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Delamination Identification for FRP Composites with Emphasis on Frequency-Based Vibration Monitoring-A Review

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Abstract: Fibre reinforced polymer (FRP) composite laminates are now commonly used in many structural applications, especially in the aerospace industry, where margins of safety are kept low in order to minimise weight. Timely detection and assessment of damage (in particular delaminations) in composite laminates are therefore critical, as they can cause loss of structural integrity affecting the safe operation of the composite structures. The current trend is towards implementation of structural health monitoring (SHM) systems which can monitor the structures in situ without down time. In this paper, first, the current available SHM techniques for delamination detection in FRP composites are briefly reviewed, including acoustic emission, fibre optic sensors, Lamb wave-, impedance- and vibration-based methods. Among different vibration-based methods, frequency monitoring is the simplest to implement, requiring only single point measurement, and is relatively accurate and reliable, thus it becomes the main focus of present paper. A comprehensive review of frequency-based vibration monitoring is conducted in terms of the various aspects of delamination identification in FRPs through frequency shifts, including review of theoretical models for free vibration of delaminated FRP beams, survey of finite element modelling of delaminated composite structures, summary of experimental modal analyses on FRP composites with delaminations, and inverse algorithms for frequency-based delamination assessment. This paper aims to help the readers to get an overview of the available SHM techniques for monitoring the integrity of FRP composites, with a special emphasis on delamination assessment through frequency-based vibration monitoring.

Keywords: Structural health monitoring, delamination detection, FRP, composite structures, vibration-based method, inverse algorithms.

1 Introduction

The use of fibre-reinforced plastics (FRPs) as alternatives to conventional materials such as metallic alloys has been growing steadily in the aeronautical, naval and automotive industries due to their excellent mechanical properties, low density and ease of

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manufacture. Laminated composite materials are unique in their micromechanical interactions and failure modes which include matrix cracking, fibre breakage and interlaminar cracking or delaminations. Delamination, or the debonding of adjoining plies, is a commonly occurring damage in laminated composites [Zou and Tong (2000)] which is due to the low inter-laminar strengths of the FRP laminates.

Delamination can be caused by improper lamination and curing during manufacture caused by either air entrapment or insufficient resin flow, or by low- and medium-energy impacts during service caused by, for instance, the accidental dropping of tools, runway debris, hailstones, bird strike, ground service vehicles, etc. on aircraft components. Since delamination is an internal damage, it is often not visible from the outside; however, as it reduces the bending stiffness of a laminate, it can give rise to severe loss of structural integrity [Albuquerque, Tavares and Durão (2010)]. With the increasing use of FRP laminates in load-bearing structural applications, especially in the aircraft industry, it has become critical to be able to detect the presence and assess the severity of any delamination in them as early as possible. This been the subject of extensive research for many years [Wang (1980); Garg (1988); Kageyama and Kimpara (1991); Bolotin (1996); Tay (2003); Mathews (2007)].

Delamination identification methods currently used include visual inspection, ultrasonic testing, radiography and thermography [Doherty (1987)]. However, these traditional Non-Destructive Inspection (NDI) methods are off-line, time consuming and labour intensive, and require equipment to be taken out of operation which can be very costly in the case of aircraft (although thermographic inspections are whole-field and relatively quick, their effectiveness is limited in terms of delaminations close to the laminate surface). Most importantly, since inspections are carried out only periodically, off-line NDI techniques may not detect damage until it is too late. Therefore, the current trend in the aerospace industry is to implement Structural Health Monitoring (SHM) techniques which can monitor structural integrity on-line, *in situ*, continuously and without grounding the aircraft. In general, SHM involves integrating appropriate sensors in structural components to facilitate the continuous monitoring of structural integrity through analysing data from them. Over the past few decades, research into SHM has focused on developing its various aspects, such as sensors, sensor integration, and data collection, processing, transmission and automated analysis [Doebling and Farrar (1996); Doebling and Farrar (1998); Chang, Flatau and Liu (2003); Sohn, Farrar and Hemez (2003); Montalvao, Maia and Ribeiro (2006)]. Due to increasing demand, there are many different types of SHM techniques being developed and investigated, with some of the techniques such as fibre optic sensors (FOSs), acoustic emission (AE), and Lamb wave-, impedance- and vibration-based methods showing good promise for monitoring delaminations in composite structures. All these methods have their own benefits and drawbacks and are often promoted as the most useful or reliable by their developers and exponents. The following section provides brief overviews of these major SHM techniques in which their possible disadvantages are highlighted, especially in comparison with frequency-based vibration monitoring which is the main focus of this review paper.

2 SHM techniques for delamination assessment

2.1 Fibre optic sensors (FOS)

The earliest use of optical fibre technology in sensor applications dates back more than 30 years [Zhou and Sim (2002)]. Optical fibres are usually embedded in a host structure during manufacture and interrogated using coherent light from lasers to continuously monitor either its transmitted or reflected light, a sudden drop in the intensity of which indicates that it has been damaged possibly due to structural damage in its vicinity [Barazanchy, Martinez, Rocha et al. (2014)]. Therefore, FOS had been used for monitoring the integrity of composite structures extensively with strain, temperature or pressure being the measurands [Akhavan, Watkins and Chandrashekhara (1995); Bhatia, Schmid and Murphy (1995); Watkins, Sanders, Akhavan et al. (2002)]. Despite its wide success in applications of SHM, one of the main limitations of a FOS is that, as its optical fibres have to be embedded during manufacture, it cannot be applied to existing composite structures and, as it requires a fairly fine grid of optical fibres and has to be monitored continuously to cover all possible locations of damage, its implementation is quite labour intensive. Also, optical fibres typically have diameters ranging from 100 μm to 250 μm, significantly larger than those of the reinforcing fibres (5-10 μm) in FRP laminates [Ling (2006)] which may cause stress concentrations and geometric discontinuities in their host structures. It is also to be noted that, while a FOS is successful in detecting crack-like defects, such as reinforcement fractures and transverse cracks which break its optical fibres, it is reported to be insensitive to delaminations which occur in the interfaces between the laminate layers [Kawiecki (2001)].

2.2 Acoustic emission (AE)

AE is the phenomenon whereby transient elastic waves are generated by the rapid release of energy within a material due to damage initiation or growth [Hamstad (1985)]. Materials in which AE has been successfully applied range from wood, rocks, metals and concrete to composites [Silversides, Maslouhi and LaPlante (2013); Crivelli, Guagliano, Eaton et al. (2015)] and the first application of AE to monitoring bridges was reported in the early 1970s [Eaton, Pullin, Holford et al. (2009)]. Proponents of AE have suggested applications for monitoring rotating machinery, tool wear, pipes and weld analysis, pressure vessels and aerospace structures. AE monitoring is a passive technology which uses AEs generated by damage growth and enables the real-time monitoring of a structure as signals originate as soon as damage occurs and is also regarded as highly sensitive and able to detect even the smallest defects. Despite its advantages, SHM using an AE technique has its limitations: (1) It cannot be used to assess an existing defect unless the damage propagates and generates AE signals; (2) Due to the sampling rate needed for data capture, as a large amount of data is usually generated during AE testing, an effective data analysis strategy for data storage and transmission has been a major challenge [Kaphle, Tan, Thambiratnam et al. (2011)]; (3) AE methods need to filter out the electrical interference and ambient noise from emission signals to improve identification accuracy; and (4) Finally, real-time AE monitoring requires several trial monitoring sessions and experienced personnel for a signal to be distinguished from background noise.

2.3 Lamb wave-based methods

The application of Lamb waves to composite materials began during the late 1980s and early 1990s [Kessler, Spearing and Soutis (2002)]. They are elastic guided waves in traction-free thin plates introduced to a composite structure by a transducer at one point and sensed by another transducer at a different point [Raghavan and Cesnik (2007)]. Since the entire thickness of a laminate can be interrogated by various Lamb wave modes, it offers the possibility of detecting internal delamination as well as surface damage [Rose (2002)]. The application of Lamb waves for NDI of composite laminates is regarded as promising for long-range monitoring since Lamb waves can propagate large distances and a large region can therefore be interrogated with each transducer position [Guo and Cawley (1993); Su, Ye and Lu (2006); Sohn, Swenson, Olson et al. (2011)]. Detection using Lamb waves is reported to be sensitive to multiple defects and has high identification accuracy. However, a Lamb wave-based detection method in an anisotropic viscoelastic material is notoriously complicated. At a very fast velocity, waves reflected from boundaries may easily conceal damage scattered components in the signals. To ensure precision, the structure under inspection may have to be relatively large, with a relatively small area for detection. Usually, multiple wave modes exist and their dispersive properties throughout the thickness of the medium are not identical. Also, the monitoring strategy based on Lamb waves for damage detection requires knowledge and experience regarding the choices and locations of transducers, excitation frequencies and other parameters.

2.4 Impedance-based methods

In the impedance-based SHM, a piezoelectric transducer (PZT), either bonded to the surface of or embedded in a structure, is used to actuate and sense structural responses [Lopes, Park, Cudney et al. (2000)] through which the impedance of the structure is monitored, with any variation in it indicating a variation in structural integrity caused by delaminations, surface cracks or other forms of damage [Park, Cudney and Inman (2000); Yan and Chen (2010)]. Impedance domain methods are particularly suitable for detecting planar defects, such as delamination [Bois, Herzog and Hochard (2007)], and are reliable except when the layer above the delamination is thin and the lower sub-laminate is relatively stiff [Cawley (1984)]. However, due to the difficulty of developing an analytical model at high frequencies, correlating the changes in electrical impedance with specific changes in structural properties is very complicated. Therefore, this method provides only limited information on the nature of damage [Lopes, Park, Cudney et al. (2000)].

It may be observed that all the above methods, FOS, AE, and Lamb wave- and impedance-based approaches, require several transmission lines or transducers to be deployed to exactly identify the location and size of damage.

2.5 Vibration-based methods

The principle behind vibration-based damage detection techniques is that damage causes local discontinuities in structural stiffness and/or variations in damping properties which affect a structure’s dynamic characteristics and modal parameters, namely, natural frequencies, mode shapes and damping, by monitoring which the incidence of damage in a structure can be identified [Montalvao, Maia and Ribeiro (2006)].

