Finite Element Analysis of Three-dimensional Periodic Notched Plates

R. Afshar¹ and **F.** Berto^{1,2}

Abstract: Using the finite element method, three-dimensional models of a number of periodic blunt and sharp notches subjected to tension loading are carried out. The aim of this research is to investigate the thickness effect on the location of maximum stress and notch stress intensity factor (NSIF) of corresponding blunt and sharp periodic notches respectively. With this aim, a wide range of notch geometries are examined. While for two-dimensional plates weakened by periodic notches some results are available in the literature, this paper first faces with the problem of three-dimensional cases. The total of about 100 geometrical configurations are investigated.

It is found that, the effect of plate thickness of periodic notched components can be characterized by its relative value with respect to the depth of the notch (H/t). For the blunt periodic notches with relatively higher values of H/t ratio, the value of the maximum tensile stress is located near the free surface. On the contrary for lower values of H/t, it is placed at the middle plane. The same behaviour is observed for sharp periodic notches in terms of notch stress intensity factors.

Keywords: Periodic notches, finite element analysis, three-dimensional, notch stress intensity factor, tension loading.

Nomenclature

- 3D Three-dimensional
- E Young's modulus
- FE Finite Element
- H Plate thickness
- L Plate length
- N Number of notches

¹ University of Padova, Department of management and engineering, Stradella San Nicola 3, 36100, Vicenza, Italy.

² corresponding author email: berto@gest.unipd.it, phone number: +39 0444 998754.

NSIF	Notch stress intensity factor
Р	Pitch of the notch
SCF	stress concentration factor
SED	Strain energy density
t	Notch depth
\mathbf{U}_{x}	Applied displacement
W	width of the plate
Z	Coordinate along the thickness of plate
Z _{max}	Maximum value of plate thickness (H)
2α	Notch opening angle
σ_n	Equivalent applied remote stress
ρ	Notch radius
ν	Poisson's ratio
σ_{max}	Maximum stress value

1 Introduction

The fasteners such as bolts and nut are the well known examples of periodic notches. In particular, bolt-nut connection play an important role in the safety and reliability of structural systems. In [Pedersen (2013)] the reduction of stress concentration is achieved by optimization of bolt thread geometry. Reduction of stress concentration in bolt-nut connection by using a two dimensional axisymmetric finite element model is studied in [Venkatesan and Kinzel (2005)].

Due to convenience and relative simplicity of solutions of plane theory of elasticity, they still are popular and serve a basis for many engineering design procedures, standards and failure assessment codes. In terms of numerical costs, twodimensional models, based on plane stress or plane strain assumption, are much more computationally efficient, much easy to build and verify in comparison with the corresponding three-dimensional counterparts. Furthermore, three-dimensional equations of elasticity are not very amenable to analytical techniques.

Dealing with crack problems, in order to evaluate the dominant state of stress, a simple empirical rule as being thin and thick enough is based for plane stress and plane strain condition respectively. On the other hand, until now there is no generally accepted criterion for identifying what thicknesses correspond to planestress or to plane-strain conditions.

Addressing the above mentioned issue, there are a large amount of publications in the recent literatures [Kotousov (2005, 2007); Kotousov, Lazzarin, Berto, Harding (2010); Lazzarin, Zappalorto (2012); Pook (1990, 1992, 2000, 2003)]. By considering some realistic crack shapes in three-dimensional (3D) finite element (FE)

