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Article

Acceptance Sampling Plans with Truncated Life Tests for the Length-Biased Weighted Lomax Distribution

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Abstract: In this paper, we considered the Length-biased weighted Lomax distribution and constructed new acceptance sampling plans (ASPs) where the life test is assumed to be truncated at a pre-assigned time. For the new suggested ASPs, the tables of the minimum samples sizes needed to assert a specific mean life of the test units are obtained. In addition, the values of the corresponding operating characteristic function and the associated producer's risks are calculated. Analyses of two real data sets are presented to investigate the applicability of the proposed acceptance sampling plans; one data set contains the first failure of 20 small electric carts, and the other data set contains the failure times of the air conditioning system of an airplane. Comparisons are made between the proposed acceptance sampling plans and some existing acceptance sampling plans considered in this study based on the minimum sample sizes. It is observed that the samples sizes based on the proposed acceptance sampling plans are less than their competitors considered in this study. The suggested acceptance sampling plans are recommended for practitioners in the field.

Keywords: Acceptance sampling plan; producer's risk; truncated life tests; operating characteristic function; length-biased weighted lomax distribution; consumer's risk

1 Introduction

The ASP is a useful tool that can be used to conclude whether to accept a product or reject it by utilizing a sample chosen randomly from the product. The ASP procedure firstly starts by defining the smallest size of the sample needed to assert a specific mean life when the life test time is truncated at a determined time. These types of tests are known as truncated lifetime tests.

The ASP, in terms of truncated life tests, is discussed by numerous researchers. For example, Al-Nasser et al. [1] developed an ASP based on life test truncation for the exponentiated Fréchet distribution; Al-Nasser et al. [2] developed single ASPs based on a truncated lifetime test for the

Ishita distribution, Al-Nasser et al. [3] considered double ASPs based on Quasi Lindley distribution, Recently, Al-Nasser et al. [4] introduced ASPs for the Tsallis q-exponential distribution, Al-Omari [5] considered ASPs for Sushila distribution, Al-Omari [6] developed ASPs based on time truncated tests for the transmuted inverse Rayleigh distribution, Al-Omari [7] developed ASPs based on time truncated test for the generalized inverted exponential distribution, and latter, Al-Omari [8] applied similar ASPs but for the generalized inverse Weibull distribution, Al-Omari [9,10] studied the Garima and transmuted generalized inverse Weibull distributions in ASPs, Al-Omari et al. [11] introduced ASPs based on truncated life tests for the extended Exponential distribution, and, in Al-Omari et al. [12] they used the ASPs for the Rama distribution. Also, for more examples of ASP see Al-Omari et al. [13] for Akash distribution with an application to electric carts data, Aslam et al. [14] for considering the generalized exponential distribution, Baklizi et al. [15] for the Birnbaum Saunders model, Braimah et al. [16] for Weibull product life distributions, Gui et al. [17] considered ASPs for the weighted Exponential distribution, Gupta et al. [18] for the Gamma distribution, Kantam et al. [19] used log-logistic distribution and considered the truncated life tests for this distribution, Malathi et al. [20] considered Fréchet distribution, Sobel et al. [21] studied the Exponential distribution and investigated the life test dependent on it. A similar life test is used by Tsai et al. [22] but for the generalized Rayleigh distribution. Rao et al. [23] for applying truncated tests on the inverse Rayleigh distribution; a similar idea is considered by Sriramachandran et al. [24] but for the exponentiated inverse Rayleigh distribution. Sudamani et al. [25] for time truncated chain sampling plan for Weibull distribution. ASPs using percentiles for the exponentiated Fréchet distribution was developed by Rao et al. [26]. Al-Omari et al. [27] proposed double acceptance sampling plan for time truncated life tests based on the transmuted generalized inverse Weibull distribution. Based on our knowledge, the ASPs have not been considered for the Length-biased weighted Lomax distribution. This paper will focus mainly on this distribution and proposed new ASPs based on it.

The structure of this paper is prepared as follows. Section 2 affords the Length-biased weighted Lomax distribution (LBWLD) and introduces some of its statistical properties. Section 3 is devoted to clarify the proposed sampling plans for the LBWLD distribution with its properties, including the minimum sample size, the operating characteristic function values and the corresponding producer's risk. The necessary tables of the new plan with illustrated examples are presented in Section 4. The usefulness of the LBWLD acceptance sampling plans is investigated to real data sets in Section 5. Findings and summarized results are presented in Section 6.

2 The Length-Biased Weighted Lomax Distribution

Ahmad et al. [28] suggested the LBWLD with the probability density function (pdf) and the cumulative distribution function (cdf), respectively, given by

$$f_{LBWLD}(x) = \frac{\alpha(\alpha-1)}{\varphi^2} x \left(1+\frac{x}{\varphi}\right)^{-(\alpha+1)}, \quad x>0,\ \alpha,\ \varphi>0 \tag{1}$$

$$F_{LBWLD}(x) = 1-\left(1+\frac{x}{\varphi}\right)^{-\alpha}\left(1+\frac{\alpha x}{\varphi}\right), \quad x>0,\ \alpha,\ \varphi>0. \tag{2}$$

The rth non central moment of the LBWLD is

$$\mu_r' = E\left(X^r\right) = \alpha\left(\alpha-1\right)\varphi^r \sum_{k=0}^{r+1} (-1)^{k+1}\, C_k^{r+1} \frac{1}{r-\alpha-k+1}. \tag{3}$$

The mean, variance, and coefficient of variation (CV), respectively, are

$$\mu_{LBWLD} = \frac{2\varphi}{\alpha-2}, \quad \sigma_{LBWLD}^2 = \frac{2\alpha\varphi^2}{(\alpha-2)^2\,(\alpha-3)}, \quad CV_{LBLWD} = \frac{1}{2}\sqrt{\frac{2\alpha}{\alpha-3}}.$$

