

Flower Pollination Heuristics for Nonlinear Active Noise Control Systems

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Abstract: Abstract In this paper, a novel design of the flower pollination algorithm is presented for model identification problems in nonlinear active noise control systems. The recently introduced flower pollination based heuristics is implemented to minimize the mean squared error based merit/cost function representing the scenarios of active noise control system with linear/nonlinear and primary/secondary paths based on the sinusoidal signal, random and complex random signals as noise interferences. The flower pollination heuristics based active noise controllers are formulated through exploitation of nonlinear filtering with Volterra series. The comparative study on statistical observations in terms of accuracy, convergence and complexity measures demonstrates that the proposed meta-heuristic of flower pollination algorithm is reliable, accurate, stable as well as robust for active noise control system. The accuracy of the proposed nature inspired computing of flower pollination is in good agreement with the state of the art counterpart solvers based on variants of genetic algorithms, particle swarm optimization, backtracking search optimization algorithm, fireworks optimization algorithm along with their memetic combination with local search methodologies. Moreover, the central tendency and variation based statistical indices further validate the consistency and reliability of the proposed scheme mimic the mathematical model for the process of flower pollination systems.

Keywords: Active noise control; computational heuristics; volterra filtering; flower pollination algorithm

1 Introduction

The trend of exploiting the potential of bio/nature-inspired soft computing techniques is growing in the research community due to their extensive use in optimization problems arising in engineering, science and technology [1–5]. For instance, heat transfer model [6],



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magnetohydrodynamics [7], nonlinear system identification [8], atomic physics [9], nonlinear optics [10], plasma physics [11], and scheduling problem [12,13]. Recently, a new meta-heuristic name as flower pollination algorithm (FPA) is introduced by Yang [14] for efficiently solving nonlinear, constrained, single/multi-objective optimization problems [15–18]. The mathematical model for the process of flow pollination of the flowering plants is used to develop the FPA meta-heuristic. Few potential applications of FPA include photovoltaic system optimization [19], dimension improvement [20], truss structures [21], wireless sensor network [22,23], feature selection [24], control of power systems [25], biometric systems [26], wind speed forecasting [27], image segmentation [28,29], antenna synthesis [30], power flow problem [31,32], neural network optimization [33], chaotic systems identification [34] and bio-impedance models [35]. These illustrative applications are motivations for the authors to exploit the potential of FPA based meta-heuristic for solving the optimization problems of nonlinear active noise control (ANC) systems.

The ANC is a fundamental problem in control engineering and has been studied extensively with both traditional and different local/global optimization techniques [36]. The well-known local search methods used in ANC systems are based on the least mean squares approach [37–44]. The local search algorithms are easy to implement but suffer from premature convergence, i.e., local minima issues. To overcome these issues, different global search based soft computing techniques are proposed such as, genetic algorithms (GAs) [45,46], particle swarm optimization (PSO) [47–49], backtracking search heuristics [50], fireworks algorithm [51], and artificial neural networks [52]. The optimization strength of FPA looks promising to be exploited for ANC problems as an alternate, accurate, reliable, and robust computing paradigm. The innovative contributions of the current study are given as:

- The design of FPA based intelligent computing paradigm is presented for an effective solution of nonlinear ANC systems.
- Mean squared error based merit function with nonlinear Volterra series filtering is formulated.
- The accurate and robust performance of the FPA based ANC for various noise interferences in the case of different primary and secondary path scenarios prove the efficacy of the approach.
- Central tendency and variation based statistical indices validate the consistency and reliability of the proposed scheme.

The rest of the manuscript is prepared as: ANC model is given in Section 2. The design approach is described in Section 3. Section 4 presents the results and the comparative studies with state of the art counterparts, and conclusions are given in Section 5.

2 System Model: ANC

The conventional block diagram of ANC based controller is given in Fig. 1, and the proposed model of nonlinear ANC with FPA is shown in Fig. 2. The algorithm used for filter's coefficients update belongs to a class of nature-inspired heuristics named FPA. In the proposed mechanism, the reference microphone detects the source noise and error microphone measures the output response of a system. When anti-noise and source noise signals combine silence zone is created. The proposed mechanism works on the principle of superposition theorem [53]. Related information for ANC system model can be seen in [54–56] and citations mentioned therein.

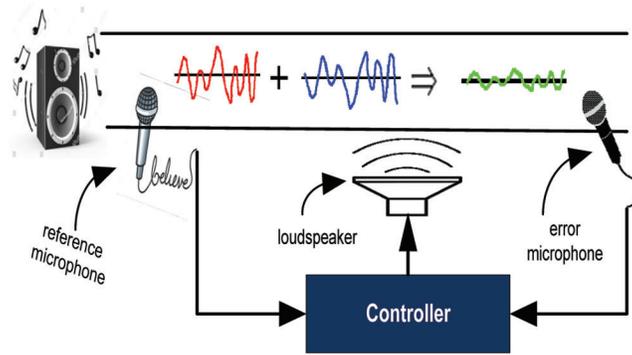


Figure 1: Graphical representation of general analogy of ANC system

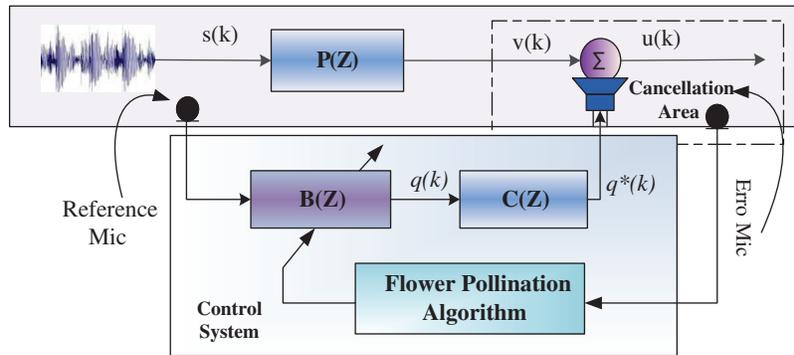


Figure 2: Process block structures of proposed flower pollination algorithm based ANC system

3 System Methodology

The methodology for ANC modeling with FPA consists of two phases; (1) formulating fitness function (2) presenting optimization mechanism based on FPA. The detailed flowchart in terms of process block structure is shown in Fig. 3.

3.1 Modeling for ANC

Block diagram of proposed ANC controller is given in Fig. 2 while the adjustable parameter L -tap weights, i.e., decision variables of optimization algorithm, for ANC system based on nonlinear filtering with Volterra series is given mathematically as:

$$b(k) = [b(0, k), b(1, k), \dots, b(L-2, k), b(L-1, k)] \tag{1}$$

here $b(k)$ represents coefficients Volterra filter at instance k . Let B be a set of contestant solutions of ANC systems, i.e., elements of FPA, a set of k numbers of b as in (1) construct B as follows

$$B(k) = \begin{bmatrix} b_1(k) \\ b_2(k) \\ \vdots \\ b_n(k) \end{bmatrix} = \begin{bmatrix} b_1(0, k) & b_1(1, k) & \dots & b_1(L-1, k) \\ b_2(0, k) & b_2(1, k) & \dots & b_2(L-1, k) \\ \vdots & \vdots & \ddots & \vdots \\ b_n(0, k) & b_n(1, k) & \dots & b_n(L-1, k) \end{bmatrix} \tag{2}$$

