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Joint Time-Frequency Analysis of Seismic Signals: A Critical Review

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Abstract: This paper presents an evaluation of time-frequency methods for the analysis of seismic signals. Background of the present work is to describe, how the frequency content of the signal is changing in time. The theoretical basis of short time Fourier transform, Gabor transform, wavelet transform, S-transform, Wigner distribution, Wigner-Ville distribution, Pseudo Wigner-Ville distribution, Smoothed Pseudo Wigner-Ville distribution, Choi-William distribution, Born-Jordan Distribution and cone shape distribution are presented. The strengths and weaknesses of each technique are verified by applying them to a particular synthetic seismic signal and recorded real time earthquake data.

Keywords: Time-frequency distribution, Seismic signals, Cross-term interference, Auto-term, Gabor transform, Wigner-Ville distribution.

1 Introduction

The traditional method for analysis of the seismic signal is the discrete Fourier transform (DFT), which determines the frequency components of a time domain seismic signal. But, this method becomes inadequate when data point increases in the DFT. Further, Cooley and Tukey explained an efficient algorithm to compute the DFT and this algorithm is known as fast Fourier transform (FFT) [Lathi (1998); Proakis and Manolakis (2006)]. In the present scenario, it has become one of the most prominent tools to analyze the spectral components of the seismic signal in earthquake engineering. In the last few years, scientist and researchers have become attentive to the limitations of the Fourier transform. Spectral analysis using Fourier transform measures the frequency components of a signal, but does not provide the temporal information of time varying signal [Huang and Yu (2011)], [Black (1998)]. Another problem of the Fourier transform is that the same frequency of vibrations at two distinct time interval is not able to distinguish; such information shows as a single peak in the Fourier transform plots.

Based on above inference, the joint time-frequency representation (JTFR) of the signals has been developed. JTFR is a prominent tool to analyze the spectral components of a signal and their temporal information. A few commonly employed time-frequency

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techniques are short time Fourier transform (STFT), Gabor transform, Wavelet transform (WT), Wigner-Ville distribution, (WVD), Pseudo Wigner-Ville distribution and Choi-William distribution (CWD). STFT is employed for frequency analysis of seismic signals. And, it can be defined as Fourier transform of the product of the signal and shifted version of the window function, which can be rectangular, Hanning, and Hamming window in 1998 [Black (1998)] was applied this technique to the data recorded on the instrumented building to detect the change in frequency. The result of STFT suffers from the windowing effects due to the limitation is given by the uncertainty principle [Black (1998); Lingyu and Vector (2005); Trifunac, Ivanovic and Todorovska (2001); Chandra and Barai (2009, 2014); Roshan, Sumiti and Ashok (2015)]. And, another problem associated with STFT is the possible amount of leakage [Black (1998); Douglas and Thomas (1987)]. For this reason, Gaussian window is introduced in place of Hanning and Hamming window and it turns out to be a Gabor transform. The application of the Gaussian window in Gabor transform provides better resolution. Therefore, Gabor transform was used to determine the instantaneous frequency and damage detection of Imperial County services building for Imperial Valley earthquake (1979) [Todorovska and Trifunac (2007)]. But, the drawbacks associated with the Gabor transform is that the time and frequency resolution cannot achieve simultaneously since window function is involved.

To overcome the fixed window length problem as discussed in case of STFT and GT, wavelet transform has been introduced, which is having variable window length. Because of this specialty i.e. adaptive window length, some of the researchers applied this technique in several fields such as structural damage detection, health monitoring in Taha e al. [Taha, Noureldin, Lucero et al. (2006)] detection of damage locations in a beam using the wavelet analysis in Rucka [Rucka (2011)]. The further wavelet transform analysis has also been used for the diagnosis of incipient faults in ball bearings, measurement of seismic attenuation [Li, Zhao, Cao et al. (2006)] and recently in Tsunami warning system [Kuenza (2010)] Similar to the STFT and GT, wavelet transform also suffer from time and frequency resolution problem since window function is involved [Li and Zheng (2007); Amy and Wiesław (2007); Christopher and Kyle (1998); Zhu, Wang and Shen (2012); Sinha, Routh, Anno et al. (2005)]. The S-transform is a hybrid combination of STFT and wavelet transform [Pinnegar and Mansinha (2003); Zhu, Wang and Shen (2012)] that takes the advantage of both. The prior research suggests that clarity of S-transform is worse than Wigner-Ville distribution which achieves higher resolution [Yang and Sergey (2013); Tobback, Steeghs, Drijkoningen et al. (1996)], but it is seldom used in practice due to the presence of cross-term interference. These cross-term interferences are undesirable, which are present in the middle of the two actual frequency components. And, it creates the misleading information [Shie and Chen (1999); Boashash (2003)]. This unwanted signal i.e. cross-term interference can be reduced by applying the low pass filter called Pseudo Wigner-vile distribution (PWVD), smoothed Pseudo Wigner-Vile distribution (SPWVD), Choi-William distribution, Born-Jordon distribution and Cone shape distribution. PWVD has been used for detection of events in seismic time series and detection problem consists in identifying the actual seismic signal [Gabarda and Cristoba (2010)] and this technique has also been applied in Electrical engineering for analysis of power quality [Szmajdaand, Górecki and Mroczka (2010)]. A smoothed pseudo-Wigner-Ville distribution is a precise time-frequency method has been applied to the data recorded on the instrumented building to detect the frequency variation [Clotaire and Philippe (2010)].