The functions of hazard rate and reliability for the LBWLD, respectively, are:

$$h_{LBWLD}\left(x\right) = \frac{f_{LBWLD}\left(x\right)}{1-F_{LBWLD}\left(x\right)} = \frac{\alpha\left(\alpha-1\right)}{\varphi^2} x \left(1+\frac{x}{\varphi}\right)^{-1} \left(1+\frac{\alpha x}{\varphi}\right)^{-1}, \tag{4}$$

$$R_{LBWLD}\left(x\right) = 1-F_{LBWLD}\left(x\right) = \left(1+\frac{x}{\varphi}\right)^{-\alpha} \left(1+\frac{\alpha x}{\varphi}\right), \quad x>0,\ \alpha,\ \varphi>0.$$

The reverse hazard function for the Length-biased weighted Lomax distribution is

$$h_{r-LBWLD}\left(x\right) = \frac{f_{LBWLD}\left(x\right)}{F_{LBWLD}\left(x\right)} = \frac{\alpha\left(\alpha-1\right)}{\varphi^2} x \left(1+\frac{x}{\varphi}\right)^{-(\alpha+1)} \left[1-\left(1+\frac{x}{\varphi}\right)^{-\alpha}\left(1+\frac{\alpha x}{\varphi}\right)\right]^{-1}.$$

Details of the LBWLD distribution are available in Ahmad et al. [28] and the reliability for the LBWLD, in case the outliers exist, is investigated by Karimi et al. [29].

3 The Proposed Acceptance Sampling Plans

The suggested ASPs are illustrated, here, as in the following steps:

1) The number of units, m, on a test.
2) An acceptance number, say c, where if c or fewer failures occur within the test time t, the lot is accepted.
3) A ratio t/μ_0, where μ_0 is the identified average lifetime.

The probability of accepting a bad lot, the one for which the real mean life (μ), is less than the specified mean life (μ_0). The latter is usually known as the consumer's risk and it is determined to be at most $1-P^*$, i.e., that the real mean life μ is less than μ_0, not exceeds $1-P^*$.

3.1 Minimum Sample Size

In this proposed ASP, the size of the lot is supposed to be large enough. In this case, we can apply the theory of binomial distribution. Therefore, for $0<P^*<1$, t, c, and μ_0, we aim to determine the minimum size of the sample, m, that is necessary to satisfy

$$\sum_{i=0}^{c} \binom{m}{i} p^i\,(1-p)^{m-i} \le 1-P^*. \tag{5}$$

Here, $p = F(t; \mu_0)$ denotes the likelihood of observing a failure within the time t which is depend on t/μ_o and $\mu_0 = \dfrac{2\varphi_0}{\alpha_0 - 2}$. The researcher is usually interested in determining the ratio t/μ_o. If the number of observed failures, within the time t, is at most c, then based on Eq. (6) with the probability of P, we can confirm that $F(t; \mu) \le F(t; \mu_0)$, which involves $\mu_0 \le \mu$.

The minimum sizes of the sample that satisfying Eq. (6), with values of $P^* = 0.75$, 0.90, 0.95, 0.99, are calculated for the values $t/\mu_0 = 0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712. These choices of the ratio t/μ_0 and P^* are harmonic with that of values presented in Gupta et al. [18], Al-Omari et al. [11], Baklizi et al. [15], Al-Nasser et al. [1] and Kantam et al. [19]. The results displayed in Tab. 1 are for the LBWLD parameter $\alpha = 3$, where the results for $\alpha = 6$, are presented in Tab. 4.

3.2 Operating Characteristic Function

The function of operating characteristic, denoted by $OC(p)$, of the sampling plan $(m, c, t/\mu_0)$ is known as the probability of accepting a lot. For the ASP given in Subsection 3.1, the $OC(p)$ is determined as

$$OC(p) = \sum_{i=0}^{c} \binom{m}{i} p^i (1-p)^{m-i} = 1 - B_p(c+1, m-c). \tag{6}$$

Here, $p = F(t; \mu)$ is a function of μ (and usually used to judge the quality of this parameter) and $B_p(c+1, m-c)$ refers to the function of an incomplete beta. It is of interest to note that the latter function is an increasing function of the probability p, while the operating characteristic function is a decreasing function of p. For fixed c and $\alpha = \alpha_0$, from Eq. (7), the values of the operating characteristic function, based on the LBWLD, for the ASP $(m, c = 2, t/\mu_0)$ are presented in Tab. 2 for $\alpha = \alpha_0 = 3$ while they are given in Tab. 5 for $\alpha = \alpha_0 = 6$.

3.3 Producer's Risk

The producer's risk denoted by $PR(p)$ is defined as the probability of rejecting a lot when $\mu \ge \mu_0$ with the formula defined as

$$PR(p) = P(\text{Rejecting a lot}) = \sum_{i=c+1}^{m} \binom{m}{i} p^i (1-p)^{m-i}. \tag{7}$$

For an assumed value of the producer risk, ε, a researcher may want to know the minimum amount of μ/μ_0 that will asserts that the $PR(p)$ is less than or equal to ε in case of adopted this sampling plan. This value of μ/μ_0, is the minimum positive number for which $p = F[(t/\mu_0)(\mu_0/\mu)]$ fulfills the inequality

$$\sum_{i=c+1}^{m} \binom{m}{i} p^i (1-p)^{m-i} \le \varepsilon. \tag{8}$$

For a specified ASP $(m, c, t/\mu_0)$ considering the LBWL distribution at an identified level of confidence, P^*, Tabs. 3 and 6 summarize the minimum values of the ratio μ/μ_0 that satisfying Eq. (9) for $\alpha = \alpha_0 = 3$ and $\alpha = \alpha_0 = 6$, respectively.