models and using automatic crack growth techniques, the extent of surface regions in cracked bodies under tension loading condition was investigated in [Branco, Antunes, Ricardo, Costa (2012)]. Based on their findings, The extent of both regions (extent of surface and near surface) was related with the elastic stress concentration factor (SCF) of the corresponding uncracked geometries and a linear relationship was found. In [May, Kilchert, Hiermaier (2012)] by using a user defined material model based on analytical approach, simulation of brittle fracture in isotropic materials in 3D space was performed. In another work, some detailed 3D FE models of spot welds for different ratio values of sheet metal thickness and spot-weld diameter are investigated in [Deng, Chen, Shi (2000)] with the aim of investigating the mechanical behavior of spot welds under tensile-shear and symmetric coach-peel loading conditions. In [Darwish, Gharaibeh, Tashtoush (2012)] by using 3D FE models, a modified equation for the SCF of an isotropic plate with a centered countersunk hole is proposed. Factorial and multi parameter fit analyses were conducted on the FE results to formulate a general parametric equation for the maximum SCF in terms of the four dimensionless geometric parameters. The advantage of proposed equations is demonstrated by comparing with other method in the literature.

In [Kotousov, Lazzarin, Berto, Harding (2010)] an extensive review of some recent analytical, numerical and experimental results is performed to investigate the effect of plate thickness on elastic deformation as well as quasi-brittle fracture of plate components. For a plate under in-plane loading condition, two basic assumption for the state of stress in the frame of plane theories of elasticity, namely as plane stress (zero transverse stress) and plane strain (zero transverse strain), are commonly used [Berto, Lazzarin, Kotousov, Pook (2012)].

Dealing with periodic blunt notches and by using the strain energy density (SED) approach, plane strain condition is assumed to evaluate the SCFs of a number of flat plates and round bars with periodic U- and V-notches. Tension, bending and torsion loading conditions have been considered [Afshar, Berto (2011)].

In the presence of sharp periodic notches and by means of SED, the plane theory of elasticity is assumed to study the variability of the notch stress intensity factors (NSIFs) of periodic sharp notches in Ref. [Afshar, Berto, Lazzarin, Pook (2013); Lazzarin, Afshar, Berto (2012)]. A new model of depth reduction factor for different ratios of relative depth of the notch is proposed to match the results from the SED approach. In the case of shallow notches, the results are compared with some semi-analytical solutions provided in [Savruk, Kazberuk (2008)]. In addition, based on best fit of numerical data from the SED approach, some polynomials for non-dimensional NSIFs in the case of intermediate and deep notches are presented. In another work by the authors [Berto, Lazzarin, Afshar (2012)], some very simple expressions were derived for the direct evaluation of the SED and the NSIF of

plates with infinite width as a function of the notch spacing in the case of narrow sharp notches.

In this study, for the first time, an attempt is made to investigate the thickness effect on the location of maximum stress and notch stress intensity factor (NSIF) of corresponding blunt and sharp periodic notches in three-dimensional plates weakened by periodic blunt and sharp notches. A number of sophisticated three-dimensional finite element models are built with this aim.

In addition, different number of periodic notches as well as different notch opening angles are examined.

2 Geometry and boundary conditions

In the present paper, both types of 3D blunt and sharp periodic notches are analyzed by means of the FE method within ANSYS software. The six models used in this study are summarized in Table 1. The total of about 100 geometrical configurations are investigated.

The geometrical parameters, namely as notch depth (t), notch radius (ρ), pitch of the notch (p), notch opening angle (2 α), plate thickness (H), width of the plate (W) and plate length (L) and boundary conditions are shown in Fig. 1, in which E is Young's modulus of elasticity, v is Poisson's ratio, U_x is the applied displacement and σ_n is the equivalent applied remote stress. For all the FE models the elastic isotropic material with E=206 GPa, v=0.3 is used. The value of σ_n =100 MPa for all the models is assumed. Of course, the value of applied U_x is varied, depending on the length of the plate (L), as it is shown in Fig.1.

Table 1: Different models with geometrical parameters (2α : notch opening	g angle;
ρ : notch radius; p: pitch of the notch; t: notch depth; H: thickness of the p	late; N:
number of notches).	