The input noise interference or source signal $s(k)$ and output of nonlinear adaptive Volterra filtering $b(k)$ with length $L = 20$, i.e., VF-T1, for the population B, then ANC system using (1) and (2) is written as:

$$\begin{bmatrix} q_1(k) \\ q_2(k) \\ \vdots \\ q_{P-L/4}(k) \\ q_{P-L/4+1}(k) \\ q_{P-L/4+2}(k) \\ \vdots \\ q_P(k) \end{bmatrix} = \begin{bmatrix} b_1(0,k) & b_1(1,k) & \dots & b_1(L-1,k) \\ b_2(0,k) & b_2(1,k) & \dots & b_2(L-1,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_{P-L/4}(0,k) & b_{P-L/4}(1,k) & \dots & b_{P-L/4}(L-1,k) \\ b_{P-L/4+1}(0,k) & b_{P-L/4+1}(1,k) & \dots & b_{P-L/4+1}(L-1,k) \\ b_{P-L/4+2}(0,k) & b_{P-L/4+2}(1,k) & \dots & b_{P-L/4+2}(L-2,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_P(0,k) & b_P(1,k) & \dots & b_P(L-1,k) \end{bmatrix} \begin{bmatrix} s(k) \\ s(k-1) \\ \vdots \\ s(k-L/4+1) \\ s^2(k) \\ s^2(k-1) \\ \vdots \\ s^2(k-L/4+1) \end{bmatrix} \quad (3)$$

Accordingly, Volterra filtering of type 2 (VF-T2) with $L = 35$ for the ANC system using Eqs. (1) and (2) is written as:

$$\begin{bmatrix} q_1(k) \\ q_2(k) \\ \vdots \\ q_{P-L/7}(k) \\ q_{P-L/7+1}(k) \\ q_{P-L/7+2}(k) \\ \vdots \\ q_P(k) \end{bmatrix} = \begin{bmatrix} b_1(0,k) & b_1(1,k) & \dots & b_1(L-1,k) \\ b_2(0,k) & b_2(1,k) & \dots & b_2(L-1,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_{P-L/7}(0,k) & b_{P-L/7}(1,k) & \dots & b_{P-L/7}(L-1,k) \\ b_{P-L/7+1}(0,k) & b_{P-L/7+1}(1,k) & \dots & b_{P-L/7+1}(L-1,k) \\ b_{P-L/7+2}(0,k) & b_{P-L/7+2}(1,k) & \dots & b_{P-L/7+2}(L-2,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_P(0,k) & b_P(1,k) & \dots & b_P(L-1,k) \end{bmatrix} \begin{bmatrix} s(k) \\ s(k-1) \\ \vdots \\ s(k-L/7+1) \\ s^2(k) \\ s^2(k-1) \\ \vdots \\ s^2(k-L/7+1) \end{bmatrix} \quad (4)$$

Similarity, for Volterra filtering of type 3 (VF-T3) in case of the length of the Volterra filter $L = 65$ in ANC system is given by:

$$\begin{bmatrix} q_1(k) \\ q_2(k) \\ \vdots \\ q_{P-L/10}(k) \\ q_{P-L/10+1}(k) \\ q_{P-L/10+2}(k) \\ \vdots \\ q_P(k) \end{bmatrix} = \begin{bmatrix} b_1(0,k) & b_1(1,k) & \dots & b_1(L-1,k) \\ b_2(0,k) & b_2(1,k) & \dots & b_2(L-1,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_{P-L/10}(0,k) & b_{P-L/10}(1,k) & \dots & b_{P-L/10}(L-1,k) \\ b_{P-L/10+1}(0,k) & b_{P-L/10+1}(1,k) & \dots & b_{P-L/10+1}(L-1,k) \\ b_{P-L/10+2}(0,k) & b_{P-L/10+2}(1,k) & \dots & b_{P-L/10+2}(L-2,k) \\ \vdots & \vdots & \vdots & \vdots \\ b_P(0,k) & b_P(1,k) & \dots & b_P(L-1,k) \end{bmatrix} \begin{bmatrix} s(k) \\ s(k-1) \\ \vdots \\ s(k-L/10+1) \\ s^2(k) \\ s^2(k-1) \\ \vdots \\ s^2(k-L/10+1) \end{bmatrix} \quad (5)$$

In case of $[c_1, c_2, \dots, c_L]^T$ are the response of secondary path transfer function $C(z)$ with L -tap weights/coefficients is written as:

$$\begin{bmatrix} q_1^*(k) \\ q_2^*(k) \\ \vdots \\ q_p^*(k) \end{bmatrix} = \begin{bmatrix} q_1(k) & q_1(k-1) & \dots & q_1(k-L+1) \\ q_2(k) & q_2(k-1) & \dots & q_2(k-L+1) \\ \vdots & \vdots & \ddots & \vdots \\ q_p(k) & q_p(k-1) & \dots & q_p(k-L+1) \end{bmatrix} \begin{bmatrix} c_1(k) \\ c_2(k) \\ \vdots \\ c_L(k) \end{bmatrix} \quad (6)$$

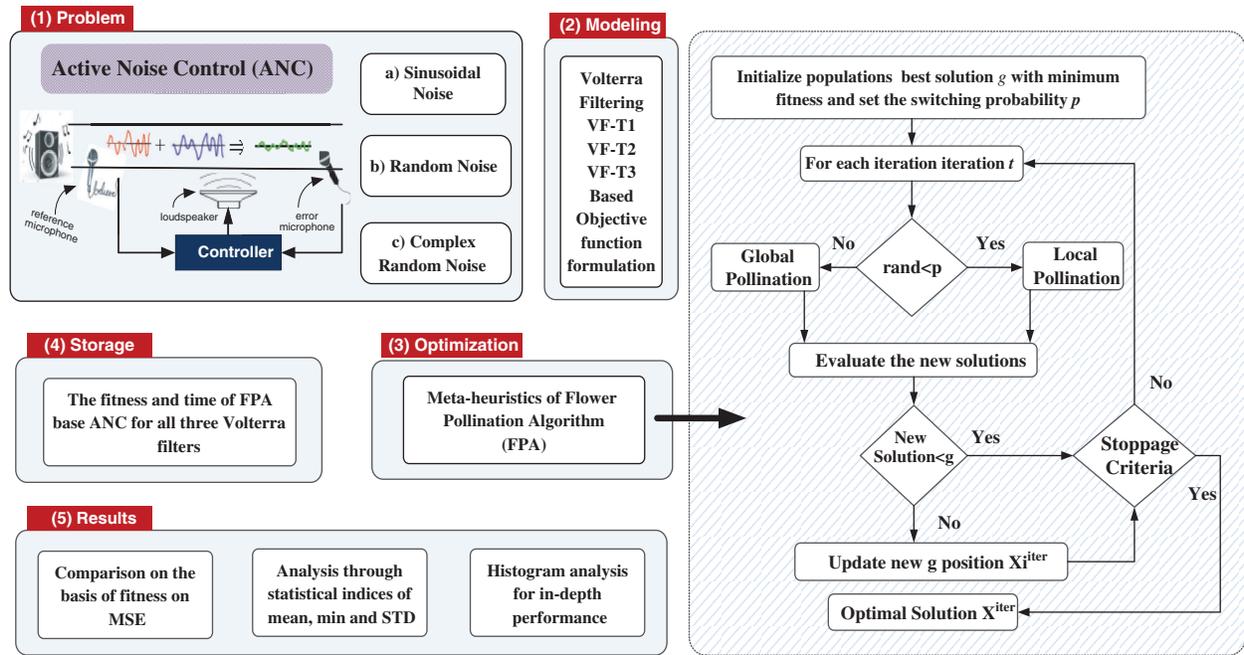


Figure 3: Process blocks for flower pollination based heuristics for ANC system

The fitness or merit function for ANC model is given as:

$$u_j = \frac{1}{L} \sum_{k=0}^{L-1} U_j(k), \quad j=1,2,\dots,p \quad (7)$$

for

$$U_j(k) = \left(v(k) - q_j^*(k) \right)^2, \quad j=1,2,\dots,p \quad (8)$$

Here $v(k)$ and $q^*(k)$ are the desired and estimated responses of the primary and secondary paths, respectively. and is the response of the secondary path. Eq. (7) equivalently represented as:

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} = \frac{1}{L} \begin{bmatrix} U_1(k) + U_1(k-1) + \dots + U_1(k-L+1) \\ U_2(k) + U_2(k-1) + \dots + U_2(k-L+1) \\ \vdots \\ U_p(k) + U_p(k-1) + \dots + U_p(k-L+1) \end{bmatrix} \quad (9)$$

In the case of perfect model, one has fitness function $u=0$, so optimization mechanism is exploited for tuning of fitness (7), such that the magnitude of residual error of the ANC system is reduced substantially. In the next section, optimization of ANC system with FPA is presented.