4 Descriptions of Tables and Examples

The proposed ASPs performance based on values of minimum sample size values, operating characteristic function and the values of minimum ratio are now analyzed. Various values for the parameters of the LBWLD are investigated to see their effect on the proposed ASPs.

4.1 Results for $\alpha = 3$ *in LBWLD*

The lifetime distribution is assumed to follow a Length-biased weighted Lomax distribution with $\alpha = 3$. Also, the mean life μ_0 is assumed to be at least 1000 h having a probability value of $P^* = 0.95$. Assume that the researcher wants to terminate the lifetime test at $t_0 = 1257$ h, that is $t_0/\mu_0 = 1.257$, and set the number of acceptance to be $c = 2$. Therefore, the requested sample size m is the entry presented in Tab. 1, that corresponds to $t_0/\mu_0 = 1.257$, $c = 2$, $P^* = 0.95$, is $m = 6$. That is, 6 units have to be out on test. If two or fewer failures are detected during t_0, then the mean life μ of the items can be emphasized to be at least 1000 h with a level of confidence equal to 0.95. The minimum samples sizes obtained here are less than their counterparts in Al-Omari [5] for Sushila distribution.

Includes the values of the OC for the suggested ASPs based on LBWLD distribution obtained from Tab. 1 for different values of t/μ_0 and P^* with acceptance number $c = 2$. For the previous example and based on the results in Tab. 2, the values for the operating characteristic function considering the sampling plan $(m, c, t/\mu_0) = (6, 2, 1.257)$ are:

μ/μ_0	2	4	6	8	10	12
OC	0.200531	0.683262	0.888350	0.957350	0.981937	0.991603

This above table displays that if the real mean life is double the specified mean life, the producer's risk is about 0.79947, when ($\mu/\mu_0 = 2$), and then goes to zero when $\mu/\mu_0 > 2$.

The values presented in Tab. 3 are for the minimum ratio of μ/μ_0 when the producer's risk less than or equal to $\varepsilon = 0.05$. Therefore, for our example with $P^* = 0.95$, $t/\mu_0 = 1.257$ and $c = 2$ the ratio value is 7.655. That is, the mean life of the product should be 7.655 times the identified mean life of 1000 h; hence, for the above ASP and with a probability of 0.95, the product can be accepted.

4.2 Results for $\alpha = 6$ *in LBWLD*

Based on the results in Tab. 1 and Tab. 4, it can be seen that the obtained sizes of the minimum sample in Tab. 1 for $\alpha = 3$ are less than their counterparts for $\alpha = 6$ in Tab. 4. Also, the results presented in both tables are less than that given in Kantam et al. [19], Aslam et al. [14], and Al-Omari [9,10]. When $\alpha = 6$ and $P^* = 0.99$, for the ASP $(m, c, t/\mu_0) = (8, 2, 1.257)$, the operating characteristic value equals 0.995775. Hence, the producer risk is equal to 0.004225, and equal to zero for lagre values when $\mu/\mu_0 \geq 4$; therefore, Tab. 6 shows that the value of μ/μ_0 is 5.760.

5 Applications of Real Data Sets

Two sets of real data, in this section, are used to investigate the efficiency of the suggested ASPs based on the LBWL distribution. The product lifetime is assumed to follow the LBWLD. First, we have to check whether the LBWL distribution can be used or not. The LBWLD distribution is fitted to both data sets and the following criteria are used to check the goodness of fitting:

Table 1: Minimum sizes of sample with P^* probability and acceptance number c for $\alpha = 3$ in the LBLWD

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	2	2	1	1	1	1	1	1
0.75	1	4	3	3	3	2	2	2	2
0.75	2	6	5	4	4	3	3	3	3
0.75	3	8	6	6	5	5	4	4	4
0.75	4	10	8	7	6	6	5	5	5
0.75	5	12	9	8	8	7	7	6	6
0.75	6	14	11	10	9	8	8	7	7
0.75	7	16	12	11	10	9	9	9	8
0.75	8	17	14	12	11	10	10	10	9
0.75	9	19	15	13	13	11	11	11	10
0.75	10	21	17	15	14	13	12	12	12
0.90	0	3	2	2	2	1	1	1	1
0.90	1	5	4	4	3	3	3	2	2
0.90	2	8	6	5	5	4	4	4	3
0.90	3	10	7	6	6	5	5	5	4
0.90	4	12	9	8	7	6	6	6	6
0.90	5	14	11	9	8	8	7	7	7
0.90	6	16	12	11	10	9	8	8	8
0.90	7	18	14	12	11	10	9	9	9
0.90	8	20	15	13	12	11	10	10	10
0.90	9	22	17	15	14	12	12	11	11
0.90	10	24	18	16	15	13	13	12	12
0.95	0	4	3	2	2	2	2	1	1
0.95	1	6	5	4	4	3	3	3	3
0.95	2	9	7	6	5	4	4	4	4
0.95	3	11	8	7	6	6	5	5	5
0.95	4	13	10	9	8	7	6	6	6
0.95	5	15	12	10	9	8	7	7	7
0.95	6	17	13	11	10	9	9	8	8
0.95	7	19	15	13	12	10	10	9	9
0.95	8	21	16	14	13	12	11	10	10
0.95	9	23	18	16	14	13	12	12	11
0.95	10	25	20	17	16	14	13	13	12
0.99	0	6	4	3	3	2	2	2	2
0.99	1	9	6	5	5	4	3	3	3
0.99	2	11	8	7	6	5	5	4	4
0.99	3	14	10	8	8	6	6	6	5
0.99	4	16	12	10	9	8	7	7	6
0.99	5	18	14	12	10	9	8	8	8
0.99	6	20	15	13	12	10	9	9	9
0.99	7	23	17	15	13	11	11	10	10
0.99	8	25	19	16	15	13	12	11	11
0.99	9	27	20	17	16	14	13	12	12
0.99	10	29	22	19	17	15	14	13	13