Model	$2\alpha(^{\circ})$	ρ (mm)	p (mm)	t (mm)	H (mm)	N
1	60	0.1	2.5	0.5	10, 5.0, 2.0, 1.0	3-9
2	60	1.0	25	5.0	10, 5.0, 2.0, 1.0	3-5
3	120	0.1	2.5	0.5	10, 5.0, 2.0, 1.0	3-9
4	120	1.0	25	5.0	10, 5.0, 2.0, 1.0	3-5
5	60	0.0	2.5	0.5	10, 5.0, 2.0, 1.0	3-9
6	120	0.0	2.5	0.5	10, 5.0, 2.0, 1.0	3-9

As it can be seen from Table 1, the first four models correspond to blunt notches, whereas the last two represent the sharp notch configurations. Furthermore, the



Figure 1: The geometrical parameters of 3D periodic notched plate and boundary conditions.

models 2 and 4 are the scaled-up geometries (only in xy-plane) of the corresponding models 1 and 3 respectively. The sample mesh pattern of both types (blunt and sharp periodic notches) have been depicted in Fig. 2.

3 Results and discussions

In order to present the results in a more systematic manner, the selected results for each FE model (tabulated in Table 1) are presented and discussed in that order. For the case of blunt notches, the stresses shown in figures are notch tip stresses which could be converted into stress concentration factors.

3.1 Model 1: 2α=60°; ρ=0.1mm; p=2.5mm; t=0.5mm (H=10, 5.0, 2.0 and 1.0 mm; N=3-9)

Fig. 3 shows the variation of tensile stress at the middle notch of periodic notched plates along the thickness of the plate (H=10 mm) for different number of notches. From Fig. 3, two essential trends can be observed. The first feature belongs to periodic notches, which shows the effect of number of notches on the variation of tensile stress. In fact, as it is expected, increasing the number of notches leads to decrease in the value of tensile stress at the middle notch of periodic notched plate. The second characteristic is that the maximum tensile stress is not placed on the



Figure 2: Sample mesh pattern used for: (a) blunt periodic notches ($\rho \neq 0.0$); (b) sharp periodic notches ($\rho=0.0$).



Figure 3: The variation of tensile stress at the middle notch of periodic notched plates along the thickness of the plate (H=10 mm) for different number of notches (Model 1).

surface nor at the middle of the plate (2D assumption), but very close to the surface (at a distance $z\approx 0.5$ mm).

The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate (N=3) as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) is depicted in Fig. 4 (z varies along the thickness of the plate and z_{max} is equal to H)



Figure 4: The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) (Model 1).

According to Fig. 4, the effect of thickness of plate on the tensile stress is clearly evident. For the relatively thinner plates, the maximum stress appears to be at the middle of the plate and that confirms the plane stress assumption of 2D FE models. However, increasing the thickness of the plate leads to generate the maximum tensile stress at the near surface location. In particular, for the case of thicker plates (H=5 and 10 mm), the shift of maximum stress from the mid plane to the closer location of the free surfaces is clearly evident.

3.2 Model 2: $2\alpha = 60^{\circ}$; $\rho = 1.0 \text{ mm}$; p = 25 mm; t = 5.0 mm (H=10, 5.0, 2.0 and 1.0 mm; N=3-5)

As it is mentioned earlier (Section 2), Model 2 is the scaled-up geometry (only in xy-plane) of Model 1.

Fig. 5 shows the variation of tensile stress at the middle notch of periodic notched

plates along the thickness of the plate (H=10 mm) for different number of notches (N=3-5).



Figure 5: The variation of tensile stress at the middle notch of periodic notched plates along the thickness of the plate (H=10 mm) for different number of notches (Model 2).

As it can be seen from Fig. 5, although the plate seems thick (H=10 mm), no effect of number of notches on the tensile stress is observed. In addition, in contrast to Model 1, the maximum tensile stress is located at the mid-plane. This behavior will be discussed in Fig. 6 by comparing the different thicknesses of the plates with periodic notches.

The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate (N=3) as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) is depicted in Fig. 6 (z varies along the thickness of the plate and z_{max} is equal to H).