In addition, SPWVD has also been used in the detection of characteristics of seismic signal whether it is a seismic signal or not [Rivero-Moreno and Escalante-Ramires (1996)]. Further, the author applied several time-frequency distributions on seismic signals namely WVD, PWVD, SPWVD, and CWD, to detect and predict the seismic wave through TFD and showed the efficiency of the SPWVD [Tseng (2014)]. CWD employed to identify the reflection pattern [Steeghs and Drijkoningen (1996)]. Further, the author showed a comparison between several signal analysis techniques including STFT, Pseudo Wigner-vile distribution, smoothed Pseudo Wigner-vile distribution, Choi-William distribution, Born-Jordon distribution and Cone shape distribution [Samuel (2006); Saldaña (2008)]. They applied several methods to the data recorded on the instrumented buildings. The aim of this paper is to examine the theoretical basis of some of the methods and then apply them to synthetic seismic signals to show the merits and demerits of time-frequency methods.

2 Time-frequency mathods

2.1 Short time fourier transform

Short time Fourier transform is used to analysis the seismic signal at a desire time t and suppressing it all other times that is accomplished by multiplying the actual time signal x(t) by a window function w(t), centered at time *t* [Black (1998)]. The mathematical representation of STFT is expressed as,

$$STFT\ (t,f)=\int_{-\infty}^{\infty}x(t)w^{*}(t-\tau)e^{-j2\pi f\tau}d\tau \tag{1}$$

where x(t) is the signal to be analyzed, w(t) is a window function and τ indicates the position of wthe indow. Eq. (1) provides a time-frequency representation of the windowed signal $x(t)w^{*}(t-\tau)$.

Spectrogram or energy content of seismic signal can be defined by computing the square magnitude of the STFT [Amy and Wiesław (2007); Rucka (2011)].

The STFT spectrogram could be obtained from the following expression.

$$SP_{STFT}(t,f)=\left|STFT(t,f)\right|^{2}=\left|\int x(t)w*(t-\tau)e^{-j2\pi f\tau}d\tau\right|^{2} \tag{2}$$

While using the STFT, choice of the window size is significant as it controls the time and frequency resolution. The problem associated with STFT is that the signal energy is scrambled with the window function and the energy of the signal appears on the time-frequency plane which is not present in actual signal [Black (1998); Trifunac, Ivanovic and Todorovska (2001)]. Another problem associated with STFT is leakage due to the effect of the window. The Fourier transform of a rectangular window used in STFT is a “Sinc” function having narrow main lobe width but larger side lobes resulting in higher spectral “leakage” or the signal energy spreading from one spectral location to other by performing the convolution operation of signal spectrum and window spectrum [Douglas and Thomas (1987)]. For an ideal window, essentially the main lobe width should be very small, and side lobes should fall offs rapidly so that high detection ability could be obtained. To reduce the effect of leakage, window with smaller side lob can be used such as Hanning [Proakis and Manolakis (2006)].

2.2 Gabor transform

The mathematical representation of Gabor transform of signal x(t) is defined as,

$$GT(t,f)=\int x(t)e^{\frac{-(t-\tau)^2}{2}}e^{-j2\pi f\tau}d\tau \tag{3}$$

Where x(t) is the input signal $e^{\frac{-(t-\tau)^2}{2}}$ is Gaussian window and τ indicates the time-shift of the window. Like STFT, Gabor transform also uses the running window for analysis of sthe ignal. The window being used in Gabor transform is Gaussian in nature, is broadly used due to its less spectral leakage. It has been shown in the application of Gaussian window in Gabor transform provides the better resolution [Todorovska and Trifunac (2007); Janssen (1991); Tomaz (2001)]

Seismic signal energy is obtained by computing the square magnitude of the Gabor transform. The GT spectrogram is given by the equation,

$$SP_{GT}(t,f)=\left|GT(t,f)\right|^2=\left|\int x(t)e^{\frac{-(t-\tau)^2}{2}}e^{-j2\pi f\tau}d\tau\right|^2 \tag{4}$$

Similar to the STFT, the Gabor transform also suffers from the time and frequency resolution problem since window function is involved. A narrow window gives good time resolution and poor frequency resolution. A window size becomes wide, time resolution becomes worse but the frequency resolution improves.

2.3 Wavelet transform

A wavelet is limited duration signal that has an average value of zero. In order to overcome the shortcomings of the fixed window techniques (e.g. STFT, GT), Morlet and Grossman introduced the concept of wavelet transform in 1980 and proposed the theory of multi-resolution in wavelet transform which offers different resolutions at different frequencies [Christopher and Kyle (1998)]. The mathematical equation of wavelet transform defined as,

$$w(\tau,s)=\int_{-\infty}^{\infty}x(t)\frac{1}{\sqrt{s}}\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{5}$$

Where $x(t)$ is a signal to be analyzed, $\psi(t)$ is mother wavelet, τ indicates translation (or time shift) of the window and s is the scale (or dilation) of the mother wavelet. The wavelet transform is a time-scale representation called scalogram which does not provide a direct interpretation of frequency. There is a reciprocal relationship between scale and frequency that converts the time-scale to the time-frequency spectrum [Qiang (2012)]. Similar to STFT and GT, uncertainty principle also affects the wavelet transform [Li and Zheng (2007); Sinha, Routh, Anno et al. (2005)]. Since at high scale, it gives poorer time resolution and better frequency resolution, and at the low scale, it gives better time resolution and poorer frequency resolution. Hence, time and frequency resolution cannot be represented simultaneously due to window effect.

2.4 S-transform

S-transform a time-frequency technique was proposed by Stockwell and his coworkers [Pinnegar and Mansinha (2003)]. S-transform is fusions of short time Fourier transform (STFT) and wavelet transform that takes the advantage of STFT and wavelet [Pinnegar and Mansinha (2003); Zhu, Wang and Shen (2012)]. The mathematical representation of S transform is expressed as,

$$S(\tau, f) = \int_{-\infty}^{+\infty} h(t) \frac{|f|}{\sqrt{2\pi}} e^{\frac{-(\tau-t)^2 f^2}{2}} e^{-j2\pi f t} dt \tag{6}$$

S represents the S-transform of the signal, h is a continuous function of time t, the frequency is denoted by f and the quantity τ is a parameter which controls the position of the Gaussian window on the *y*-axis. Because of use of frequency-dependent window function, S-transform provides a good frequency resolution at low frequency and a good time resolution at a higher frequency. Hence, it can be said that time and frequency resolution cannot achieve simultaneously since window function is involved. And, the clarity of S-transform is poorer than the Wigner-Ville distribution [Yang and Sergey (2013)].

