed. The adopted mesh consists of 325 nodes and 288 elements with 4-node bilinear isoparametric quadrilateral elements. The numerical simulations are performed and implemented into the Finite Element Analysis Program (FEAP) (Zienkiewicz and Taylor (2005), Taylor (2008)).

In the computations we assume the following material properties: elastic modulus $E = 70 \cdot 10^3$ MPa, Poisson's ratio $\nu = 0.2$, yield limit $\sigma_{yo} = 243$ MPa, hardening moduli $H_{kin} = H_{iso} = 1.5 \cdot 10^2$ MPa. The upper edge displacement is prescribed in single steps Δu up to the final displacement $u = 20$ cm. In the computational analysis a constitutive model of the Perzyna type is assumed, with linear viscous effects.

For the analyzed problem the evolution of the plastic interface is illustrated in Fig. 2

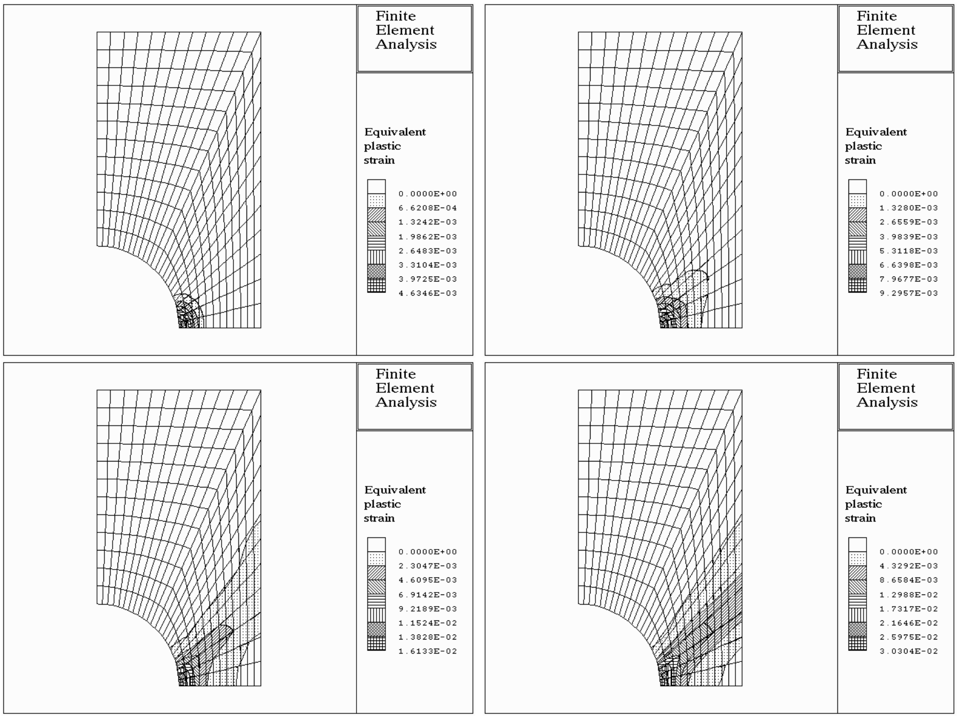

Figure 2: Evolution of the plastic process for an increasing upper edge prescribed displacement u and a rate-independent behavior. (Top left: u =4 *cm*. Top right: u =5 *cm*. Bottom left: u =6 *cm*. Bottom right: u =8 *cm*.)

by reporting the contour plots of the equivalent plastic strain in the strip for prescribed displacements equal to u =4, 5, 6, 8 *cm* and a rate-independent loading program. As the prescribed displacement at the upper edge of the strip is increased it is observed that the plastic strain originates at the rim of the hole and it evolves towards the external lateral edge of the strip.

The numerical results herein illustrated show to be in excellent agreement with the experimental results reported by Theocaris and Marketos (1964), at this regard see e.g. Fig. 12 of page 388 in Theocaris and Marketos (1964). This illustrates the trustworthiness and soundness of the numerical model. The effective reliability of the reported numerical analysis is thus verified.