By comparing the different plate thicknesses of periodic notches in Fig. 6, the location of maximum tensile stress evidently is placed at the middle of the plate. This can be explained by the value of plate thickness to the notch depth ratio (H/t). For instance, considering Model 1 (Fig.3), when the ratio H/t=1.0/0.5=2.0, the maximum stress appears to be at the middle of the plate. Similarly, for Model 2, with the same ratio of H/t=10/5.0=2.0, the same behaviour can be observed. In other words, the effect of plate thickness of notched components can be characterized by its relative value with respect to the depth of the notch (H/t). This is especially



Figure 6: The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) (Model 2).

important in investigation of one-plane scaled geometries (e.g. Model 2, which the geometry of Model 1 is scaled-up 10 times in only in xy-plane).

3.3 Model 3: 2α=120°; ρ=0.1mm; p=2.5mm; t=0.5mm (H=10, 5.0, 2.0 and 1.0 mm; N=3-9)

Model 3 is similar to Model 1, with the only difference of the notch opening angle. In Model 3 a larger notch opening angle $(2\alpha=120^{\circ})$ is considered in comparison with $2\alpha=60^{\circ}$ used in Model 1. Fig. 7 shows the variation of tensile stress at the middle notch of periodic notched plates along the thickness of the plate (H=10 mm) for different number of notches.

From Fig. 7 it is evident a similar trend of Fig. 3. The variation of the tensile stress at the middle notch of the plate along the thickness as well as the effect of number of notches on the tensile stress can be observed from the Figure.

The variation of the normalized tensile stress (σ_x/σ_{max}) at the middle notch of the periodic notched plate (N=3) as a function of the normalized thickness (z/z_{max}) of the plate for different values of plate thicknesses (H) is depicted in Fig. 8 (z varies along the thickness of the plate and z_{max} is equal to H).



Figure 7: The variation of tensile stress at the middle notch of periodic notched plates along the thickness of the plate (H=10 mm) for different number of notches (Model 3).



Figure 8: The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate (N=3) as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) (Model 3).

3.4 Model 4: $2\alpha = 120^{\circ}$; $\rho = 1.0 \text{ mm}$; p = 25 mm; t = 5.0 mm (H = 10, 5.0, 2.0 and 1.0 mm; N = 3-5)

Model 4 is also similar to Model 2, with the difference of the notch opening angle. In Model 4 a notch opening angle equal to 120° is considered in comparison with $2\alpha=60^{\circ}$ investigated in Model 2. In addition, Model 4 is the scaled-up geometry (only in xy-plane) of Model 3.

The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate (N=3) as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) is depicted in Fig. 9 (z varies along the thickness of the plate and z_{max} is equal to H).



Figure 9: The variation of normalized tensile stress (σ_x/σ_{max}) at the middle notch of periodic notched plate (N=3) as a function of normalized thickness (z/z_{max}) of the plate for different values of plate thickness (H) (Model 4).

As it can be seen from Fig. 9 and similarly to Fig. 6, the location of the maximum tensile stress clearly is placed at the middle of the plate for different plate thicknesses. The reason for this behaviour was explained in the discussion of Fig. 6.

It is worth noting that in all Figs 3-9, stresses decrease towards the surface due to the influence of corner point effects.

3.5 Model 5: $2\alpha=60^{\circ}$; $\rho=0.0 \text{ mm}$; p=2.5mm; t=0.5mm (H=10, 5.0, 2.0 and 1.0 mm; N=3-5)

Models 5 and 6 represent the sharp periodic notch configurations. The only difference between the two models is the notch opening angle. In Model 5 a relatively medium notch opening angle $2\alpha = 60^{\circ}$ is considered in comparison with a larger notch opening angle $2\alpha = 120^{\circ}$ in Model 6.

Due to existence of singularity point at the tip of sharp notches and in order to evaluate the results in different locations of the 3D FE models, in addition to the along-thickness path for study the variation of tensile stress along the plate thickness, a path for different distance from the tip of the notch is considered.

