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Matrix Crack Effects on Composite Beams with Damage Tolerant Non-Traditional Layups

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Abstract: Two traditional layups built from $0^o/45^o/90^o$ plies and two recently proposed alternative non-traditional layups built from $5^o/65^o$ plies are analyzed in this paper. It was recently shown experimentally that using such off-axis plies in a composite laminate will result in a more damage tolerant structure. A cantilever beam with two traditional layup composite laminates and two non-traditional layup composite laminates is considered in this paper. Both traditional and non-traditional layup schemes are chosen such that they are "hard" laminates, i.e, much stiffer in the longitudinal direction than the lateral direction. The damage is simulated on the beams using a matrix crack model. The reduction in extensional and flexural modulus is discussed for both traditional and non-traditional laminates with different damage levels in the beam. For traditional and non-traditional composite laminates, the tip deflection of beam with constant tip loading and natural frequencies for different damage levels is determined and presented. It is found that the non-traditional layups tend to moderate the behavior of the traditional layups and show better damage tolerance with respect to matrix cracking.

Keywords: Non-traditional laminates, Off-axis plies, Matrix crack, Composites, Cantilever beam, Frequency.

1 Introduction

Composite materials are widely used in structural applications due to their high specific strength and stiffness characteristics [Tompson and Johnson (2009)]. Tailor made directional properties and the possibility of molding into any contour shape are the other advantages of composite materials when compared to metallic structures. The most common use of composite material is in the aircraft industry due

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to low weight and high stiffness requirements. The failure analysis of composite materials is more complicated when compared to traditional metallic materials due to various damage modes in composites. viz., delamination, fiber-matrix debonding, fiber breakage, fiber pull-out and matrix cracking [Adolfsson and Gudmundson (1997)]. The first mode of failure of composite materials observed would be matrix cracking parallel to the fibres in the off-axis plies. The matrix crack density increases proportional to the applied stress and the constraints provided by neighboring plies [Talreja (1993)]. The process of cracking may continue in each layer of the laminate until the cracks in each layer have attained an equilibrium state. This equilibrium state with which the matrix cracking pattern is stabilized is called as characteristic damage state (CDS).

Since a damaged lamina within the laminate retains certain amount of load carrying capacity, it is important to predict the stiffness of the laminate as a function of the damage level. The matrix crack saturation is an indication of the starting point of other more serious forms of damage modes such as delaminations [Pawar and Ganguli (2005); Nairn and Hu (1992)] or matrix cracking in the adjacent plies of the composite laminate [Bailey, Curtis and Parvizi (1979); Jamison, Schulte, Reifsnider and Stinchcomb (1984); Charewicz and Daniel (1986)]. These delaminations may lead to fibre breakage in the primary load bearing plies and will result in loss of load carrying capacity of the entire laminate [Jamison, Schulte, Reifsnider and Stinchcomb (1984)]. Matrix cracking gradually reduces the load carrying capability, strength and stiffness of the laminate [Highsmith and Reifsnider (1982)]. This will result in changes in natural frequency of the structure [Birman and Byrd (2001)], changes in the coefficients of thermal expansion [Bowles (1984)] and moisture absorption characteristics of the structure [Lundgren and Gudmundson (1999)].

In most composite structural applications, a traditional layup sequence $[0^o/45^o/90^o]$ is used due to ease of manual fabrication. Davis, McCarthy and Schrub [Davis, McCarthy and Schrub (1964)] showed that under fatigue loading, the unnotched specimens with non-woven laminates with fibers biased at $\pm 5^o$ performed better than unidirectional laminates. Treasurer and Johnson [Treasurer and Johnson (2008)] discussed the use of off-axis plies in place of 0^o plies while studying the damage progression in open hole specimens under quasi-static loading. Singh and Talreja [Singh and Talreja (2008)] proposed a synergistic damage mechanics (SDM) approach for composite laminates having off-axis plies and used it to predict the stiffness reduction in damaged laminate. The SDM approach combines Micromechanical Damage Mechanics (MDM) and Continuum Damage Mechanics (CDM). Here, crack opening displacement (COD) i.e., average crack surface separation per unit of an applied load quantity is used in the CDM model to predict stiffness degradation of the damaged laminate. Varna, Joffe, Akshantala and Talreja [Varna, Joffe, Akshantala and Talreja (1999)] studied the damage in off-axis plies of composite laminates using continuum damage mechanics. They found that shear-degradation of the off-axis plies is responsible for the laminate stiffness change. Khashaba [Khashaba (2004)] examined the in-plane shear properties of different off-axis cross ply composite laminates. He found that the laminates with 45^o and 60^o off axis plies have maximum in-plane shear strength whereas laminates with 0^o and 90^o plies have minimum in-plane shear strength. Kashtalyan and Soutis [Kashtalyan and Soutis (2006)] discussed the theoretical modeling of matrix cracking in the off-axis plies of unbalanced symmetric composite laminates subjected to in-plane tensile loading. They used 2D shear lag analysis to find the stresses in plies. Equivalent laminate concept is used to obtain the expressions for Mode I, Mode II and total strain energy release rate associated with off-axis ply cracking. Degraded stiffness properties and strain energy release rate are related to crack density and ply orientation angle.