4 Computational applications for evaluating the effects of the loading rates

In this section the effect of different loading programs on the mechanical response of elasto/viscoplastic material behavior is investigated. In order to account for different values of the displacement rate $\Delta u/\Delta t$ and the intrinsic properties of the material in rate-sensitive loading programs, a non-dimensional loading program parameter is introduced

$$\tau = \frac{t_R}{L_c}\frac{\Delta u}{\Delta t}, \tag{29}$$

where $t_R = \eta/2G$ is the relaxation time and $L_c = L/c$ is a reduced length, being L the length of the strip and c a dimensionless constant. The dimensionless constant c is introduced in order to reduce the length L to a reduced value of the length $L_c = L/c$. In this numerical example it has been assumed $c = 2900$. The value of c has been adopted herein for having more friendly values of τ ranging from 0 to 10.

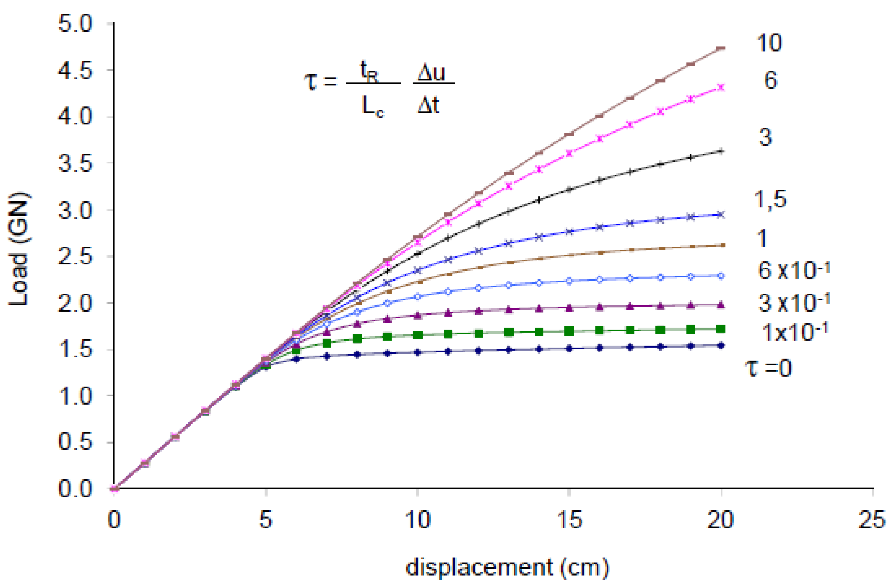

Figure 3: Perforated strip: load vs. displacement curves.

In Fig. 3 load versus displacement curves are plotted for different values of the loading program parameter τ at constant material properties, that is for different values