Table 2: Values for operating characteristic function based on the ASP ($m, c=2, t/\mu_0$) with probability P^* for $\alpha=3$ in the LBLWD

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	6	0.628	0.683790	0.957473	0.991632	0.997739	0.999236	0.999697
0.75	5	0.942	0.541931	0.904830	0.976209	0.992569	0.997236	0.998825
0.75	4	1.257	0.550493	0.890181	0.968758	0.989308	0.995751	0.998102
0.75	4	1.571	0.412449	0.808693	0.935167	0.975147	0.989315	0.994945
0.75	3	2.356	0.513396	0.823292	0.930950	0.970112	0.985822	0.992737
0.75	3	3.141	0.378877	0.707612	0.861637	0.930970	0.963500	0.979637
0.75	3	3.927	0.287544	0.602529	0.784249	0.880109	0.930935	0.958701
0.75	3	4.712	0.224398	0.513396	0.707564	0.823292	0.890945	0.930950
0.90	8	0.628	0.472471	0.904590	0.979241	0.994133	0.997969	0.999182
0.90	6	0.942	0.386619	0.843151	0.957473	0.986188	0.994750	0.997739
0.90	5	1.257	0.343613	0.792279	0.934048	0.976137	0.990207	0.995537
0.90	5	1.571	0.214096	0.664556	0.870554	0.946793	0.976151	0.988411
0.90	4	2.356	0.201796	0.590652	0.808785	0.908274	0.953582	0.975166
0.90	4	3.141	0.105365	0.412641	0.661240	0.808831	0.890323	0.935230
0.90	4	3.927	0.059144	0.286838	0.525199	0.697788	0.808748	0.877630
0.90	3	4.712	0.224398	0.513396	0.707564	0.823292	0.890945	0.930950
0.95	9	0.628	0.381433	0.871688	0.970678	0.991526	0.997032	0.998796
0.95	7	0.942	0.264918	0.773015	0.933417	0.977526	0.991271	0.996191
0.95	6	1.257	0.200531	0.683262	0.888350	0.957350	0.981937	0.991603
0.95	5	1.571	0.214096	0.664556	0.870554	0.946793	0.976151	0.988411
0.95	4	2.356	0.201796	0.590652	0.808785	0.908274	0.953582	0.975166
0.95	4	3.141	0.105365	0.412641	0.661240	0.808831	0.890323	0.935230
0.95	4	3.927	0.059144	0.286838	0.525199	0.697788	0.808748	0.877630
0.95	4	4.712	0.035422	0.201796	0.412577	0.590652	0.720088	0.808785
0.99	11	0.628	0.237835	0.796736	0.948884	0.984565	0.994466	0.997724
0.99	8	0.942	0.175913	0.698577	0.904590	0.966554	0.986730	0.994133
0.99	7	1.257	0.111470	0.574006	0.834204	0.933233	0.970833	0.986171
0.99	6	1.571	0.102962	0.522902	0.792283	0.908690	0.957375	0.978733
0.99	5	2.356	0.068773	0.386467	0.664692	0.822979	0.904708	0.946831
0.99	5	3.141	0.025071	0.214257	0.467743	0.664760	0.792517	0.870669
0.99	4	3.927	0.059144	0.286838	0.525199	0.697788	0.808748	0.877630
0.99	4	4.712	0.035422	0.201796	0.412577	0.590652	0.720088	0.808785

Table 3: Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = 3$ in the LBLWD

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	11.976	17.964	16.061	20.072	30.102	40.131	50.174	60.203
0.75	1	5.244	6.275	8.373	10.465	10.687	14.248	17.813	21.374
0.75	2	3.824	4.886	5.215	6.517	6.744	8.991	11.24	13.487
0.75	3	3.209	3.620	4.830	4.844	7.265	6.754	8.444	10.132
0.75	4	2.861	3.417	3.891	3.915	5.871	5.507	6.885	8.261
0.75	5	2.636	2.885	3.291	4.112	4.980	6.639	5.882	7.058
0.75	6	2.476	2.844	3.354	3.589	4.358	5.810	5.179	6.214
0.75	7	2.357	2.532	2.988	3.201	3.897	5.195	6.495	5.585
0.75	8	2.130	2.535	2.705	2.901	3.540	4.720	5.900	5.098
0.75	9	2.071	2.320	2.479	3.098	3.255	4.340	5.426	4.706
0.75	10	2.021	2.338	2.588	2.867	3.699	4.029	5.037	6.044
0.90	0	15.017	17.964	23.971	29.959	30.102	40.131	50.174	60.203
0.90	1	6.155	7.865	10.495	10.465	15.693	20.922	17.813	21.374
0.90	2	4.803	5.736	6.520	8.148	9.774	13.030	16.291	13.487
0.90	3	3.883	4.246	4.830	6.037	7.265	9.685	12.108	10.132
0.90	4	3.372	3.871	4.559	4.863	5.871	7.828	9.786	11.742
0.90	5	3.045	3.621	3.850	4.112	6.167	6.639	8.301	9.960
0.90	6	2.817	3.152	3.795	4.192	5.382	5.810	7.263	8.715
0.90	7	2.648	3.060	3.378	3.734	4.800	5.195	6.495	7.793
0.90	8	2.518	2.766	3.055	3.380	4.351	4.720	5.900	7.080
0.90	9	2.414	2.731	3.096	3.497	3.993	5.323	5.426	6.510
0.90	10	2.329	2.523	2.861	3.234	3.699	4.932	5.037	6.044
0.95	0	17.584	22.526	23.971	29.959	44.929	59.898	50.174	60.203
0.95	1	6.968	9.232	10.495	13.116	15.693	20.922	26.158	31.386
0.95	2	5.239	6.502	7.655	8.148	9.774	13.03	16.291	19.547
0.95	3	4.190	4.813	5.666	6.037	9.053	9.685	12.108	14.529
0.95	4	3.608	4.292	5.165	5.698	7.293	7.828	9.786	11.742
0.95	5	3.236	3.953	4.359	4.811	6.167	6.639	8.301	9.960
0.95	6	2.977	3.440	3.795	4.192	5.382	7.174	7.263	8.715
0.95	7	2.786	3.303	3.741	4.222	4.800	6.399	6.495	7.793
0.95	8	2.639	2.985	3.382	3.819	5.069	5.800	5.900	7.080
0.95	9	2.521	2.922	3.377	3.497	4.646	5.323	6.654	6.510
0.95	10	2.425	2.868	3.120	3.576	4.300	4.932	6.166	6.044
0.99	0	21.893	26.376	30.058	37.566	44.929	59.898	74.887	89.857
0.99	1	9.039	10.452	12.319	15.397	19.67	20.922	26.158	31.386
0.99	2	6.035	7.204	8.676	9.567	12.219	16.290	16.291	19.547
0.99	3	5.023	5.824	6.423	8.027	9.053	12.069	15.089	14.529
0.99	4	4.256	5.058	5.727	6.456	8.544	9.723	12.156	11.742
0.99	5	3.767	4.567	5.275	5.448	7.215	8.222	10.279	12.334
0.99	6	3.426	3.975	4.591	5.256	6.287	7.174	8.970	10.763
0.99	7	3.297	3.758	4.407	4.676	5.600	7.465	8.000	9.600
0.99	8	3.089	3.589	3.983	4.612	5.726	6.758	7.252	8.701
0.99	9	2.924	3.283	3.644	4.220	5.244	6.194	6.654	7.985
0.99	10	2.790	3.190	3.601	3.899	4.850	5.732	6.166	7.398