Matrix crack modeling and experimental work has typically addressed traditional layup sequence $[0^o/45^o/90^o]$. It is important to see if the damage tolerant layups obtained in the literature also show good matrix crack resistance properties. These damage tolerant layups have been subjected to many tests [Treasurer and Johnson (2008)] and the results are encouraging. In this paper, the effect of matrix cracking on composite beams with non-traditional layup sequence i.e., $[5^o/65^o]$ is studied. A mathematical model of the matrix cracking [Gudmundson and Zang (1993)] is used in conjunction with a finite element model of a cantilever beam to investigate the four layups (two traditional and two non-traditional) proposed in [Tompson and Johnson (2009); Tompson and Johnson (2011)].

2 Matrix Crack Model

Matrix crack in the composite beam is modeled by a change in the A,B and D matrices. The extensional stiffness matrix A, bending stiffness matrix D and bending-extensional coupling stiffness matrix B of the composite laminate are computed from

$$A = \sum_{k=1}^{N} t^k \overline{Q}^k \tag{1}$$

$$B = \sum_{k=1}^{N} t^k z^k \overline{Q}^k \tag{2}$$

$$D = \sum_{k=1}^{N} t^k \left[(z^k)^2 + \frac{(t^k)^2}{12} \right] \overline{Q}^k \tag{3}$$

where $t^k$is the thickness of the k^{th} laminate, $z^k = \frac{t^k + t^{k-1}}{2}$, $\overline{Q}^k$ is the plane stress stiffness matrix of ply k and N is the number of plies. The transformed reduced stiffness matrix $\overline{Q}^k$ is given as

$$\overline{Q}^k = \mathrm{T}^{-1} Q^k \mathrm{T}^{-\mathrm{T}} \tag{4}$$

where Q^k represents the reduced stiffness matrix of the ply k, T is the transformation matrix and the superscripts -1 and T denote the matrix inverse and matrix transpose of the transformation matrix T, respectively. The reduced stiffness matrix Q^k of the ply k is written as

$$Q^k = \begin{bmatrix} \frac{E_L}{1-\nu_{LT}\nu_{TL}} & \frac{\nu_{LT}E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ \frac{\nu_{LT}E_T}{1-\nu_{LT}\nu_{TL}} & \frac{E_T}{1-\nu_{LT}\nu_{TL}} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \tag{5}$$

where E_L is the Young's modulus in the longitudinal direction of the ply, E_T is the Young's modulus in the transverse direction of the ply, G_{LT} is the shear modulus in L-T plane, ν_{LT} and ν_{TL} are the Poisson's ratios. The transformation matrix T for the ply angle θ is given as

$$\mathrm{T} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{6}$$

The reduced stiffness matrices of the laminate due to the presence of matrix cracks $A^{(c)}$, $B^{(c)}$ and $D^{(c)}$ are obtained by subtracting the ΔA, ΔB and ΔD from stiffness matrices A,B and D of the virgin laminate.

$$A^{(c)} = A - \Delta A \tag{7}$$

$$B^{(c)} = B - \Delta B \tag{8}$$

$$D^{(c)} = D - \Delta D \tag{9}$$

The reduction in stiffness of the laminate due to matrix cracking is a function of crack density ρ. The crack density ρ^k is defined as

$$\rho^k = \frac{t^k}{s^k} \tag{10}$$

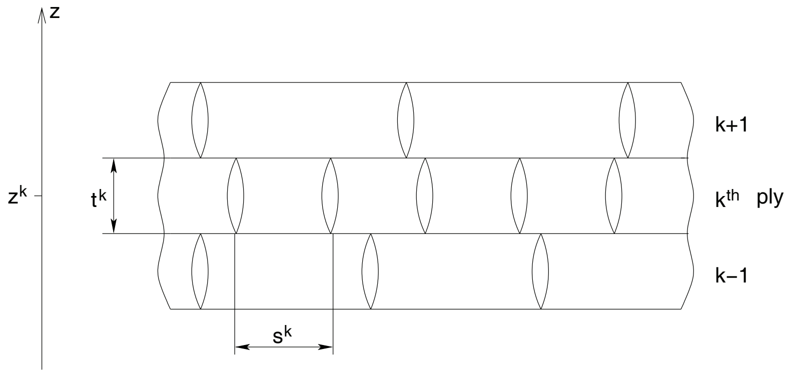


Figure 1: Representation of matrix crack density ($\rho^k = t^k/s^k$) in k^{th} ply

where t^k refers to thickness of the k^{th} ply and s^k refers to average crack spacing shown in Fig. 1. The stiffness matrices $A^{(c)}$, $B^{(c)}$ and $D^{(c)}$ reduce with the increase in crack density ρ. Based on Adolfsson and Gudmundson [Adolfsson and Gudmundson (1997)] model, the strain increment produced by an array of cracks can be related to the local crack face displacement. The changes in stiffness matrices can be given as

$$\Delta A = \sum_{k=1}^{N}\sum_{l=1}^{N}\sqrt{t^k\rho^k t^l\rho^l}C_{EE}^{kl} \tag{11}$$