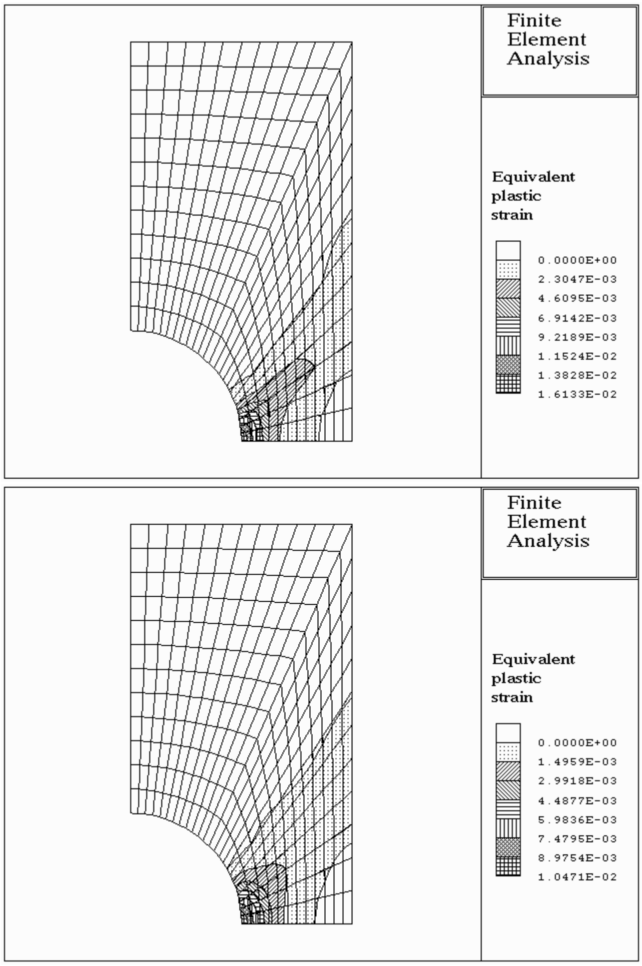

Figure 4: Contour plots of the equivalent plastic strain for the same prescribed displacement $u = 6\ cm$ and with different prescribed loading rates. Top: $\tau = 0$ (rate-independent behavior). Bottom: $\tau = 0.1$ (prescribed loading rate).

of the upper edge displacement rate. In Fig. 3 the load is the sum of the nodal reactions on the bounded upper edge and the displacement is the prescribed displacement at the bounded upper edge. It is observed that the plastic rate-independent behavior is recovered for $\tau = 0$, which corresponds to a static imposition of the load. Nonzero values for τ correspond to a rate-dependent behavior with nonzero prescribed displacement rates $\Delta u/\Delta t$.

In Fig. 4 a straightforward assessment of the structural response of the elasto/visco-

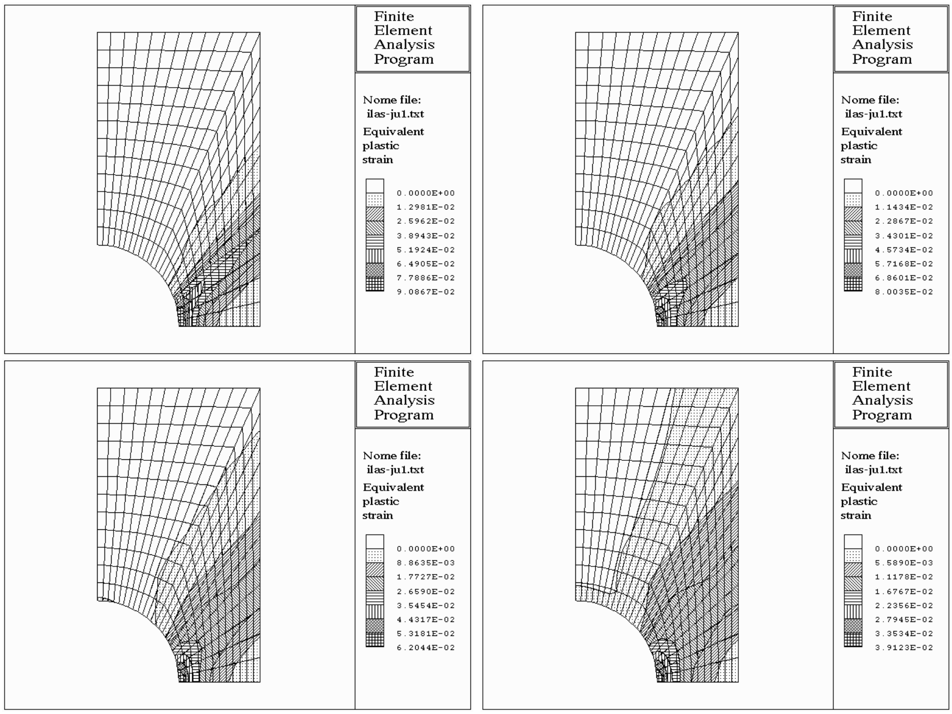

Figure 5: Contour plots of the equivalent viscoplastic strain in the strip for the final prescribed displacement $u = 20$ *cm* and for increasing values of the prescribed loading rate. Top left: τ =0 (rate-independent behavior). Top right: τ =0.1. Bottom left: τ =0.3. Bottom right: τ =1.

plastic material behavior is readily obtained by comparing the contour plots of the equivalent plastic strain for the same prescribed displacement $u = 6$ *cm* with two different loading rates, corresponding respectively to $\tau = 0$ (rate-independent behavior) and $\tau = 0.1$ (prescribed loading rate).