3.2 Optimization: Flower Pollination Algorithm

The FPA is a mathematical model inspired by the process of pollination dynamics in flowers during the reproduction mechanism [14]. Yang et al. [15] introduced FPA in early 2012 as an alternate optimization solver for both global and local search. Most of the flower plants reproduction strategy is based on the pollination process in which pollen is transferred from one plant to another plant of flowers by butterflies, insects, birds, and bees. The pollination process is segmented into biotic and abiotic types. Biotic type flower pollination is also called cross-pollination, i.e., the main form of flowering pollination, in which pollens are transferred by insects and birds. The majority of flowering plants use biotic pollination procedures for pollen spread over a long distance via Lévy flights. While in abiotic pollination, the flowering plants does not required pollinators and 10% of total flowering plants follow such pollination. In abiotic, the distance covered by the pollinators is short and such types of actions are considered as local search. Biotic and abiotic characteristics of pollinators are used to design an optimization algorithm called FPA. The four basics rules of FPA based heuristic are introduced by Yang in 2012 as follows:

- Rule 1.** Global pollination carried out via biotic/cross pollination procedures with the help of insects, birds and bees to transport the pollens.
- Rule 2.** Abiotic or self-pollination process is adapted for efficient local search.
- Rule 3.** Flower fidelity process based reproduction probability.
- Rule 4.** Switching probability between 0 and 1 is exploited for feasible local and global pollination process [21].

The impressive swarm based optimization characteristics of FPA is exploited by the scholars from different fields [57–60]. The mathematical mechanism of FPA bases of these four rules are given as follows [25]:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + L(\lambda)(\mathbf{x}_i^t - \mathbf{g}^*) \quad (10)$$

where, \mathbf{x}_i^t stands for pollen vector for i th candidate solution at iteration t , \mathbf{g}^* represents best solution at current iteration and L stands for Lévy flight represented as:

$$L(\lambda) = \frac{\lambda \Gamma(\lambda) \sin(\lambda/2)}{\pi s^{1+\lambda}}, \quad (s \gg s_0 > 0), \quad (11)$$

here $\Gamma(\lambda)$ represents the gamma function while distribution is effective for $s > 0$ and $\lambda = 1.5$. The local search with FPA is represented as:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mu(\mathbf{x}_j^t + \mathbf{x}_k^t), \quad (12)$$

here, μ be the uniform distribution between 0 and 1, \mathbf{x}_j^t and \mathbf{x}_k^t are j th and k th pollens vectors from different flowers of the same plant, respectively. In this study, the meta-heuristics of FPA based ANC is implemented as shown in Fig. 3, while the pseudocode is presented in Algorithm 1.

4 Results and Discussion

The results of detailed ANC experimentations are presented here for multiple independent executions of the FPA. Three ANC problems are implemented based on different lengths (L) of

Volterra filter (VF), i.e., $L=20,35$, and 65 in the case of VF-T1, VF-T2, and VF-T3, respectively. The FPA based ANC system are evaluated for sinusoidal/random/complex random noise interferences having linear primary path (LPP), nonlinear primary path (NPP), linear secondary path (LSP) and nonlinear secondary path (NSP). The transfer function for LPP is:

$$P(z) = z^{-5} - 0.3z^{-6} + 0.2z^{-7} \quad (13)$$

While, in case of LSP, the transfer function is defined as

$$C(z) = z^{-2} + 1.5z^{-3} - z^{-4} \quad (14)$$

The NPP transfer function is given as:

$$x(k) = s(k-3) - 0.3s(k-4) + 0.2s(k-5) \quad (15)$$

$$v(k) = x(k-2) + 0.08[x(k-2)]^2 + 0.04[x(k-2)]^3 \quad (16)$$

Let $q^*(k)$, i.e., an anti-noise signal is generated by the NSP as:

$$r(k) = 0.06 \tanh(1.5q(k)) \quad (17)$$

$$q^*(k) = r(k-2) + 1.5r(k-3) - r(k-4) \quad (18)$$

The simulations are conducted in Matlab R2017b running under Windows 10 environment on DESKTOP-73HVB7M, with Intel(R) Core(TM) i7-4790 CPU@3.60 GHz, 16-GB RAM.

4.1 Problem 1: ANC Model for Sinusoidal, Random and Complex Random Signals of VF-T1

In this problem, FPA based ANC system is exploited for Case 1: ANC for LPP and NSP (ANC-LPP-NSP), Case 2: ANC for NPP and LSP (ANC-NPP-LSP) and, Case 3: ANC for NPP and NSP (ANC-NPP-NSP). The ANC primary/secondary paths are defined in Eqs. (13)–(18).

Reliable inferences on the outcome of ANC are presented for hundred independent trials of the FPA and result in the form of graphical representation of the statistics are given in Fig. 4 for different cases of sinusoidal noise interference. While the results in case of random and complex random noise scenarios are presented in Fig. 5. The results illustrated in Figs. 4 and 5 show that the proposed FPA based outcomes are effective for reliable treatment of ANC systems having LPP, LSP, NPP and NSPs.

The performance of the FPA is further examined through histogram plots and statistical measures of minimum (MIN), mean, and standard deviation (STD). The histogram plots are provided in Fig. 6 for all cases of Problem 1. While the statistical operators are given in Tab. 1 and one may observe that the results of random VF-T1 are relatively better than that of sinusoidal, but a bit degraded to complex random. Moreover, the small STD values further validate the precision of the proposed FPS based ANC controllers.

The computational complexity of the FPA based ANC controllers is evaluated via mean time of execution required for the optimization and results for mean along with STD are tabulated in Tab. 2. It is observed that the average time lies around 100 ± 50 , 85 ± 80 and 80 ± 50 for sinusoidal VF-T1, random VF-T1, and complex random VF-T1 cases.