$$\Delta B = \sum_{k=1}^{N}\sum_{l=1}^{N}\sqrt{t^k\rho^k t^l\rho^l}z^l C_{EE}^{kl} \tag{12}$$

$$\Delta D = \sum_{k=1}^{N}\sum_{l=1}^{N}\sqrt{t^k\rho^k t^l\rho^l}\left[z^k z^l C_{EE}^{kl} + \frac{t^k t^l}{4}C_{BB}^{kl}\right] \tag{13}$$

where the matrix C takes into account the elastic properties and crack orientation relative to applied stress and this can be written as

$$C_m^{kl} = \overline{Q}^k (N^k)^T \beta_m^{kl} N^l \overline{Q}^l \qquad m = EE, BB \tag{14}$$

where EE and BB denote pure extension and bending, respectively. The matrix N^k defined from the constant unit normal vectors n^k for crack surfaces of ply k and can be written as

$$N^k = \begin{bmatrix} n_1^k & 0 & n_2^k \\ 0 & n_2^k & n_1^k \end{bmatrix} \tag{15}$$

Therefore, damage matrices are functions of crack density and crack displacement vector β_m^{kl}. Adolfsson and Gudmundson [Adolfsson and Gudmundson (1997)] give the crack opening displacement matrix with the assumption that the different modes of crack opening displacements and tractions are independent and can be written as

$$\beta_m^{kl} = \begin{bmatrix} \beta_{11(m)}^{kl} & 0 \\ 0 & \beta_{22(m)}^{kl} \end{bmatrix} \qquad m = EE, BB \tag{16}$$

Due to the above assumption, it can be seen that there will be no coupling between the crack opening displacements of different plies, hence

$$\beta^{kl} = 0 \qquad \forall \quad k \neq l$$

In the present work, the crack surfaces in a ply are subjected to mode I and mode III type tractions (i.e., extensional and bending) [Adolfsson and Gudmundson (1997)]. The traction vectors in a ply consist of two components such as constant tractions over the crack surface for extensional component and a linearly varying part of the tractions for bending component. In general, the β^{kk} matrix can be derived for both surface and interior cracks. In the present study of interior cracks, there will be no coupling between the extensional and bending components of the ply traction vector, i.e., β_{EB}^{kk} and β_{BE}^{kk} both vanish [Adolfsson and Gudmundson (1997)]. Due to this reason, the subscript m in Eq. (14) and (16) does not contain the coupling terms EB and BE, which are present otherwise. The components of β^{kk} are derived using the relation between the stress intensity factors and energy release rate. The β^{11} components relate to crack face displacement in mode III anti-plane strain. Moreover, β^{22} relates to mode I crack opening and can only be evaluated numerically by a series expression. The resulting components of the β^{kk} matrices are given by

$$\beta_{11(EE)}^{kk} = \frac{\pi}{2}\gamma_1^k \frac{8}{(\pi\rho^k)^2} \ln\left[\cosh\left(\frac{\pi\rho^k}{2}\right)\right] \tag{17}$$

$$\beta_{22(EE)}^{kk} = \frac{\pi}{2}\gamma_2^k \sum_{j=1}^{10} \frac{a_j}{(1+\rho^k)^j} \tag{18}$$

for the components connected with pure extension (EE) and

$$\beta_{11(BB)}^{kk} = \frac{\pi}{16}\gamma_2^k \sum_{j=1}^{10} \frac{b_j}{(1+\rho^k)^j} \tag{19}$$

$$\beta_{22(BB)}^{kk} = \frac{\pi}{2}\gamma_2^k \sum_{j=1}^{10} \frac{c_j}{(1+\rho^k)^j} \tag{20}$$

for the components which must be added to take bending (BB) into account. The quantities γ_1^k and γ_2^k are defined from the material properties of ply k as

$$\gamma_1^k = \frac{1}{2G_{LT}^k} \tag{21}$$

$$\gamma_2^k = \frac{1-\mu_{LT}^k \mu_{LT}^k}{E_T^k} \tag{22}$$

The results of the components connected to pure extension are obtained from Gudmundson and Zang [Gudmundson and Zang (1993)] and the components required to take bending into account are obtained from Adolfsson and Gudmundson [Adolfsson and Gudmundson (1997)] using the least square fit to the results from numerical integration. The matrix crack model outlined here is relatively easy to include in composite structural analysis [Gayathri, Umesh and Ganguli (2010); Umesh and Ganguli (2009)].