2.5 Wigner distribution

The Wigner distribution (WD) has been introduced to overcome the limitations of time-frequency distribution as discussed before. The advantage of Wigner distribution over other transforms, such as the wavelet transform STFT, and GT is having sharp localization properties in the time-frequency plane [Tobback, Steeghs, Drijkoningen et al. (1996)]. WD is the Fourier transform of the input signal's autocorrelation [Qian and Chen (1999); Boashash (2003)]. WD could be expressed by the following equation,

$$WVD_X(t, f) = \int_{-\infty}^{+\infty} X(t+\frac{\tau}{2}) + X^*(t-\frac{\tau}{2}) e^{-j2\pi f\tau} d\tau \tag{7}$$

Where, $x(t)$ is a real signal.

WD increases the time-frequency resolution, but the unwanted signal i.e. cross-term interference creates due to the non-linearity of the WD causes interaction between the positive and negative frequency components. These cross-term interference, which reduces the worth of the WD for real signals, can be reduced by modifying the WD with the analytic signal [Qian and Chen (1999); Boashash (2003)].

2.6 Wigner-Ville distribution

WVD is an expansion of WD [Boashash (2003, 1996)]. WVD is the Fourier transform of the input signal's autocorrelation. WVD could be expressed by the following equation,

$$WVD_z(t,f)=\int_{-\infty}^{+\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau \tag{8}$$

Where, $z(t)$ is analytic signal of $x(t)$. The real signal first converted into an analytic form by computing the Hilbert transform in order to give a positive spectrum. The benefit of analytic signal is that it removes the negative frequency component and reduces the cross term interference arises due to the positive and negative frequency terms. But, the cross term interference created between the actual frequency components cannot be removed. For example,

WVD of the signal $z(t)=x_1(t)+x_2(t)$ from Eq. (8) is

$$WVD_z(t,f)=\int_{-\infty}^{+\infty}\left[x_1(t+\frac{\tau}{2})+x_2(t+\frac{\tau}{2})\right]\left[x_1^*(t-\frac{\tau}{2})+x_2^*(t-\frac{\tau}{2})\right]e^{-j2\pi f\tau}d\tau \tag{9}$$

$$\begin{aligned}&=\int_{-\infty}^{+\infty} x_1(t+\frac{\tau}{2})x_1^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau+\int_{-\infty}^{+\infty} x_2(t+\frac{\tau}{2})x_2^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau\\&+\int_{-\infty}^{+\infty}\left[x_1(t+\frac{\tau}{2})x_2^*(t-\frac{\tau}{2})+x_2(t+\frac{\tau}{2})x_1^*(t-\frac{\tau}{2})\right]e^{-j2\pi f\tau}d\tau\end{aligned} \tag{10}$$

$$WVD_X(t,f)=WVD_{x_1}(t,f)+WVD_{x_2}(t,f)+2\,\mathrm{Re}\left[WVD_{x_1x_2}(t,f)\right] \tag{11}$$

The WVD of the signal z (t) does not simply sum of the WVDs of the signals $x_1(t)$ and are the actual signal components and, there is present one more signal component, which is the cross term $2\,\mathrm{Re}\left[WVD_{x\,x_2}(t,f)\right]$. The cross term interference can have a peak value of as high as twice of that of the actual signal. . If a signal consists of two sinusoids at frequencies x_1 and x_2, and cross-term interference occurs over the Wigner time-frequency distribution at a frequency equal to $(x_1+x_2)/2$. In general, for *n* sinusoids there exists *n* (*n*−1)/2 such interference terms. Note that, cross term interference increases quadratically with a number of signal components. This cross term creates the misleading information particularly if WVD outcome is to be visually analyzed by a human analyst. This distribution provides the best time-frequency resolution, but due to the presence of cross term interference make the interpretation of WVD is impossible [Qian and Chen (1999); Boashash (2003)].

2.7 Pseudo Wigner-Ville distribution

Pseudo Wigner Ville distribution (PWVD) is a windowed version of WVD [Gabarda and Cristobal (2010); Szmajdaand, Górecki and Mroczka (2010)]. On the other hand, PWVD is sliding version of WVD obtained by inserting a time domain window, $w(\tau)$. PWVD could be expressed by the following equation

$$WVD_z(t,f)=\int_{-\infty}^{+\infty} w(\tau)\; z(t+\frac{\tau}{2})+z^*(t-\frac{\tau}{2})e^{-j2\pi f \tau}\,d\tau \tag{12}$$

The window function determines how to suppress the cross-terms. By selecting the appropriate window function, it can be reduced the cross-term interference and keep some constructive properties of the WVD. But the drawback is that it also reduces the time-frequency resolution [Yang and Sergey (2013)].

2.8 Choi-William distribution

Choi-William distribution (CWD) overcomes the WVD limitation by reducing the cross term interferences to a large extent. CWD adopts the exponential kernel [Samuel (2006); Saldaña (2008)] to suppress the cross-term interference of WVD.

The Choi-Williams distribution could be expressed by the following equation,

$$C_z(t,f)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_z(\eta,\tau)\phi(\eta,\tau)e^{(j2\pi(\eta t-\tau f))}\,d\eta\,d\tau \tag{13}$$

Where $\phi(\eta,\tau)$ is kernel function.

