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Structural System Identification Using Quantum behaved Particle Swarm Optimisation Algorithm

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Abstract: Development of efficient system identification techniques is highly relevant for large civil infrastructure for effective health monitoring, damage detection and vibration control. This paper presents a system identification scheme in time domain to estimate stiffness and damping parameters of structures using measured acceleration. Instead of solving the system identification problem as an inverse problem, we formulate it as an optimisation problem. Particle swarm optimisation (PSO) and its other variants has been a subject of research for the past few decades for solving complex optimisation problems. In this paper, a dynamic quantum behaved particle swarm optimisation algorithm (DQPSO) is proposed for the solution of the complex nonlinear optimisation problem associated with the system identification. If the uniqueness of the parametric solution is guaranteed for the assumed model, this heuristic method allows finding a solution without incurring restrictions of other classical optimisation methods, like the need for reliable initial estimates and convergence to local optima. In order to solve large size problems, we propose to use reduced order models. Proper orthogonal decomposition (POD) is used for the model reduction instead of traditional modal analysis. The validity of the proposed method is demonstrated by a numerical simulation study on a simply supported beam, 50 storey shear building model and a truss bridge. The robustness of the proposed system identification technique is investigated using a noise sensitivity study.

Keywords: Structural system identification, proper orthogonal decomposition, meta-heuristic algorithm, Quantum particle swarm optimisation, measurement noise.

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1 Introduction

System identification is the process of constructing or updating the mathematical model of a dynamical system based on input and output observations. Structural system identification (SSI) is an important research topic and has greater significance for civil engineering applications. Apart from updating numerical models for better response prediction, System identification can be applied to health monitoring of structures and even detect damages based on changes in identified parameters. For active vibration control of structures, actual parameter values of the structure, rather than the assumed or design values, are required for effective control.

Due to rapid advances in computer and instrumentation capabilities, the use of structural identification methods has increasingly become a feasible option for non-destructive structural assessment. Research interest in this subject has been intense over the last two decades. Various system identification (SI) schemes [Ghanem and Shinozuka (1995)] have been developed to verify structural models or to assess damage in a structure during last few decades. Based on the types of measured response, SI algorithms can be classified into static SI [Hjelmstad and Shin (1997); Lee *et al.,* (1999); Yeo *et al.,* (2000); Park *et al.,* (2001)] frequency-domain SI [Hjelmstad and Shin (1996); Shi *et al.,* (2000); Vestroni and Capecchi (2000)] and time-domain SI [Hjelmstad *et al.,* (1995); Banan *et al.,* (1995); Ge and Soong (1998); Huang (2001)]. Frequency-domain SI or time-domain SI may be more practical in real applications. Even if frequency-domain SI algorithms utilize the same source of dynamic responses as time-domain SI, the amount of data dealt with is remarkably reduced through transformation. Due to the ease in handling data, frequency-domain SI algorithms have been more widely developed and applied. However, local damage may influence higher modes that are usually difficult to measure from experiments [Raghavendrachar and Aktan (1992)]. Moreover, damping properties of structures cannot be estimated in frequency-domain SI. To overcome these drawbacks of the frequency-domain SI and to yield more meaningful identification results, the time-domain SI schemes are an attractive alternative. In developing a time-domain SI algorithm, the incomplete measurements in space and state should be considered in addition to measurement noise [Hjelmstad *et al.,* (1995)]. The incompleteness in space occurs when structural responses are not measured at all degrees-of-freedom (dof) corresponding to its numerical model. Some SI algorithms circumvent this difficulty by including the unmeasured dof as system parameters to be estimated in SI [Hjelmstad *et al.,* (1995)]. The incompleteness in state also occurs in most dynamic measurements because only one state of acceleration, velocity, or displacement time history is usually measured. Numerical schemes for integrating or differentiating the measured state vector [Hjelmstad *et* *al.,* (1995)] are applied to compute unmeasured state vectors. Since the numerical schemes naturally develop computational error and amplify noise in measured responses, the most desirable way may be to avoid computing unmeasured responses using measured data in formulating a SI algorithm.

The majority of the earlier system identification methods come under the purview of classical methods and they invariably use least square methods like Extended Kalman Filter (EKF), recursive least squares, instrumental variable, and maximum likelihood methods etc. These methods [Ghanem and Shinozuka (1995)], in one way or other, search for the optimal solution by exploiting the previous solution. In the present work, the system identification problem is formulated as an optimisation problem and a meta-heuristic algorithm based on swarm intelligence is employed for solving the resulting complex non-linear optimisation problem. Although using evolutionary algorithms for solving system identification problems is not new, it is not very frequent. Moreover, most of the earlier works invariably use genetic algorithms (GA) for SSI [Cunha *et al.*, (1999); Franco *et al.*, (2004); Perry et al., (2006); Chou and Ghaboussi (2001); Koh *et al.,* (2000. 2000a, 2003); Rama Mohan Rao and lakshmi (2011)]. GA [Goldberg (1989)] is a stochastic optimisation algorithm employed for combinatorial and continuous optimisation problems. GA can efficiently search large solution spaces due to its parallel structure and the probabilistic transition rules employed in the operators. However, a basic GA has two main drawbacks: lack of good local search ability and premature convergence. On the other hand, swarm intelligence based algorithms like particle swarm optimisation algorithms [Kennedy and Eberhart (1995); Eberhart and Kennedy (1995)] are gaining popularity over other meta-heuristic algorithms due to simplicity in parameter setting and also high adaptability for fine tuning. PSO has some attractive characteristics and in many cases proved to be more effective when compared with GA and other similar evolutionary techniques [Hassan et al., (2005)] .Keeping these things in view, an advanced particle swarm optimisation algorithm called dynamic quantum particle swarm optimisation algorithm (DQPSO) is proposed in this paper, for solving the optimisation problem associated with system identification.

In most of the earlier investigations, the identification of system matrices has been achieved by traditional modal analysis techniques. For low-frequency vibration problems, only a few modes contribute to the total response of the system and thus modal analysis can be used to identify the mass, stiffness and damping matrices from the measured response [Ewins (2000)]. Thus, the accuracy of the identified parameters depends on the presence of distinct peaks in the measured frequency response functions (FRFs). This problem is inherent to the conventional modal analysis. If the peaks in the measured FRFs are not distinct or are closely spaced, the modal parameter extraction procedure is difficult to apply. Thus, the identified system matrices obtained using the extracted modal parameters become erroneous. Apart from this, identification of complex modes poses a serious challenge in the presence of non-proportional damping [Srikantha Phani and Woodhouse (2002)]. Keeping these things in view, the proper orthogonal modes are extracted by Proper Orthogonal Decomposition (POD) of the response and used these modes for structural system identification. Recent innovation in the field of data-acquisition hardware allows us to acquire highly resolved spatio-temporal vibration data. For example, [Dionysius and Yozo (2009)] made use of the Laser Doppler vibrometer (LDV) to conduct contact-free measurement for operational modal analysis.

The major issue with the population based stochastic algorithms for structural system identification is the high computational time for convergence. In this paper, the focus is on improving the computational performance of structural system identification algorithms, by substantially reducing the forward simulation time during evolutionary process through reduced order models. Apart from that a new meta-heuristic algorithm called DQPSO with better convergence characteristics is developed and employed to improve the computational performance of structural system identification algorithms.

During experimental investigations, dynamic response is normally measured by using accelerometers. Error is incurred to obtain velocity and displacement signals by integration. Hence, direct use of acceleration signals is preferred over velocity and displacement signals. Keeping this view, it is proposed to use only acceleration measurements in the proposed system identification algorithm.

It is assumed that mass is known *a priori* and structures behave linearly. Since structural dynamic behaviours are not only dependent on mass and stiffness but also on damping properties, estimation of damping parameters may be important for correct identification of structural systems. Nevertheless, most time-domain SI schemes have assumed damping as known and thus dealt solely with stiffness parameters [Banan et al., (1995); Ge and Soong, (1998)]. In this paper, the structural damping is modeled by the Rayleigh damping, and two Rayleigh damping coefficients are estimated together with the unknown stiffness parameters.

To evaluate the proposed method, numerical simulation studies are carried out on a simply supported beam, a fifty storey framed structure and a truss bridge. To examine the developed algorithm with noisy measured data, random noise is added to the generated time history of acceleration in the simulation study in the form of signal to noise ratio (SNR). Discussions on numerical results and behaviours of the proposed method are presented.