3 Composite Beam Finite Element Model

The different layups are evaluated in this paper for a cantilever beam structure. The cantilevered composite beam is modeled using the finite element method. Each finite element has two end nodes with two degrees of freedom per node (vertical displacement and rotation). The governing differential equation of motion for bending of a symmetrically laminated beam is given by Reddy [Reddy (1997)]

$$\frac{\partial^2}{\partial x^2}\left[E_{xx}^b I_{yy}\frac{\partial^2 w_0}{\partial x^2}\right] - b\hat{N}_{xx}\frac{\partial^2 w_0}{\partial x^2} - \hat{q} + \hat{I}_0\frac{\partial^2 w_0}{\partial t^2} - \hat{I}_2\frac{\partial^4 w_0}{\partial x^2 \partial t^2} = 0 \tag{23}$$

where w_0 is the deflection of the beam, $\hat{N}_{xx}$ is the axial load, E_{xx}^b is the longitudinal modulus of the beam, I_{yy} is the area moment of inertia of the beam about lateral direction y as shown in Fig. 2 and

$$\hat{q} = bq,\ \ \hat{I}_0 = bI_0,\ \ \hat{I}_2 = bI_2$$

Here, b is the width of the beam, $q(x,t)$ is the distributed transverse load and I_0 and I_2 are mass inertias

$$I_0 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho dz, \qquad I_2 = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho z^2 dz \tag{24}$$

where h is the total thickness of the laminate and ρ is the density of the lamina. The element mass and stiffness matrix used for finite element modeling of the beam

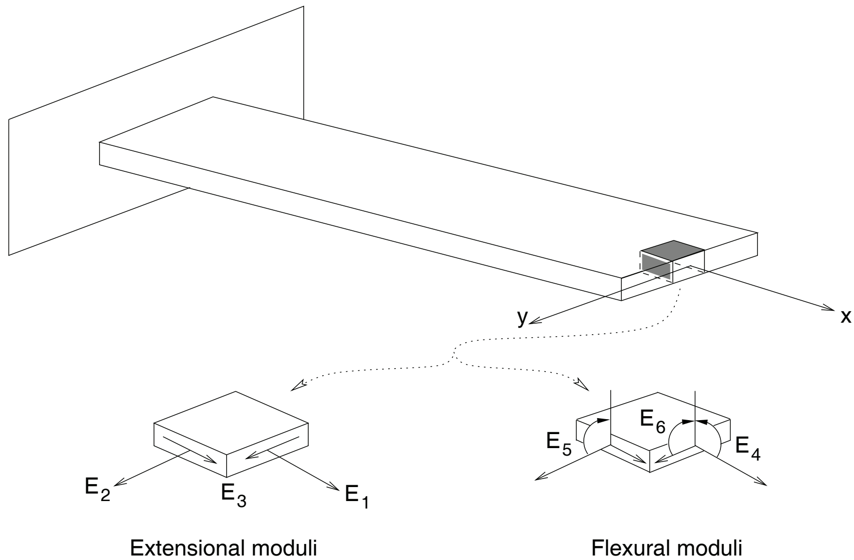


Figure 2: Representation of extensional and flexural stiffnesses

correspond to a symmetrically laminated composite structure. Typically, symmetric layups are used in design. The element mass and stiffness matrices are given in Appendix. This finite element model is validated with commercial software package ANSYS using the beam geometrical and material properties given in Table 1. The material properties are corresponding to AS4/8522 carbon epoxy composite system [Lopes, Seresta, Abdalla, Gürdal, Thuis and Camanho (2008)]. The layup sequence is chosen as $[45/90/-45/0_2/45/0_2/-45/0]_s$. Number of elements used in finite element model and ANSYS software package is 20. Twenty elements were found to be sufficient as shown in Fig. 3. In ANSYS software package, the composite beam is modeled using four node SHELL181(layered) element which has six degrees of freedom at each node (three translations and three rotations). The first three natural frequencies obtained from finite element model are 70.3 Hz, 440.3 Hz and 1232.4 Hz whereas from ANSYS software package, the natural frequencies obtained are 70.3 Hz, 440.4 Hz and 1235.0 Hz, respectively. The first three mode shapes of the composite cantilever beam obtained using ANSYS software package is given in Fig. 4.

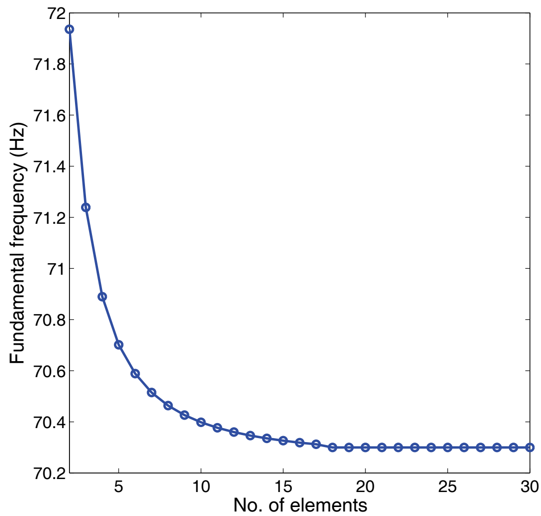
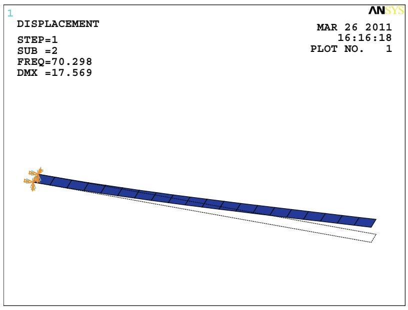


Figure 3: Convergence study of the finite element model

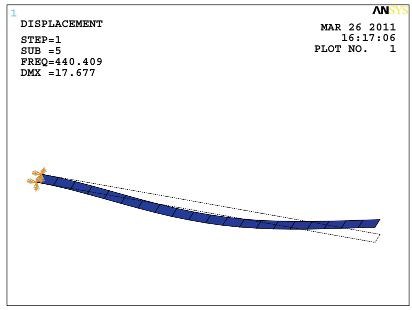
4 Numerical Results

The mechanical and geometrical properties used for analysis of the beam are given in Table 1. Symmetrically laminated composite beam with a total of 20 layers, with ply thickness 0.182mm is used.