The kernel function is expressed as follows,

$$\phi(\eta,\phi)=e^{\left[-\beta(\eta\tau)^2\right]} \tag{14}$$

The exponential kernel function includes a beta parameter to suppress the cross-terms away from the horizontal axis and the vertical axis. Therefore, the CWD reduces the cross-terms generated by two auto-terms with different time centers and frequency centers. However, CWD preserves the cross-terms on the horizontal axis and the vertical axis. In other words, the CWD does not suppress the cross-terms that two auto-terms with the same time center or frequency center generate.

2.9 Smoothed pseudo Wigner-Ville distribution

SPWVD is a windowed version of WVD, which employs windowing operation to suppress the undesired cross-term interference. The mathematical expression for SPWVD is as follows,

$$P(t,f;g,h)=\int_{-\infty}^{+\infty} h(\tau)\int_{-\infty}^{+\infty} g(u-\tau)\; e^{-j2\pi f \tau}\; x_a(u+\frac{\tau}{2})\; x_a^{\,*}(u-\frac{\tau}{2})\,du\,d\tau \tag{15}$$

Where function *g* allows the smoothing of the cross term along the time axis (time smoothing) and the function *h* allows the smoothing of the cross term along the frequency axis (frequency smoothing). The amount of smoothing in the time domain and in the frequency domain can be controlled by the length of window function (for example

Hanning window). This windowing operation is used to reduce the effect of cross term interference [Amirtaha and Hansen (2017); Samuel (2006); Saldaña (2008)].

2.9 Born-Jordan distribution

Born-Jordan a time-frequency technique provides temporal and spectral information of a signal. This distribution employs a kernel function is $\sin c(\pi\eta t)$ to reduce the effect of cross-term interference [Samuel (2006); Saldaña (2008)]. BJD suppress the effect of cross term partially in the center of time and frequency axis. However, it removes the effect of cross the erm in the different time and frequency axis.

2.10 Cone shape distribution

The Cone shape distribution (CSD) could be expressed by the following equation

$$C_z(t,f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_z(\eta,\tau)\phi(\eta,\tau)e^{(j2\pi(\eta t-\tau f))}\,d\eta\,d\tau \tag{16}$$

Where, $A_z(\eta,\tau)$ is ambiguity function of the signal, and $\phi(\eta,\tau)$ is kernel function. Here, the kernel function is defined as Saldana [Saldana (2008)],

$$\sin c(\pi\eta\tau)\times\exp(-2\pi\beta\tau^2) \tag{17}$$

The reason why this distribution is so named is that its kernel function in (t,τ) domain looks like two cones. This kernel function is also known as butterfly function The CSD suppresses the cross-terms that two auto-terms with different time centers and frequency centers generate. Additionally, the CSD also suppresses cross-terms that two auto-terms with the same frequency center generate. However, the cross-term components with the same time center cannot be removed by the cone-shape kernel [Samuel (2006); Saldaña (2008)].

Table 1: Comparison of time-frequency methods

S. No	Methods	Applications	Remarks
1	STFT	Damage analysis of buildings [Black (1998); Trifunac, Ivanovic and Todorovska (2001)]	Provides time and frequency information, but suffers from windowing effect
2	GT	Analysis of Geophysical signal [Bradford (2006)]	Improve time and frequency resolution as it employs Gaussian window. Although, it is limited to the uncertainty principle
3	WT	Damage analysis of buildings [Todorovska and Trifunac (2007)] Damage identification of EUSR system [Yu and Giurgiutiu (2005)] Structural health monitoring [Taha, Noureldin, Lucero et al. (2006)] Processing of earthquake records [Heidari and Salajegheh (2008)] Detection of damage location in the beam [Rucka (2011)] Tami warning system [Chew and Kuenza (2007, 2009)]	Spectral smearing problem

4	S transform	Oil and Gas detection [Liu and Fomel (2013)] Enhancement of seismic signals [Pinnegar and Mansinha (2003)] Structural health monitoring [Vikram and Bidisha (2009)] Seismic exploration. Denoising of seismic signals [Parol (2009)]	Suffers from the poor energy concentration and it is also restricted to the uncertainty principle
5	WVD	Analysis of earthquake damaged structurer [Brasdford (2006)] etermine the seismic reflection pattern [Steeghs and Drijkoningen (1996)]	It provides the highest resolution for non-stationary signal in the time-frequency plane. However, due to the cross-terms, it is difficult to analysis the signals
6	PWVD	Damage detection of structures [Cano and Cruzado (2007); Trifunac, Ivanovic and Todorovska (2001); Trifunac and Todorovska (1999)] Detection of events in seismic time series [Gabarda and Cristobal (2010)] Reef, shoal carbonate reservoir characterization [Pinnegar and Mansinha (2003)]	The presence of the cross term does decrease as compared with the WVD, but at the cost of a slight increase in the broadness of the frequency values
7	SPWVD	Estimating the attenuation of seismic signals [Yandong and Xiaodong (2007)] Detection of transition of seismic signal [Moreno and Boris (1996)] Damage analysis of the instrumented buildings [Michel and Philippe (2010)]	Energy bands of frequencies distributions are thick, so it is very difficult to detect the small variation of signals
8	RSPWVD	Analysis of earthquake signal [Demetriu and Trandafir (2003)] Detection of gas reservoir [Ning et al. (2012] Damage analysis of instrumented buildings [Michel and Philippe (2010)]	It involves the filtering operation so it affects the smoothing of the auto terms
9 .	CWD	Seismic reflection pattern [Steeghs and Drijkoningen (1996)] Detection of the arrival time of P-wave [Moriya and Niitsuma (1996)] Damage assessment of structures. Damage assessment of a frame structure [Chandra and Barai (2014)]	CWD does suffer from an inherent cross-term problem
10	CSD	Applied on the 2D seismic data to extract attributes.	The cross-term components with the same time center cannot be removed by the cone-shape kernel