2 Parameter estimation in time domain using reduced order models

The forced vibration of a linear, discrete viscously damped system with 'n' degrees of freedom, can be represented by

$$M_n\ddot{u}_n(t) + C_n\dot{u}_n(t) + K_n u_n(t) = f_n(t) \tag{1}$$

$u(0) = 0$ and $\dot{u}(0) = 0$

where, $M_n \in \Re^{nxn}$ is the mass matrix, $C_n \in \Re^{nxn}$ is the damping matrix and $K_n \in \Re^{nxn}$ is the stiffness matrix. The damping matrix is computed using Rayleigh damping, which can be related to damping ratios for any two selected modes. The displacement, velocity, acceleration and force vectors at time '$\boldsymbol{t}$' are represented by $\mathrm{u}_n(\mathrm{t})$, $\dot{u}_n(t)$, $\ddot{u}_n(t)$ and $\mathrm{f}_n(t)$ respectively and are of size $\Re^n$.

The system parameters include stiffness and damping properties of a structure, which need to be identified. In the formulation, it is assumed that mass properties, load history and the initial conditions for Eq. (1) are known *a priori*, and that the system parameters are invariant in time. The unknown system parameters are identified through the following minimization of the least-squared error between calculated and measured accelerations at observation points from the beginning up to current time t.

$$\text{Min } \Psi(x,nt) = \frac{\sum_{k=1}^{k=nt}\sum_{i=1}^{np}\left({}^{i}\ddot{u}_k^m - {}^{i}\ddot{u}_k^e(x)\right)^2}{nt} \tag{2}$$

where $\Psi(x,nt)$ represents the function to be minimized, x is the estimated system parameters , nt is the number of time steps, np represents the degrees of freedom where the measured acceleration time history response is available, ${}^{i}\ddot{u}_k$ represents the acceleration vector at i^{th} measured degree of freedom for k^{th} time step. The superscripts 'm' and 'e' represent measured and estimated values.

Considering member stiffness parameters and damping ratios as design variables, the optimisation can be performed using an evolutionary approach to arrive at identified parameters. However, for large structures, population based evolutionary optimisation process takes enormous time as the time integration with estimated parameters to arrive at $\ddot{u}_k^e$ is computationally intensive. In order to overcome this problem we use reduced order models. Instead of identifying parameters using the physical equations of motion (searching in a domain of usually a high order of dimensions), the evolutionary search is conducted using the reduced (transformed) equations of motion (searching in a number of smaller domains). Physical parameters are then recovered by making use of the orthogonal properties of the basis vectors.

The most important consideration for any kind of reduced-order modelling is the selection of a good reduced-order basis that represents the response of the high-dimensional system. POD [Kerschen et al. (2005); Liang et al. (2002); Kerschen and Golinval (2002); Feeny and Kappagantu (1998); Feeny (2002); Azeez and Vakakis (2001)] provides a basis for the spectral decomposition of a spatio-temporal signal and its property of mean-square optimality provides an efficient means of capturing the dominant components of a high-dimensional signal through a few dominant scales of fluctuations called Proper Orthogonal Modes.

The response function of acceleration at the ith (i = 1 to r time steps) snapshot obtained at n locations typically looks like

$$\ddot{u}(t_i) = \begin{bmatrix} \ddot{u}_1(t_i) \\ \vdots \\ \ddot{u}_n(t_i) \end{bmatrix} . \tag{3}$$

The response correlation matrix $R_{uu} \in \Re^{nxn}$ in the time domain is found to be $R_{uu} = \langle \ddot{u}(t)\ddot{u}^T(t) \rangle$, where $\langle . \rangle$ is the time averaging operator. Further, its spectral decomposition is obtained as $R_{uu} = \sum_{i=1}^{n} \lambda_i \phi_i \phi_i^T$ where λ_i are the eigenvalues (average energy contributed by mode ϕ_i) of R_{uu} and ϕ_i are the corresponding eigenvectors which form an orthonormal basis. The first few dominant modes known as the proper orthogonal modes contain the greatest amount of energy and need to be selected. If $E = \sum_{j=1}^{n} \lambda_j$ represents the total energy content in the data, then $\sum_{j=1}^{p} \frac{\lambda_j}{E} \geq \kappa$ represents the p modes required to capture κ energy of the measured accelerations. The transformation matrix containing the first p dominant POD eigenvectors can now be written as

$$\Sigma = [\phi_1, \ldots, \phi_p] \in \Re^{nxp} \tag{4}$$

Using this POD transformation matrix $\sum$, the reduced order representation of the system in equation (1) can be written as

$$M_p \ddot{u}_p(t) + (\alpha M_p + \beta K_p)\dot{u}_p(t) + K_p u_p(t) = f_p(t), \;\; p = 1, 2, \ldots \bar{N} \tag{5}$$

where

$$M_p = \Sigma^T M_n \Sigma \in \Re^{pxp}; \quad C_p = \Sigma^T C_n \Sigma \in \Re^{pxp}; \quad K_p = \Sigma^T K_n \Sigma \in \Re^{pxp} \tag{6}$$

are the reduced-order mass, damping and stiffness matrices respectively. The reduced-order acceleration and force vectors are respectively given by

$$\ddot{u}_p = M_P^{-1} \Sigma^T M_n \ddot{u}_n \tag{7}$$

$$f_p(t) = \Sigma^T f_n(\text{t}) \tag{8}$$

The modal response is computed in the time domain numerically using Newmark's time integration scheme.

3 Cost function for system identification

As mentioned earlier, instead of working in the physical domain, the search for optimal parameters is carried out in the modal domain. Accordingly, the cost function given in equation (2) can be modified as

$$\text{Min } \psi(\text{x, nt}) = \frac{\sum_{p=1}^{p=nm} \sum_{k=1}^{k=nt} \left({}^k\ddot{u}_p^a - {}^k\ddot{u}_p^e\right)^2}{nt} \tag{9}$$

where *'nt'* are the number of samples, *'nm'* are the number of POD modes ${}^k\ddot{u}_p^e$ is the modal acceleration responses corresponding to the estimated system and ${}^k\ddot{u}_p^a$ corresponding to the modal acceleration responses of the actual system.

An advanced evolutionary algorithm called dynamic quantum particle swarm optimisation (DQPSO) algorithm is proposed for solving the resulting complex nonlinear optimisation problem. Comparisons have also been made with other variants of swarm intelligence algorithms and also hybrid genetic algorithm to demonstrate the superiority of the proposed approach.

4 Particle swarm optimisation algorithm

Particle swarm optimisation (PSO) is a population based stochastic optimisation technique developed by Eberhart and Kennedy (Kennedy and Eberhart (1995), Eberhart, and Kennedy (1995)), inspired by social behaviour of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). However, unlike GA, PSO has no evolution operators such as crossover and mutation and PSO is also easy to implement. PSO has been successfully applied in many areas: function optimisation, artificial neural network training, fuzzy system control, and other areas where GA can be applied.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating these solutions iteratively. Each particle is updated by following two "best" values in every iteration. The first one is the best solution (fitness) it has achieved so far. This value is called *pbest*. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called *gbest*. After finding the two best values, the particle updates its velocity and positions with following equations.

$$v_{ij}^{k+1} = \omega\, v_{ij}^{k} + c_1 r_1 (\text{pbest}_{ij} - x_{ij}^{k}) + c_2 r_2 (\text{gbest}_j - x_{ij}^{k}) \quad (10)$$

$$x_{ij}^{k+1} = x_{ij}^{k} + v_{ij}^{k+1} \quad (11)$$

v_{ij} is the particle velocity, x_{ij} is the current particle (solution) and ω, c_1 and c_2 are weight coefficients, r_1 and r_2 random values between 0 and 1. The subscripts, i refer to the particle number and j refers to the element (design variable) in a typical i^{th} particle. The superscript k indicates the iteration number. In PSO, the existence of position data on the swarm-shared best solution *gbest* assures interaction among agents.