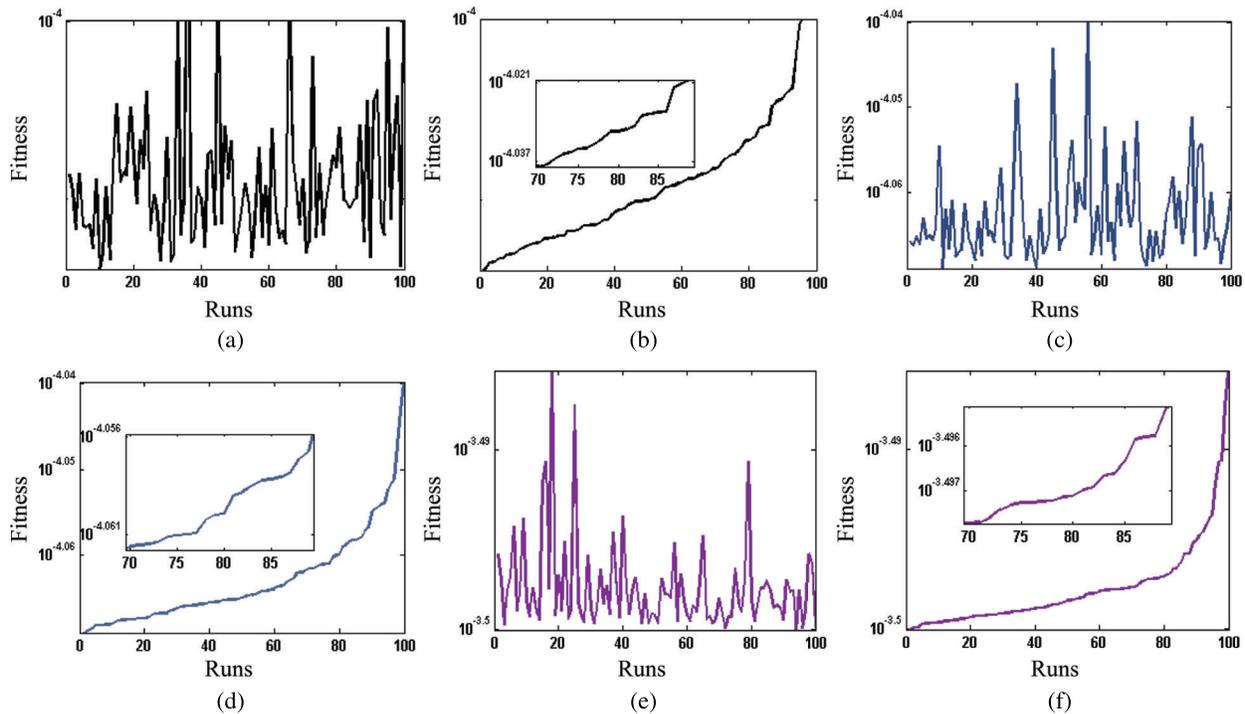


Figure 4: Results of fitness achieved for 100 runs of flower pollination algorithm for ANC with VF-T1 filtering each sinusoidal noise scenario of problem 1. (a) Un-sorted results of sinusoidal noise based ANC-LPP-NSP. (b) Sorted results for sinusoidal noise based ANC-LPP-NSP. (c) Un-sorted results of sinusoidal noise based ANC-NPP-LSP. (d) Sorted results for sinusoidal noise based ANC-NPP-LSP. (e) Un-sorted results of sinusoidal noise based ANC-NPP-NSP. (f) Sorted results for sinusoidal noise based ANC-NPP-NSP

4.2 Problem 2: ANC Model for Sinusoidal, Random and Complex Random Signals of VF-T2

In problem 2, FPA based ANC system is implemented for Case 1: ANC for LPP and NSP (ANC-LPP-NSP), Case 2: ANC for NPP and LSP (ANC-NPP-LSP) and, Case 3: ANC for NPP and NSP (ANC-NPP-NSP).

Graphical representation of the statistical outcomes for hundred independent trials of the FPA based ANC for each case of different noise interferences are given in Fig. 7. The statistical operators are given in Tab. 3. The fitness values of FPA based ANC system for ANC-LPP-NSP, ANC-NPP-LSP and ANC-NPP-NSP are around 10^{-05} to 10^{-06} for sinusoidal, 10^{-04} to 10^{-05} for random and 10^{-04} to 10^{-06} , for complex random noise interferences of VF-T2. The results for different scenarios presented in Fig. 8 show that the proposed FPA controllers are effective for the reliable treatment of ANC systems.

The performance of the FPA based ANC systems is further investigated through histogram plots and STATISTICAL operators and it is observed that the results of random VF-T2 are better than that of complex random but inferior to sinusoidal VF-T2. One may decipher that relatively better accuracy is attained for ANC system based sinusoidal and random noise signals. While the results of ANC with sinusoidal noise are consistently found better than random noise scenarios.

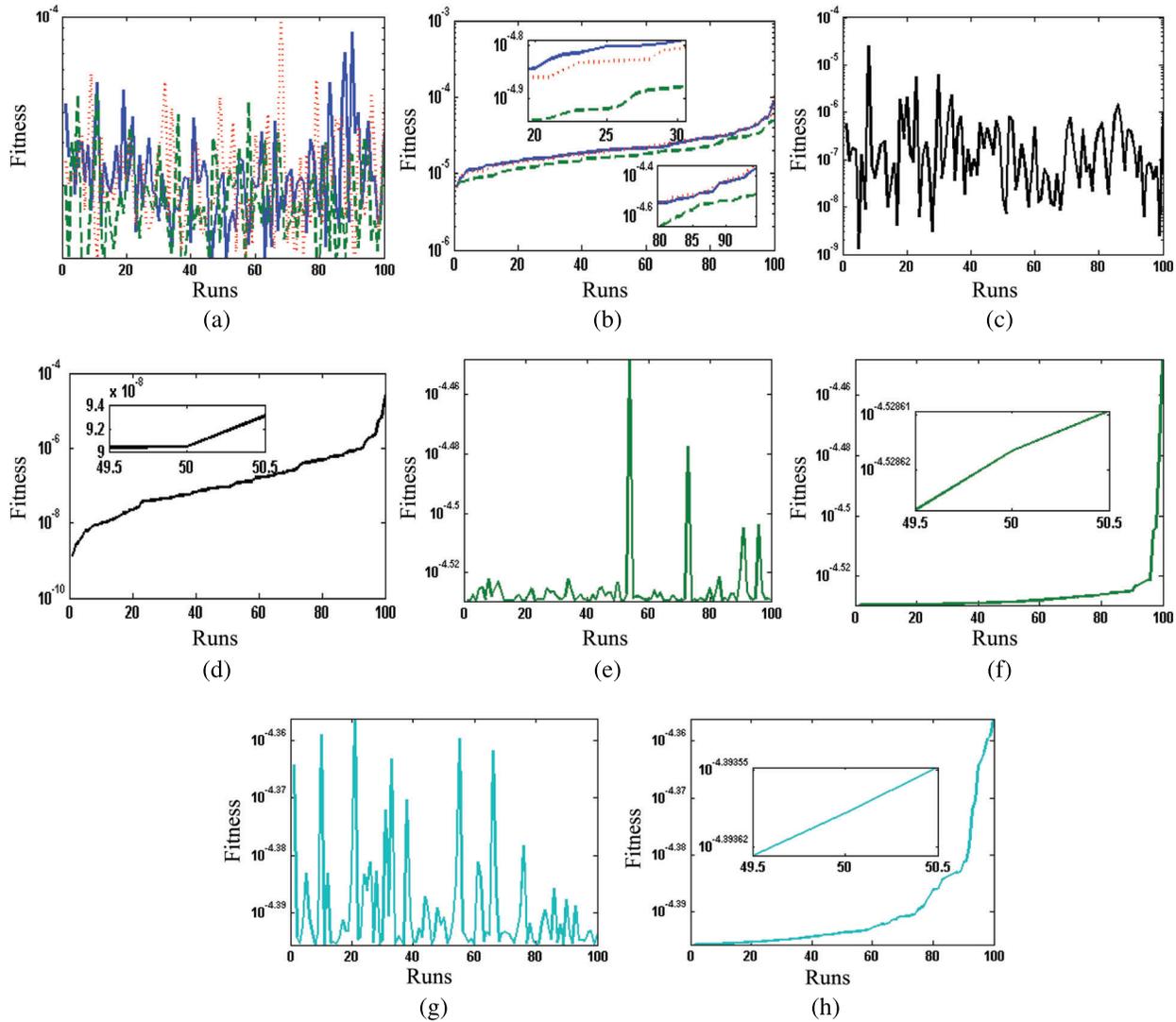


Figure 5: Results of fitness achieved for 100 runs of flower pollination algorithm for ANC with VF-T1 filtering in case of random and complex random. (a) Un-sorted results of all three scenarios of random noise. (b) Sorted results of all three scenarios of random noise. (c) Unsorted results of complex random noise based ANC-LPP-NSP. (d) Sorted results of complex random noise based ANC-LPP-NSP. (e) Unsorted results of complex random noise based ANC-NPP-LSP. (f) Sorted results of complex random noise based ANC-NPP-LSP. (g) Unsorted results of complex random noise based ANC-NPP-NSP. (h) Sorted results of complex random noise based ANC-NPP-NSP