Vibration-based methods have the advantage that, in general, dynamic parameters can be easily and continuously extracted from the vibrations of operating machines, especially aircraft [Carden and Fanning (2004); Fritzen (2005)]. Vibration monitoring has been used in the aerospace and offshore industries since the late 1970s [Farrar, Doebling and Nix (2001)] and, in particular, frequency-based vibration methods have been routinely employed for condition monitoring and fault diagnosis in rotating machinery, gear boxes, wind turbines, gas turbine components and automobile engines for several decades [Salawu (1997); Goyal and Pabla (2016)]. Most modern aircraft, particularly military helicopters such as the MRH90, are fitted with vibration-based health monitoring systems [Tuck, McGregor, Buller et al. (2011)]. Another area where vibration monitoring has shown immense potential is in wind turbines where Tcherniak and Molgaard have demonstrated the use of vibration based SHM to detect damage while the turbine was still operating [Tcherniak and Mølgaard (2017)]. The interest in vibration based SHM has been steadily increasing and this is clearly evident from the research output in this topic. Scopus search term of “Vibration SHM” shows 1210 articles published since 1998 with approximately 160 articles published in 2017 alone, Fig. 1 displays the record of publications since 1998. This review focuses on some of the topics involved within the vibration based structural health monitoring.

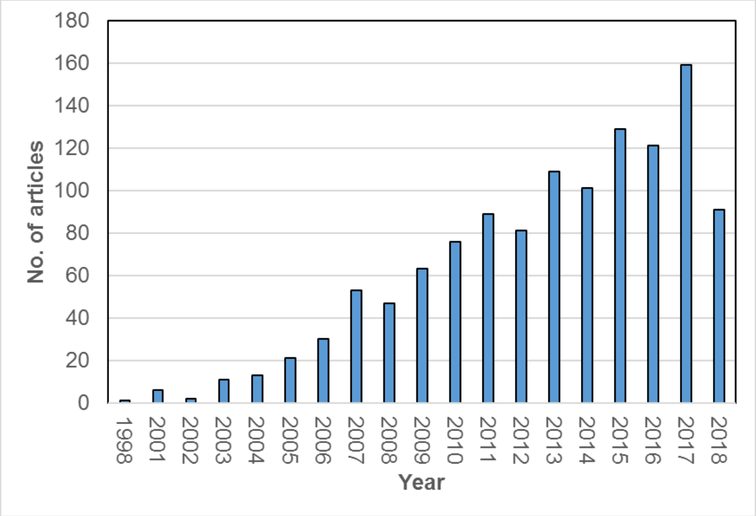

Figure 1: Scopus search term of “Vibration SHM” published since 1998

There are many different vibration-monitoring methods, depending on the parameters to be monitored and the way in which the data is to be analyzed to extract the damage parameters. In general, they can be broadly classified into time domain methods, in which the dynamic response of a structure is monitored in the time domain and analyzed for sudden variations, and methods based on modal parameters (frequencies, mode shapes and damping). An overview of these different categories is presented in the sub-sections below, noting that the current paper is focused on frequency-based vibration monitoring.

2.5.1 Time response methods

Techniques based on time domain data are extensively used as a means of damage detection since their measurements are always taken in the time domain [Majumder and Manohar (2003); Fassois and Sakellariou (2007); Garcia, Palazzetti, Trendafilova et al. (2015)]. These methods are regarded as highly sensitive to damage since they use raw data and can detect damaged situations both globally and locally by changing their input frequencies [Thwaites and Clark (1998)]. Usually, time-domain methods for system identification use models of stochastic processes and the common ones are the Auto-regressive [Sohn and Farrar (2001)], Auto-Regressive Exogenous [Lei, Kiremidjian, Nair et al. (2003)], Eigen Realisation Algorithm [Chiang and Lin (2010)], Auto-regressive Moving Average [Lu and Gao (2005)] and Auto-regressive Moving Average Vector [KIM and LEE (1998)]. Apart from input into system identification, time domain data can also be broken down into a two-dimensional function of time and frequency, and use wavelet analysis for fault detection [Liu and Ling (1999); Peng and Chu (2004)], or as a signal processing tool for filtering noise [Lin and Qu (2000)].

2.5.2 Mode shape methods

As a mode shape contains spatial information about structural changes, a damage location may be detected straightforwardly from its measurement. Early mode shape-based methods directly compared the mode shape deflections of a structure before and after damage using the Modal Assurance Criteria (MAC) [West (1986)] or other related assurance criteria [Allemang (2002)] to determine the level of correlation between the mode shapes from intact and damaged structures. Instead of mode shape deflections, comparison of mode shape curvatures is reported to be more reliable and useful [Cao, Radzieński, Xu et al. (2014)], since curvature is directly proportional to bending stiffness and discontinuities in stiffness directly show up as discontinuities in curvatures. Identifying both the severity and location of damage, Pandey et al. [Pandey, Biswas and Samman (1990)] first proposed a curvature mode shape to identify and locate damage in structures in 1990. Since curvature is proportional to the bending strain, mode shape curvatures can be obtained by directly measuring strains instead of differentiating displacement values which is subject to numerical errors.

Despite the advantage of providing spatial information regarding the location of damage, mode shape methods have several drawbacks. Firstly, as measurements have to be taken over the entire surface of a structure, this requires either a grid of sensors or the movement of a single sensor from point to point between measurements which can be quite time consuming and labour intensive. Secondly, as variations in mode shape deflections due to damage can be quite small, damage signatures can be lost in the noise and scatter in the measurements [Cao, Xu, Ostachowicz et al. (2014)]. Thirdly, in practical SHM, obtaining mode shapes from ambient response measurements is particularly difficult because the excitation force is difficult to determine.

2.5.3 Methods based on damping

The initiation of damage as well as its growth can cause significant damping changes in a structure. Damping ratio measurements have been found to be highly sensitive to very small abnormalities which is of interest to the SHM community [Modena, Sonda and Zonta (1999); Montalvao, Ribeiro and Maia (2007)]. However, damping mechanisms in composites are entirely different from those in traditional materials such as metals and alloys. Ways of dissipating energy in FRP composites include [Chandra, Singh and Gupta (1999)]: Viscoelastic damping in the matrix/fibre; damping at the inter-phase between the fibre and matrix; viscoplastic damping; thermoelastic damping due to cyclic heat flowing from regions with compressive stress to those with tensile stress; and, for FRP laminates with defects, energy dissipation through matrix-fibre inter-phase slip as well as friction between the sub-laminates in the delaminated region. The complex nature of damping in composites has been the subject of several theoretical studies [Yim (1998); Adams and Maheri (2003); Berthelot (2006)], finite element analyses [Lin, Ni and Adams (1983); Maheri and Adams (1995); Kyriazoglou and Guild (2005); Berthelot, Assarar and Sefrani (2008); Kiral (2009)] and experimentation [Berthelot and Sefrani (2004); Chrysochoidis and Saravanos (2004); Kyriazoglou, Page and Guild (2004); Yam, Wei and Cheng (2004); Kyriazoglou and Guild (2005); Berthelot (2006); Kiral, Malgaca and Akdag (2009)]. Damping coefficients are influenced by boundary conditions and environmental factors such as humidity and temperature and, in laminated composites, damping is a function of the material properties of the fibre and matrix as well as the ply orientation, stacking sequence, frequencies and mode shapes [Maheri and Adams (1995)]. Due to the complexities of modelling damping, as well as the influence of numerous factors on its measurements, damping has been used limitedly for delamination detection in composites.

2.5.4 Frequency-based methods

Shahdin et al. [Shahdin, Morlier and Gourinat (2010)] reported that one of their beam specimens impacted at 4 *J* did not produce visible damage on the surface but that the average change in frequency for the first four bending modes between the undamaged and damaged cases was 14% which proved that the beam specimen had a notable loss of rigidity without any signs of damage on its surface.

As mentioned in the foregoing subsections, mode shape methods require measurements at multiple locations and are susceptible to measurement noise which may drown the damage signature while damping measurements are notoriously difficult and sensitive to variations in environmental conditions. In comparison, frequency-based vibration techniques facilitate global monitoring, mostly require only a single point measurement, can be implemented easily even on existing structures, and provide reliable and accurate data [Salawu (1997)]. Furthermore, in structures such as aircraft, they do not need an external source for excitation. For these reasons, frequency-based methods are the most promising for implementation as SHM systems. As previously mentioned, frequency monitoring is already utilised in many applications for condition monitoring and fault detection, especially in rotating machinery [Farrar, Doebling and Nix (2001)]. However, its application for the detection and assessment of delaminations in composite structures is still under development.

Since damage causes a reduction in stiffness, any significant level of damage causes appreciable and permanent reductions in the natural frequencies of a structure [Kessler, Spearing and Atalla (2002)], as illustrated in Fig. 2 for measurements taken from a carbon fibre-reinforced beam with artificially introduced delamination damage. Thus, detection of damage using frequency monitoring is straightforward and relatively easy to accomplish. However, frequency shifts do not directly indicate either the location or size of damage since different combinations of damage type, size and location can produce the same amount of frequency shift in any particular mode [Kessler (2002)]. However as, by using the frequency shifts from multiple modes, information regarding the location and size of damage can be extracted, research into frequency-based methods has focused mainly on this aspect [Adams and Cawley (1978); Cawley and Adams (1979); Valdes and Soutis (2000); Kim, Ryu and Cho (2003); Xia and Hao (2003); Kannappan (2008)]. The assessment of damage parameters from measured frequency shifts is often referred to as the 'solution to the inverse problem'. Employing theoretical or finite element models to generate forward data, i.e. frequency shifts in multiple modes caused by a range of damage with known parameters, the solution to the inverse problem essentially consists of comparing the measured frequency shifts with their calculated values in the database to predict the geometric parameters of the damage being assessed. Early applications of damage assessment using frequency measurements which focused on assessing cracks in metallic beams and rods employed graphical approaches to solve the inverse problem [Cawley and Adams (1979)]. More recent developments, especially for assessing delaminations in composites, involve the use of artificial intelligence techniques, such as genetic algorithms (GAs) and artificial neural networks (ANNs) [Su, Ling, Zhou et al. (2005)].

In the later sections, a comprehensive review of various aspects of applying frequency-based vibration monitoring to detect delamination in FRP composites are conducted, including: Section 3-review of theoretical models for free vibration of delaminated FRP beams, Section 4-survey of finite element modelling of delaminated composite structures, Section 5-summary of experimental modal analyses on FRP composites with delaminations, and Section 6-inverse algorithms for frequency-based delamination assessment.