It can be shown that, PSO algorithm can be made adaptive by suitably tuning the weight coefficients c_1 and c_2 using the information acquired in the course of the search. For this purpose, a parameter P is defined as

$$P = \frac{2\,.|\text{gbest} - x|}{c_1.|\text{pbest} - x| + c_2.|\text{gbest} - x|} \quad (12)$$

It can be easily verified that by adjusting the single parameter P adaptively [Rama Mohan Rao and Ganesh Ananda kumar (2007)] during the course of the search based on the current state of swarm, PSO can be made as an adaptive algorithm. The parameters c_1 and c_2 can be determined uniquely as

$$c_2 = \frac{2}{P} \text{ when x = pbest} \quad (13)$$

The value of c_1 has no significance here, as (*pbest* - *x*) = 0 in equation (10)

$$c_2 = \frac{1}{P} \text{ and } c_1 = c_2 \frac{|\text{gbest} - x|}{|\text{pbest} - x|} \text{ when } x \neq \text{ pbest} \quad (14)$$

Numerical experiments [Rama Mohan Rao and Ganesh Ananda kumar (2007)] indicate that while, $P = 0.10$ diversifies the search, $P = 0.50$ intensifies. Thus by varying a single parameter, P adaptively, a good balance between intensification and diversification can be maintained.

4.1 Neighbourhood search algorithm

Although PSO algorithm provides promising result, it remains clear that metaheuristic algorithms, in many cases, cannot compete with specialized neighbourhood search algorithms [Johnson and McGeoch(1997)]. However, neighbourhood search algorithms often suffer from the *initialization problem*. That is, the performance of a local optimizer is often a function of the initial solution to which it is applied. Therefore hybridization of meta-heuristic algorithm with an effective neighbourhood search algorithm is likely to provide much superior solutions as metaheuristic algorithms generates good initial solutions for the local search algorithm to explore further to provide a good local optimised solution. Keeping this in view, a neighbourhood search algorithm is embedded into the adaptive PSO algorithm to search in the better local areas. For this purpose, the Nelder-Mead algorithm [Nelder and Mead (1965)] is employed as neighbourhood search algorithm in our adaptive PSO algorithm.

The Nelder-Mead algorithm is given in Figure 1 and it will be discussed briefly in the subsequent sections. However, detailed implementation of Nelder-Mead algorithm in self configurable PSO can be found in Rama Mohan Rao and Ganesh (2007). After every L iterations, the swarms are sorted according to their fitness values and best 80 % of the particles in the swarm are refined using Nelder-Mead algorithm. The self configurable PSO augmented with Nelder-Mead Algorithm is found to be more effective for sensor optimisation applications [Rama Mohan Rao and Ganesh Ananda kumar (2007)] and it is termed as hybrid adaptive PSO algorithm.

In the present work, efforts are made to further improve the convergence characteristics by employing quantum behaved PSO algorithm [Sun et al. (2004, 2004a)] and also by incorporating dynamic reconfigurable features in the quantum PSO algorithm.

4.2 Quantum PSO algorithm (QPSO)

In the quantum physics, the state of a particle with momentum and energy can be depicted by its wave function $\psi(x,t)$ instead of the position and velocity used in traditional PSO. The dynamic behavior of the particle is widely divergent from that of the particle in traditional PSO systems. In this context, the probability of a particle appearing in a certain position x_i can be obtained from probability density function $|\psi(x,t)|^2$, the form of which depends on the potential field in which the particle lies. The particle move according to the following equation

$$x_{ij}^{t+1} = p_{ij}^{t} \pm \beta \left|\mathrm{mbest}_{ij}^{t} - x_{ij}^{t}\right| * \ln(1/u_{ij}) \tag{15}$$

where mbest_{ij} is the mean best of all the particles in j^{th} dimension and u_{ij} is a random number uniformly distributed in (0,1). The subscripts i and j refers to the particle and design variable respectively. This equation is implemented by using

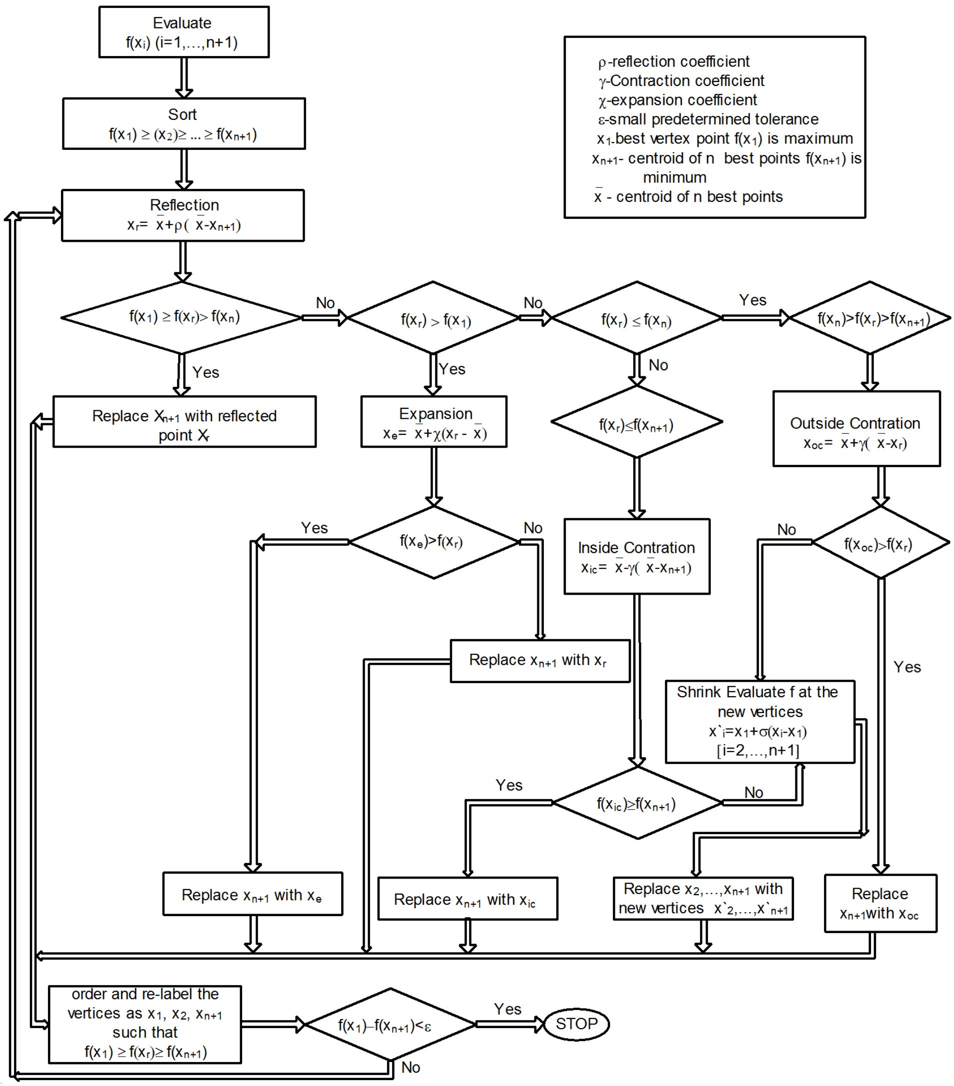


Figure 1: Nelder Mead Algorithm

Monte-Carlo technique as

$$\begin{aligned} x_{ij}^{t+1} &= p_{ij}^{t} + \beta \left| \text{mbest}_{ij}^{t} - x_{ij}^{t} \right| * \ln(1/u_{ij}) \ if \ k > 0.50 \\ x_{ij}^{t+1} &= p_{ij}^{t} - \beta \left| \text{mbest}_{ij}^{t} - x_{ij}^{t} \right| * \ln(1/u_{ij}) \ if \ k \leq 0.50 \end{aligned} \tag{16}$$

where k is a random number in the range [0, 1]. The most commonly used controlled strategy of β is to initially setting it to 1.0 and reducing it linearly to 0.30. In the present work, the parameter β varied linearly from 1.0 to 0.30 with the iteration as

$$\beta^{t} = \beta_{\max} - \frac{(\beta_{\max} - \beta_{\min})}{\text{max\_iterations}} \ x \ t \tag{17}$$

p_{ij}^{t} is the local attractor and defined as:

$$p_{ij}^{t} = \phi_{ij}^{t} \ \text{Pbest}_{ij}^{t} + (1 - \phi_{ij}^{t}).\text{gbest}_{j}^{t} \tag{18}$$

where ϕ_{ij}^{t} is a random number uniformly distributed in (0, 1). β is called the contraction-expansion coefficient, which can be tuned to control the convergence speed of the algorithm. The 'mbest' is the mean best position and is defined as the center of *pbest* positions of the swarm and it can be written as:

$$\begin{aligned} &\text{mbest}_{ij}^{t} = (\text{mbest}_{1}^{t}, \text{mbest}_{2}^{t}, \text{mbest}_{3}^{t} \ldots\ldots \text{mbest}_{D}^{t}) = \\ &\left(\frac{1}{M} \sum_{i=1}^{M} P_{i1}^{t}, \frac{1}{M} \sum_{i=1}^{M} P_{i2}^{t}, \frac{1}{M} \sum_{i=1}^{M} P_{i3}^{t}, \ldots\ldots\ldots\ldots, \frac{1}{M} \sum_{i=1}^{M} P_{iD}^{t}, \right) \end{aligned} \tag{19}$$

where M is population size and P_i is the personal best position of particle i. The details of the QPSO algorithm are given in Figure 2.