The computational complexity analyses for the optimization of FPA based ANC is evaluated based on mean time and STD. The results of complexity are given in [Tab. 4](#) and analysis show that the average time lies around 75 ± 65 for sinusoidal, 115 ± 110 for random and 80 ± 70 for complex random noise interferences for ANC system with VF-T2 filtering cases.

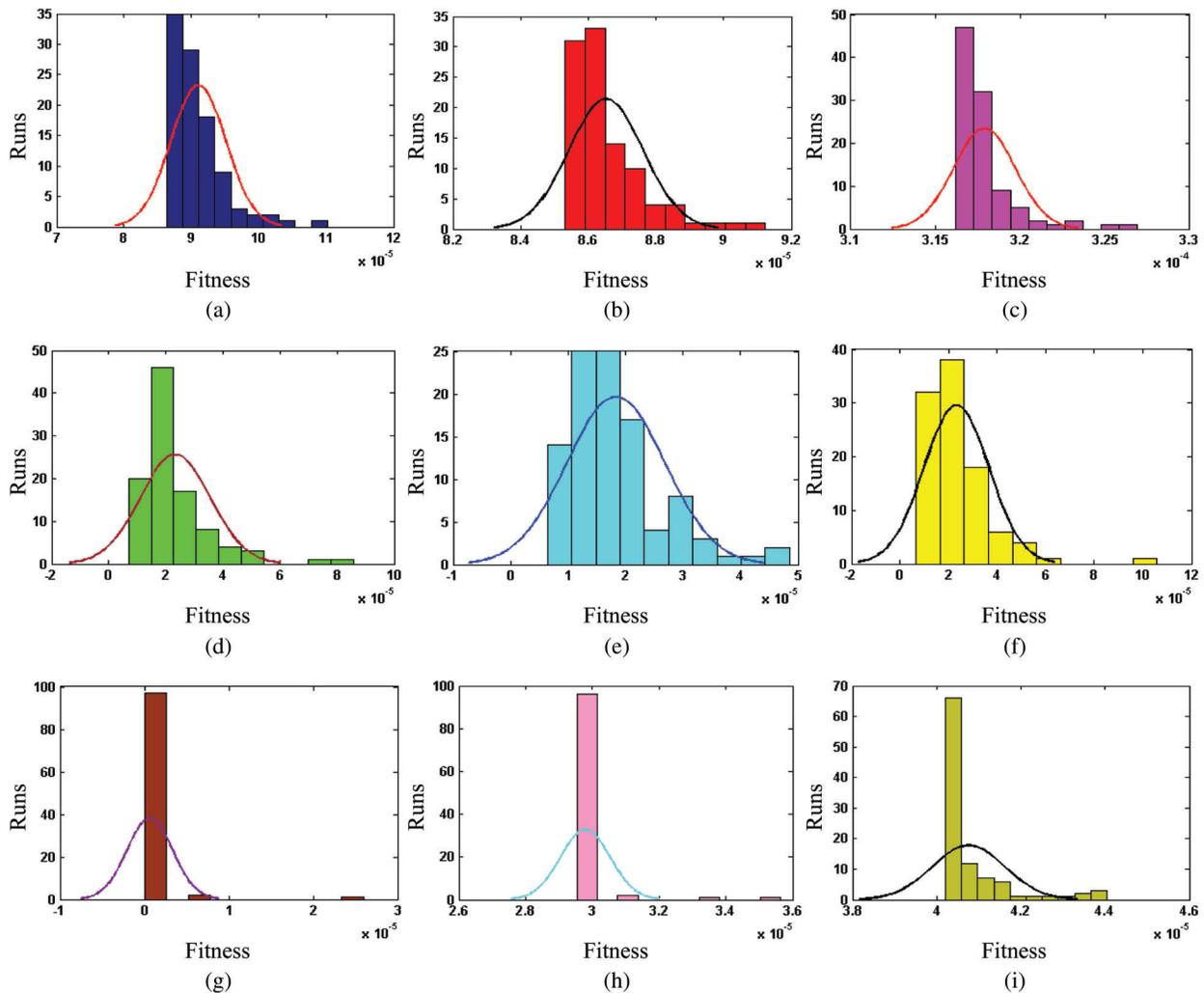


Figure 6: Comparison with histogram analysis for 100 runs of flower pollination algorithm for ANC system for each noise scenario of problem 1. (a) Results of sinusoidal noise based ANC-LPP-NSP with VF-T1. (b) Results of sinusoidal noise based ANC-NPP-LSP with VF-T1. (c) Results of sinusoidal noise based ANC-NPP-NSP with VF-T1. (d) Results of random noise based ANC-LPP-NSP with VF-T1. (e) Results of random noise based ANC-NPP-LSP with VF-T1. (f) Results of random noise based ANC-NPP-NSP with VF-T1. (g) Results of complex random noise based ANC-LPP-NSP with VF-T1. (h) Results of complex random noise based ANC-NPP-LSP with VF-T1. (i) Results of complex random noise based ANC-NPP-NSP with VF-T1

4.3 Problem 3: ANC Model for Sinusoidal, Random and Complex Random Signals of VF-T3

In this problem, FPA based ANC system is exploited for different primary/secondary path scenarios. The proposed FPA based ANC are conducted for hundred independent trials and graphical representation of the statistics in sort and unsorted plots are given in Fig. 9. The histograms are provided in Fig. 10 for each case of problem 3, while the statistics are provided in Tab. 5. It is observed that the results of random VF-T3 are better than that of complex random but inferior to sinusoidal VF-T3. Relatively better outcomes in term of accuracy are observed for

FPA based ANC system in case of sinusoidal and random noise interferences. The comparison shows that sinusoidal noise interference based ANC with VF-T3 are consistently superior than each random noise scenario.

Table 1: Comparison through statistical operators for flower pollination algorithm based ANC system for each scenarios of problem 1

ANC system with VF-TI	Index	Statistical indices		
		Min	Mean	STD
Sinusoidal noise	Case 1	8.64E-05	9.10E-05	4.10E-06
	Case 2	8.53E-05	8.65E-05	1.10E-06
	Case 3	3.16E-04	3.18E-04	1.83E-06
Random noise	Case 1	6.97E-06	2.32E-05	1.23E-05
	Case 2	6.42E-06	1.83E-05	8.55E-06
	Case 3	6.55E-06	2.33E-05	1.34E-05
Complex Random noise	Case 1	1.32E-09	6.43E-07	2.71E-06
	Case 2	2.95E-05	2.98E-05	7.43E-07
	Case 3	4.02E-05	4.08E-05	8.62E-07

Table 2: Complexity of flower pollination algorithm for ANC cases of problem 1 under different noise interferences

Index	Sinusoidal		Random		Complex random	
	Mean	STD	Mean	STD	Mean	STD
Case 1	93.479	0.333	85.145	0.386	76.974	2.959
Case 2	52.696	0.349	85.864	0.498	56.501	0.371
Case 3	50.839	0.297	80.377	0.217	56.708	0.355

The computational complexity analyses for the optimization of FPA based ANC is also evaluated based on mean execution time and STD, and results are provided in [Tab. 6](#). The analysis show that the average time lies around 15 ± 10 for sinusoidal, 30 ± 15 for random and 60 ± 40 for complex random noise interferences of ANC with VF-T3 cases, respectively.