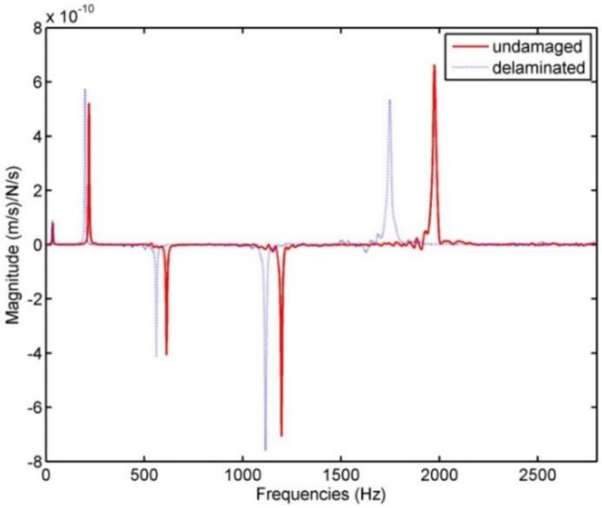

Figure 2: Frequency shifts due to delamination in CFRP beam

3 Theoretical models of delamination

When using vibration-based method, delaminations are assessed from measured frequency shifts using inverse algorithms such as the graphical method, artificial neural network (ANN) and genetic algorithms to solve the inverse problem. These inverse algorithms require a database of frequency shifts versus delamination parameters in which the search is made to extract the delamination parameters (size, x-location and interface in the case of a composite beam). This database can be generated by solving the forward problem, i.e. predicting frequency shifts caused by the presence of delamination of a given size at a given axial and interface location using either theory or finite element simulation. Development of the theoretical models employed for the forward problem requires a good knowledge of current strategies about this topic, thus a review of current literature on theoretical models of composite beams with delaminations is presented in this section.

The earliest model for vibration analyses of composite beams with delaminations is recognized as the one proposed by Ramkumar et al. [Ramkumar and Kulkarni (1979)]. In it, a cantilever beam with one through-width delamination was modelled using four Timoshenko beams joined together with appropriate boundary and matching conditions, and the natural frequencies and mode shapes of free vibration solved by a boundary Eigenvalue problem. The natural frequencies predicted with this model were consistently lower than those reported from experimental measurements. Ramkumar et al. [Ramkumar and Kulkarni (1979)] attributed this discrepancy to the effect of contact between the delaminated 'free' surfaces during vibrations being ignored. They suggested that including the contact effect may improve analytical predictions since it would increase the stiffness and, hence, the frequencies.

Instead of adding the contact effect, Wang et al. [Wang and Liu (1982)] improved the analytical solution by including coupling between the flexural and axial vibrations of delaminated sub-laminates. By modelling an isotropic beam as four Euler-Bernoulli beam segments and considering the effect of longitudinal motion, they found that the calculated natural frequencies were closer to the experimental results. However, as their model allowed the delaminated parts to vibrate freely without interference from each other, it was referred to as a 'free model' and criticised by later researchers for allowing possible inter-penetration between the sub-laminates which cannot happen in reality.

Mujumdar et al. [Mujumdar and Suryanarayan (1988)] pointed out that, although some of the delamination opening mode shapes predicted by Wang et al. were mathematically admissible, they were physically impossible, especially when delamination is off-mid-plane. To avoid this kind of incompatibility and retain a linear model, their model imposed a normal pressure between the delaminated sub-laminates to constrain the two segments and force them to vibrate with identical flexural deformations. This model was referred to as the 'constrained model' and was used to study both mid-plane and off-mid-plane delaminations in isotropic beams. Experimental verification of this model was conducted on beam specimens made by bonding identical thin isotropic strips together with a very thin adhesive layer. Good agreement with the experimental results confirmed that their constrained model was a simple and accurate linear model for studying the vibration behaviour of beams with delaminations and it was later extended for bi-material beams [Shu and Fan (1995)], sandwich beams [Hu and Hwu (1995); Shu (1995)] and fibre-reinforced composite beams [Shu and Della (2004); Shu and Della (2004); Grouve, Warnet, Boer et al. (2008)].

Tracy et al. [Tracy and Pardoen (1989)] proposed a similar constrained model by forcing the delaminated sub-laminates of a simply supported composite beam to have consistent deflections during vibration. However, their model was restricted to beams with only mid-plane delaminations. By verifying their model through experiments on a quasi-isotropic laminated beam, they found that the simplified analytical model (using the Euler-Bernoulli beam theory) was able to predict stiffness degradation in an anisotropic composite beam very well even though bending-extensional coupling was not included [Tracy and Pardoen (1988)]. This model has been employed by Okafor et al. [Okafor, Chandrashekhara and Jiang (1996)] and Watkins et al. [Watkins, Sanders, Akhavan et al. (2002)] to generate datasets for training neural networks for delamination detection.

However, the constrained model fails to explain the 'delamination breathing' found in experiments by Shen et al. [Shen and Grady (1992)] and Lestari et al. [Lestari and Hanagud (1999)]. Shen and Grady took photos of vibrating delaminated beams and found that, in certain cases, opening and closing of delamination occurred even in the first bending mode. Their mathematical models (Models A and B) essentially followed the constrained model of Mujumdar et al. [Mujumdar and Suryanarayan (1988)] and free model of Wang et al. [Wang and Liu (1982)], respectively. The discrepancy between the results predicted by the two models was significant even in cases in which the mode shapes did not show any opening in the delamination region. Furthermore, in cases in which opening delamination modes were observed in the experiment, the frequencies predicted by the constrained model were closer to the corresponding experimental results even though delamination was always forced to be closed in the model. Shen and Grady's experimental results have been widely used by later researchers to validate their theoretical and numerical models.

Luo et al. [Luo and Hanagud (2000)] stated that, to the best of their knowledge, there was no analytical model which could consistently explain the opening and closing behaviour of sub-laminates observed in experiments. Therefore, they tried to develop a model which could capture this experimental phenomenon without introducing contradictions by using a piecewise-linear spring model to simulate contact between delaminated surfaces. When a delaminated beam vibrated with a small amount of relative displacement between its sub-laminates, the spring model was simplified by using a linear spring model in which, for a real problem, the exact values of the spring constant needed to be determined by special experiments. Their proposed model included the free model (spring stiffness equal to zero) and constrained model (spring stiffness equal to infinity) as special cases, with shear and rotary inertia effects, as well as bending-extension coupling, included in the governing equations. By setting the values of the spring stiffness, K, to 0, 0.1 and ∞, they found that, for a case in which delamination did not open, both models converged to predict the same frequencies. This indicated that, when there was no opening in the delamination region, both models were essentially the same which differed from Shen and Grady's conclusion [Shen and Grady (1992)]. Additionally they found that, when a delamination was large and close to the beam surface, an opening existed and the vibration response of the delaminated beam was a non-linear combination of the vibrations from the free and constrained models.

They claimed that this non-linear response simulated by their model showed good agreement with experimental test data. Later, Lestari et al. [Lestari and Hanagud (1999)] presented a similar piecewise-linear spring model based on the Euler-Bernoulli theory for analysing multiple delaminations in woven composite beams.

Wang et al. [Wang and Tong (2002)] pointed out that, in the model developed by Luo et al. [Luo and Hanagud (2000)], as it was necessary to specify whether the sub-laminates were in contact beforehand in order to nominate the spring constant, it did not reflect the essential nature of the contact problem. They introduced a non-linear anti-inter-penetration model in the Timoshenko beam theory in which the possible contact force was brought about naturally as a function of the relative transverse displacements of the sub-laminates. It automatically gave a zero contact force for the opening mode and a contact force following a given function for the overlapping mode without artificially specifying in advance the status of the relative displacement. Since the contact forces caused non-linearity in the equations of motion, the model had to be solved using the finite difference method. However, the physically impossible inter-penetration of the crack faces was not always prevented in this model because no constraint to account for this phenomenon was provided.

Lee [Lee (2000)] tried to include interactions between the opening and closing modes by using a layerwise theory to analyse the free vibration of a laminated beam with a breathing delamination. He derived the equations of motion from Hamilton's principle, developed a finite element (FE) method for formulating the problem and compared the results with those of Wang et al. [Wang and Liu (1982)] which evidenced excellent agreement between the two theories. He also investigated delamination in a cantilever beam with a $[0/\theta]_s$ stacking sequence and found that the delamination occurring in the mid-plane had a greater effect on natural frequencies than those located between the 0 and θ layers and that, as θ increased, this effect increased.

All the constrained models mentioned so far were based mainly on simple engineering beams without considering other effects, such as shear deformation, rotary inertia, the Poisson's effect and friction between the contact surfaces. However, researchers have put in a great deal of effort to improve the accuracy of the models and investigate these effects. To study the effect of shear deformation, Chandrashekhara et al. [Chandrashekhara, Krishnamurthy and Roy (1990)] compared the frequencies of a composite beam obtained using shear deformation theory (SDT) with those from the classical laminate theory (CLT) and concluded that for long thin beams (in their case L/h=120, L-beam length, h-beam thickness), both theories gave the same results, indicating no significant effect of shear deformation. Shu et al. [Shu (1995); Shu and Fan (1995)] also pointed out that the four beam segments of a delaminated beam could be treated as classical Euler-Bernoulli beams provided that $L>>h$.

Valoor et al. [Valoor and Chandrashekhara (2000)] extended Tracy and Pardoen's constrained model to include shear deformation and the Poisson's effect for thick delaminated angle-ply beams due to the high ratios of their in-plane to transverse shear moduli. They studied thick beams with L/h=15 and found that including the Poisson's effect had little effect on the results for unidirectional or cross-ply beams but significantly affected those for angle-ply beams (a report in 2013 by Jafari-Talookolaei et al.

[Jafari-Talookolaei, Kargarnovin and Ahmadian (2013)] further pointed out that, especially for layout angles between 30° and 60°, considering Poisson's effect or not leads to significant deviation in the fundamental frequency in angle-ply laminate). They used their model to generate frequencies to train a back-propagation neural network to predict the location and size of delamination and found that prediction errors were highest for delaminations located near the beam end which they suspected was due to an insufficient number of training data. For symmetrical structures, their neural networks could predict the location of delamination only in each symmetrical segment while additional information, such as mode shapes, was required to differentiate other possible locations.

Qiao et al. [Qiao and Chen (2011)] pointed out that the previously mentioned theoretical models did not consider friction between sub-laminates and that delamination tips were modelled as rigid joints with the displacements of the delaminated sub-layers and intact substrates restricted to being identical in terms of their continuity conditions. This restriction obviously violated the real continuity conditions of the deformed delamination tips as delaminated sub-layers rotate (deform) differently at the delamination tip in the opening mode. Therefore, they proposed an improved dynamic analysis of delaminated bi-layer beams which included transverse shear and delamination tip deformations as well as the effects of friction and contact ahead of the delaminated tips. The piecewise-linear spring model and linear bridging model were employed to simulate the contact and friction behaviours, respectively, of the delaminated sub-layers. Comparisons with previous models and experimental results showed that their proposed model agreed well with existing analytical, experimental and numerical FE results which reflected that the effects of the contact, friction and delamination tip deformations considered in their model changed frequencies by only a small amount. The authors claimed that their model could be used with confidence for accurate evaluations of the dynamic responses of delaminated composite structures.