The characteristics of QPSO algorithm are reflected mainly in two ways. First of all, the introduced exponential distribution of positions makes QPSO search in a wide space. Furthermore, the introduction of Mean Best Position into QPSO is another improvement. In the standard PSO, each particle converges to the global best position independently. In the QPSO with mean best position GP, each particle cannot converge to the global best position without considering its colleagues, because the distance between the current position and GP determines the position distribution of the particle for the next iteration.

4.3 Dynamic quantum PSO (DQPSO) algorithm

The dynamic quantum particle swarm optimizer is constructed based on the QPSO algorithm with a new neighborhood topology. It has been reported in the literature

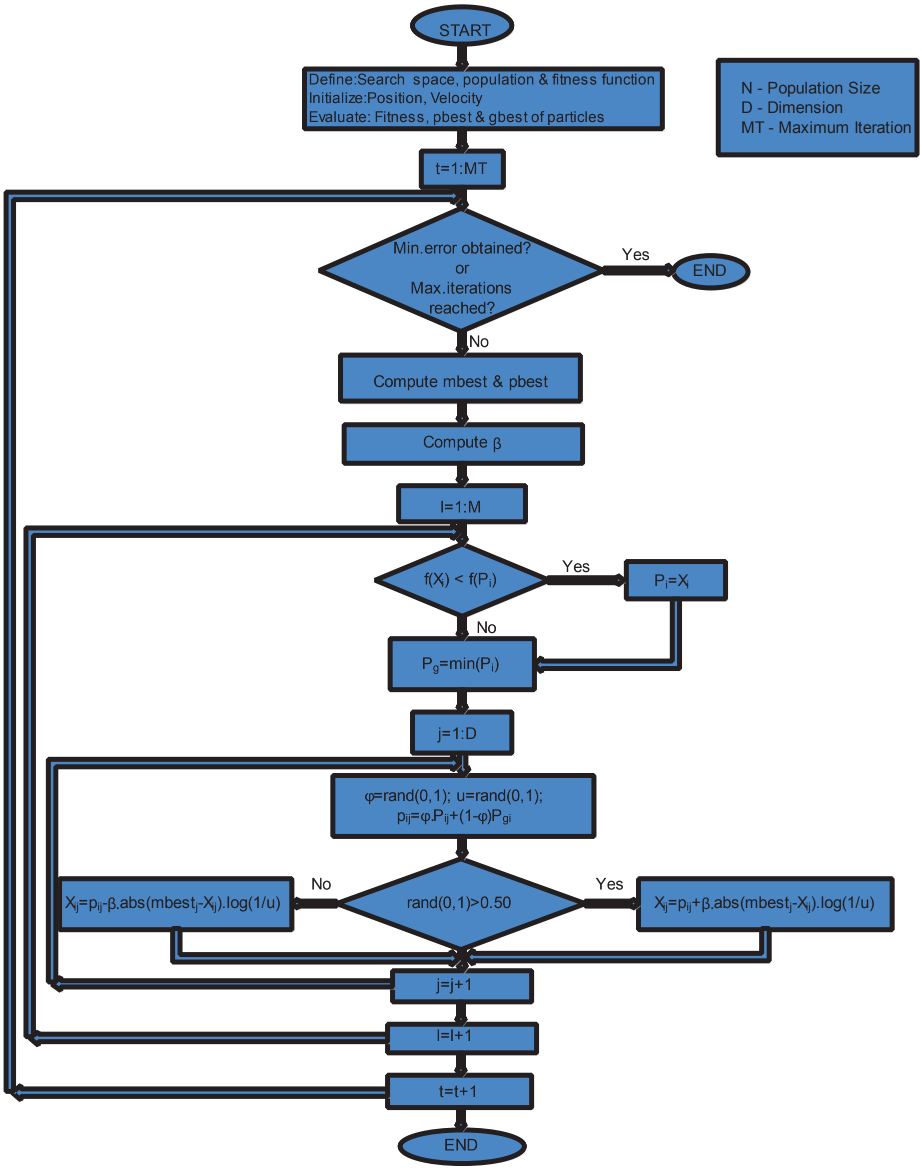


Figure 2: Quantum PSO algorithm

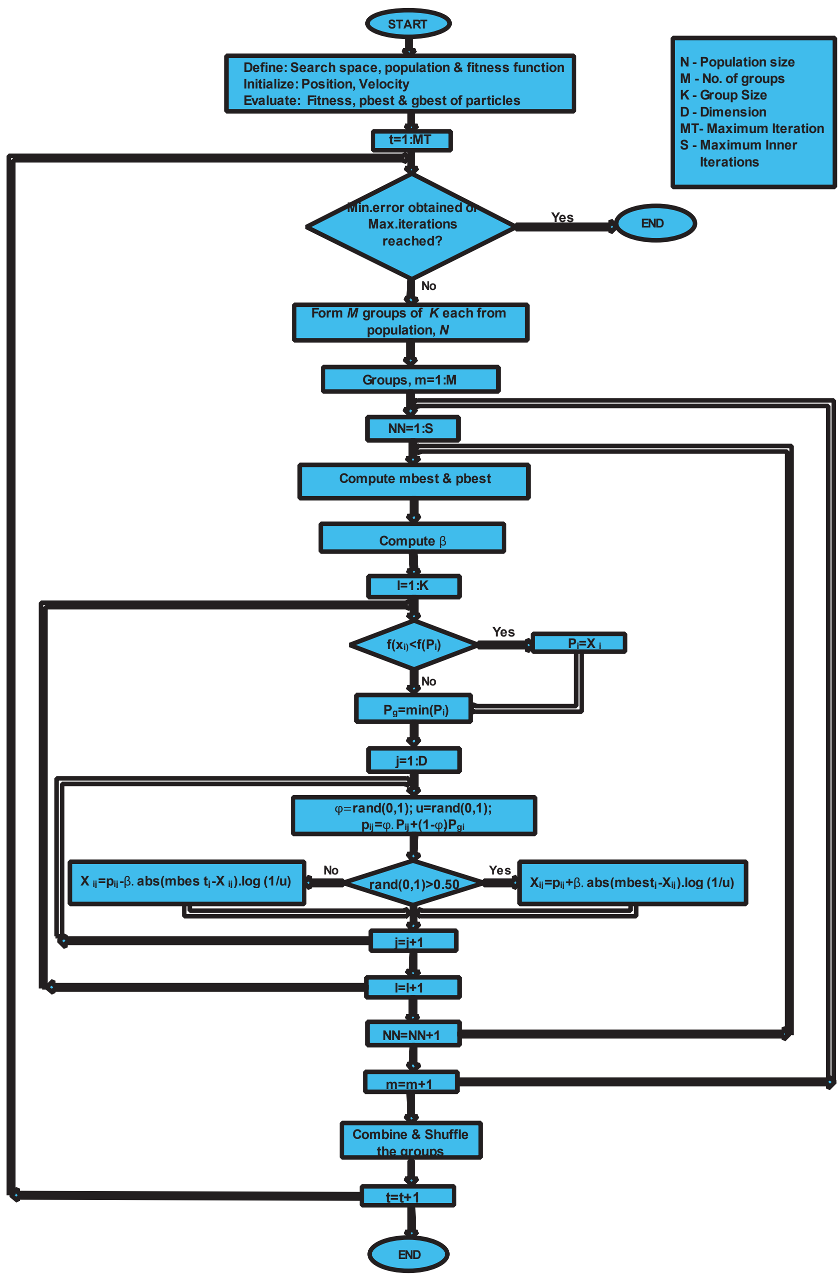


Figure 3: Dynamic Quantum PSO algorithm

[Clerc and Kennedy (2002)] that, PSO with small neighborhoods performs better on complex problems. Hence, in order to increase the diversification to achieve better results especially on multimodal problems, the proposed DQPSO uses small neighborhoods. The swarm is divided into small sized swarms called sub-swarms. Each sub-swarm uses its own members to search for better area in the search space. Since the small sized swarms are searching by using their own best historical information, they are likely to converge to a local optimum, because of typical PSO's convergence characteristics. In order to prevent the convergence to sub-optimal solution, the information needs to be exchanged among the swarms. While exchanging information among sub-swarms, it is necessary to exercise sufficient care to maintain larger diversity in sub-swarms. In order to accomplish this, we have proposed a shuffling schedule to have a dynamically changing neighborhood structure for the particles. After every user defined 'S' generations, the population is shuffled and the search will be continued using a new configuration of small swarms. In the proposed DQPSO algorithm, search is based on quantum principles (QPSO) in each sub-swarm and dynamic mixing of the results obtained through this parallel searches contributes to move towards a global solution. The details of the algorithm are as follows:

1. In DQPSO, each possible solution X_i= (x_{i1}, x_{i2}, x_{i3}, ...,x_{iD}), where D is the number of design variables is considered as a particle. The initial population of N particles (solutions) is generated randomly and it constitutes the swarm. The fitness of each of the solution (frog) is evaluated and the particles are then sorted in descending order according to their fitness.