The effects of the different loading rates on the structural response of the elasto/viscoplastic material behavior are also illustrated in Fig. 5 and Fig. 6. The contour plots of the equivalent viscoplastic strain are reported in Fig. 5 for the same final prescribed displacement $u = 20$ *cm* and for increasing values of the prescribed loading rate, corresponding respectively to τ =0, 0.1, 0.3, 1.

In Fig. 6 the contour plots of the equivalent viscoplastic strain in the strip are reported for the same final prescribed displacement $u = 20$ *cm* and for increasing imposed loading rate, corresponding respectively to τ =1.5, 3, 6, 10.

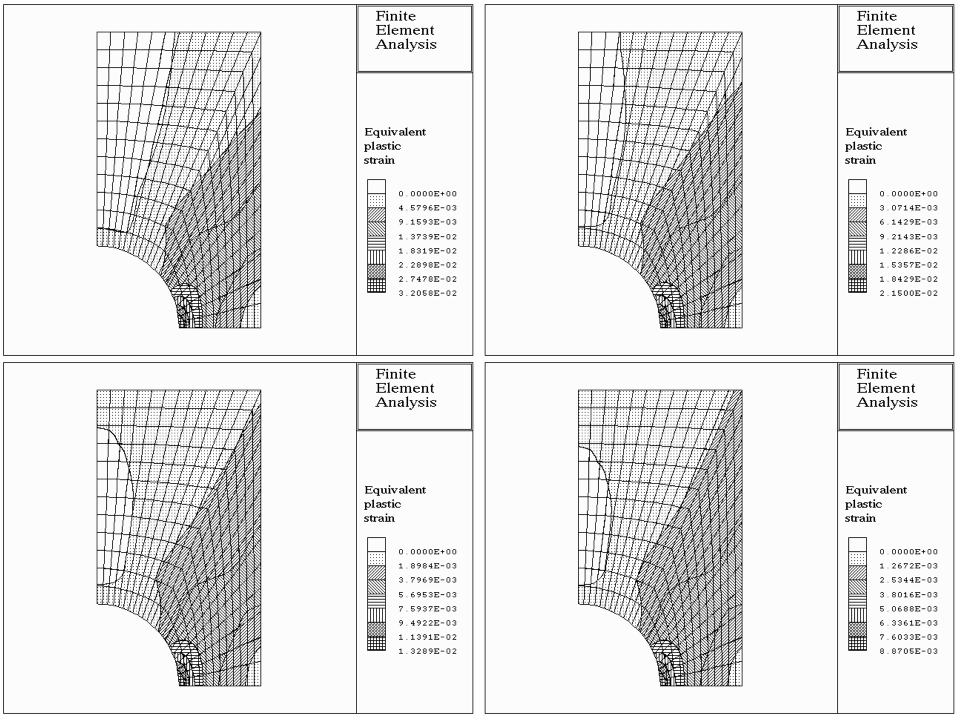

Figure 6: Contour plots of the equivalent viscoplastic strain in the strip for the final prescribed displacement $u = 20\ cm$ and for increasing values of the prescribed loading rate. Top left: τ =1.5. Top right: τ =3. Bottom left: τ =6. Bottom right: τ =10.

From Fig. 5 and Fig. 6 it is possible to assess the influence of an increasing loading rate on the mechanical response of elasto/viscoplastic materials. In particular the plots of Fig. 5 and Fig. 6 are all related to the end points of the curves of Fig. 3, corresponding to the final prescribed displacement $u = 20\ cm$, but with an increasing loading program parameter τ. In a rate-independent loading process (τ =0) the plastic strain evolves in an area of restricted width. In a rate-dependent loading process it is shown in Fig. 5 and Fig. 6 that for the same prescribed displacement and for increasing loading rates, the areas of the strip interested by viscoplastic deformations are more spread over the strip. Part of the areas that initially in a rate-independent loading process were not experiencing inelastic strains, with the increasing of the loading rate become involved in the plastic straining process. With the increasing of the loading program parameter τ, the feature of having more areas interested by plastic strains is associated with a decrease of the maximum value of the equivalent plastic strain experienced at the rim of the hole in the strip. The feature of having more solid material and more areas involved in the dissipation is beneficial with regard to the limitation of the maximum value of the equivalent plastic strain in the solid. In this sense the material shows to act for the benefit of security when subject to loading programs with increasing loading rates.