Table 4: Minimum sizes of the sample to be tested for time t with probability P^* and acceptance number c where $\mu \geq \mu_0$ for $\alpha = 6$ in the LBLWD

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	3	2	2	1	1	1	1	1
0.75	1	6	4	3	3	2	2	2	2
0.75	2	8	6	5	4	3	3	3	3
0.75	3	11	7	6	5	5	4	4	4
0.75	4	14	9	8	7	6	5	5	5
0.75	5	16	11	9	8	7	6	6	6
0.75	6	19	13	10	9	8	7	7	7
0.75	7	21	14	12	11	9	9	8	8
0.75	8	24	16	13	12	10	10	9	9
0.75	9	26	18	15	13	11	11	10	10
0.75	10	28	20	16	14	12	12	11	11
0.90	0	4	3	2	2	1	1	1	1
0.90	1	8	5	4	3	3	2	2	2
0.90	2	11	7	6	5	4	4	3	3
0.90	3	14	9	7	6	5	5	4	4
0.90	4	16	11	9	8	6	6	5	5
0.90	5	19	13	10	9	7	7	7	6
0.90	6	22	15	12	10	9	8	8	7
0.90	7	25	17	13	12	10	9	9	8
0.90	8	27	18	15	13	11	10	10	9
0.90	9	30	20	16	14	12	11	11	11
0.90	10	33	22	18	16	13	12	12	12
0.95	0	6	4	3	2	2	1	1	1
0.95	1	9	6	5	4	3	3	2	2
0.95	2	12	8	6	5	4	4	4	3
0.95	3	16	10	8	7	5	5	5	4
0.95	4	18	12	10	8	7	6	6	6
0.95	5	21	14	11	10	8	7	7	7
0.95	6	24	16	13	11	9	8	8	8
0.95	7	27	18	14	12	10	9	9	9
0.95	8	30	20	16	14	11	10	10	10
0.95	9	32	22	17	15	13	12	11	11
0.95	10	35	24	19	16	14	13	12	12
0.99	0	8	5	4	3	2	2	2	2
0.99	1	12	8	6	5	4	3	3	3
0.99	2	16	10	8	7	5	4	4	4
0.99	3	19	12	10	8	6	6	5	5
0.99	4	23	15	11	10	8	7	6	6
0.99	5	26	17	13	11	9	8	7	7
0.99	6	29	19	15	13	10	9	8	8
0.99	7	32	21	16	14	11	10	10	9
0.99	8	35	23	18	15	12	11	11	10
0.99	9	38	25	20	17	14	12	12	11
0.99	10	41	27	21	18	15	14	13	12

Table 5: Values of the OC for the ASP ($m, c=2, t/\mu_0$) with a specified probability P^* for $\alpha=6$ in the LBLWD