3 Synthetic signal analysis

3.1 Simulate signal

A synthetic seismic signal is considered for analysis and results are presented in this section from several time-frequency methods. Simulation has been carried out for the particular synthetic seismic signal. For examination purpose, a synthetic seismic signal is generated, which consists of modulated sine waveforms with Gaussian signals of frequencies 10 Hz, 5 Hz and 5 Hz, is sampled at 100 Hz of frequency. The frequency components are labeled as F1, F2, and F3, and are indicated by the arrow in all the plots. The generated synthetic seismic signal is shown in Fig. 1 and it could be expressed as

$$x(t)=\begin{cases} 0, & if\ \ 0\le t<1 \\ [t\ e^{-t^2}\sin(2\pi 5t)]+0.5\times[t\ e^{-t^2}\sin(2\pi 10t)], & if\ \ 1\le t<3 \\ 0, & if\ \ 3\le t<4 \\ t\ e^{-t^2}\times 0.35\times\sin(2\pi 5t), & if\ \ 4\le t\le 10 \end{cases} \quad (18)$$

$$y(t)=x(t)+n(t)$$

Where $x(t)$ is clean signal and $n(t)$ is additive white Gaussian noise. The mathematical model was used to generate such signal is composed of the product of the Gaussian signal and sine signal with different frequency components. The synthetic seismic signal has been generated using Eq. (18), by adding Gaussian noise to the signal. The resulting synthetic seismic signal with Gaussian noise is shown in Fig. 1.1.

The two-dimensional plots of the application of time-frequency distributions on the synthetic seismic signal for the methods such as STFT, GT, WT, S-transform, WD, WVD, PWVD, SPWVD, CWD, Born-Jordan, and CSD are shown in Figs. 1.2-1.12. The result of short time Fourier transform on the application of synthetic seismic signal is shown in Fig. 1.2. The use of such windowed method needs a compromise between frequency and time precision. From Fig. 1.2, it appears that energy of individual frequency component obtained from STFT spectrograms are not localized properly over the time-frequency plane and also energy in the time-frequency plane appears before the true temporal onset of the transient signals. In order to get a better result, analysis of seismic data has been performed with GT. The result obtained with GT is shown in Fig. 1.3, and it has been observed that the energy of the individual frequency component obtained from GT spectrogram is confined to a smaller area in comparison to the STFT. Since Gaussian signal is more concentrated than the rectangular function in the frequency domain and provides less spectral leakage, and GT offers a better resolution over the time-frequency plane as compared to STFT as can be seen in Fig. 1.3. As STFT and GT have limitations of having a fixed resolution that is why, analysis has been carried out by wavelet transform. And, it has been observed that this transform provides better time resolution and blurry frequency information on the time axis in between 1 s to 2 s as can be seen in Fig. 1.4. The frequency components are present on the time axis between 1 s to 2 s, not easily able to discriminate. However, this method also suffers from the time-frequency resolution problem since window function is involved. The analysis of the seismic signal is continued with S transform and the result obtained with S transform is presented in Fig. 1.5. The result shows that S transform has poor energy distribution of signal over the time-frequency plane. Because of this blurring information, the exact frequency distribution with respect to time is unpredictable.

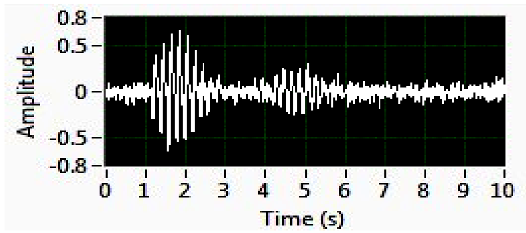


Figure 1.1: Synthetic seismic signal that consists of modulated sine curves with frequencies of 10, 5 and 2 Hz