5 Conclusions

In the present paper the evolutive problem of the elasto/viscoplastic material behavior has been considered and the consequences of different loading programs and different loading rates have been analyzed in order to illustrate their effects on the structural response of elasto/viscoplastic materials. The treatment has been developed by resorting to an internal variable theory and within the framework of the generalized standard material model. The associated problem of evolution in elasto/viscoplasticity has been presented by considering a formulation which is capable to be specialized to different constitutive models. Different loading programs and different loading rates have been considered and their influence on the structural response of the elasto/viscoplastic material behavior has been described. Numerical results for both rate-independent and rate-dependent loading programs have been presented. Computational examples have been illustrated which describe the rate-dependency of the elasto/viscoplastic material behavior. Computational applications have been developed for assessing the influence of different loading programs and different loading rates on the structural response of elasto/viscoplastic materials. Some characteristical features have been highlighted with regard to the structural response of elasto/viscoplastic solids subjected to rate-dependent loadings and it has been noted that the material shows to act for the benefit of security when subject to a loading process with increasing loading rates. Finally, the significance of the different types of loading programs and loading rates and the effects that they produce on the structural response of elasto/viscoplastic materials have been illustrated in detail with specific numerical examples.

References

Alfano, G.; DeAngelis, F.; Rosati, L. (2001): General solution procedure in elasto/viscoplasticity. *Computer Methods in Applied Mechanics and Engineering*, vol. 190, pp. 5123–5147.

Chaboche, J.; Cailletaud, G. (1996): Integration methods for complex plastic constitutive equations. *Computer Methods in Applied Mechanics and Engineering*, vol. 133, pp. 125–155.

Crisfield, M. (1997): *Non-linear Finite Element Analysis of Solids and Structures*, volume I-II. Wiley, Chicester.

DeAngelis, F. (1998): *Constitutive Models and Computational Algorithms in Elasto-Viscoplasticity*. Ph.D. Thesis, february 1998, University of Naples, Naples.

DeAngelis, F. (2000): An internal variable variational formulation of viscoplasticity. *Computer Methods in Applied Mechanics and Engineering*, vol. 190, pp. 35–54.

DeAngelis, F. (2007): Multifield potentials and derivation of extremum principles in rate plasticity. *Materials Science Forum*, vol. 539-543, pp. 2625–2630.

DeAngelis, F. (2007): A variationally consistent formulation of nonlocal plasticity. *Int. Journal for Multiscale Computational Engineering*, vol. 5 (2), pp. 105–116.

DeAngelis, F. (2012): A comparative analysis of linear and nonlinear kinematic hardening rules in computational elastoplasticity. *Technische Mechanik*, vol. 32 (2-5), pp. 164–173.

DeAngelis, F. (2012): Evolutive laws and constitutive relations in nonlocal viscoplasticity. *Applied Mechanics and Materials*, vol. 152-154, pp. 990–996.

DeAngelis, F.; Cancellara, D. (2012): Implications due to different loading programs in inelastic materials. *Advanced Materials Research*, vol. 422, pp. 726–733.

DeAngelis, F.; Cancellara, D. (2012): Results of distinct modes of loading procedures in the nonlinear inelastic behavior of solids. *Advanced Materials Research*, vol. 482-484, pp. 1004–1011.

DeAngelis, F.; Cancellara, D.; Modano, M.; Pasquino, M. (2011): The consequence of different loading rates in elasto/viscoplasticity. *Procedia Engineering*, vol. 10, pp. 2911–2916.