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	8	0.628	0.817578	0.987869	0.998308	0.999624	0.999888	0.999959
0.75	6	0.942	0.703391	0.970902	0.995216	0.998839	0.999634	0.999862
0.75	5	1.257	0.611343	0.949008	0.990377	0.997476	0.999165	0.999674
0.75	4	1.571	0.625048	0.945103	0.988793	0.996906	0.998940	0.999575
0.75	3	2.356	0.629341	0.930079	0.983010	0.994738	0.998046	0.999169
0.75	3	3.141	0.441386	0.842561	0.95219	0.983017	0.993084	0.996857
0.75	3	3.927	0.303908	0.736964	0.903998	0.961578	0.983005	0.991795
0.75	3	4.712	0.210040	0.629341	0.842518	0.930079	0.966695	0.983010
0.90	11	0.628	0.649128	0.969152	0.995367	0.998939	0.999679	0.999882
0.90	7	0.942	0.597721	0.953749	0.992034	0.998028	0.999373	0.999762
0.90	6	1.257	0.461736	0.912269	0.982241	0.995198	0.998386	0.999363
0.90	5	1.571	0.425103	0.888805	0.975037	0.992808	0.997478	0.998975
0.90	4	2.356	0.316132	0.806733	0.945145	0.981699	0.992917	0.996909
0.90	4	3.141	0.145728	0.625281	0.860817	0.945165	0.976289	0.988808
0.90	3	3.927	0.303908	0.736964	0.903998	0.961578	0.983005	0.991795
0.90	3	4.712	0.210040	0.629341	0.842518	0.930079	0.966695	0.983010
0.95	12	0.628	0.592390	0.960825	0.993971	0.998606	0.999576	0.999844
0.95	8	0.942	0.497358	0.932735	0.987869	0.996939	0.999016	0.999624
0.95	6	1.257	0.461736	0.912269	0.982241	0.995198	0.998386	0.999363
0.95	5	1.571	0.425103	0.888805	0.975037	0.992808	0.997478	0.998975
0.95	4	2.356	0.316132	0.806733	0.945145	0.981699	0.992917	0.996909
0.95	4	3.141	0.145728	0.625281	0.860817	0.945165	0.976289	0.988808
0.95	4	3.927	0.066364	0.453458	0.747867	0.885207	0.945128	0.972146
0.95	3	4.712	0.210040	0.629341	0.842518	0.930079	0.966695	0.983010
0.99	16	0.628	0.388198	0.917778	0.986068	0.996651	0.998961	0.999613
0.99	10	0.942	0.327384	0.880632	0.976441	0.993823	0.997976	0.999217
0.99	8	1.257	0.237568	0.817128	0.957599	0.987825	0.995775	0.998301
0.99	7	1.571	0.166135	0.741434	0.930422	0.978212	0.992010	0.996663
0.99	5	2.356	0.139689	0.661671	0.888882	0.960156	0.983941	0.992815
0.99	4	3.141	0.145728	0.625281	0.860817	0.945165	0.976289	0.988808
0.99	4	3.927	0.066364	0.453458	0.747867	0.885207	0.945128	0.972146
0.99	4	4.712	0.030920	0.316132	0.625203	0.806733	0.898928	0.945145

Table 6: Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = 6$ in the LBLWD

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	8.595	10.341	13.798	11.713	17.566	23.418	29.278	35.131
0.75	1	4.091	4.687	5.064	6.328	6.674	8.897	11.124	13.347
0.75	2	2.878	3.492	4.022	4.107	4.441	5.920	7.402	8.881
0.75	3	2.533	2.654	3.069	3.159	4.737	4.641	5.802	6.961
0.75	4	2.333	2.442	2.915	3.170	3.943	3.920	4.900	5.880
0.75	5	2.101	2.300	2.513	2.742	3.431	3.451	4.315	5.178
0.75	6	2.023	2.198	2.231	2.442	3.072	3.120	3.901	4.680
0.75	7	1.892	1.982	2.244	2.526	2.804	3.738	3.59	4.307
0.75	8	1.854	1.940	2.060	2.322	2.595	3.460	3.346	4.015
0.75	9	1.767	1.904	2.083	2.160	2.428	3.237	3.149	3.779
0.75	10	1.696	1.874	1.948	2.026	2.290	3.053	2.986	3.583
0.90	0	10.03	12.892	13.798	17.245	17.566	23.418	29.278	35.131
0.90	1	4.891	5.453	6.254	6.328	9.490	8.897	11.124	13.347
0.90	2	3.568	3.922	4.660	5.026	6.158	8.210	7.402	8.881
0.90	3	3.001	3.267	3.541	3.835	4.737	6.315	5.802	6.961
0.90	4	2.571	2.901	3.258	3.643	3.943	5.256	4.900	5.880
0.90	5	2.389	2.666	2.802	3.140	3.431	4.574	5.719	5.178
0.90	6	2.260	2.500	2.715	2.788	3.662	4.095	5.120	4.680
0.90	7	2.165	2.378	2.451	2.804	3.328	3.738	4.673	4.307
0.90	8	2.033	2.173	2.422	2.574	3.068	3.460	4.326	4.015
0.90	9	1.979	2.109	2.243	2.389	2.860	3.237	4.047	4.856
0.90	10	1.935	2.056	2.237	2.435	2.689	3.053	3.817	4.580
0.95	0	12.44	15.045	17.203	17.245	25.862	23.418	29.278	35.131
0.95	1	5.251	6.137	7.277	7.816	9.490	12.652	11.124	13.347
0.95	2	3.775	4.317	4.660	5.026	6.158	8.210	10.265	8.881
0.95	3	3.282	3.542	3.967	4.425	4.737	6.315	7.896	6.961
0.95	4	2.793	3.111	3.575	3.643	4.753	5.256	6.572	7.885
0.95	5	2.567	2.834	3.069	3.501	4.112	4.574	5.719	6.862
0.95	6	2.409	2.641	2.933	3.102	3.662	4.095	5.120	6.143
0.95	7	2.291	2.499	2.645	2.804	3.328	3.738	4.673	5.607
0.95	8	2.200	2.388	2.588	2.807	3.068	3.460	4.326	5.190
0.95	9	2.079	2.300	2.395	2.603	3.238	3.812	4.047	4.856
0.95	10	2.024	2.228	2.371	2.435	3.038	3.584	3.817	4.580
0.99	0	14.472	16.943	20.076	21.5	25.862	34.479	43.107	51.724
0.99	1	6.218	7.337	8.189	9.095	11.722	12.652	15.818	18.980
0.99	2	4.518	5.027	5.760	6.541	7.537	8.210	10.265	12.316
0.99	3	3.669	4.045	4.726	4.958	5.751	7.667	7.896	9.474
0.99	4	3.292	3.681	3.871	4.468	5.463	6.337	6.572	7.885
0.99	5	2.973	3.300	3.557	3.836	4.709	5.482	5.719	6.862
0.99	6	2.751	3.034	3.336	3.666	4.180	4.882	5.120	6.143
0.99	7	2.586	2.838	3.004	3.306	3.787	4.436	5.546	5.607
0.99	8	2.460	2.686	2.899	3.027	3.482	4.090	5.113	5.190
0.99	9	2.359	2.566	2.814	2.994	3.583	3.812	4.766	4.856
0.99	10	2.276	2.468	2.624	2.796	3.356	4.050	4.481	4.580