Other than the abovementioned models for single delamination in a composite laminate, considering that, in a real situation, there may be more than one delamination present, such as that due to a low-velocity impact by a foreign object, researchers have investigated multiple and overlapping delaminations [Tay (2003)]. Since 2004, Shu et al. [Shu and Della (2004)] have published an analytical study of vibrations of multiple delaminated beams, a vibration analysis of composite beams with double delaminations [Della and Shu (2005)], the vibration behaviours of beams with two non-overlapping delaminations [Shu and Della (2004)] and vibrations of delaminated multi-layer beams [Della and Shu (2006)]. In 2007, Della et al. [Della and Shu (2007)] presented a comprehensive review of the vibrations of delaminated composite laminates. Besides Della et al. [Della and Shu (2003)], Lee et al. [Lee, Park and Voyiadjis (2003)] proposed an analytical solution using the 'free mode' assumption for multiple delaminations in laminated beams with arbitrary lateral, longitudinal locations. Their frequency equations for multiple delaminated beams were obtained by dividing a global delaminated beam into beam segments and establishing a recurrence relationship from the continuity conditions on each sub-beam which were determined by assuming constant curvatures at delamination junctions. The results obtained from their analytical model for single delamination were compared with experimental data and, for multiple delaminations, FE test data were used for verification.

In summary, theoretical models of delaminated beams have evolved, in line with increasing understanding of the phenomenon captured in their vibration responses, from free to constrained to piecewise-linear spring models. The main difference between the free and constrained models is that, in the latter, a normal contact pressure is added between the sub-laminates to prevent inter-penetration. The piecewise model was developed mainly to simulate the non-linear opening and closing of delaminations during vibration. Of these three models, the constrained model has been widely used as the analysis tool for the inverse detection of delaminations since it can prevent the inter-penetration which occurs in the free model and is easier to implement unlike the piecewise model in which the frequencies are dependent on the spring stiffness, K, which is hard to determine. Based on these basic models, researchers have tried to improve accuracy by adding different effects, such as those of shear deformation, the Poisson's effect and friction [Jafari-Talookolaei, Kargarnovin and Ahmadian (2013)], which have been found to have some impact on the frequencies and mode shapes of delaminated beams. However, the extent of these effects on a beam's dynamic parameters depends on factors such as the beam dimensions, laminate layup, delamination characteristics and even mode number. Therefore, it is necessary to select a theoretical model suited to the specific problem being studied.

4 Finite element modelling of delamination

To use theoretical delaminated composite beam model for generating a database of frequency shifts against known delamination parameters has its limitations and restrictions, such as its neglect of shear deformation, Poisson's effect, and in particular, for the complex FRP structures, there is no theoretical model available. However, finite element analysis (FEA) is more versatile in this regard as, using the FEA software currently available, all the structural stiffness properties, including in-plane and extensional-bending couplings, can be computed and employed by selecting appropriate types of layered elements (unlike the beam theory in which only the effective axial stiffness is used). Furthermore, aspects such as shear deformation, the Poisson's effect and rotary inertia can be included in the analysis by choosing higher-order shell or solid elements for the FE model. In fact, even the influence of friction between sub-laminates in the delaminated region can be included in the FE model by choosing appropriate contact elements. Thus, finite element analysis has been widely used to model delaminations in composite laminates.

The current literature on FEAs of delaminated composites can be classified into two themes: One focusing on developing finite elements to model delaminations; and the other using commercial FE software, such as ANSYS or ABAQUS, to model composites with delaminations. Regardless of the way in which a FE model (FEM) of delaminations is constructed, modelling of delaminations in composite laminates must be performed efficiently for the purpose of generating database to be used in inverse algorithms. Therefore, this summary of the literature on numerical analyses of delaminated composites focuses on learning the strategies to model delaminations in FRP structures with an attention on model efficiency.

4.1 Development of finite elements

According to the literature, vibration problems in delaminated beams or plates have been analysed by numerous researchers using FEA based on the classical beam model [Babu and Hanagud (1990)], first-order shear deformation theory (FSDT) model [Zak, Krawczuk and Ostachowicz (2000)] and higher-order shear model based on either the layer-wise or simplified zig-zag theory [Cho and Kim (2001)]. The following summary emphasises the strategies employed for delamination modelling in the reported FEMs as well as how such a FEM is used after it is built, for instance, in a forward analysis of the vibration responses of delaminated structures.

One assumption for modelling delamination in a FEM is called the 'free model' which, like the theoretical model, has no contact elements added in the delamination interface or the nodes in the delaminated section are not constrained or coupled [Chakraborty and Mahapatra (2002)]. Zak et al. [Zak, Krawczuk and Ostachowicz (2000)] modelled an edge delamination (at the end of a beam) based on the FSDT using three beam elements connected at the tip of the delamination. The continuities of the transverse displacement, axial displacements and rotations were satisfied at the delamination junctions. However, as no contact between the elements was added in the delaminated region, it belonged to the 'free model' category. Krawczuk et al. [Krawczuk, Ostachowicz and Zak (1997)] modelled delamination in a composite plate in a similar manner and their FEM results were validated using experimental data from glass/epoxy composite beams and plates.

Gadelrab [Gadelrab (1996)] developed two types of elements for modelling the undamaged and delaminated regions in a composite beam. As, in the undelaminated element, all lamina had the same lateral and longitudinal displacements in a typical cross-section while, in the delaminated element, the lateral and axial displacements were only the same at both its ends (the rotation is different along the delaminated element length), this FEM basically followed the free model assumption. It was used to study the forward problem of delamination in E-glass/polyester laminated beams considering fixed-free, fixed-fixed and hinged-hinged boundary conditions.

Based on the third-order shear deformation theory (TSDT), Chattopadhyay et al. [Chattopadhyay, Radu and Dragomir (2000)] presented a FEM for examining the dynamic stability of composite plates with delaminations based on the free model assumption, with the continuity conditions at the delamination junctions modelled using the penalty function approach. The effects of delamination on the natural frequencies, critical buckling loads and instability regions of a composite plate subjected to dynamic loads were studied. Later, Radu et al. [Radu and Chattopadhyay (2002)] presented a similar FE model for investigating delamination in a composite plate subjected to dynamic compressive loads.

Ju et al. [Ju, Lee and Lee (1994)] developed a FE formulation based on the Mindlin plate theory to analyse the free vibrations of composite plates with multiple delaminations. In the delaminated segments, the sub-laminates were meshed separately by plate elements and the displacement continuities at the connecting boundaries obtained by matrix transformations of the stiffness and mass matrices. As, due to its complexity, their model did not consider the effects of either delamination growth or contacts in the delaminated segments, it was based on the free model assumption. It was used to investigate how the natural frequencies and mode shapes of a delaminated plate are affected by various sizes, locations and numbers of delaminations with different boundary conditions. Although it was found that decreases in the natural frequencies had an interesting relationship with the distribution of modal strain energies, this FEM was not validated by the experimental results.

By using a FEM based on the three-dimensional (3D) theory of linear elasticity, Tenek et al. [Tenek, Henneke and Gunzburger (1993)] studied the impact of delaminations on the natural frequencies of composite plates as well as delamination dynamics over a broad frequency range. Two sets of nodes were placed at the interfaces between layers, one in the upper and the other in the lower layer. In the integral regions, these nodes were constrained to vibrate together by imposing the continuities of the displacements while, in the delamination region, they were allowed to vibrate independently (a free model).

Shiau et al. [Shiau and Hwang (2007); Shiau and Zeng (2010)] studied the effect of delamination on the natural frequencies and mode shapes of a simply supported rectangular symmetric composite plate with a through-width delamination using the finite strip method. No contact was added between the delaminated segments and, to avoid contact/impact behaviour, only mid-plane delamination was considered in their FEM. The effects of the delamination length, location, fibre angle, stacking sequence and aspect ratio of the laminate on the natural frequencies were studied, with those of all factors found to be influential; in particular, the aspect ratio had a significant effect on the second mode frequency of the cross-ply laminate.

Since a free model allows the nodes in the delaminated region to vibrate freely, the two sub-laminates possibly penetrate each other. Therefore, the 'constrained model', which has contact added or ties the nodes in the delaminated region to have the same flexural displacement, was developed later by researchers. As mentioned above, Zak et al. [Zak, Krawczuk and Ostachowicz (1999)] improved the free FE model in Krawczuk et al. [Krawczuk, Ostachowicz and Zak (1997)] by including contact forces between the delaminated layers which helped to meet the compatibility conditions of the vertical displacements of the contacting nodes using a penalty parameter approach. The experiments conducted verified the numerical results from their FEM.

Using the FSDT, Ju et al. [Ju, Lee and Lee (1994)] presented a FEM for composite beams with multiple delaminations discretised using three-noded isoparametric beam elements, with both the free and constrained models presented. For the free model, all the nodal degrees of freedom in the delamination region, except those connecting nodes at the delamination junctions, were independent whereas, for the constrained model, the transverse nodal deflections of the elements in the delamination region were equal.

Hu et al. [Hu, Fukunaga, Kameyama et al. (2002)] used a higher-order plate theory to study the vibration responses of moderately thick delaminated composite plates, with the continuity conditions at the delamination junction satisfied using the penalty function method and the natural frequencies determined using the assumptions of both the free and constrained models. For the constrained model, high-stiffness springs were placed between the node pairs of the two delaminated layers so that the upper and lower delaminated layers vibrated together without opening. They used this model to analyse the influence of delamination on the vibration properties of composite laminates, especially curvature of the vibration mode. It was found that there were obvious sudden changes in the curvature if the delamination size was relatively large, with the most significant observed when the delamination was located in the mid-plane of the laminates. The vibration data from the model was extensively compared with results from previous FEMs and experiments.

A 3D FEM of delaminated fibre-reinforced composites for analysing the dynamics of multi-layer composite plates with internal delaminations was proposed by Yam et al. [Yam, Wei, Cheng et al. (2004)]. The FE was an eight-node rectangular thin plate element with three degrees of freedom at each node, the element thickness the same as that of the corresponding individual lamina, with the displacements and their variations in each pair of coincident nodes of two adjacent laminates assumed to be equal, except in the delamination region, and virtual elements adopted in the region of delamination to prevent element penetration. The natural frequencies, modal displacements and modal strains of samples with different delamination dimensions were analysed, with the results from the FEM showing good agreement with the available experimental data and improved accuracy.

Apart from the FEMs developed by dividing the delaminated region into two sub-laminates, a FEM using the layer-wise theory has also been reported [Barbero and Reddy (1991)]. Their basic assumption was that the same displacement distributions in individual layers are capable of representing the displacement discontinuities at the interfaces between the layers, with delamination modelled by the jump discontinuity conditions at the interface. This layer-wise model was claimed to be able to accurately predict distributions of strain energy release rates along the boundaries of delaminations of arbitrary shapes as well as multiple delaminations through the thickness of a plate. However, its computational cost makes it relatively unattractive for predicting global behaviour.