The computational complexity of FPA based ANC is examined with counterpart optimization solvers. The computational complexity on mean execution time index of BSA and BSA-SQP results for sinusoidal noise signal are lie around 800 ± 50 , 1000 ± 60 and 780 ± 30 for FIR, VF-1 and VF-2, respectively, while 750 ± 100 and 900 ± 90 for random and complex random noise signals, respectively [50]. The complexity of variants of GAs and its moments combination of IPA, i.e., GA-IPM-1 to 12, for ANC with FIR filter with sinusoidal, random and complex random noise interference is 40 ± 10 , 60 ± 5 , and 90 ± 5 , respectively. Computational complexity of nature inspired heuristics of PSO and its hybridized methodologies with PSO-IP, PSO-AS, PSO-SQP, and PSO-NM for ANC system based on FIR filtering for all three noise variations is around 7 ± 4 [49]. The computational complexity on mean values of respective FWA, enhanced FWA and adaptive FWA are around 250 ± 40 , 145 ± 40 , and 100 ± 20 for sinusoidal noise signal, 530 ± 20 , 340 ± 20 , and 220 ± 10 for random noise signal and 835 ± 17 , 540 ± 25 , and 352 ± 5 for complex random

signal [51]. One can quite evidently observe that the complexity requirements of FPA based ANC system is relatively superior from GAs, BSA and FWA along with their memetic combination with local search methodologies. While the results of PSO based variants are efficient from rest but these results are for ANC systems based on FIR filtering having relatively inferior in accuracy from FPA based ANC.

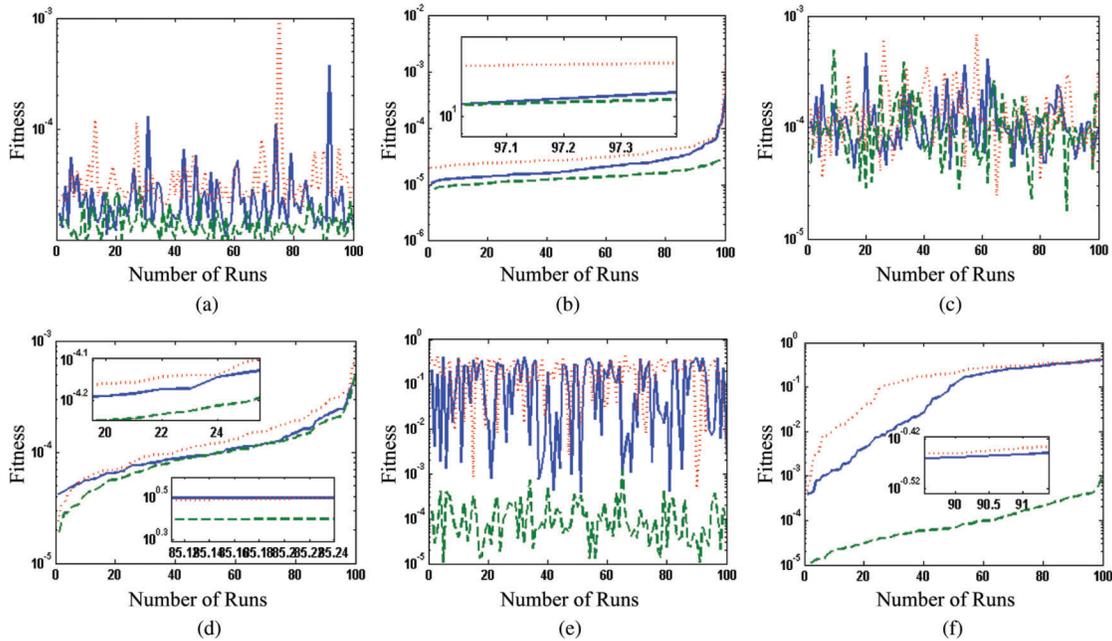


Figure 7: Results of fitness achieved for 100 runs of flower pollination algorithm for ANC with VF-T2 filtering each noise scenario of problem 2. (a) Sinusoidal noise all three cases. (b) Sinusoidal sorted all three cases. (c) Random noise all three cases. (d) Random sorted all three cases. (e) Complex random all three cases. (f) Complex random sorted cases

Table 3: Comparison through statistical operators for flower pollination algorithm based ANC system for each scenarios of problem 2

ANC system with VF-T2	Index	Statistical indices		
		Min	Mean	STD
Sinusoidal noise	Case 1	1.07E-05	2.73E-05	3.91E-05
	Case 2	8.19E-06	1.42E-05	4.32E-06
	Case 3	2.00E-05	4.79E-05	1.39E-04
Random noise	Case 1	4.19E-05	1.17E-04	7.34E-05
	Case 2	1.79E-05	1.07E-04	7.15E-05
	Case 3	2.44E-05	1.45E-04	1.04E-04
Complex random noise	Case 1	3.96E-04	1.49E-01	1.42E-01
	Case 2	1.08E-05	1.37E-04	1.71E-04
	Case 3	4.78E-04	2.03E-01	1.31E-01