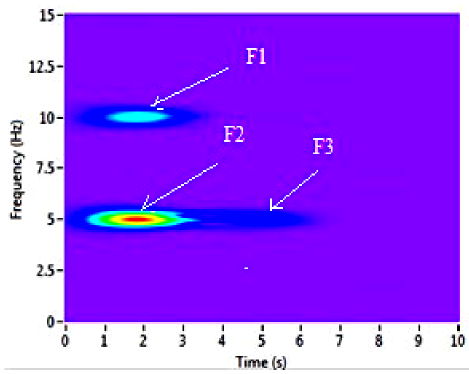


Figure 1.2: The intensity graph obtained from STFT

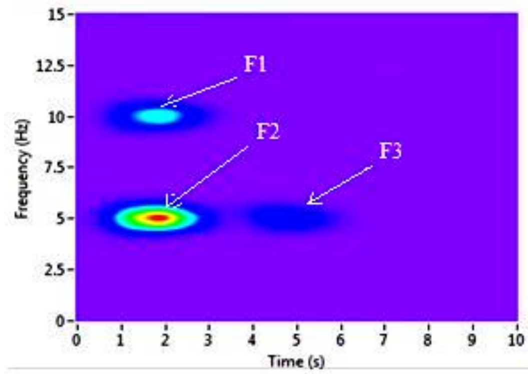


Figure 1.3: The intensity graph obtained from Gabor transform

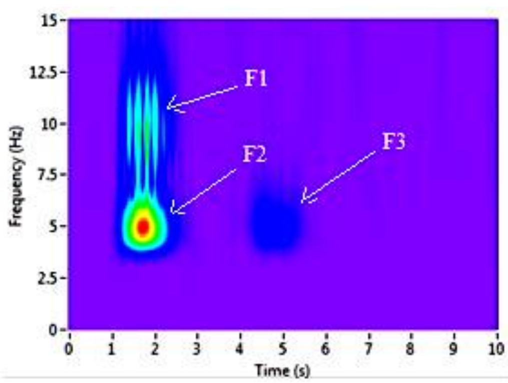


Figure 1.4: The intensity graph obtained from wavelet transform

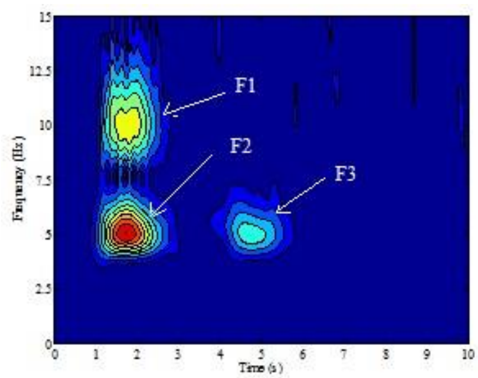


Figure 1.5: The intensity graph obtained from S transform

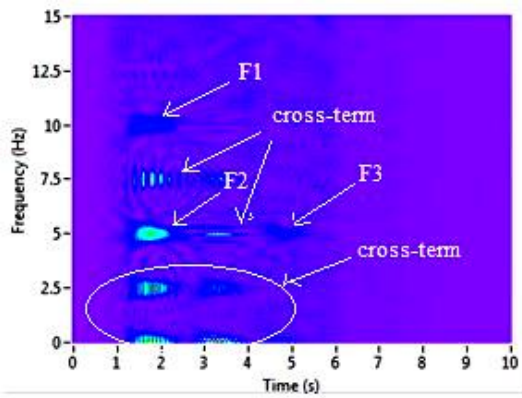


Figure1.6: The intensity graph obtained from WD

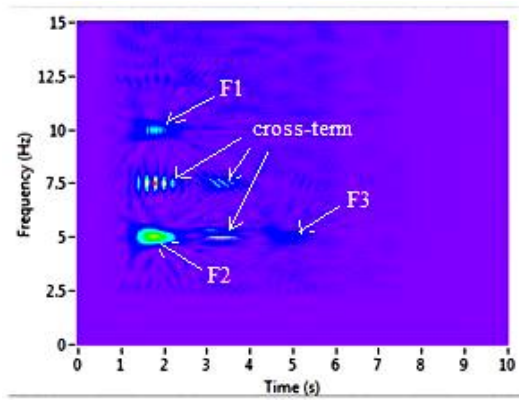


Figure 1.7: The intensity graph obtained from WVD

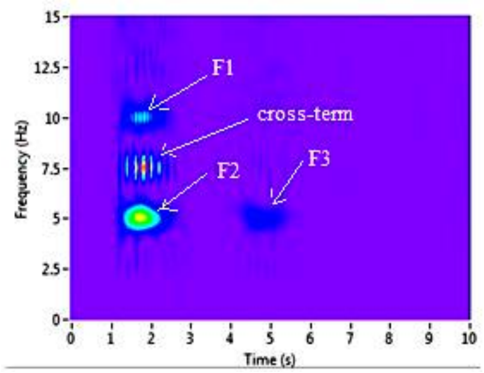


Figure 1.8: The intensity graph obtained from WVD

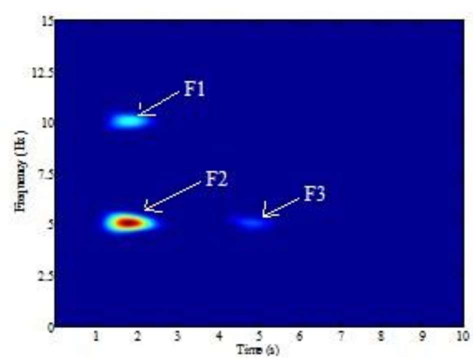


Figure 1.9: The intensity graph obtained WVD

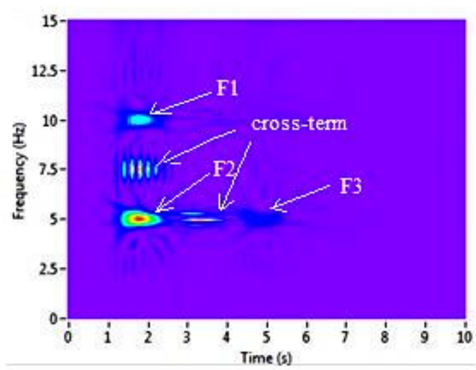


Figure 1.10: The intensity graph obtained from from WVD

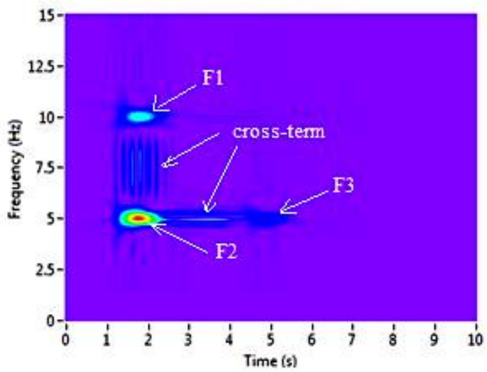


Figure 1.11: The intensity graph obtained from WVD

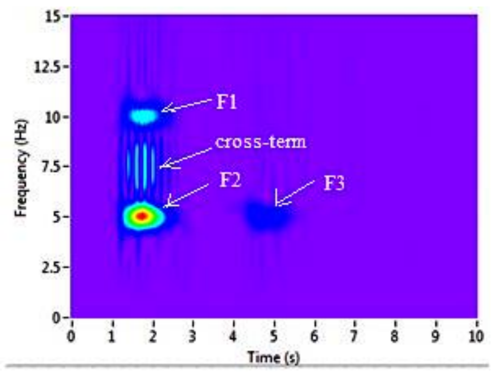


Figure 1.12: The intensity graph obtained from WVD

To overcome the effect of the window function in time-frequency method as discussed above, the analysis is carried out with WD. The result of WD on synthetic seismic data is provided in Fig. 1.6. It could be noticed from the Fig. 1.6 that the presence of cross terms in between each pair of the auto term, which is not present in STFT, GT, WT and S-transform plots. Although, this method increases time and frequency resolutions, due to the occurrence of cross terms, characterization of seismic signal and identification of their frequency components become very difficult. In Fig. 1.6, those cross-term interferences are generated due to the interaction between the positive and negative frequency components as appears in the lower frequency band (i.e. below of frequency 5 Hz) in the time-frequency plane. Those cross terms are present in the lower frequency band; it can be removed by converting the real signal into the analytic signal as suggested by researcher Ville [Boashash (2003)]. This new distribution is named as WVD. The WVD removes the cross terms which are present in the lower frequency band. But, the cross terms which were generated between the actual frequency components are not removed as shown in Fig. 1.7. The signal to noise ratio (SNR) of seismic signal is further decreased due to cross-term interference. At the low signal, to noise ratio, a rapid degradation in performance is observed. The PWVD overcomes the WVD limitations reducing the effect of cross-terms. It removes the effect of cross term interference diagonally of the actual frequency components as well as it removes the effect of the cross term, those are present in the same frequency axis. But, the cross-terms are present on the same time axis of true frequency components, cannot be eliminated as can be seen in Fig. 1.8. Moreover, the analysis is carried out with SPWVD. From Fig. 1.9, it is clear that SPWVD suppressing the effect of cross-terms and make the time-frequency plane free from the cross- terms. But, the drawbacks of PWVD and SPWVD are that the energy band of this distribution is generally thick and makes the small frequency variations difficult to distinguish. Since then, there is continuous developed in time-frequency method to get better resolution. Additionally, the