Cho et al. [Cho and Kim (2001)] proposed a third-order zig-zag theory for analysing the static and dynamic behaviours of composite plates with multiple delaminations because of its accuracy and efficiency in a ply-level analysis. This theory requires transverse shear stress continuities at the layer interfaces and traction-free boundary conditions at the top and bottom faces of the laminates, with delamination modelled by a very thin layer with compliant material stiffness and placed between the laminae. This method is called the compliant layer concept or embedded layer approach and is valid for modelling delaminations provided they remain closed. By comparing its results with reported frequencies, this model showed its accuracy in predicting frequencies. However, in the case of a thick plate with large delaminations, its predictions of buckling loads were overestimated compared with the elasticity solutions.

In summary, there are three main strategies for modelling delamination, the ‘free model’, ‘constrained model’ and ‘layer-wise theory’, with a large proportion of FEMs based on the free model assumption. Although FEMs have been used extensively to study buckling behaviour under dynamic loads and vibration responses, such as the frequencies and mode shapes of delaminated structures, only some have been validated using experimental test data.

4.2 Use of commercial FE softwares

Compared with developing FEs, it is quite straightforward to use commercial FE software, such as ABAQUS and ANSYS, to model delaminated composites. The element types adopted in those commercial softwares and the strategies used to model delaminations are extracted.

Islam et al. [Islam and Craig (1994)] used ABAQUS to model delaminated beams to verify their theoretical model. Two solid beams were connected (one at the top and the other at the bottom) by rigid elements in the undamaged region and gap elements in the delaminated region. The authors agreed that their models did not truly represent a delaminated section because they did not account for the non-linearities due to interferences of the top and bottom parts of delamination but claimed that they were adequate for predicting the shift in natural frequencies of the beam with a mid-plane delamination.

Rajendran et al. [Rajendran and Song (1998)] and Lee et al. [Lee, Park and Voyiadjis (2003)] used SHELL 99 elements in ANSYS, 3D eight-node layered elements to model composite laminates. Both models were built under the free model assumption, i.e. for the undelaminated region, the nodes from the top and bottom of the two separately modelled sub-laminates were constrained together using the *Coupling/Constraint Equation* while the nodes in the delaminated region were left free.

Chakraborty [Chakraborty (2005)] modelled a $[0]_{20}$ square graphite/epoxy plate with an embedded delamination at the interface layer in ANSYS 5.4A, employing SOLID46, an eight-node isoparametric element, with orthotropic material properties for the analysis. Two sub-laminates were modelled individually and the interface between them modelled with a thin resin rich layer. In the delaminated region, the corresponding nodes of the top and bottom sub-laminates were connected by contact elements (CONTAC52 in ANSYS5.4A) to simulate the contact force between the sub-laminates, and the first ten natural frequencies extracted using the sub-space iteration method for modal analysis.

Grouve et al. [Grouve, Warnet, Boer et al. (2008)] used the 2D structural element PLANE42 in ANSYS to model delamination in cantilever fibre-reinforced composite beams with contact elements added between the sub-laminates to permit frictionless sliding but not separation. Modal analysis was employed to extract the resonant frequencies which were compared with those obtained from their analytical model.

Orłowska et al. [Orłowska, Kołakowski and Holnicki-Szulc (2008); Orlowska and Kolakowski (2009)] used eight-node 3D layered elements in ANSYS to model an eight-layer graphite-epoxy cantilever composite beam with delamination. Damping and friction were neglected but contact pairs added (CONTA171 and the associated TARGE169) to prevent penetration of the sub-laminates. The coefficients of the normal and sliding contact stiffness factors (FKN and FKT, respectively) were set to 0.1 and 1.0, respectively, in their model.

Montalvão et al. [Montalvao, Ribeiro and Duarte-Silva (2009)] used a total of 1750 SHELL 63 elements, a 2D four-node quadrilateral linear structural element, to model a CFRP rectangular plate with six delaminations, each of which covered 16 elements, approximately 1% of the total plate area. However, no details of how the delaminations were modelled were provided.

Besides the common way of modelling a delaminated region as two beam segments, one at the top and the other at the bottom, there are alternative techniques, such as reducing either the beam thickness [Sahin and Shenoi (2003)] or the Young's modulus at the particular element's damage location [Bayissa, Haritos and Thelandersson (2008)].

It is to be noted that, in the literature, quite a few papers have reported using a cohesive zone or interface element to simulate the delamination interface between sub-laminates [Bruno, Greco and Lonetti (2005); Iannucci (2006); Harper and Hallett (2008)]. However, they have focused mainly on the fracture mechanics or propagation of delamination rather than the dynamic responses of structures with existing delaminations. Thus, they are not elaborated.

In summary, the reported FEA of delaminated composite structures have used the assumptions of the 'free model', 'constrained model' and 'layer-wise theory', following the same lines as those of theoretical models. Of the three, the free model appeared more frequently than the others probably due to its simplicity although it could not truly represent real delamination.

5 Modal testing on delaminated composites

The assessment of delaminations in composite beams and plates using the inverse algorithms for damage detection requires a database of measured frequency shifts against known delamination parameters generated from theoretical or FE models. As the accuracy of this database is critical, experimental testing of composite laminates with delaminations is required to validate and verify the results obtained from the numerical models. This section provides a review of the reported experimental work of measuring the vibration response of composite laminates with delamination.

Experimental modal analyses (EMA) for measuring the natural frequencies of composite beams and plates with delaminations have previously been conducted by a number of researchers. The majority of these studies were conducted to validate theoretical [Ramkumar, Kulkarni and Pipes (1979); Lin, Ni and Adams (1983); Tracy and Pardoen (1988); Shen and Grady (1992); Keilers and Chang (1995); Saravanos and Hopkins (1996); Chrysochoidis and Saravanos (2004); Grouve, Warnet, Boer et al. (2008)] or numerical models [Campanelli and Engblom (1995); Zak, Krawczuk and Ostachowicz (1999); Zak, Krawczuk and Ostachowicz (2000); Kessler, Spearing and Atalla (2002); Orlowska and Kolakowski (2009); Ullah and Sinha (2011)] of the forward problem (predictions of the natural frequencies of laminates with known delaminations), with some performed to validate solutions to the inverse problem, i.e. techniques for assessing delamination parameters [Islam and Craig (1994); Keilers and Chang (1995); Luo and Hanagud (1995); Okafor, Chandrashekhara and Jiang (1996); Harrison and Butler (2001); Su, Ling, Zhou et al. (2005); Ullah and Sinha (2011)]. A literature survey of the experimental studies reported by previous researchers was conducted mainly to gain knowledge and understanding of their manufacturing methods, including details of the materials and layups used interfaces at which delaminations were introduced, methods of simulating delaminations and boundary conditions, and techniques for modal testing. Tab. 1 summarise the findings from this survey for experimental modal analyses on composite laminates.

Table 1: Summary of experimental modal analyses of delaminated composites by previous researchers

Reference	Composite material	Layup	Delamination interface	Method used to simulate delamination	Boundary conditions
[Ramkumar, Kulkarni and Pipes (1979)]	AS/3501 graphite/epoxy	$[0_4/\pm45_2]_{2s}$	-	Collapsed Teflon tube	Cantilever beams
[Tracy and Pardoen (1988)]	AS4/3501-6 graphite/epoxy prepreg	$[90/\pm45/0]_{2s}$	Mid-plane, centred	Insertion of fluorinated ethylene propylene tape	Simply supported beams
[Shen and Grady (1992)]	T300/934 graphite/epoxy prepreg	$[0/90]_{2s}$	Interfaces 1 to 4	Teflon strip insert	Cantilever beams
[Islam and Craig (1994)]	3M Kevlar epoxy prepreg	$[0/90/90/0]_s$	Mid-plane	FEP insert	Cantilever beams
[Keilers and Chang (1995); Keilers and Chang (1995)]	T300/976graphite/epoxy	$[0]_{12}$	Mid-plane	Teflon insert	Cantilever beams
[Saravanos and Hopkins (1996)]	T300/934 graphite/epoxy	$[0/90/\pm45]_s$ $[\pm45/90/0]_s$	Mid-plane	Teflon insert	Cantilever beams
[Okafor, Chandrashekhara and Jiang (1996)]	Glass/epoxy	$[0/90/90/0]_s$	Mid-plane	Teflon insert	Cantilever beams
[Harrison and Butler (2001)]	T300/913C	[0/90]2s $[0/\pm45/0]_s$	Mid-plane and 2/6 interface*	Insertion of thin strip of PTFE film	Cantilever beams
[Kessler, Spearing and Atalla (2002)]	Graphite/epoxy	$[90/\pm45/0]_s$	Mid-plane, centred	1. Slot created with thin blade on one side 2. Teflon insert	Cantilever beams
[Chrysochoidis and Saravanos (2004)]	T300/934 graphite/epoxy plies (fibre to volume ratio between 0.57 and 0.63)	$[0/90/\pm45]_s$	Mid-plane	Teflon tape insert	Free-free beams supported by strings
[Su, Ling, Zhou et al. (2005)]	Glass/epoxy	10-ply weave fabric	Mid-plane and 2/8 interface	Insertion of Teflon film	Cantilever beams
[Grouve, Warnet, Boer et al. (2008)]	Carbon/poly-etherimide	$[90/0/90/0_2/90/0/90]_s$	Mid-plane	Insertion of thin polyimide foil	Cantilever beams
[Zak, Krawczuk and Ostachowicz (2000)]	Glass/epoxy (fibre volume fraction 0.27-0.31)	$[\pm45]_s$	Mid-plane	Insertion of thin Teflon film	Cantilever beams
[Orlowska and Kolakowski (2009)]	Glass/epoxy	-	Mid-plane	-	Cantilever beams
[Luo and	E-glass/epoxy	0/90 woven cloth	(1) 6/10	Teflon film	Cantilever

Hanagud (1995)]			interface in 16-layer woven cloth (2) 2/10, 5/7 interface in 12-layer woven cloth	insert	plate (CFFF)
[Campanelli and Engblom (1995)]	AS4/Peek	$[0/90]_{6s}$	Mid-plane: mid-edge and corner delaminations	Insertion of foil	All sides simply supported plate (SSSS)
[Zak, Krawczuk and Ostachowicz (1999)]	Carbon/epoxy (volume fraction of carbon fibre 0.3)	-	Mid-plane	Insertion of thin Teflon film	All sides clamped (CCCC)
[Zak, Krawczuk and Ostachowicz (2000)]	Glass/epoxy (fibre volume fraction 0.27-0.31)	$[\pm 45]_s$	Mid-plane	Insertion of thin Teflon film	Cantilever plates
[Ullah and Sinha (2011); Ullah and Sinha (2011)]	E-glass/epoxy	$[0/90/0/90]_s$	3/5 interface	-	FFFF plates

*Interface a/b indicates laminate with total of a+b plies and delamination located at interface between ath and (a+1)th plies