2. Divide the swarm into 'M' sub-swarms each holding 'K' particles such that N = K $\times$ M. The division is done in round robin fashion i.e., the first particle is assigned to the first sub-swarm. Second one is assigned to the second sub-swarm, the M^{th} particle to the M^{th} sub-swarm and $(M+1)^{th}$ particle back to the first sub-swarm. This way of distributing particles to sub-swarms preserves diversity among frogs within each sub-swarm.

3. Each sub-swarm works independently in achieving the goal of exploring the search space for optimum solution. Various steps involved in each of the sub-swarm of DQPSO algorithm are same as QPSO algorithm discussed earlier.

After user specified number (say 'S') of evolutions in each of the sub-swarm, the particles are regrouped and are sorted again. Repeat steps (2) and (2) till the convergence criteria are satisfied. With the randomly regrouping schedule, particles from different swarms are grouped in a new configuration, so that each small swarms search space is enlarged and better solutions are possible to be found by the new small swarms. Figure 3 clearly depicts the proposed dynamic QPSO implementation.

4.4 Hybrid GA algorithm

A modified version of genetic algorithm usually known in the literature as a memetic algorithm [Moscato (1999)] is also employed for solving the optimisation problem associated with the proposed system identification technique. Basically, they are genetic algorithms that apply a separate neighbourhood search process to refine individuals. It is usually being applied after crossover and mutation and before the selection. One big difference between memes and genes is that memes are processed and possibly improved by the people that hold them - something that cannot happen to genes. Experimental results show that the memetic algorithms have better results over simple genetic or evolutionary algorithms [Moscato (1999)]. The memetic algorithm employed here is devised by introducing a neighbourhood search algorithm to improve the intensification mechanism of the algorithm by way of searching around a good solution and adopting a better solution if found.

Nelder-Mead algorithm [Nelder and Mead (1965)] is one of the most popular derivative-free nonlinear optimisation algorithms. Instead of using the derivative information of the function to be minimized, the Nelder-Mead algorithm maintains at each iteration a non-degenerate simplex, a geometric figure in n dimensions of nonzero volume that is the convex hull of n+1 vertices, x_1; x_2;. . . . . ; x_{n+1}, and their respective function values. In each iteration, new points are computed, along with their function values, to form a new simplex. Four scalar parameters must be specified to define a complete Nelder-Mead algorithm; coefficients of reflection (ρ), expansion (χ), contraction (γ), and shrinkage (σ): These parameters are chosen to satisfy: $\rho > 0$, $\chi > 1$, $0 < \gamma < 1$, and $0 < \sigma < 1$ The Nelder-Mead algorithm is given in Figure 1. The implementation of Nelder-Mead algorithm in the float encoded GA algorithm is as follows:

1. After every user specified number of iterations, the population is sorted according to their fitness values and chooses best twenty percent of the total number of population for refinement using neighbourhood search algorithm.

2. After several years of studying and applying the Nelder-Mead method, McKinnon (1999) shows that the Nelder-Mead algorithm can stagnate and converge to a non-optimal point even for very simple problems. However, Kelley [Kelley (1999a), (1999)] proposes a test for sufficient decrease which, if passed for all iterations, will guarantee convergence of the Nelder-Mead iteration to a stationary point under some appropriate conditions. The Kelley's

modification [Kelley (1999a), (1999)] of the Nelder-Mead method is employed in the final stage of our method i.e. at the end of the search, the best solution is refined using the Kelly's modification of Nelder-Mead algorithm.

The resulting memetic algorithm is termed in this paper as hybrid genetic algorithm. Numerical studies have been carried out using hybrid GA, hybrid adaptive PSO, quantum PSO algorithms to compare and evaluate the performance of the proposed DQPSO algorithm.

5 Numerical studies

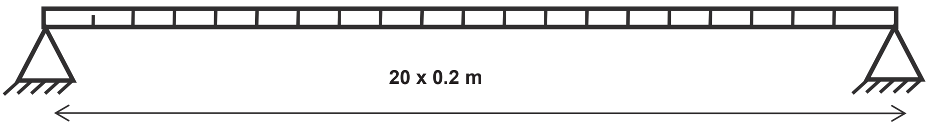
The effectiveness of the proposed structural system identification technique combining proper orthogonal decomposition technique with dynamic hybrid particle swarm optimisation algorithm is demonstrated by solving three different structural systems. The first numerical example is a simply supported beam, the next one is a 50-DOF shear building model. The third example considered is a truss bridge. The data of all DOFs of the numerical model is first calculated in the form of acceleration time history using Newmark's time integration method.

One of the main issues related to structural system identification techniques, when applied to real situations, is their sensitivity to noise. In view of this, it was decided to add white Gaussian noise to the acceleration time history response generated by the finite element code. The white Gaussian noise is added to the acceleration time history before it is processed. The white Gaussian noise is added in the form of 'SNR' (signal-to-noise ratio) that defines the 'amplitude' of the noise with respect to that of the clean signal. When the noise level is given by a particular value of 'SNR' it means that a noisy signal with such an SNR' has been added to the time series of each node. Moreover the noisy sequences affecting different nodes are uncorrelated, in this way severe experimental conditions were simulated.

Numerical Example 1: Simply supported beam

The first numerical experiment considered is a simply supported beam girder shown in Figure 4. For the purpose of numerical simulation studies the beam is discretised into 20 elements as shown in Figure 4. The material and geometrical properties are also shown in the Figure. The first four natural frequencies of the structure are 51.33 Hz, 205.1Hz, 462 Hz, and 648 Hz respectively. The two damping coefficients of the Rayleigh damping model are computed assuming 5% damping ratio for the first two modes of vibration. Accordingly the Rayleigh coefficients α and β are taken as 4.119 and 0.389 e-03 respectively.

The beam is excited using a known excitation force. The acceleration time history response is computed using finite element analysis and Newmark's time marching



Elastic Modulus = 205 GPa ; Cross sectional Area = 0.225m^2

Mass Density = 7850 Kg/m^{3}; Moment of Inertia = 0.000562 m^4

Figure 4: Simply supported beam

scheme. The acceleration time history response thus obtained is used for computation of POM and POV as outlined in the earlier sections. The time step size is chosen based on the sampling rate requirements during online vibration monitoring. The sampling rate is chosen as 2500 Hz for all the numerical examples considered in the present work. This sampling rate was enough according to the Nyquist criterion to capture all the frequency content of the vibration responses in the range of 0–500Hz. Accordingly, the time step length is chosen in the numerical simulations as inverse of the sampling rate. Eventhough the acceleration time history response is available in all nodes, we have considered only vertical acceleration time history responses at alternative nodal points i.e. at 9 nodes (i.e., at node numbers 1, 2, 5, 8, 11, 14, 17, 19, 20) in order to simulate the practical situation of limited measurements. The numerical simulations have been carried out for 4 seconds i.e. 10000 samples.

The mass is assumed to be known and the unknown parameters are the stiffness parameters and the Rayleigh damping coefficients. These unknown parameters are considered as design variables in the DQPSO algorithm. The cost function given in equation (9) is employed in the proposed system identification algorithm using DQPSO. The total number of particles considered as 30 and the number of swarms are taken as five. The swarms are regrouped after every 5 evolutions. The solution is said to have been converged if there is no improvement in the solution for the last thirty continuous evolutions.