By examining the different distances from the notch tip, the distance d=0.01 mm from notch tip is considered to present the results in terms of NSIF along the thickness for both Models 5 and 6.

Fig. 10 shows the variation of NSIF at the middle notch of periodic notched plates along the thickness of the plate (H=5 mm) for different number of notches.



Figure 10: The variation of NSIF at the middle notch of periodic notched plates along the thickness of the plate (H=5 mm) for different number of notches (Model 5).

According to Fig. 10, the maximum NSIF appears to be at the location of very close to the surface of the plate with periodic notches ($z\approx0.5$ mm). This behaviour is similar to the one found in Fig. 2 (Model 1) in terms of tensile stress for the blunt notches. Furthermore, the NSIF decreases as the number of notches increase.

3.6 Model 6: $2\alpha = 120^{\circ}$; $\rho = 0.0 \text{ mm}$; p = 2.5 mm; t = 0.5 mm (H=10, 5.0, 2.0 and 1.0 mm; N=3-5)

As it is mentioned in section 3.5, Model 6 is the same of Model 5 with the only difference of the notch opening angle which is $2\alpha = 120^{\circ}$ in the latter case.

Fig. 11 shows the variation of the NSIF at the middle notch of periodic notched plates along the thickness of the plate (H=5 mm) for different number of notches.



Figure 11: The variation of NSIF at the middle notch of periodic notched plates along the thickness of the plate (H=5 mm) for different number of notches (Model 6).

A similar trend of Fig. 10 in both in terms of NSIF variation and the effect of number of notches is observed in Fig. 11.

It should be noted that on the contrary of blunt notches, the strength of the sharp notch tip singularity is a function of the included angle as it is shown in Figs 10 & 11. In addition and similar to the case of blunt notches, the notch stress intensity factors decrease towards the surface due to the influence of corner point effects.

4 Comparison with 2D plane solution

In order to validate the 3D FE analyses, Model 2 (see Table 1) with following geometrical dimensions is selected:

 $-2\alpha = 60^{\circ}$; $\rho = 1.0 \text{ mm}$; p = 25 mm; t = 5.0 mm; H = 10 mm; W = 290 mm; N = 3.

The results in terms of σ_{max} at the middle notch of plate with periodic notches have

been compared with the those from 2D plane element models, as it is shown in Fig. 12.



Figure 12: Comparison of 3D and 2D models in term of σ_{max} .

As it can be seen from Fig. 12, the σ_{max} =461.8 MPa of plane model (a) is about 5% lower than σ_{max} =484.8 MPa of 3D model (b). The location of σ_{max} in 3D model is placed in this case at the middle of the plate, in contrast to some other cases where the maximum of the principal stress is placed close to the surfaces (see Fig. 3).

5 Conclusions

This study is aimed to face for the first time with three-dimensional effects in plates weakened by periodic notches. While some recent papers provide useful information for plane plates with periodic notches (blunt and sharp), no results have been presented up to now to discuss the 3D effects due to finite values of the plate thickness. This paper is mainly focused to close this gap and to present some results from 3D plates weakened by periodic notches of both sharp and blunt types. Different geometrical parameters have been varied in the models to have results form about one hundred new accurate 3D models. In particular the thickness of the plate, the notch opening angle, the pitch of the notch and the notch depth have been varied to investigate a wide number of geometrical configurations.

The main result of this study is that the effect of the plate thickness in periodic notched components can be characterized by the relative value between the plate thickness and the notch depth (H/t). For blunt periodic notches under tension with relatively higher values of H/t ratio (H/t>2.0), the value of maximum tensile stress is located near the free surfaces while, on the contrary, for lower values (H/t<2.0) it is placed at the mid-plane. The same behaviour is observed for sharp periodic notches subjected to tension loading, in terms of notch stress intensity factors. The number of notches has been also investigated showing that a saturation effect is always found for N=9.