Please noted that publications which dealt with only experimental work on undamaged composite laminates and those which did not provide details of their experimental procedures are not included in Tab. 1. As can be seen, the most common materials used to measure vibrations of delaminated structures have been carbon and glass fibre-reinforced epoxy composites. Many studies have employed 0 and/or 90 degree fibre reinforcements (including woven fabrics which may be considered 0/90 degree combinations). Quasi-isotropic layups with ±45° reinforcements have also been studied as typical laminates although not as frequently as 0/90 degree combinations. Regarding transverse locations of delamination, experimental research has focused mainly on those in the mid-plane, possibly because they cause the largest shifts in frequencies. As the basic principle for introducing artificial delaminations into specimens is to prevent the prepreg lamina at the selected interface from bonding during curing, either Teflon (the most common) or other films such as FEP have been employed to separate the sub-laminates. For modal testing, the favoured boundary condition appears to be clamp-free which may be because others (except the free-free) are not easy to implement experimentally [Zak, Krawczuk and Ostachowicz (2000)]. The experimental modal analysis consists mainly of two parts, exciting vibrations and recording the time-domain data for its conversion by the fast Fourier transform (FFT) to the frequency response function in order to extract the natural frequencies. The majority of previous studies have employed impulse excitation using a hammer [Shen and Grady (1992); Islam and Craig (1994); Luo and Hanagud (1995); Zak, Krawczuk and Ostachowicz (1999); Zak, Krawczuk and Ostachowicz (2000); Harrison and Butler (2001); Grouve, Warnet, Boer et al. (2008)] or shaker [Ramkumar, Kulkarni and Pipes (1979); Su, Ling, Zhou et al. (2005)], others piezoelectric actuators [Keilers and Chang (1995); Okafor, Chandrashekhara and Jiang (1996); Kessler, Spearing and Atalla (2002); Chrysochoidis and Saravanos (2004); Orlowska and Kolakowski (2009);

Ullah and Sinha (2011)] and the transient force generated by an electromagnetic coil connected to a metallic chip with negligible mass adhering to the free end of a cantilever beam [Saravanos and Hopkins (1996)]. The acquisition of time-domain data for measuring displacement or acceleration has been achieved mainly by local sensors (accelerometers and PZT sensors) while Shen and Grady used strain gauges to measure flexural strain [Shen and Grady (1992)]. Only a few studies have employed a laser vibrometer to measure the vibration responses of composite specimens [Aryan, Kotousov, Ng et al. (2017); Zhang, Zhan, Shankar et al. (2017)].

Besides the studies summarised in Tab. 1, the literature also provides good information regarding practical matters to be taken into consideration during experimentation. Ramkumar et al. [Ramkumar, Kulkarni and Pipes (1979)] checked the magnified view (x400) of the implanted inter-laminar defect and Campanelli et al. [Campanelli and Engblom (1995)] used infrared thermography to verify the locations and sizes of delaminations. These are good examples which suggest that the actual size and location of a delamination should be double-checked using means other than simply taking the location and area of the inserted Teflon film as those of the delamination.

Experiments are not always perfect and some researchers have pointed out problems in conducting them. Islam et al. [Islam and Craig (1994)] found that their first batch of samples was warped due to thermal stress during the curing process. One of Lin et al.'s [Lin, Ni and Adams (1983)] plates had a lower quality than the others due to some fibre 'washing' during its final pressing which gave slightly different mode shapes from those predicted. Campanelli et al. [Campanelli and Engblom (1995)] performed experiments to verify the validity of their FE model but the measured data was limited and agreement with the numerical results not good. They mentioned that possible reasons for this mismatch could have been small variations in the sizes and thicknesses of the plates due to the manufacturing process and the use of cooling water instead of slower air cooling during manufacturing which could have caused the plates to warp. They suggested that a possible solution to differences due to manufacture could be to first test a plate with no delamination and then retest the same plate after inducing a delamination in it, although they did not indicate how a delamination could be induced in a plate after it had been cured. The last possibility for the mismatch between experimental and numerical results that Campanelli et al. [Campanelli and Engblom (1995)] identified was that the fixture designed to support the plate may not have achieved the exact simply supported conditions as assumed in the numerical model. Maio et al. [Maio, Ricci, Memmolo et al. (2018)] conducted uncertainty quantification in the velocity calculation of laser Doppler vibrometer by studying the relationship between factors such as the distance between sensors as well as the sampling frequency and the error on the velocity estimation. Luo et al. [Luo and Hanagud (1995)] pointed out that differences in conditions during the layup process and in the curing environment may also cause variations in the dynamic characteristics of specimens.

In summary, while experimental modal testing is widely accepted as a good method for validating numerical models and delamination detection techniques, the majority of experimental work in this area has concentrated on laminates with 0/90 reinforcements, cantilever boundary conditions and mid-plane delaminations. More lay-ups other than 0/90 combinations which are commonly taken in application, and delaminations in other interfaces instead of in the mid-plane, design of fixtures for all kinds of boundary conditions, and use of non-contact modal testing machine are encouraged in the future research.

6 Inverse algorithms for delamination assessment

The occurrence of damage causes a reduction in stiffness which will subsequently reduce the natural frequencies of the structure, so the incidence of damage can be detected simply by monitoring the natural frequencies. An appreciable permanent decrease in the values of the structures natural frequencies indicates the occurrence of damage [Salawu (1997); He and Zhu (2011)]. However, while a shift in frequency easily identifies the presence of damage, the determination of the location as well as the severity of the damage is not easy to accomplish. This requires the solution to the inverse problem, viz., identifying the parameters indicating the location and severity of damage from measured changes in frequencies. The highly non-linear nature of the relationship between the vibration responses and the damage parameters makes the inverse problem difficult to solve and often resort to some artificial intelligence algorithms such as least square identification, artificial neural networks (ANN), genetic algorithms (GA) and fuzzy logic etc. The least square identification is a gradient based searching method which may get stuck in local minimum. If fuzzy logic is used, fuzzified output value for the damage are generally obtained, for instance, the damage size was not obtained quantitatively but qualitatively. Therefore, in the literatures, most work is reported to have used ANN or GA to detect the damage from the shifts in frequencies. However, when only two or three parameters are to be determined, such as location along the composite beam length, the depth at which delamination occurs and the damage size, it can be easily determined using a graphical technique, which used to be employed for estimating location and size of cracks in isotropic (metallic) beams. In this section, the application of graphical technique (GT) in detecting cracks in isotropic beams, the delamination detection in composite laminates using ANN and optimisation method are reviewed and summarised.

6.1 Graphical technique

The graphical technique (GT) was first proposed by Adams et al. [Adams and Cawley (1978)] who employed it to estimate the size and location of a crack in a bar and then modelled crack in an isotropic bar as a linear spring (with its stiffness characterising the crack severity) to predict shifts in the axial frequencies of the bar due to the crack [Adams and Cawley (1978); Cawley and Adams (1979)]. They showed that, if the crack size index versus crack location for constant measured values of the first three modes of the frequency shifts were plotted, the curves all intersected at a common point and provided predictions of the damage size and location corresponding to the measured frequency shifts. Fig. 3 shows the intersection points indicating possible damage sites (*x* axis is crack in the lengthwise location) and predicted damage sizes (*y* axis is damage index with the spring stiffness, *K*, related to the crack severity) obtained by plotting the location-severity curves of the first three modes together. As pointed out by the authors, there are two possible damage sites are indicated due to the symmetric of bar. If the structures are asymmetrical, the use of any two modes will give two or more possible damage sites, while the use of additional modes will reduce this.

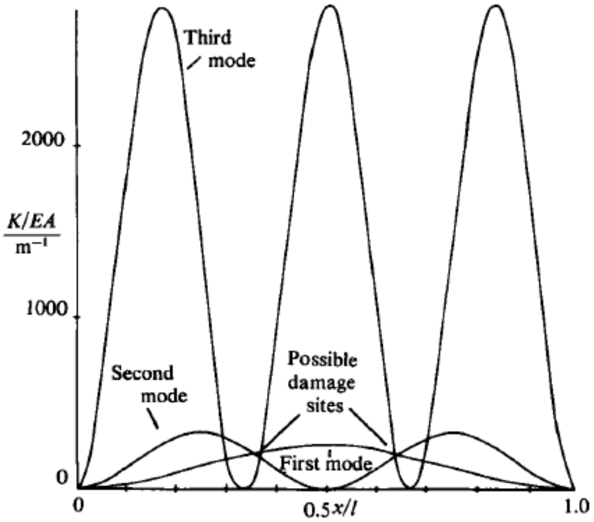

Figure 3: Damage locations in straight bar using graphical method in Adams et al. [Adams and Cawley (1978)]

Since Adams and Cawley, many researchers have employed similar graphical techniques to solve the inverse problem. Springer et al. [Springer, Lawrence and Lawley (1988)] used one to predict damage locations and sizes through shifts of the first three longitudinal natural frequencies. Due to the symmetric nature of their free-free beam, they plotted normalised locations up to only 50% of the whole beam length. Boltezar et al. [Boltezar, Strancar and Kuhelj (1998)] extended the graphical methods proposed by Adams et al. [Adams and Cawley (1978)] from axial to flexural vibrations to identify the open transverse crack location of a uniform free-free steel beam. Based on the assumption that the stiffness of the equivalent spring which models the crack must be the same for all the vibrating modes of flexural natural vibrations, by plotting the relative spring stiffness along the length of the beam for distinct natural frequencies (the first six modes in their case), the crack site was identified by the intersection of the curves. Owolabi et al. [Owolabi, Swamidas and Seshadri (2003)] plotted the contour lines, which have the same normalized frequency change resulting from a combination of different crack depths and locations, together in a plane with crack location and crack depth as its axes. The two unknowns, the location and size of the crack, were successfully assessed using the measured changes in the first three natural frequencies. However, as pointed out by the authors, more modes are required if there are more damage parameters to be determined since the function should have solutions one order higher than the number of unknowns. Similarly, Nahvi et al. [Nahvi and Jabbari (2005)] predicted the open crack in a cantilever beam from the intersection of the contour lines of normalised frequencies in terms of the normalised crack depth and location. In the case of a crack located in the vibration node, they suggested using higher modes to correctly predict it. The prediction accuracy of their graphical technique was claimed to be no more than a 1% error for medium-sized cracks.