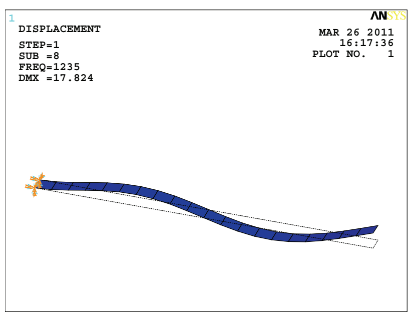
To assess the damage tolerance capability of traditional and non-traditional layup schemes, four different layup schemes given by Tompson and Johnson [Tompson and Johnson (2009); Tompson and Johnson (2011)] are considered. Now, $[0_4/45/0_3/90/0]_s$ and $[45/90/-45/0_2/45/0_2/-45/0]_s$ layups are considered as traditional schemes whereas $[\pm 5/65/(\pm 5)_2/-65/\pm 5]_s$ and $[\pm 5/65/(\pm 5)_2/-65/5/65]_s$ layups are considered as non-traditional schemes. In general, traditional layup schemes comprise of $0^o, 45^o$ and 90^o plies. In non-traditional layup schemes recently proposed in the literature [Tompson and Johnson (2009); Tompson and Johnson (2011)], 0^o plies are replaced with off-axis plies such as $\pm 5^o$ plies, while 45^o and 90^o plies are replaced by $\pm 65^o$ plies. The laminates which are considered here are labeled as "hard" laminates since their longitudinal stiffness is much greater than the transverse stiffness. It can be seen that the percentage of 0^o plies in $[0_4/45/0_3/90/0]_s$ is 80% while the 45^o and 90^o plies contribute 10% each. The two traditional laminates are typically used for cyclic loading. The non-



(a) First mode shape



(b) Second mode shape



(c) Third mode shape

Figure 4: First three mode shapes of composite cantilever beam (ANSYS)

Table 1: Mechanical and Geometrical properties of Composite beam

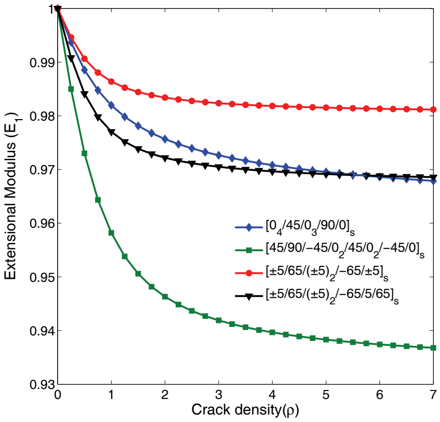
Description of properties	Values used in study
Longitudinal Modulus, $E_L(GPa)$	145
Transverse Modulus, $E_T(GPa)$	9.1
Shear Modulus, $G_{LT}(GPa)$	5.3
Poisson's ratio, ν_{LT}	0.32
Laminate density (kg/m^3)	1590
Ply thickness (mm)	0.182
Beam length (mm)	225
Beam width (mm)	10
Beam thickness (mm)	3.64
Number of layers	20

traditional layups were suggested as a hypothesis by Tompson and Johnson in the recent paper. The number of plies are same and the layups are all hard laminates. Each laminate is given by an identifier. For the traditional laminates, this means the number of $0^o/45^o/90^o$ plies. For the non-traditional laminates, they are the number of $\pm5^o$ and 65^o plies. Thus, the $[0_4/45/0_3/90/0]_s$ layup is identified by 80/10/10 and has a normalized stiffness of 1.0. The normalization is done with the stiffness value of this laminate. For the $[\pm5/65/(\pm5)_2/-65/\pm5]_s$ layup, the identifier is 80/20 and the normalized stiffness is 0.97. The traditional layup $[45/90/-45/0_2/45/0_2/-45/0]_s$ has an identifier given by 50/40/10 and a normalized stiffness of 0.75. For the layup $[\pm5/65/(\pm5)_2/-65/5/65]_s$, the identifier is 70/30 and the normalized stiffness is 0.86. The plies were selected by Tompson and Johnson based on dispersion criteria, stiffness, etc. The laminates are not identical except in terms of weight. However, they are close and can be used to replace the traditional layups.