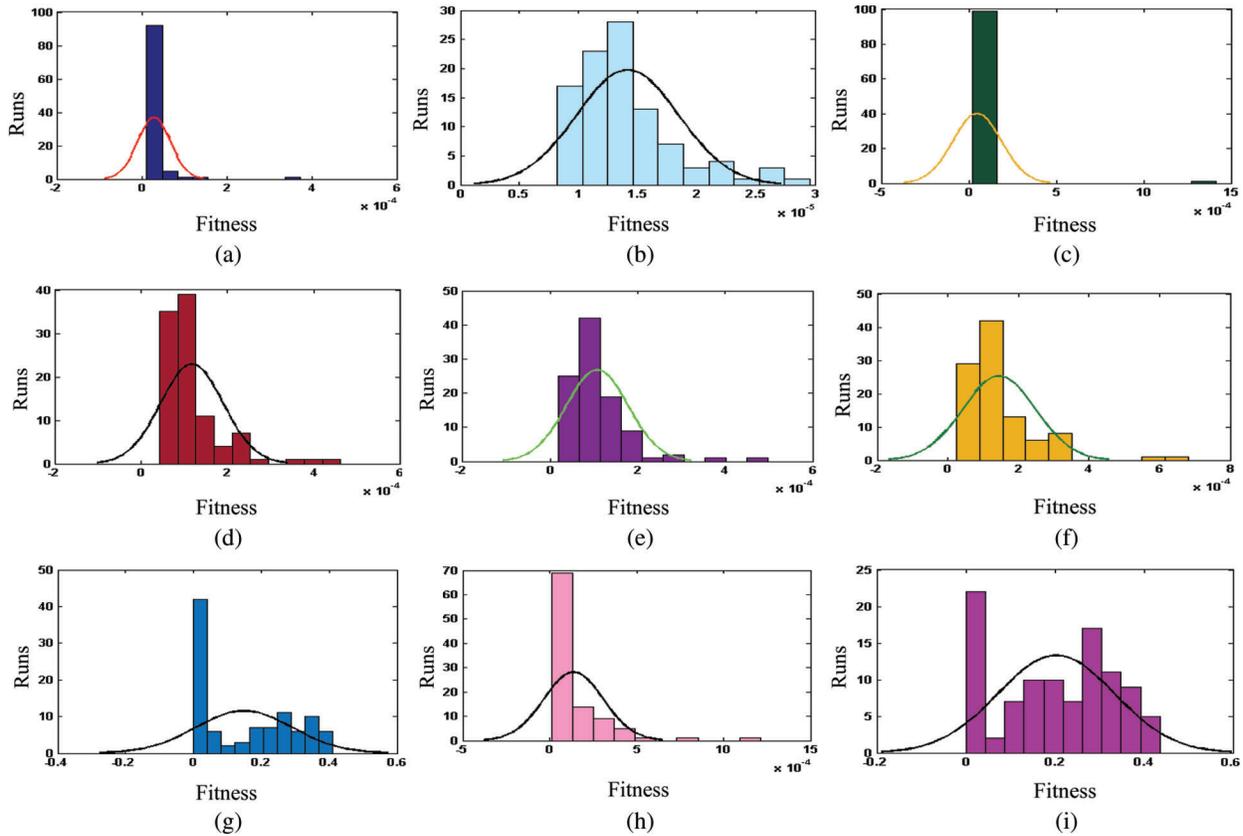


Figure 8: Comparison with histogram analysis for 100 runs of flower pollination algorithm for ANC system for each noise scenario of problem 2. (a) Results of sinusoidal noise based ANC-LPP-NSP with VF-T2. (b) Results of sinusoidal noise based ANC-NPP-LSP with VF-T2. (c) Results of sinusoidal noise based ANC-NPP-NSP with VF-T2. (d) Results of random noise based ANC-LPP-NSP with VF-T2. (e) Results of random noise based ANC-NPP-LSP with VF-T2. (f) Results of random noise based ANC-NPP-NSP with VF-T2. (g) Results of complex random noise based ANC-LPP-NSP with VF-T2. (h) Results of complex random noise based ANC-NPP-LSP with VF-T2. (i) Results of complex random noise based ANC-NPP-NSP with VF-T2

Table 4: Complexity of flower pollination algorithm for ANC cases of problem 2 under different noise interferences

Index	Sinusoidal noise		Random noise		Complex random noise	
	Mean	STD	Mean	STD	Mean	STD
Case 1	71.902	1.456	113.904	1.405	72.157	1.205
Case 2	71.008	1.381	114.081	1.615	79.680	1.264
Case 3	68.994	1.167	113.046	0.856	79.737	1.439

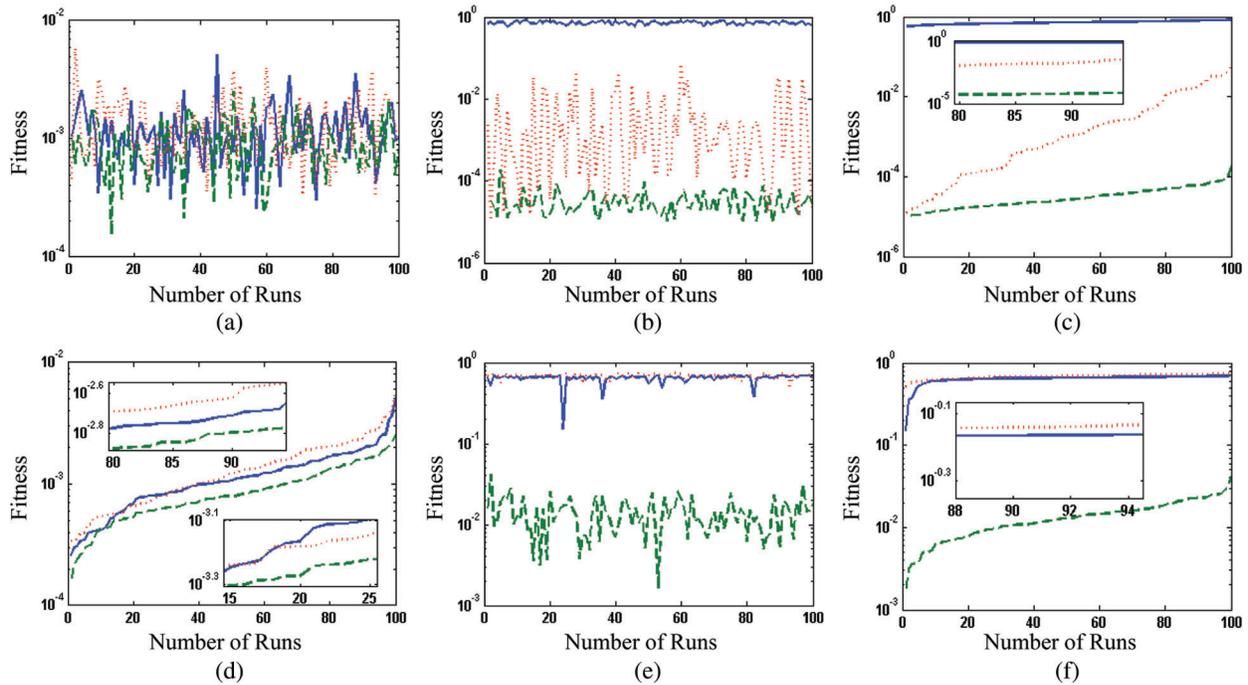


Figure 9: Results of fitness achieved for 100 runs of flower pollination algorithm for ANC with VF-T3 filtering each noise scenario of problem 3. (a) Sinusoidal noise all three cases V3. (b) Sinusoidal noise sorted all cases V3. (c) Random noise all three cases V3. (d) Random noise sorted all cases V3. (e) Complex random noise NP-LSP V3. (f) Complex random sorted NP-LSP V3

4.4 Comparative Study with Reported Results

Comparative studies of FPA results for ANC systems are made with reported studies based on adaptive genetic algorithm AGA [58], variants of memetic combination of GAs with interior-point (IP) algorithm, i.e., GA-IPA-1, to GA-IPA-12 [59], nature-inspired heuristic via particle swarm optimization (PSO) and its hybrid with IP (PSO-IP), active-set (PSO-AS), sequential quadratic programming (PSO-SQP) and Nelder-Mead (PSO-NM) methods [49], backtracking search optimization algorithm (BSA) and its hybrid with SQP (BSA-SQP) [50], and variants of fireworks algorithm (FWA) [51]. One may decipher from all these reported results and statistical observation that the accuracy and convergence of FPA are in good agreement with state of the art methodologies for all three ANC problems. An additional advantage of FPA based ANC is that optimization of decision variables is based on a standalone algorithm with the ability of both local and global search whereas most of the reported results are based on hybrid methodologies.