Other researchers have reported using graphical methods to detect damage [Liang (1992), Liang, Hu and Choy (1992); Lee and Chung (2000); Chinchalkar (2001); Lele and Maiti (2002); Kim, Ryu and Cho (2003); Kim and Stubbs (2003); Patil and Maiti (2005); Kannappan, Shankar and Sreenatha (2007); Kshirsagar and Bhuyar (2011)], the common procedure of which can be summarised as plotting the curves representing the possible combinations of crack sizes and locations together in a size-location plane to obtain the intersection points at which the coordinates indicate possible damage parameters. However, there are some differences among applications of the graphical method. One is in the number of modes employed: Chinchalkar [Chinchalkar (2001)] and Liang et al. [Liang (1992)] used the lowest three natural frequencies and Kim et al. [Kim and Stubbs (2003)] the first four natural frequencies while Boltezar et al. [Boltezar, Strancar and Kuhelj (1998)] used frequency changes in the first six modes. At least three modes are required to uniquely identify the two unknown damage parameters since any two curves will usually have multiple intersection points and, in general, reliability increases as the number of modes employed increases although not always prediction accuracy. The second difference between the studies is in the data sources employed to plot the contours of frequency changes versus crack sizes and locations. Some authors have used a theoretical model in which the crack was modelled by a rotational spring [Adams and Cawley (1978); Liang (1992); Boltezar, Strancar and Kuhelj (1998); Lele and Maiti (2002)] while others have used finite element analysis (FEA) to model a damaged beam [Nahvi and Jabbari (2005)]. A third difference is in the parameter used to represent crack severity along the y axis (in almost all cases, the x axis is the x location of the damage). Authors have used diverse styles to represent crack severity, with some simply using the crack size itself and others a damage index parameter derived from the theoretical model; for instance, if a crack is modelled as a spring, the damage index is generally related to the spring's stiffness.

All applications of the graphical technique reported above have been for assessments of surface cracks in isotropic beams which require predictions of only two variables, the axial location of the crack and its size-either the depths (in the thickness direction) of through-width cracks or transverse sizes (in the width direction) of through-thickness cracks. Zhang et al. [Zhang, Shankar, Tahtali et al. (2016)] for the first time, have reported applying the graphical technique to assess three parameters of delamination in composite beams (the additional parameter is the interface location of the delamination). Validation of the graphical technique with numerically simulated data showed that the method can identify all three parameters accurately. However, the experimental validations of the delamination detection technique, with measurements on simply supported and cantilever beams, show that, in practice, the delamination interface is hard to predict correctly, but the x-location and size can be predicted mostly with errors of less than 5% even when the interface has not been correctly predicted. Zhang et al.'s [Zhang, Shankar, Tahtali et al. (2016)] work has proved the possibility to employ the graphical technique to for delamination identification in composites.

6.2 Techniques based on artificial neural networks

Inspired by biological neurons, artificial neural networks (ANNs) are computational systems which mimic the microstructures of biological nervous systems [Bishop (1994); Fausett (2006)]. The most basic element of the human brain is the neuron, each of which can connect with up to 200,000 other neurons, and the power of the brain comes from the numbers of them and their multiple connections. Basically, a biological neuron receives inputs from other sources, combines them in some way, performs a generally nonlinear operation and then outputs the final result.

Similar to the biological NN, an ANN consists of a potentially large number of simple processing elements known as nodes or neurons, each of which computes a nonlinear weighted sum of its inputs and transmits the result over its outgoing connections to other neurons. Therefore, the behaviour of the network, which consists of several layers of neurons, namely an input layer, hidden layer or layers and output layer, depends largely on interactions among its neurons. The input layer takes the input data and distributes them to the hidden layer(s), the information in which cannot be seen by the user, which undertakes all the necessary computations and transmits the final results to the output layer for the user. It is reported that a maximum of four layers (one input, two hidden and one output) is sufficient to solve problems of any complexity [Heaton (2005)]. During the training phase, the network is presented with many cases of inputs and true outputs, and learns by adjusting the weights (initially assigned randomly) of the relative impacts of the inputs to outputs through trying many combinations of weights until a good fit to the training cases is obtained. Then, the resulting network can be used to evaluate future cases with new given situations of interest.

The greatest advantage of an ANN is its ability to model complex non-linear, multi-dimensional functional relationships without any prior assumptions about their natures. However, its principal disadvantage is the need to provide a suitable set of example data for network training and the potential problems which can arise if a network is required to extrapolate to new regions of the input space significantly different from those corresponding to the training data [Zhang and Friedrich (2003)]. In general, ANNs are particularly applicable to problems for which a significant database of information is available but it is difficult to specify an explicit algorithm. It has been successfully implemented in many applications, e.g., speech recognition, diagnosis of hepatitis, image recognition, etc. In particular, its capability to detect failures and damage in structures has been comprehensively reported [Wu, Ghaboussi and Garrett (1992); Tsou and Shen (1994); Addin, Sapuan, Mahdi et al. (2006); Budipriyanto, Haddara and Swamidas (2007); Odejobi and Umoru (2009); Albuquerque, Tavares and Durão (2010); Dackermann (2010)]. Particular applications of ANN to predictions of delamination damage in composites are summarised below.

Rhim et al. [Rhim and Lee (1995)] tried to detect damage in composite structures and identify its characteristics using ANN in conjunction with system identification. Transfer functions were extracted as the features by system identification and fed into a multi-layer perceptron as input patterns for training. The multi-layer perceptron served as a nearest neighbourhood classifier in which patterns were classified according to the location and severity of the damage. Although their numerical validation demonstrated the feasibility of using ANN to identify damage, their proposed method could only provide an overall classification of it, i.e. large damage near the root or small damage in the mid-span.

Zheng et al. [Zheng, Wang and Liu (2005)] combined computational mechanics and ANNs to predict delamination locations and sizes in composite beams. They performed a FEA to generate the first five modal frequencies of a beam to train a feed-forward back-propagation NN (BPNN) with two outputs, delamination size and lengthwise location. A piezoelectric sensor was attached to the top of the composite beam to measure its modal frequencies. Their experimental validation demonstrated that the proposed ANN could predict delamination locations and sizes with errors of less than 7% and 5%, respectively.

In a study by Rosales et al. [Rosales, Filipich and Buezas (2009)], ANN was applied to crack detection in cantilever beams with transverse cracks, with its training data set generated from finite element models (FEMs) with different crack depths and locations. It was found that using one network for both parameters provided adequate results but that better results were obtained when a separate ANN was used for the prediction of each parameter.

Valoor et al. [Valoor and Chandrashekhara (2000)] built a thick beam model to investigate the vibration responses of composite beams which accounted for the Poisson's effect and transverse shear deformation. Modal frequencies (the first five modes for thin beams and four for thick beams) obtained from the developed model were used to train a BPNN to predict a delamination's size and lengthwise location. Three boundary conditions were considered in the numerical validation: cantilever, simply supported and both ends clamped. The results showed that the errors were highest for delaminations near the beam ends which might have been due to an insufficient number of training data for these delamination locations. For symmetrical structures, to differentiate between possible locations, more information is required. While the authors considered only mid-plane delaminations, they suggested that detection of interface would be included in future work.

Islam et al. [Islam and Craig (1994)] trained a BPNN using the first five modes of frequencies for detecting damage in composite beams, with modal testing carried out using piezoceramic transducers as both sensors and actuators on the beams. The theoretical model used to generate training data sets was based on Tracy and Pardoen's constrained model [Tracy and Pardoen (1988)] and extended to include unsymmetric laminates with piezoceramic patches. The trained ANN predicted the axial locations and sizes of mid-plane delaminations from measured frequencies with maximum errors of 27% and 10%, respectively.

Okafor et al. [Okafor, Chandrashekhara and Jiang (1996)] trained BPNNs to predict only delamination sizes in composites using data obtained from the theoretical model of delaminated beams developed by Tracy and Pardoen [Tracy and Pardoen (1988)] which is restricted to mid-plane delaminations. The learning and momentum rates were studied and it was reported that they significantly affected the training of ANN as well as its subsequent performance. Their ANNs were tested numerically and experimentally by inputting the first four normalised frequencies of eight-layer ([0/90/90/0]) glass/epoxy beams with mid-plane delaminations of varying sizes. Although it was found that ANNs trained by the theoretical data performed well in predicting delamination size, interface and lengthwise locations of delamination were not assessed.

Watkins et al. [Watkins, Sanders, Akhavan et al. (2002)] trained a feed-forward BPNN using the first five modal frequencies as inputs to predict the lengthwise locations and sizes of delaminations in beams. Experimental verification using frequency measurements on eight-layer $[0/90]_{2s}$ glass/epoxy beams showed that predictions of location were generally less accurate for delaminations closer to the clamped end of a beam.

Su et al. [Su, Ling, Zhou et al. (2005)] evaluated the efficiencies of a genetic algorithm (GA) and ANN for quantitative damage identification. Both methods were validated using the measured frequencies obtained for glass fibre-reinforced epoxy composite beams from embedded fibre Bragg grating sensors, with the theoretical model proposed by Mujumdar et al. [Mujumdar and Suryanarayan (1988)] used to generate frequency shift data for training the ANN. Based on numerical and experimental validations, although both the GA and ANN were reported to provide quantitative evaluations of the location, size and interlaminar position of delamination, they predicted the interface as a continuous rather than discrete variable despite the fact that, in reality, delaminations appear only in interfaces between the layers. Furthermore, directly applying a GA for delamination detection is time consuming since the FEA has to be performed during each cycle of the iterative evaluation of the objective function.

All the above applications of ANN were for assessments of delaminations in composite beams while there have been only a few for those in composite plates which require the determination of more variables.

Chakraborty [Chakraborty (2005)] ran hundreds of FEMs of square composite plates with different sizes and shapes (with delaminations simulated to be elliptic geometries with different eccentricities) and locations of delaminations to generate the first ten modal frequencies which were used to train a BPNN with three layers (input, hidden and output). The network was tested to predict the presence of delamination along with its size shape, and location. It was observed that it could effectively learn and reasonably well predict the sizes, shapes and locations of delaminations embedded in the laminate when tested using unknown data sets. However, the limitations of Chakraborty's study include: 1) As the plate was square, the results could not be generalised to plates with other aspect ratios; 2) Delaminations were located at the mid-plane of a twenty-layer graphite/epoxy plate while predictions of delaminations at other interfaces were not attempted; 3) As the plates were orthotropic, with all their CFRP layers having reinforcements in the zero degree direction, the findings could not be generalised to plates with layers reinforced in different orientations and with unsymmetric and/or unbalanced sub-laminates; and 4) Validation was performed using only numerically simulated not experimental data.

Watkins et al. [Watkins, Akhavan, Dua et al. (2007)] used the strain profiles (which are associated with impact damage, such as matrix cracking, fibre breakage and delamination) as inputs to train a BPNN to detect and classify the type and severity of damage in a composite plate. The training data was generated using a finite element code with the NN toolbox in Matlab used to perform the NN analysis. The outputs were author-defined binary vectors with three elements related to types and severity of damage. Although the ANN was validated for classifying damage using the measured strain signatures, the authors admitted that a practical system would need to know the impact location or provide additional inputs to the ANN to accommodate this parameter.