analysis on synthetic seismic data with CWD is shown in Fig. 1.10. The CWD offers a time-frequency resolution on par with the WVD. It could be observed that CWD suppresses the cross term interference to a large extent, but the cross-terms which are present in the same time and frequency axis, cannot be removed as shown in Fig. 1.10. The analysis of seismic data with BJD is continued and yields result shown in Fig. 1.11. The result obtained with BJD shows that it removes the cross-terms in a different time and frequency axis completely, but those cross-terms are present in same time and a frequency axis, are moderately reduce the effect of cross-terms as can be seen in Fig. 1.11. The CSD is a smoothed version of BJD which filters the cross-term along the same frequency axis has also been in Fig. 1.12, but it preserves the cross-term along the time axis.

4 Northridge earthquake data recorded in 1994

4.1 Seismic signal

A Northridge earthquake data recorded in the year of 1994 can be seen in Fig. 2.1. This earthquake data recorded at the top of the building (https://www.strongmotioncenter.org/) is considered for analysis and results are presented in this section from several time-frequency methods. Further, these methods such as STFT, GT, WT, S-transform, WD, WVD, PWVD, SPWVD, CWD, Born-Jordan, and CSD are applied as shown in Figs. 2.2-2.12. The STFT was applied. Its response can be seen in Fig. 2.2. From the response, it can be observed that energy of the signal is not localized over the time-frequency plane. In order to overcome this problem, Gabor introduced the Gaussian window. Further, the response was computed from GT which provides the better resolution as could be seen in Fig. 2.3. Further, WT was also tested and attempted to detect the variation of frequency can be seen in Fig. 2.4, but it also does not provide the promising result. Furthermore, well-known transform i.e. S transform was employed, and response can be seen in Fig. 2.5. From figure it can said that energy of signal is not localized over the time-frequency plane. Hence, it is suggested that S transform cannot be used for analysis of seismic signals. Further, WD and WVD were applied to the same data as can be seen in Figs. 2.6 and 2.7 respectively. These methods improve the resolution, however in the presence of cross-terms variation of frequency components are difficult to detect. In this direction, researchers have been explored smoothing methods namely PWVD, SPWVD, CWD, BJD and CSD, and responses can be seen in Figs. 2.8-2.12 respectively. During analysis, it has been observed that smoothing methods reduce the cross-terms effect, but characterization of seismic signal and identification of their frequency components become very difficult. Hence, it can be said that time-frequency methods are strong ability to detect the variation of frequency components. Therefore, we need to explore a powerful time-frequency methods so that variation of frequency components are easily detected. In future, time-frequency methods can be an efficient technique to detect the changes in the frequency, and can be correlated to the physical structure of the systems. Further geophysics scientist can also be engaged to detect the reservoir.

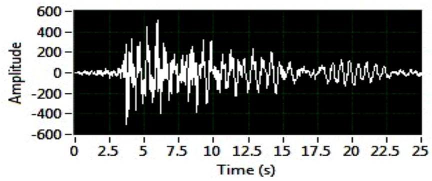
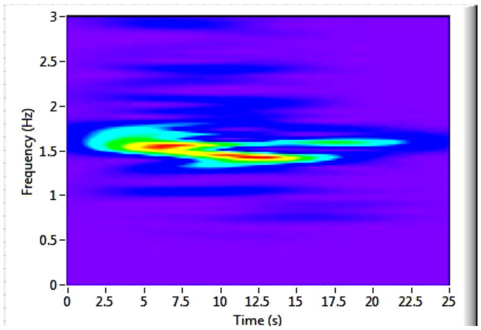