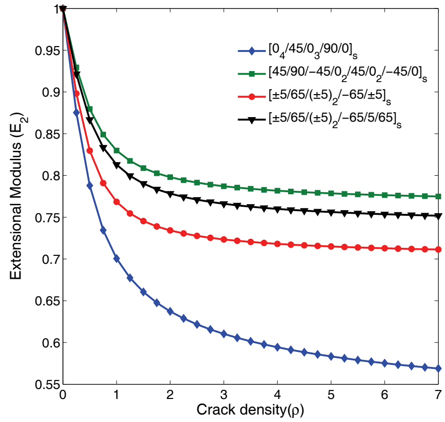
Tompson and Johnson mention with regard to their paper that "These non-traditional laminates show outstanding promise in specific applications, especially when the loading is dominated by compression. The design of composites requires a delicate balance between many properties. This paper has shown that by using off-axis $\pm5^o$ plies to replace traditional 0^o plies, the damage resistance of a joint can be increased without sacrificing too much strength or stiffness. In some cases an increase in stiffness and an increase in damage resistance can be obtained". The laminates can thus be considered as an alternative to conventional laminates for some applications where damage tolerance is more important. The damage tolerance of the laminated composite structure is assessed from parameters such as reduction of stiffness of the beam at matrix crack saturation, tip deflection of the beam for constant tip loading and natural frequency of the beam. These parameters used for damage tolerance study are discussed in next section.

Typical aerospace structures such as helicopter rotor blades and aircraft wings are modeled as beams and any change in their tip deflections due to damage can be undesirable from the structural dynamics and aeroelastic viewpoint [Umesh and Ganguli (2009)]. The extensional stiffness matrix A, bending stiffness matrix D and bending-extensional coupling stiffness matrix B for all the four layup schemes of the beam are computed. Representation of extensional and flexural stiffnesses is given in Fig. 2 [Adolfsson and Gudmundson (1997); Reddy (1997)]. The extensional stiffness E_1 is in the longitudinal direction of the beam, E_2 is in lateral direction of the beam and E_3 is the in-plane shear modulus of the beam. The flexural stiffnesses E_4 is about lateral direction of the beam as shown, whereas E_5 is about longitudinal direction of the beam and E_6 is torsional stiffness of the beam. The reduction in extensional stiffnesses and flexural stiffnesses due to damage modeling by matrix cracking for the traditional and non-traditional layup schemes are determined with different crack densities. These variations are plotted in Fig. 5(a) to Fig. 6(c) for extensional and flexural stiffnesses, respectively. The reduced stiffness values are normalized with respect to corresponding virgin laminate stiffness. Fig. 5 and 6 show the typical behavior of composites undergoing matrix cracking with a sharp fall in stiffness occurring initially and being followed by matrix crack saturation where any further stiffness reduction becomes very gradual. The magnitude and rate of stiffness decay is a useful indicator of damage tolerance with respect to matrix cracking.

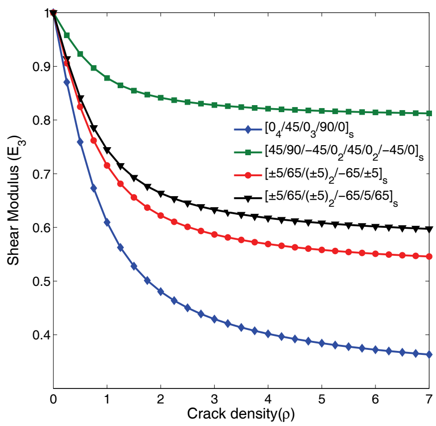
The stiffness reduction at matrix crack saturation ($\rho = 7$) from the virgin laminate is investigated. Figs. 5 and 6 clearly show that stiffness change beyond this point is negligible. From Fig. 5(a), E_1 reduction in traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ is most (6%) when compared to non-traditional layup schemes and traditional layup scheme $[0_4/45/0_3/90/0]_s$ at matrix crack saturation. From Fig. 5(b), E_2 reduction in traditional layup scheme $[0_4/45/0_3/90/0]_s$ is most (43%) when compared to non-traditional layup schemes and traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ at matrix crack saturation. From Fig. 5(c), E_3 reduction in traditional layup scheme $[0_4/45/0_3/90/0]_s$ is most (65%) when compared to non-traditional layup schemes and traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ at matrix crack saturation. The reduction in extensional stiffnesses for traditional and non-traditional layup schemes are summarized in Table 2. The non-traditional layups show a moderate level of change due to damage and avoid the large changes which occur for the traditional layups.



(a) Variation of E_1



(b) Variation of E_2



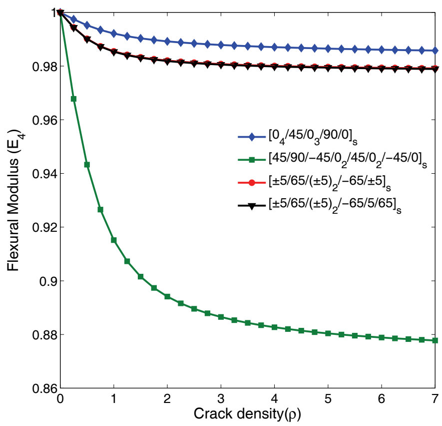
(c) Variation of E_3

Figure 5: Variation of extensional stiffness with respect to matrix crack density