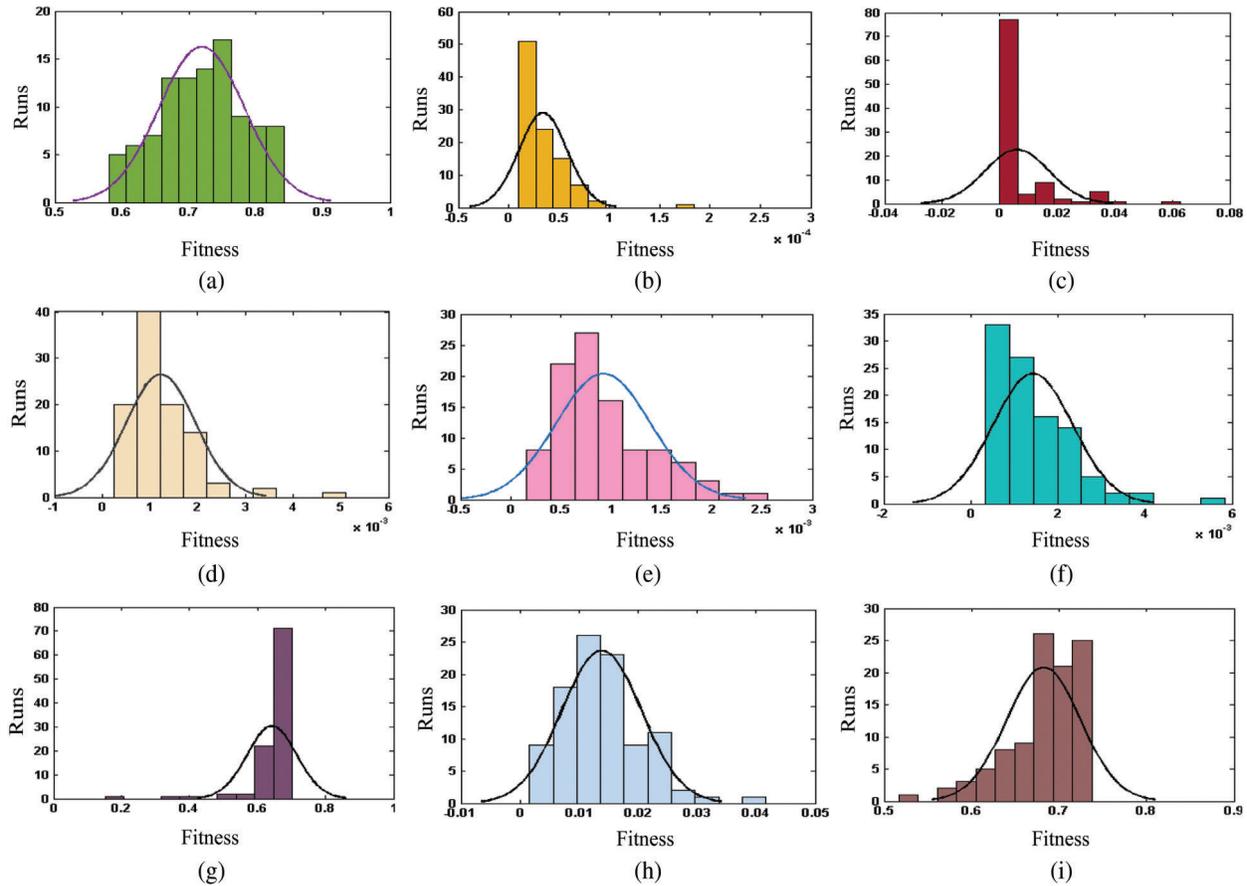


Figure 10: Comparison with histogram analysis for 100 runs of flower pollination algorithm for ANC system for each noise scenario of problem 3. (a) Results of sinusoidal noise based ANC-LPP-NSP with VF-T3. (b) Results of sinusoidal noise based ANC-NPP-LSP with VF-T3. (c) Results of sinusoidal noise based ANC-NPP-NSP with VF-T3. (d) Results of random noise based ANC-LPP-NSP with VF-T3. (e) Results of random noise based ANC-NPP-LSP with VF-T3. (f) Results of random noise based ANC-NPP-NSP with VF-T3. (g) Results of complex random noise based ANC-LPP-NSP with VF-T3. (h) Results of complex random noise based ANC-NPP-LSP with VF-T3. (i) Results of complex random noise based ANC-NPP-NSP with VF-T3

Table 5: Comparison through statistical operators for flower pollination algorithm based ANC system for each scenarios of problem 3

ANC system with VF-TI	Index	Statistical indices		
		MIN	Mean	STD
Sinusoidal noise	Case 1	5.81E-01	7.19E-01	6.40E-02
	Case 2	9.28E-06	3.40E-05	2.41E-05
	Case 3	1.26E-05	5.99E-03	1.11E-02
Random noise	Case 1	2.57E-04	1.23E-03	7.33E-04
	Case 2	1.55E-04	9.20E-04	4.70E-04
	Case 3	3.40E-04	1.43E-03	9.16E-04
Complex random noise	Case 1	1.51E-01	6.42E-01	7.25E-02
	Case 2	1.64E-03	1.38E-02	6.75E-03
	Case 3	5.16E-01	6.82E-01	4.25E-02

Algorithm 1: Pseudocode for flower pollination algorithm for nonlinear ANC

```

1 Flower pollination Algorithm
2 Begin:
3 Initialization of flower population as defined
4  $B(k)$  with  $n$  entries in equation (1) having
5 probability of switching  $P$  between 0 and 1
6 The dimensional  $d$  equal to  $L$  tap weights in
7 Volterra filters, decision variable in the the
8 objective function  $u_j$  as defined in equation (7).
9 Initialize total iterations  $T$ . The pollen vector
10 with minimum  $u_j$  is the initial best solution:  $\mathbf{g}^*$ 
11 While current iteration  $t <$  maximum iterations  $T$ .
12 do
13 for  $i = 1 : n$ 
14 if generated random number is less than  $p$ .
15 Calculate  $d$ -dimensional Lévy vector as defined in
16 equation (11) that depend on  $L$  tap weights in
17 Volterra filter.
18 Conduct global Pollination using equation (10) as
19  $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + L(\lambda) (\mathbf{x}_j^t - \mathbf{g}^*)$ 
20 else
21 conduct local Pollination using equation (12) as:
22  $\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \varepsilon (\mathbf{x}_j^t + \mathbf{x}_k^t)$ 
23 end if
24 The calculated  $\mathbf{x}_i^{t+1}$  is better than  $\mathbf{x}_i^t$  then
25  $\mathbf{x}_i^t = \mathbf{x}_i^{t+1}$ 
26 end for
27 Update pollen vector with minimum  $u_j$  current best
28 solution:  $\mathbf{g}^*$ 
29 Update iteration counter
30 end while
31 End

```

Figure 11: Pseudocode for flower pollination algorithm for nonlinear ANC**Table 6:** Complexity of flower pollination algorithm for ANC cases of problem 3 under different noise interferences

Index	Sinusoidal		Random		Complex random	
	Mean	STD	Mean	STD	Mean	STD
Case 1	10.699	0.004	14.275	0.691	45.523	3.773
Case 2	13.638	0.445	21.570	0.392	58.928	0.711
Case 3	13.131	0.447	18.866	0.377	59.225	0.785

5 Conclusions

A novel design of nature-inspired heuristic of FPA is presented for the identification problem in nonlinear ANC with interferences. Different ANC scenarios by considering linear/nonlinear and primary/secondary paths are evaluated by determining coefficients of three different Volterra filters, i.e., VF-T1, VF-T2 and VF-T3. The performance of the FPA based ANC is verified through consistently achieving reasonable gauges of statistical operators in terms of accuracy, convergence and complexity measures. The performance is further validated via histogram analysis to prove that the FPA based ANC systems are reliable, accurate, stable and robust but the performance of the VF-T3 is comparatively better. The accuracy of FPA based ANC is in good

agreement with state of the art counterpart solvers based on GA, PSO, BSA and FWA along with their hybrid with local search. In the future, one may explore to enhance the performance of ANC system by implementation of recently introduced fractional derivative definition [60–64]. Moreover, the proposed methodology can be exploited to efficiently solve various complex engineering optimization problems [65–68].

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