- Akaike Information Criterion (AIC), Akaike [30]: $\text{AIC} = -2MLL + 2\kappa$.
- Consistent Akaike Information Criterion (CAIC), Bozdogan [31]: $\text{CAIC} = -2MLL + \dfrac{2\kappa n}{n-\kappa-1}$.
- Bayesian Information Criterion (BIC), Schwarz [32]: $\text{BIC} = -2MLL + \kappa\, Log\,(n)$.
- Hannan–Quinn Information Criterion (HQIC), Hannan et al. [33]: $\text{HQIC} = 2\text{Log}\{\text{Log}\,(n)\,[\kappa - 2MLL]\}$, where κ indicates the number of penalized parameters and n refers to the size of the sample.
- Cramér–von Mises test, (CM), Cramer [34]: $CM = \dfrac{1}{12n} - + \sum_{j=1}^{n}\left[\dfrac{2j-1}{2n} - F\,(x_i)\right]^2$.
- Anderson–Darling test (AD), Stephens [35]:

$$AD = -n - \frac{1}{n}\sum_{j=1}^{n}\frac{2j-1}{n}\left[\ln\left(F\,(x_i)\right) + \ln\left(1 - F\,(x_{n+1-i})\right)\right].$$

- The Kolmogorov–Smirnov (K–S) test: $K - S = \text{Sup}_n\,|F_n\,(x) - F\,(x)|$, where $F_n\,(x) = \dfrac{1}{n}\sum_{i=1}^{n} I_{x_i \le x}$, is the function of the empirical distribution and $F(x)$ is the cdf.

We used the maximum likelihood method (MLE) for estimating the parameters of the LBWL distribution, and the negative maximized log-likelihood values (MLL) are also obtained.

The first data

The first data set is: 23, 261, 87, 7, 120, 14, 62, 47, 225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14, 71, 11, 14, 11, 16, 90, 1, 16, 52, 95. These values are the failure times of the air conditioning system of an airplane in hours. Certainly, this data is already studied by Linhart et al. [36] and Shanker et al. [37]. The descriptive statistics for this data are given in Tab. 7. Fig. 1 illustrates the estimated pdf of the air conditioning system data. The results presented in Tab. 9 show that LBWL distribution strongly fits well the air conditioning system data.

Table 7: The descriptive statistics for the airplane data

Min	1	Q_1	12.5	Skewness	1.61
Max	261	Q_3	83	Kurtosis	1.64
Mean	50.6	Standard Deviation	71.89	Range	260
Median	22	Variance	5167.42	Standard error	13.12

The MLEs for α and φ, respectively, are $\hat{\alpha} = 2.4$ and $\hat{\varphi} = 17$, and; hence, $\hat{\mu}_{LBWLD} = \dfrac{2\hat{\varphi}}{\hat{\alpha} - 2} = 85$. Let the needed mean life is demanded to be $\mu_0 = 85$ h and the testing time is $t_0 = 53$ h. Therefore, for $P^* = 0.95$, we have to make a decision about the acceptance of the lot. For the ratio $t/\mu_0 = 0.628$ and $P^* = 0.95$ in Tab. 4, we obtain $m = 30$ when $c = 8$. So, if the number of observed failures before $t_0 = 53$ h is less than or equal to 8, the lot can be accepted with the assured mean lifetime 85 h with probability of 0.95. We found that the number of failures before $t_0 = 53$ h is more than 8. Hence, we reject the lot.

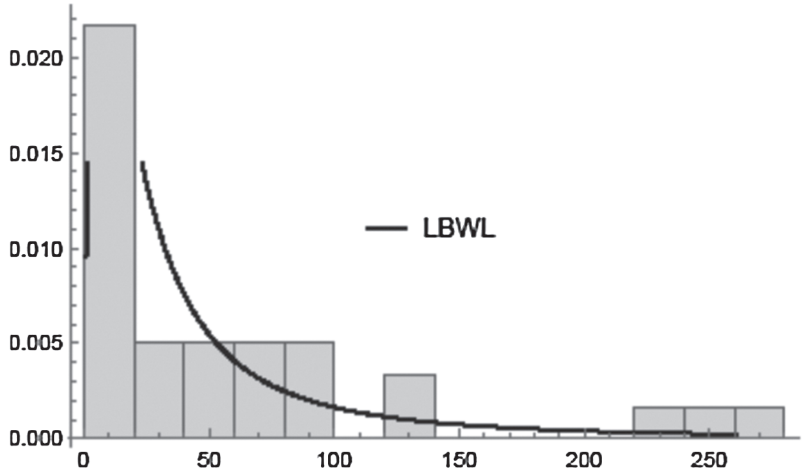

Figure 1: Estimated pdf of the air conditioning system data

The second data

The data set is 0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53.0. The data represents the monthly lifetime to the first failure of 20 little electric carts utilized for delivery and transportation inside a manufacturer with a large facility. This data is also considered by Zimmer et al. [38] and Lio et al. [39] and its asymmetrical distribution is discussed by Al-Omari et al. [13]. The descriptive statistics of the data are given in Tab. 8. Also, Fig. 2 shows the estimated pdf of the electric carts data. However, Tab. 9 revealed that the LBWL distribution strongly fits well this data; the Kolmogorov–Smirnov distance between the observed and fitted distribution functions is 0.077 with the probability of 0.999.