References

Afshar, R.; Berto, F. (2011): Stress concentration factors of periodic notches determined from the strain energy density. *Theoretical and Applied Fracture Mechanics,* vol. 56, pp. 127-39.

Afshar, R.; Berto, F.; Lazzarin, P.; Pook, L. P. (2013): Analytical expressions for the notch stress intensity factors of periodic V-notches under tension by using the strain energy density approach. *Journal of Strain Analysis for Engineering Design*, vol. 48, pp. 291-305.

Berto, F.; Lazzarin, P.; Afshar, R. (2012): Simple new expressions for the notch stress intensity factors in an array of narrow V-notches under tension. *International Journal of Fracture*, vol. 176 pp. 237-44.

Berto, F.; Lazzarin, P.; Kotousov, A.; Pook, L. P. (2012): Induced out-of-plane mode at the tip of blunt lateral notches and holes under in-plane shear loading. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 35, pp. 538-55.

Branco, R.; Antunes, F. V.; Ricardo, L. C. H.; Costa, J. D. (2012): Extent of surface regions near corner points of notched cracked bodies subjected to mode-I loading. *Finite Elements in Analysis and Design*, vol. 50, pp. 147-60.

Darwish, F.; Gharaibeh, M.; Tashtoush, G. (2012): A modified equation for the stress concentration factor in countersunk holes. *European Journal of Mechanics - A/Solids*, vol. 36, pp. 94-103.

Deng, X.; Chen, W.; Shi, G. (2000): Three-dimensional finite element analysis of the mechanical behavior of spot welds. *Finite Elements in Analysis and Design*, vol. 35, pp. 17-39.

Kotousov, A. On stress singularities at angular corners of plates of arbitrary thickness under tension. *International Journal of Fracture*, vol. 132, pp. 29-36.

Kotousov, A. (2007): Fracture in plates of finite thickness. *International Journal of Solids and Structures*, vol. 44, pp. 8259-73.

Kotousov, A.; Lazzarin, P.; Berto, F.; Harding, S. (2010): Effect of the thickness on elastic deformation and quasi-brittle fracture of plate components. *Engineering Fracture Mechanics*, vol. 77, pp. 1665-81.

Lazzarin, P.; Zappalorto, M. (2012): A three-dimensional stress field solution for pointed and sharply radiused V-notches in plates of finite thickness. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 35, pp. 1105-19.

Lazzarin, P.; Afshar, R.; Berto, F. (2012): Notch stress intensity factors of flat plates with periodic sharp notches by using the strain energy density. *Theoretical and Applied Fracture Mechanics*, vol. 60, pp. 38-50.

May, M.; Kilchert, S.; Hiermaier, S. (2012): 3D modeling of fracture in brittle isotropic materials using a novel algorithm for the determination of the fracture plane orientation and crack surface area. *Finite Elements in Analysis and Design*, vol. 56, pp. 32-40.

Pedersen, N. (2013): Optimization of bolt thread stress concentrations. *Arch Appl Mech.*, vol. 83, pp. 1-14.

Pook, L. P. (1990): Stress intensity factor expressions for regular crack arrays in pressurized cylinders. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 13, pp. 135-43.

Pook, L. P. (1992): A note on corner point singularities. *International Journal of Fatigue*, vol. 53, pp. 3-8.

Pook, L. P. (2000): Finite element analysis of corner point displacements and stress intensity factors for narrow notches in square sheets and plates. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 23, pp. 979-92.

Pook, L. P. (2003): A finite element analysis of cracked square plates and bars under antiplane loading. *Fatigue & Fracture of Engineering Materials & Structures,* vol. 26, pp. 533-41.

Savruk, M.; Kazberuk, A. (2008): A plane periodic boundary-value problem of elasticity theory for a half-plane with curvilinear edge. *Materials Science*, vol. 44, pp. 461-70.

Venkatesan, S.; Kinzel, G. L. (2005): Reduction of Stress Concentration in Bolt-Nut Connectors. *Journal of Mechanical Design*, vol. 128, pp. 6.