Table 1 gives the identified parameters of the simply supported beam. The twenty stiffness values and two damping parameters are identified using the proposed DQPSO based system identification algorithm. Table 1 also presents the identified parameters using Hybrid adaptive PSO, QPSO and hybrid GA algorithms. In order to have fair comparison, the maximum number of iterations and convergence criteria are maintained as same for all the four algorithms evaluated here. A close look at the results presented in Table 1, clearly indicates that the errors in parameter identification is marginal in DQPSO algorithm and it is closely followed by QPSO algorithm. The hybrid PSO and hybrid GA algorithms are also able to identify the parameters reasonably accurately. However, DQPSO appears to be a clear winner among all algorithms.

Since all the optimisation algorithms considered here are stochastic algorithms, each algorithm is executed thirty times with the same data and the consistent solution obtained for each of the algorithm is shown in Table 1. Here the 'consistent solution' means that the optimal solution which is obtained in maximum number of independent runs of the same stochastic algorithm and happens to be same or nearly same i.e., with a variation of 0.01%. It can be observed from the data furnished in Table 1 that DQPSO generally performs better than the other three algorithms considered here for the measurements without noise. The maximum, minimum and absolute average percentage of errors in identified element stiffness with DQPSO algorithm is found to be 4.41%, 0.14% and 1.802% respectively. Similarly, with QPSO, the maximum, minimum and absolute mean errors found to be 6.29%, 0.16% and 2.81% respectively. For hybrid adaptive PSO, the maximum, minimum and absolute mean errors are found to be 9.23%, 1.57% ad 3.96% respectively. The maximum, minimum and absolute mean errors in hybrid GA are 6.78, 2.32%, and 5.12 % respectively.

Since the optimisation algorithms used in the proposed system identification algorithm are stochastic algorithms, it is not ensured that final solutions obtained in each run are same. In order to ensure the consistency of each of the algorithm used, the concept of practical reliability is used. Practical reliability is given by the percentage of converged solutions obtained with the same stiffness and damping coefficients using the stochastic algorithm under consideration. However a variation of one percent in the solutions obtained is considered as same. The practical reliability is obtained by running 30 different instances of each stochastic algorithm and determining ratio of the maximum number of final solutions that satisfy the above requirement to the total number of independent executions of the same algorithm.

To investigate the effectiveness of the proposed parameter identification technique with noisy measurements, SNR values of 30, 40, and 50 are considered. Table 1 also shows the identified stiffness parameters and damping coefficients with various SNR values. Figure 5 shows the identified stiffness of each element for various SNR values of measurement noise and has been compared with the results obtained without measurement noise. A close look at the results presented in Table 1 and also Figure 5, clearly indicates that the proposed system identification algorithm with DQPSO performs rather well even with noisy measurements. For the cases with the measurement noise, the maximum error with SNR 30, 40 and 50 using

Table 1: Stiffness ratios of identified simply supported beam using the proposed system identification algorithm

	DQPSO				QPSO				Hybrid adaptive PSO				Hybrid GA			
Element No	No Noise	SNR =30	SNR =40	SNR =50	No Noise	SNR =30	SNR =40	SNR =50	No Noise	SNR =30	SNR =40	SNR =50	No Noise	SNR =30	SNR =40	SNR =50
1	1.04	0.935	0.961	0.965	1.023	0.9624	0.9734	1.0191	1.0623	0.9423	0.9401	0.94542	1.0623	1.0834	0.9231	1.0227
2	0.98	1.063	1.02	1.003	0.962	1.1027	1.08197	0.9234	0.9572	1.1024	1.09 97	1.0827	0.9432	1.0825	1.11 97	1.0382
3	1.018	1.053	1.02	1.028	0.982	1.1027	1.0325	0.9623	1.0322	1.1236	1.0821	1.0627	1.0462	1.1427	0.8821	1.0612
4	0.99	1.040	1.104	1.041	0.962	1.2322	1.0344	0.9542	0.9821	1.1782	1.0344	1.1402	1.0232	1.2268	1.0444	1.0529
5	0.97	1.033	1.107	1.023	1.013	1.0658	1.0475	1.0367	0.9524	1.1829	1.0975	0.8929	0.9341	1.1929	1.1175	1.0392
6	1.032	1.033	0.929	1.019	1.042	1.2278	0.9328	1.0323	1.0287	1.2021	1.1298	1.1828	1.0421	1.2438	1.1098	0.9628
7	0.98	0.929	0.975	0.989	0.973	0.96856	0.9329	0.9834	0.9734	0.9123	1.0449	0.98856	0.9645	0.8086	1.0691	0.9529
8	1.014	0.874	1.103	1.074	1.032	0.8833	1.0333	1.0463	1.0223	0.6813	1.0533	0.67443	1.0443	0.7843	1.0822	1.0633
9	1.03	0.900	0.990	0.926	1.063	0.9329	0.89061	1.0829	1.0349	0.8878	1.0561	0.96039	1.0619	0.7609	1.0661	0.8061
10	1.02	0.892	1.082	0.952	1.042	0.8112	1.0681	0.9542	1.0231	0.8912	1.0342	0.98192	1.0421	0.8019	1.0567	1.1281
11	0.974	0.881	0.896	0.960	0.962	0.8645	0.8328	0.9611	0.9627	0.8924	0.8938	0.9305	0.9636	0.8705	1.1611	1.0723
12	1.012	1.109	1.020	1.020	1.002	1.1587	1.1013	1.0016	1.0824	1.1923	1.1013	1.0987	1.0636	1.1687	1.1456	1.0811
13	1.014	1.129	1.057	1.033	1.022	1.2289	1.0723	1.0221	1.0284	1.2627	1.1565	1.0489	1.0418	1.0989	1.1678	1.0675
14	1.01	1.108	1.079	1.047	1.032	1.2466	1.0711	1.0321	1.0211	1.0567	1.1785	1.0276	1.0521	1.1676	1.1897	0.8676
15	1.016	1.048	1.028	0.99	1.023	1.1484	1.0475	1.0228	1.0187	1.2623	1.2175	1.0484	1.0628	1.2084	0.7624	1.1132
16	1.014	1.070	0.990	1.02	1.020	1.1997	0.9676	1.0198	1.0392	1.1652	1.08976	1.0697	1.0542	1.1997	1.1123	0.8828
17	0.985	0.977	1.031	0.963	0.968	0.9542	1.0132	0.9681	0.9623	0.9828	1.0732	1.0478	0.9322	0.8437	1.1722	0.8489
18	1.001	1.065	0.89516	1.0351	1.012	1.1034	0.84516	1.01227	1.0157	1.1616	0.8916	1.0451	1.0572	1.1851	0.8916	1.0233
19	1.006	0.894	1.0357	0.9735	1.012	0.8045	1.0557	1.0123	1.0816	0.8129	1.0757	1.0535	1.0356	0.8632	1.0857	0.8061
20	1.014	0.897	1.01045	0.9575	1.033	0.8997	1.03045	1.0328	1.0923	0.8822	1.1245	0.9677	1.0524	0.8877	1.1045	1.0421
α	4.119	4.087	4.1418	4.11	4.131	4.1724	4.0909	4.1221	4.1324	4.1269	4.2957	4.0731	4.1539	4.0677	4.2941	4.0864
β x E-04	3.89	3.868	3.8574	3.8765	3.880	3.7515	3.8987	3.8822	3.8807	3.7847	3.6517	3.8796	3.8613	3.8153	3.7252	3.9394
R	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.96	0.96	0.96	0.96	0.93	0.93	0.93	0.93
R: Practical Reliability																

DQPSO is found to be 12.89%, 10.67%, 7.44% respectively. Similarly, the absolute mean error with SNR 30, 40 and 50 is found to be 7.85%, 5.32% and 3.36% respectively. The maximum errors with QPSO for SNR values of 30, 40, 50 is found to be 24.66%, 10.13% and 8.29% and absolute mean errors are 13.68%, 6.57% and 3.33% respectively. For system identification algorithm with hybrid adaptive PSO, the maximum errors are 26.27%, 21.75%, 18.28% and absolute mean errors are 15.02%, 9.62% and 7.83% for the measurement data with SNR as 30, 40 and 50 respectively. Finally, hybrid GA rather yield inferior results when compared to the other three algorithms and the maximum errors with SNR 30, 40 and 50 are found to be 24.38%, 23.76%, 19.39% and the corresponding absolute mean errors are found to be 16.90%, 11.73% and 8.39% respectively.