Table 8: The descriptive statistics for the first failure of 20 small electric carts

Min	0.90	Q_1	4.725	Skewness	1.25
Max	53	Q_3	20.125	Kurtosis	0.96
Mean	14.68	Standard Deviation	13.66	Range	52.1
Median	10.75	Variance	186.697	Standard error	3.06

For this second data, we aim to determine the minimum sample sizes, the OC and the minimum ratios based on the estimated parameters from the data.

The MLEs for α and φ, respectively, are $\hat{\alpha} = 4.36445$ and $\hat{\varphi} = 17.99296$; hence, $\hat{\mu}_{LBWLD} = \frac{2\hat{\varphi}}{\hat{\alpha} - 2} = 15.2196$. Let the specified mean lifetime and the testing time are $\mu_0 = 15.2196$ and $t_0 = 9.558$ months, respectively. Therefore, for $P^* = 0.90$ and $d = t_0/\mu_0 = 0.628$, the acceptance number and the corresponding minimum sample sizes are given in Tab. 10, which are found to be $c = 6$ and $m = 20$, respectively. Hence, if the number of failures before $t_0 = 9.558$ months, is less than or equal to 6, we can accept the lot with the assured mean lifetime 15.2196 months with probability 0.90. Since the number of failures before $t_0 = 9.558$ months is 9, then the lot is rejected.

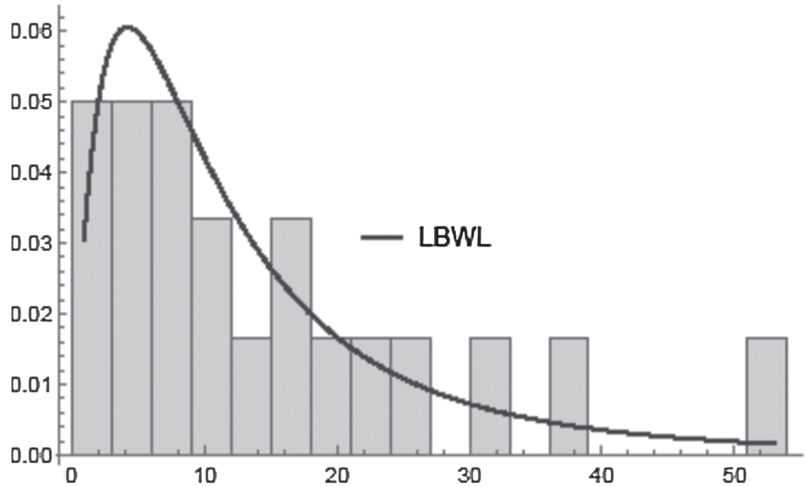

Figure 2: Estimated pdf of the electric carts data

Table 9: The AIC, CAIC, BIC, HQIC, W, A, K–S (P-Value), −2MLL, and the MLEs for the electric carts and air conditioning data

First data	AIC	BIC	CAIC	HQIC	W
	308.493	308.937	311.295	309.390	0.079
	A–D	−2MLL	K–S (P-V)	MLEs	
	0.457	152.247	0.109 (0.867)	$\hat{\alpha} = 2.35254$	$\hat{\varphi} = 17.03789$
Second data	AIC	BIC	CAIC	HQIC	W
	151.966	153.958	152.672	152.355	0.016
	A–D	−2MLL	K–S (P-V)	MLEs	
	0.120	73.983	0.077 (0.999)	$\hat{\alpha} = 4.36445$	$\hat{\varphi} = 17.99296$

Table 10: The minimum sample sizes for $t/\mu_0 = 9.558$, $P^* = 0.90$, $\hat{\alpha} = 4.36445$ and $\hat{\varphi} = 17.99296$ for the electric carts data

c	0	1	2	3	4	5	6	7	8	9	10
n	4	7	10	12	15	17	20	22	25	27	29

The values of OC for the ASP ($n = 20$, $c = 6$, $t/\mu_0 = 0.628$) and the corresponding producer's risk are presented in Tab. 11, while the minimum ratios for this example are given in Tab. 12.

Table 11: Values for the function of the operating characteristic and the corresponding producer's risk for the ASP ($n=20$, $c=6$, $t/\mu_0=0.628$) with $P^*=0.90$, $\hat{\alpha}=4.36445$ and $\hat{\varphi}=17.99296$ for the electric carts data

μ/μ_0	2	4	6	8	10	12
OC	–	0.8767536	0.9993404	0.9999898	0.9999996	1
Producer's risk	–	0.1232464	0.0006596	0.0000102	0.0000004	0

Table 12: Minimum ratio of μ/μ_0 for the acceptability of a lot with producer's risk of 0.05 with $P^*=0.90$, $c=6$, $\hat{\alpha}=4.36445$ and $\hat{\varphi}=17.99296$ for the electric carts data

P^*	c	t/μ_0							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.90	6	2.412	2.669	2.786	3.112	4.057	4.485	5.608	6.729

6 Conclusions

In this article, new ASPs are established based on the Length-biased weighted Lomax distribution for life test truncated at a pre-determined time. The necessary tables of the minimum sample sizes required to guarantee a particular mean life of the test units are obtained. The operating characteristic function values, as well as the related producer's risks are also calculated. The usefulness of the proposed ASPs are investigated based on two real data sets. The applications of the real data sets used in this study showed the usefulness of the proposed acceptance sampling plans. Therefore, the new ASPs are recommended to the researchers. For future study, other ASPs such as group sampling plans and double sampling plans can be considered for the Length-biased weighted Lomax distribution. Also, the suggested ASPs can be considered based other sampling methods as ranked set sampling and its modifications, for more details see Jemain et al. [40], Zamanzade et al. [41], and Haq et al. [42,43].

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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