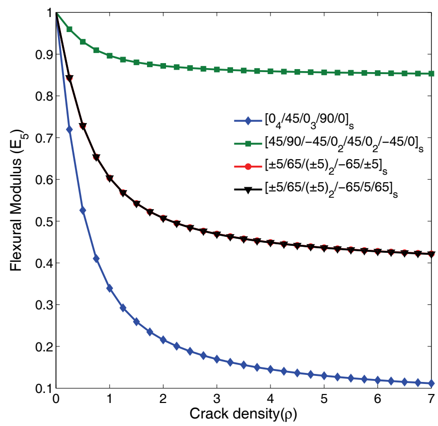
From Fig. 6(a), E_4 reduction in traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ is much more (12%) when compared to non-traditional layup schemes and traditional layup scheme $[0_4/45/0_3/90/0]_s$ at matrix crack saturation. From Fig. 6(b), E_5 reduction in traditional layup scheme $[0_4/45/0_3/90/0]_s$ is much more (90%) when compared to non-traditional layup schemes and traditional layup scheme

$[45/90/-45/0_2/45/0_2/-45/0]_s$ at matrix crack saturation. From Fig. 6(c), E_6 reduction in traditional layup scheme $[0_4/45/0_3/90/0]_s$ is much more (85%) when compared to non-traditional layup schemes and traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ at matrix crack saturation.

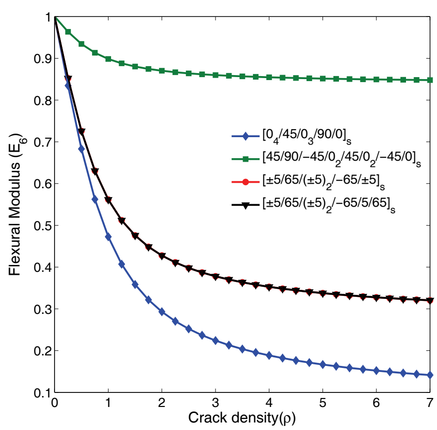
To evaluate these results for structural design, the following results are obtained



(a) Variation of E_4



(b) Variation of E_5



(c) Variation of E_6

Figure 6: Variation of flexural stiffness with respect to matrix crack density

for the cantilever beam. The cantilever beam is subjected to constant tip loading of $0.1N$. The tip deflection (δ) is determined for different crack densities and plotted for traditional and non-traditional layup schemes in Fig. 7. Here, the tip deflection is normalized with respect to the value for the virgin laminate. Also, δ_{ud} represents tip deflection of the beam corresponding to undamaged or virgin laminate. The tip deflections of the traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ is much more when compared to non-traditional layup schemes and traditional layup scheme $[0_4/45/0_3/90/0]_s$ at matrix crack saturation. This is in agreement with the E_4 reduction for traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$. The reduction of first natural frequency (ω_1) of the beam with different crack density level is shown in Fig. 8. Here, ω_1 is normalized with respect to the value for the virgin laminate. Also, ω_{1ud} represents the first natural frequency of the beam corresponding to undamaged or virgin laminate. The natural frequency reduction for the traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$ is much more when compared to non-traditional layup schemes and traditional layup scheme $[0_4/45/0_3/90/0]_s$ at matrix crack saturation. This result is also in agreement with the E_4 reduction for traditional layup scheme $[45/90/-45/0_2/45/0_2/-45/0]_s$.

The reduction in stiffnesses and natural frequencies for traditional and non-traditional layup schemes are summarized in Table 2 and Table 3, respectively. In Table 2 and 3, the damaged state corresponds to crack density $\rho = 7$. The non-traditional layups tend to moderate the behavior of the traditional layups and show better damage tolerance with respect to matrix cracking. Since matrix crack saturation is typically followed by more serious damage mechanisms such as delamination and fibre breakage, the lower stiffness loss suffered by the non-traditional layups can help in increasing the useful life of composite structures. This simulation of matrix cracking numerically strengthens the outcome of the study of Tompson and Johnson [Tompson and Johnson (2009); Tompson and Johnson (2011)], where they showed that the non-traditional layups show a better damage tolerant behavior over the traditional composite laminates under fatigue loading. We have also normalized the stiffness, tip deflection and frequency values of each laminate with the corresponding values for the same undamaged laminate. This has brought out the effect of matrix crack saturation on each laminate and exposed its damage tolerant characteristics.