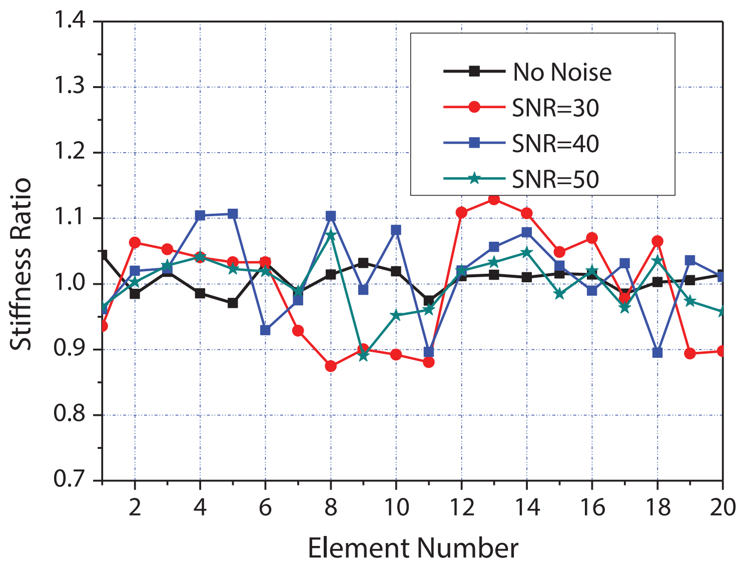


Figure 5: Identified stiffness ratios of simply supported beam using DQPSO algorithm

The damping parameters are also identified with reasonable accuracy. In order to study the convergence characteristics of the proposed DQPSO algorithm, the convergence plots are drawn and compared with hybrid GA, hybrid PSO and QPSO algorithms. The details of these convergence plots are shown in Figure 6. A close look at Figure 6 clearly indicates that the proposed DQPSO algorithm converges faster and also solution obtained is optimal. Further, the DQPSO algorithm converges in less number of evolutions thus makes it fastest among all the algorithms considered in this paper. The practical reliability of the proposed DQPSO algorithm is also very encouraging. The consistency coupled with faster convergence makes this algorithm highly suitable for complex optimisation problems associated with structural system identification.

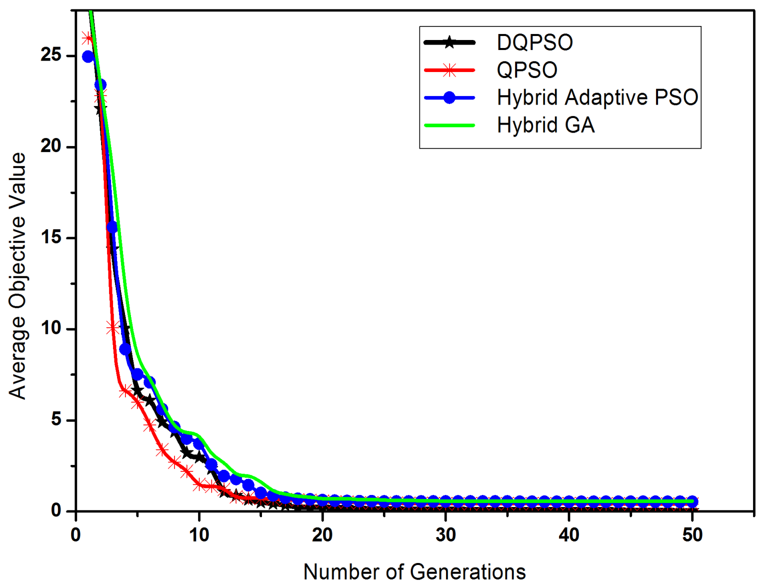


Figure 6: Convergence characteristics of DQPSO

Numerical Example 2: 50-DOF shear building model

The second numerical example considered is a 50-DOF framed structure shown in Figure 7. The frame is idealized as a shear building model . The exact stiffness is 700 kN/m for each level, while mass is 600 kg for the first level and 300 Kg for others. The first four natural frequencies of the structure are 0.2392 Hz, 0.7174Hz, 1.1945 Hz, 1.6698 Hz respectively. The two damping coefficients of the Rayleigh damping model are computed assuming 5% damping ratio for the first two modes of vibration. Accordingly the Raleigh coefficients α and β are taken as 0.1127 and 0.0167 respectively. The time history response is assumed to be available only at thirteen equally spaced locations in the shear building model (i.e., at 1^{st}, 5^{th} ,9^{th} , 13^{th} , 17^{th} , 21^{st} , 25^{th} , 29^{th} , 33^{rd} , 37^{th} , 41^{st} , 45^{th} , 50^{th} floor).

The maximum, minimum and mean errors for each of the algorithm with no measurement noise and also with noise measurements considering SNR values 30, 40

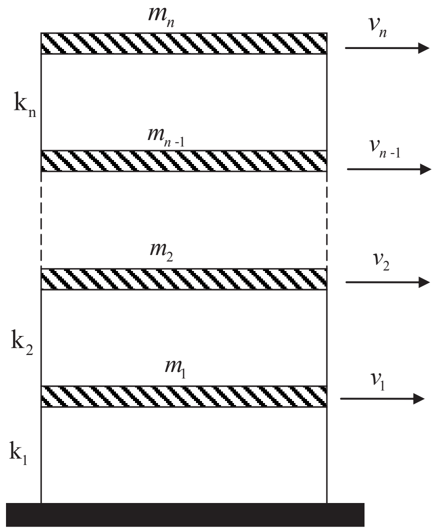


Figure 7: n-DOF framed Structure

Table 2: Performance of the proposed structural system identification algorithm for 50 storey framed structure

Details	DQPSO	QPSO	Hybrid adaptive PSO	Hybrid GA
With no measurement noise				
Minimum error (%)	0.0	0.17	0.39	0.44
Maximum error(%)	2.89	3.14	5.66	8.23
Absolute mean error(%)	1.64	1.98	3.91	4.34
With noise SNR=30				
Minimum error(%)	1.34	1.77	1.61	1.82
Maximum error(%)	6.18	6.94	7.63	11.01
Absolute mean error(%)	4.89	6.01	6.98	9.46
With noise SNR=40				
Minimum error(%)	1.02	1.06	1.31	1.08
Maximum error(%)	4.12	4.67	5.03	7.88
Absolute mean error(%)	2.94	3.56	4.08	6.04
With noise SNR=50				
Minimum error(%)	0.09	0.12	0.56	0.78
Maximum error(%)	3.36	3.46	4.02	4.67
Absolute mean error(%)	2.05	2.87	3.31	3.86

and 50 are shown in Table 2.

A close look at the results presented in Table 2, clearly indicates that the errors in parameter identification is marginal in DQPSO algorithm and it is closely followed by QPSO algorithm. Using DQPSO, the maximum and absolute mean errors in identification of member stiffness without measurement noise are 2.89% and 1.64%, respectively. For the cases with the measurement noise with SNR as 50, the maximum and absolute mean errors are found to be 3.36% and 2.05%, respectively. Similarly, with the QPSO algorithm, the maximum and absolute mean errors without noise are found to be 3.14% and 1.98%, respectively and with noise(SNR=50), the errors are 0.12% and 2.87%. The hybrid adaptive PSO and hybrid GA algorithms are also able to identify the parameters with reasonable accuracy. The maximum and absolute mean errors for hybrid adaptive PSO are 5.66%, 3.91%, and hybrid GA algorithms are 8.23%, 4.34% respectively. The details furnished in Table 2 related to measurement noise clearly indicate that, the performance of DQPSO is impressive even with noisy measurements.

Figure 8 shows the time history responses of original 50 storey shear building model and the structure with the identified stiffness and damping ratios using the proposed system identification algorithm. It is clearly evident from Figure 8 that the responses are exactly matching.

Numerical Example 3: Truss bridge

The proposed system identification method is evaluated using truss bridge with 55 elements, 24 nodes and 44 DOFs. The detailed geometrical configuration of the truss bridge is shown in Figure 9. The structure is subjected to vertical harmonic excitation at $F_1$and F_2. It is considered here for the identification of axial rigidity of the substructure members and the two Rayleigh damping coefficients. The elastic modulus E, cross-sectional area $A$and mass density of all the elements are 210GPa, 0.03m^2 and 8000 kg/m^3, respectively. The two damping coefficients α and β are chosen as 2.177 and 0.0011, respectively, resulting in a 5% damping ratio for the first two modes. The first two natural frequencies of the structure are 5.9Hz and 8.3Hz. The maximum, minimum and mean errors for each of the algorithm with no measurement noise and also with noise measurements considering SNR values 30, 40 and 50 are shown in Table 3. Figure 10 shows the stiffness ratio i.e. the ratio of identified axial rigidity to the actual value of members for the case studies carried out without noise and also with varied SNR values of noise using the proposed dynamic quantum behaved PSO algorithm.