Duvaut, G.; Lions, J. (1972): *Les inequations en mecanique et en physique*. Dunot, Paris.

Freed, A.; Walker, K. (1992): Exponential integration algorithms applied to viscoplasticity. In *3rd Int. Conf. on Computational Plasticity*, pp. 533–537. Barcelona.

Freed, A.; Walker, K. (1993): Viscoplasticity with creep and plasticity bounds. *International Journal of Plasticity*, vol. 9, pp. 213–242.

Halphen, B.; Nguyen, Q. (1975): Sur les matériaux standards généralisés. *Jour. Mech.*, vol. 14, pp. 39–63.

Hill, R. (1950): *The Mathematical Theory of Plasticity*. Clarendon Press, Oxford.

Hughes, T.; Taylor, R. (1978): Unconditionally stable algorithms for quasi-static elasto/viscoplastic finite element analysis. *Computers & Structures*, vol. 8, pp. 169–173.

Ju, J. (1990): Consistent tangent moduli for a class of viscoplasticity. *Journal of Engineering Mechanics*, vol. 116 (8), pp. 1764–1779.

Lemaitre, J.; Chaboche, J. (1990): *Mechanics of solid materials*. Cambridge University Press.

Luenberger, D. (1973): *Introduction to Linear and Non-Linear Programming*. Addison-Wesley, Reading.

Naghdi, P.; Murch, S. (1963): On the mechanical behaviour of viscoelastic/plastic solids. *J. Applied Mechanics*, vol. 30, pp. 321–328.

Ortiz, M.; Popov, E. (1985): Accuracy and stability of integration algorithms for elastoplastic constitutive relations. *International Journal for Numerical Methods in Engineering*, vol. 21, pp. 1561–1576.

Peric, D. (1993): On a class of constitutive equations in viscoplasticity: formulation and computational issues. *International Journal for Numerical Methods in Engineering*, vol. 36, pp. 1365–1393.

Perzyna, P. (1963): The constitutive equations for rate sensitive materials. *Quart. Appl. Math.*, vol. 20, pp. 321–332.

Simo, J. (1991): Nonlinear stability of the time-discrete variational problem of evolution in nonlinear heat conduction, plasticity and viscoplasticity. *Computer Methods in Applied Mechanics and Engineering*, vol. 88, pp. 111–131.

Simo, J.; Govindjee, S. (1991): Non-linear b-stability and symmetry preserving return mapping algorithms for plasticity and visco-plasticity. *International Journal for Numerical Methods in Engineering*, vol. 31, pp. 151–176.

Simo, J.; Hughes, T. (1998): *Computational inelasticity*. Springer-Verlag, New York.

Simo, J.; Kennedy, J.; Govindjee, S. (1988): Non-smooth multisurface plasticity and viscoplasticity. loading/unloading conditions and numerical algorithms. *International Journal for Numerical Methods in Engineering*, vol. 26, pp. 2161–2185.

Skrzypek, J.; Hetnarski, R. (1993): *Plasticity and Creep*. CRC Press, Boca Raton.

Szabo, L. (1990): Tangent modulus tensors for elastic-viscoplastic solids. *Computers & Structures*, vol. 34 (3), pp. 401–419.

Taylor, R. (2008): A finite-element analysis program. Technical report, University of California at Berkeley, 2008. User Manual, Vers: 8.2, http://www.ce.berkeley.edu/rlt.

Theocaris, P.; Marketos, E. (1964): Elastic-plastic analysis of perforated thin strips of a strain-hardening material. *Jour. Mech. Phys. Solids*, vol. 12, pp. 377–390.

Yosida, K. (1980): *Functional Analysis*. Springer Verlag, Berlin, 6th ed. edition.

Zienkiewicz, O.; Cormeau, I. (1974): Visco-plasticity plasticity and creep, a unified numerical solution approach. *International Journal for Numerical Methods in Engineering*, vol. 8, pp. 821–845.

Zienkiewicz, O.; Taylor, R. (2005): *The Finite Element Method for Solid and Structural Mechanics*. Elsevier, Oxford, 6th edition.