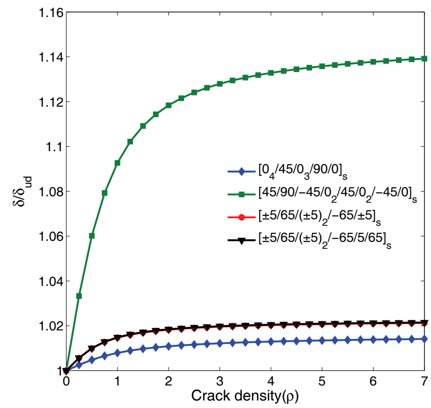


Figure 7: Tip deflection of beam with respect to matrix crack density

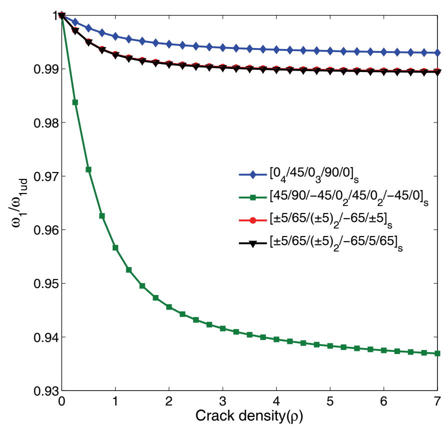


Figure 8: First natural frequency of beam with respect to matrix crack density

5 Conclusion

The effect of non-traditional layups with ply angles $5^o/65^o$ on damage tolerance of the composite laminate structure is studied using a matrix crack model and finite element simulations. The elastic and dynamic characteristics of the beam such as modulus in different directions, natural frequencies and tip deflection of the beam under external loading are studied with different crack densities upto matrix crack

Table 2: Stiffness reduction $(\%)$ for traditional and non-traditional layup schemes $(\rho = 7)$

Type	layup scheme	E_1	E_2	E_3	E_4	E_5	E_6
Traditional	$[0_4/45/0_3/90/0]_s$	3	42.5	65	1	90	85
Traditional	$[45/90/-45/0_2/45/0_2/-45/0]_s$	6	22.5	20	12	15	15
Non-traditional	$[\pm5/65/(\pm5)_2/-65/\pm5]_s$	2	30	45	2	60	70
Non-traditional	$[\pm5/65/(\pm5)_2/-65/5/65]_s$	3	25	40	2	60	70

Table 3: Frequency (Hz) for traditional and non-traditional layup schemes $(\rho = 7)$

layup scheme	ω_1		ω_2	
	Undamaged	damaged	Undamaged	damaged
$[0_4/45/0_3/90/0]_s$	106.2	105.4	665.2	660.6
$[45/90/-45/0_2/45/0_2/-45/0]_s$	70.3	65.9	440.3	412.6
$[\pm5/65/(\pm5)_2/-65/\pm5]_s$	99.8	98.8	625.6	619.0
$[\pm5/65/(\pm5)_2/-65/5/65]_s$	99.8	98.7	625.2	618.6

layup scheme	ω_3		ω_4	
	Undamaged	damaged	Undamaged	damaged
$[0_4/45/0_3/90/0]_s$	1861.7	1848.7	3645.8	3620.3
$[45/90/-45/0_2/45/0_2/-45/0]_s$	1232.4	1154.7	2413.4	2261.2
$[\pm5/65/(\pm5)_2/-65/\pm5]_s$	1750.7	1732.4	3428.5	3392.6
$[\pm5/65/(\pm5)_2/-65/5/65]_s$	1749.7	1731.2	3426.5	3390.3

saturation. A comparative study of traditional layups ($[0_4/45/0_3/90/0]_s, [45/90/-45/0_2/45/0_2/-45/0]_s$) with the non-traditional layups ($[\pm5/65/(\pm5)_2/-65/\pm5]_s, [\pm5/65/(\pm5)_2/-65/5/65]_s$) is conducted. From the study, it is observed that the non-traditional layups show a moderate behaviour and avoid the large changes in stiffness due to matrix crack in the traditional layups. For example, at matrix crack saturation, the reduction in bending stiffness (E_4) in non-traditional layups is only 2% as against 12% reduction in traditional layup $[45/90/-45/0_2/45/0_2/-45/0]_s$. Similarly, the reduction in torsional stiffness (E_6) in non-traditional layups is 70% as against 85% reduction in traditional layup $[0_4/45/0_3/90/0]_s$. Therefore, the non-traditional layup laminates can perform better in multi-axial loading conditions as the loss in stiffness is moderated across the different directions.

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Appendix

Element stiffness matrix:

$$[K^e] = \frac{2E^e_{xx}I^e_{yy}}{h_e^3}\begin{bmatrix} 6 & -3h_e & -6 & -3h_e \\ -3h_e & 2h_e^2 & 3h_e & h_e^2 \\ -6 & 3h_e & 6 & 3h_e \\ -3h_e & h_e^2 & 3h_e & 2h_e^2 \end{bmatrix}$$

Element mass matrix:

$$[M^e] = \frac{\hat{I}^e_0}{420}\begin{bmatrix} 156 & -22h_e & 54 & 13h_e \\ -22h_e & 4h_e^2 & -13h_e & -3h_e^2 \\ 54 & -13h_e & 156 & 22h_e \\ 13h_e & -3h_e^2 & 22h_e & 4h_e^2 \end{bmatrix} + \frac{\hat{I}^e_2}{30h_e}\begin{bmatrix} 36 & -3h_e & -36 & -3h_e \\ -3h_e & 4h_e^2 & 3h_e & -h_e^2 \\ -36 & 3h_e & 36 & 3h_e \\ -3h_e & -h_e^2 & 3h_e & 4h_e^2 \end{bmatrix}$$

where h_e refers to length of the element and other notations are described in Eq. (23-24) with the superscript 'e' representing element.