Zhang et al. [Zhang, He, Liu et al. (2018)] have trained a BPNN to predict five parameters of delamination (two locations, two sizes, and one interface) in FRP plate using 12 modes of frequency shifts. ANN was validated using numerical data from FE simulations and the predicted delamination was with good accuracy. However, experimental validation conducted on both cantilever (Clamped-Free-Free-Free, CFFF) and free boundary condition (FFFF) of carbon/epoxy and glass/epoxy composite panel specimens showed that ANN failed to predict delaminations in FRP specimens with acceptable accuracy which may indicates that it is sensitive to the measurement errors.

In summary, there have been several applications of ANNs for the assessment of delaminations in composite beams, a few for those in plates and even fewer for more complex structures. Application of ANN to predict damage involves the following steps: Determining the inputs and outputs; using either theory or a FEM to generate training data sets; determining the architecture of the ANN by a parametric study; validating the trained ANN by numerical or experimental data other than the training data; and, finally, verifying the capability of the network to accept new measurement data as inputs to predict the damage parameters. An ANN can deal with continuous variables very well once it is well trained and it predicts damage parameters quickly [Sahin and Shenoi (2003)]. But since interface is a discrete variable, the predicted interfaces by ANN have to be rounded off to the nearest integer since ANN treats the discrete variable to be continuous, and this will increase inaccuracy in prediction. Besides, training a good ANN is not easy and can be time consuming as it requires sufficient training data and many trials to determine the number of hidden layers or neurons, and adjustments of training parameters such as the learning rate [Poggio and Girosi (1990)]. Therefore, it depends greatly on experience and requires careful validation of the network before being used for predictions.

6.3 Techniques based on optimization

To minimize discrepancies between the computed frequencies from mathematical models and the measured values from real structures, optimization techniques have played a significant role in structural damage detection and identification problems [Keilers and Chang (1995)]. As conventional optimization methods, such as the least square algorithm and gradient-based local search (GBLS), employ gradient information of the objective function to determine the directions of subsequent search points from a given start point, they usually lead to only a local minimum. To derive a more accurate and reliable solution, a global optimization technique is required and the GA, first introduced by Holland in 1975 [Holland (1975)], has been widely used as a global optimum searching technique.

GA is a stochastic global search method which mimics Darwin's theory of evolution and natural selection. It operates on a population of potential solutions to iteratively produce increasingly better approximations to a solution by applying the principle of survival of the fittest. In each generation, individuals are chosen for reproduction based on their levels of fitness (derived from objective function values). The 'more fit individuals' have high probabilities of being selected for breeding to create a new set of approximations. A further genetic operator, mutation, is then applied to the new individuals to change their genetic representations according to a probabilistic rule. As this process continues through subsequent generations, the average performance of the individuals in a population is expected to increase, with good ones preserved and bred with each other and the less fit dying out. After a pre-specified number of generations, or when a particular point in the search space is encountered, the GA is terminated.

Compared with traditional optimisation and search algorithms, a GA searches from a population of points in the region of the whole solution space, rather than a single point, and can obtain the global optimum. Moreover, it has the advantage of easy implementation because only an objective function is required, with derivatives or other auxiliary information not necessary [Hao and Xia (2002)]. As a powerful and widely applicable optimisation algorithm, it has been increasingly used in all kinds of engineering optimisation problems, such as structural design, parameter estimation, optimal control in smart structures and damage detection [Forrester and Keane (2009)]. The literature survey below focuses on applications of GA to damage detection with emphasis on delamination detection in composites.

Chou et al. [Chou and Ghaboussi (2001)] used a GA to detect damage location in a truss bridge structure. Instead of dynamic parameters, they used the static measurements of displacements at a small number of locations. Their results showed that the proposed method was capable of detecting the location and severity of damage as well as correctly determining the unmeasured nodal displacement. They also numerically simulated measurement noise by adding a random noise with a zero mean and a specified variance to the measured data. As their case study showed that, with a 5% random noise introduced, it was still possible to detect the damaged element in the beam, the authors therefore concluded that their GA was robust in terms of measurement noise.

Harrison et al. [Harrison and Butler (2001)] located delaminations in composite beams using a two-stage procedure: The first uses a gradient-based optimisation to update the material properties and boundary conditions in the theoretical model to make the predicted frequencies and mode shapes match measured results; and the second locates the delamination using either a gradient-based method or the GA. Experimental validation showed that both could successfully predict the delamination parameters but the interface and size predictions were less accurate than that of the lengthwise location. Furthermore, in most cases, GAs provided better predictions of delaminations in composite beams.

Krawczuk et al. [Krawczuk and Ostachowicz (2002)] identified delaminations in composite beams using a GA via changes in natural frequencies, with the lengthwise location and size of delamination predicted by minimising an error function which expressed the discrepancy between its measured and theoretical frequencies. Their method was demonstrated using a numerical model of a cantilever composite beam and the results showed that the GA could predict the delamination location and magnitude at high levels of accuracy and speed via changes in natural frequencies. The authors applied a similar method to detect the addition of a concentrated mass in a rectangular composite plate using the first four measured natural frequencies [Ostachowicz, Krawczuk and Cartmell (2002)].

Nag et al. [Nag, Mahapatra and Gopalakrishnan (2002)] identified only mid-plane delaminations in composite beams using a combination of spectral estimation and a GA, with a spectral FEM consisting of a damaged spectral element used to predict the damaged structural response in the frequency domain. Unidirectional graphite-epoxy composite beams with cantilever and clamped-clamped boundary conditions were employed in their case studies and they used simulated data from the spectral FEM for comparison with the target response data for the fitness function. The results showed that, by introducing a damaged spectral element through modelling, efficient simulations and predictions of delaminations in composite beams could be obtained for online structural health monitoring.

Zheng et al. [Zheng, Li and Wang (2011)] presented a hybrid algorithm which combined fuzzy logic theory, radial basis function NNs (RBFNNs) and a GA, in which the GA played the main role. FEMs of delaminated glass/epoxy beams were built and run to generate 288 sets of modal frequencies of which 283 were used to train an ANN and the rest used as test cases. The authors claimed that the hybrid algorithm was numerically validated in terms of successfully predicting the lengthwise location and size of delamination.

All the studies mentioned above are applications of conventional GAs combined with either theory or finite element modelling to predict numerical values, such as natural frequencies, for comparison with target values from real or numerical test cases. However, this can be quite expensive in terms of time and computation, especially if a FEM is employed, because evaluation of the objective function is iterative which requires the numerical model to be run each time [Gomes, Mendéz, Cunha et al. (2017)]. A solution to this is to assist the GA with a surrogate model when an outcome of interest cannot be easily measured or is not computationally cheap to simulate [Forrester and Keane (2009)]. Basically, a surrogate is a fast approximation model of the FEM [Zhang, Shankar, Ray et al. (2013)], for instance, a well-trained ANN but, to date, there have only been a few applications of surrogate-assisted optimisation (SAO) to delamination detection, as discussed below.

Chen et al. [Chen, Le, Kim et al. (2004)] and Kim [Kim (2004)] developed an approach that combined GA and ANN techniques for delamination detection in both composite beams and plates. The FE analysis based on the layerwise composite laminate theory was employed to generate the natural frequencies of laminates with given delamination patterns. 1624 data sets were generated from FE beam models to train a BPNN with three inputs, the thickness and lengthwise locations, and delamination size, and ten outputs, the first ten natural frequencies. The well-trained network had an architecture of 3-70-70-10 and was validated using 1104 data sets generated separately using FEM. It provided an approximation of the numerical model and could quickly determine the natural frequencies of the delaminated laminate. Then, the GA was customised with this ANN approximation model to detect delaminations in laminated composite beams and plates. Although the authors did not point this out, this method was actually a typical SAO in which the ANN was the surrogate model. Using an ANN instead of directly running a FEM to predict numerical frequencies helps to save computational time during objective function evaluation. However, their approach failed to accurately predict small delamination sizes and those near the surface and, furthermore, they did not conduct any experimental validation of their methodology.

Deenadayalu et al. [Deenadayalu, Chattopadhyay and Chen (2004)] also used a hybrid method of an ANN and GA to detect delaminations in composite plates which adopted a multi-layer feed-forward network to train a ANN using 294 data sets. The trained ANN model provided an approximation of the numerical model and was basically a surrogate model. Based on a damage index distribution which quantified the difference in modal strains between the damaged and undamaged laminates, an objective function was formulated. With the combination of an ANN (as the function approximator) and modified GA, delaminations in composite plates were detected after 80 000 function evaluations. The authors concluded that, in general, the results from their investigation showed that agents constructed from ANNs can be used as tools for the effective detection of delamination. However, no experimental validation was conducted to test the efficacy of their approach.

Compared with other artificial intelligence (AI) algorithms, such as ANN, a GA can predict discrete variables, such as the interface of delamination, since it is not dependent on continuity of the parameter space [Zheng, Li and Wang (2011)]. However, due to it being an iterative process which requires a large number of objective function evaluations, using it for direct optimisation may be computationally expensive and time consuming. An alternative approach is to use SAO which, as has been reported in a few cases, has not yet been extensively applied in delamination detection. It is also to be noted that reports of detecting delaminations in composite plates (or more complex FRP structures) using frequency shifts are rare mainly because of the complexity of the problem since there are five delamination parameters in a composite plate compared with only three in a beam. Another reason may be that, unlike for a delaminated beam, as there is no existing theoretical model for delaminated plates and other structures, a FEM is the only way to generate dynamic response data for comparison with the measured values. Since FEMs require a great deal of computation and time, it is not very practical to use a GA to detect delaminations without assistance from a surrogate model.

7 Conclusions

This review was undertaken with the aim of advancing the detection and assessment of delamination damage in fibre reinforced composite laminates using structural health monitoring techniques, in particular using the frequency based vibration monitoring. Currently, an immense number of papers exist for frequency-based identification of damage, but those reporting the detection of damages other than delaminations in composites (for instance cracks) are generally not covered in this article. Some typical SHM techniques currently available for delamination detection in FRP composites are briefly reviewed with short summary of how the technique works, its advantages and disadvantages, which has covered acoustic emission, fibre optic sensors, Lamb wave-, impedance- and vibration-based methods. Then this paper focus on a comprehensive review of various aspects related to applying frequency-based vibration monitoring to identify delamination in FRPs, which has included review of theoretical models for free vibration of delaminated FRP beams, survey of finite element modelling of delaminated composite structures, summary of experimental modal analyses on FRP composites with delaminations, and inverse algorithms for frequency-based delamination assessment. It is to be noted all the techniques have their own pros and cons, there is no general SHM technique that allows the resolution of all kinds of problems in all kinds of structures. This actually explains why the research in SHM techniques for composites laminates flourishes for so many years, and will continue to attract constant attention from academic communities.

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