A close look at the results presented in Table 3 clearly indicate that the proposed system identification technique with DQPSO performs rather well even for noisy measurements. The maximum and absolute mean errors in identification of axial

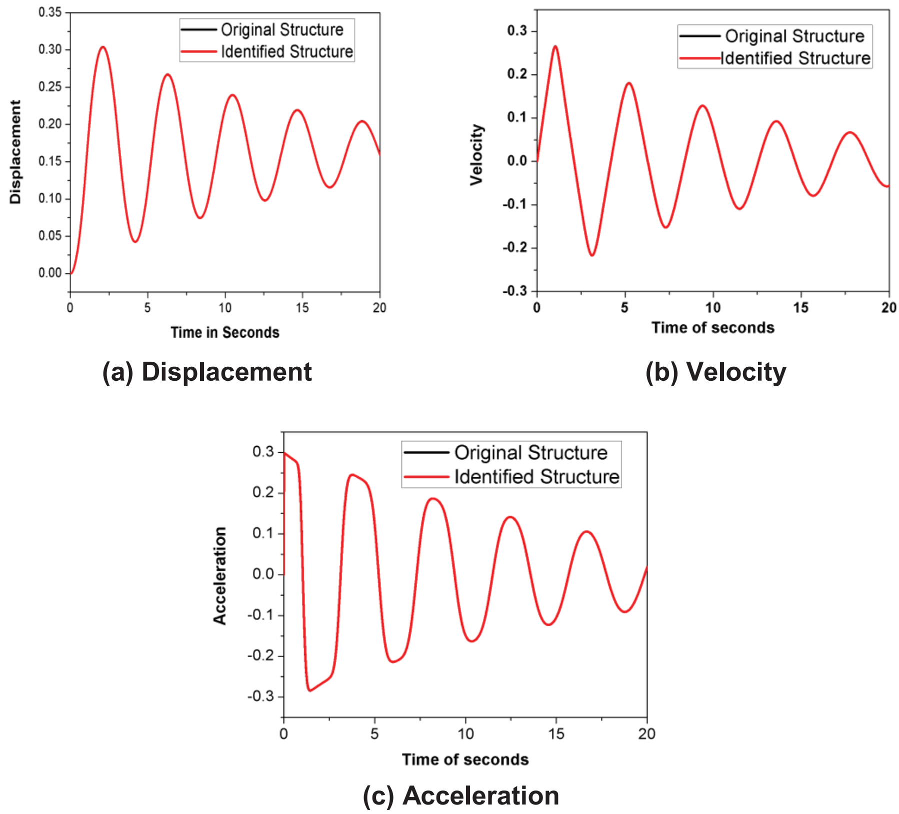


Figure 8: Displacement, velocity and acceleration time history of 50 storey shear building model subjected to harmonic excitation on top storey

rigidity are 3.46% and 1.94%, respectively, for the noise free case. The absolute mean errors with measurement noise are found to be 4.99%, 2.72% and 1.99% respectively for SNR values of 30, 40 and 50.

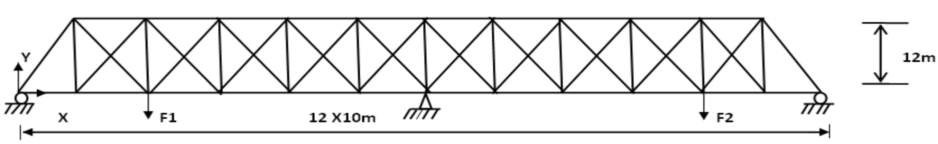


Figure 9: Truss bridge

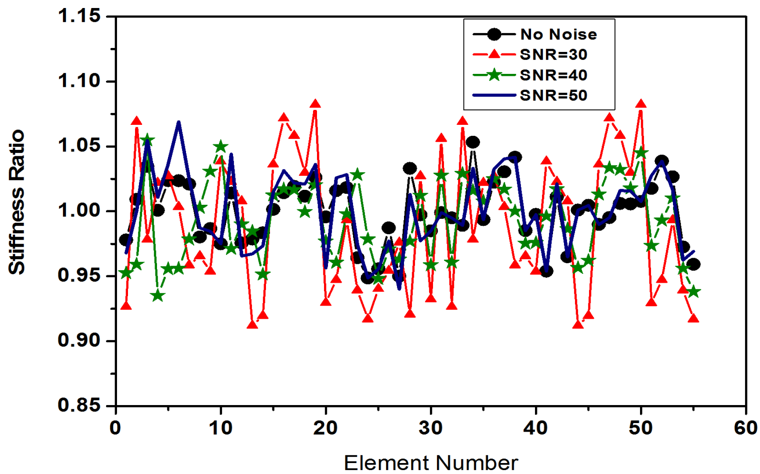


Figure 10: Identified stiffness ratios of Truss bridge using DQPSO algorithm

Table 3: Performance of the proposed structural system identification algorithm for truss bridge

Details	DQPSO	QPSO	Hybrid adaptive PSO	Hybrid GA
With no measurement noise				
Minimum error(%)	0.04	0.12	0.52	0.69
Maximum error(%)	3.46	5.16	7.02	4.78
Absolute mean error(%)	1.94	2.17	3.31	3.89
With noise SNR=30				
Minimum error(%)	2.15	2.59	2.88	2.42
Maximum error(%)	5.33	6.43	7.26	7.31
Absolute mean error(%)	4.99	5.61	6.98	8.24
With noise SNR=40				
Minimum error(%)	1.01	1.12	1.29	1.23
Maximum error(%)	4.12	4.88	6.16	6.74
Absolute mean error(%)	2.72	3.74	4.27	5.91
With noise SNR=50				
Minimum error (%)	0.002	0.07	0.18	0.17
Maximum error(%)	6.18	6.73	7.22	7.47
Absolute mean error(%)	1.99	2.25	4.01	4.19

6 Conclusions

This paper explored the feasibility of identifying a reduced-order model of linear dynamical system using proper orthogonal decomposition. POD is used for the model reduction strategy. Such a reduced-order model circumvents the limitations of traditional modal analysis. The inverse problem relating to the identification of the damping and stiffness matrices was tackled by formulating it as an optimisation problem and solved by using a meta- heuristic algorithm based on swarm intelligence. A new variant of particle swarm optimisation algorithm called dynamic quantum particle swarm optimisation algorithm is developed and implemented in the proposed system identification algorithm.

Numerical studies have been carried out to study the efficacy of the proposed algorithm by solving three problems. In order to demonstrate the effectiveness of the proposed DQPSO algorithm for system identification, we have compared with the performance of hybrid adaptive PSO, hybrid GA and Quantum PSO algorithms. The studies carried out in this paper clearly indicate that the proposed algorithm is effective in identifying the system matrices accurately. Eventhough all the four stochastic optimisation algorithms are effective in identifying the system parameters accurately, the performance of DQPSO appears to be more superior. The convergence plots drawn clearly indicate that the proposed system identification algorithm with DQPSO convergences faster and much more accurate when compared to other three stochastic optimisation algorithms employed in this paper.

For the numerical examples investigated in this paper, it is demonstrated that POD can be successfully applied for the reduced-order modelling. The dimension of the reduced model may be an order of magnitude smaller than the corresponding comprehensive model. The predictions from the identified reduced-order models match reasonably well with the original system response.

In the present work we have used the acceleration measurement as they are more commonly used for system identification. However, for ambient vibration measurements, the acceleration response will usually be very small. In such situations it is preferable to use velocity measurements. The proposed algorithm can be used straightaway without many modifications. Proper orthogonal modes can be generated using the measured velocity data in the similar way explained earlier for acceleration measurements. In the objective function given in equation (9) we need to use velocity instead of acceleration

Studies using measurement noise indicate that the noise influences the results. The proposed POD based system identification algorithm combined with DQPSO exhibits superior performance and exhibits low sensitivity to noisy measurements. The experimental verification of the proposed system identification algorithm will be taken up in future.

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