Figure 2.1: Northridge earthquake recorded in 1994



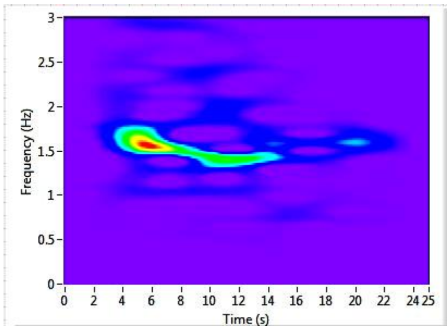


Figure 2.2: STFT **Figure 2.3:** GT

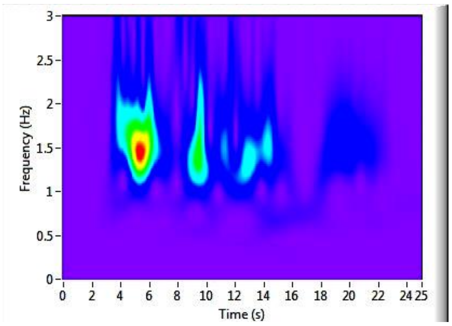


Figure 2.4: WT

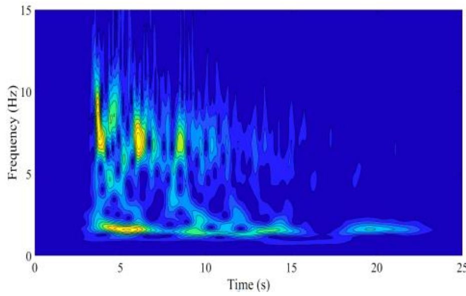


Figure 2.5: S-transform

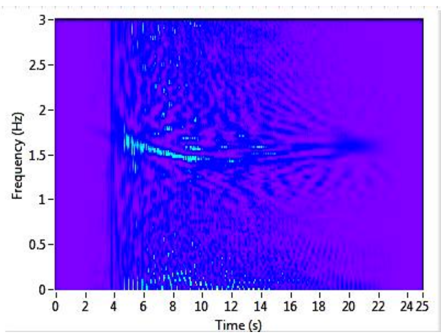


Figure 2.6: WD

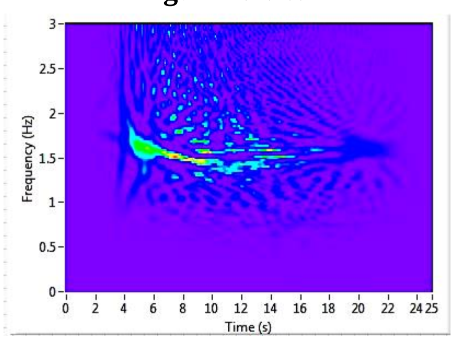


Figure 2.7: WVD

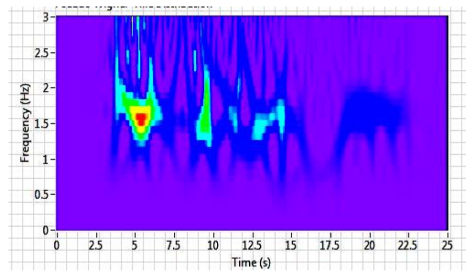


Figure 2.8: PWVD

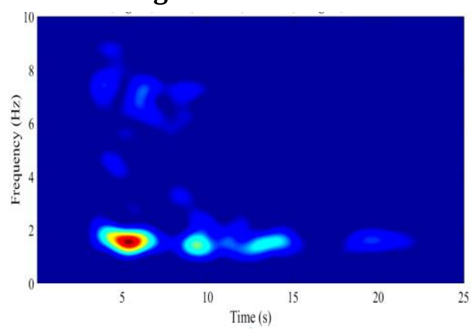


Figure 2.9: SPWVD

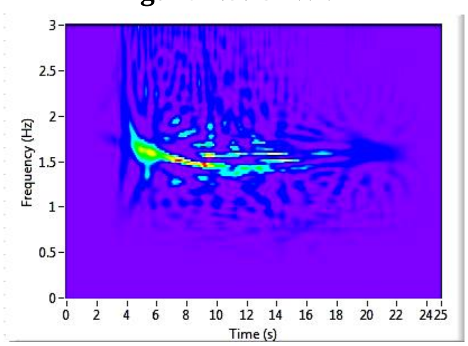


Figure 2.10: CWD

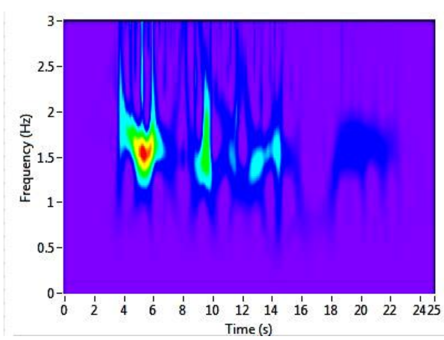


Figure 2.11: BJD

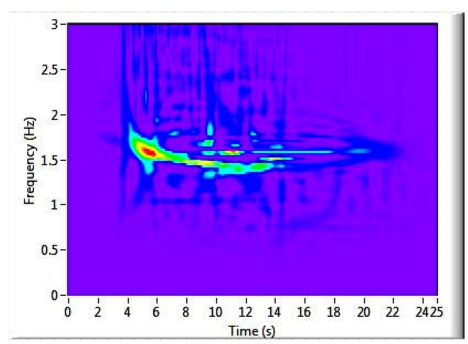


Figure 2.12: CSD

5 Conclusion

In this paper, the joint time-frequency analysis has been carried out for the synthetic seismic signal and recorded real-time data. It has been observed that better time and frequency resolutions cannot be achieved simultaneously due to window function involved in STFT, GT, WT, and S transform time-frequency distributions. The effort made to increase the time and frequency resolution introduces cross term. Further, it is proved that the PWVD, SPWVD, CWD, BJD, and CSD reduce the effect of cross terms by introducing the different sort of kernel function, but these methods limit its actual time-frequency resolution features. In smoothing method, different kinds of the kernel have been studied and selection of the best kernel function is an unsolved problem. Many time-frequency methods have been discussed in this paper and it was observed that time-frequency method performs well for some application and poorly for others. A successful application of time-frequency representations requires some degree of expertise on the user side. The prior knowledge of the signal is necessary in order to select the most suitable method for analysis of the signal. Moreover, no Cohen class distributions can significantly measure the energy of signals in the joint time-frequency plane. Tab. 1 shows the brief summary of several time-frequency distributions.

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