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Article

Two-Phase Flow of Blood with Magnetic Dusty Particles in Cylindrical Region: A Caputo Fabrizio Fractional Model

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Abstract: The present study is focused on the unsteady two-phase flow of blood in a cylindrical region. Blood is taken as a counter-example of Brinkman type fluid containing magnetic (dust) particles. The oscillating pressure gradient has been considered because for blood flow it is necessary to investigate in the form of a diastolic and systolic pressure. The transverse magnetic field has been applied externally to the cylindrical tube to study its impact on both fluids as well as particles. The system of derived governing equations based on Navier Stoke's, Maxwell and heat equations has been generalized using the well-known Caputo–Fabrizio (C–F) fractional derivative. The considered fractional model has been solved analytically using the joint Laplace and Hankel (L&H) transformations. The effect of various physical parameters such as fractional parameter, *Gr, M* and γ on blood and magnetic particles has been shown graphically using the Mathcad software. The fluid behaviour is thinner in fractional order as compared to the classical one.

Keywords: Two-phase blood flow; dusty fluid; Brinkman type model; magnetic dusty particles; heat transfer; C–F derivative

1 Introduction

Biomagnetic fluid dynamic (BFD) is a new area in fluid mechanics. It focuses on the usage of the magnetic particles as drug carriers in magnetic drug targeting, cancer tumor treatment and many more [1–3]. The Biomagnetic fluid occurs in all living organisms and for its investigation, the BFD model was initially recommended by Haik et al. [4]. Fluids that show non-linear relation between shear stress and strain are termed as non-Newtonian fluids e.g., blood. Blood is the only biological electrically conducting fluid and its mobility is influenced by an applied magnetic field. It contains plasma and red blood cells (RBC) that are oxides of iron and have hemoglobin fragments in high concentrations [5,6]. Due to the oxygenated state, blood exhibits a magnetic nature [7,8]. The non-Newtonian behaviour of blood due to

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the suspension of red blood cells in plasma and human thoracic aorta is analyzed by Caballero et al. [9]. Tripathi et al. [10] have examined the Non-Newtonian blood in a channel and attained analytical solutions for the velocity, volumetric flow rate and wavelength. In the human left ventricle (LV), the significance of the non-Newtonian blood has examined by Doost et al. [11]. Kumar et al. [12] evaluated the difference between Newtonian and non-Newtonian blood models and concluded that the non-Newtonian blood has more/less augmented wall shear stress as compared to the Newtonian blood.

Since blood is a biological fluid, biological heating is significant for metabolic heat generation [13]. The phenomenon in biological fluids was first discussed by Bernard in 1876. Afterwards, bioheat transfer became a topic applied in the practice of biology in a wide variety of applications such as chemotherapy [14,15], human thermoregulation system [16] and others [17]. Sharifi et al. [18] investigated the heat transfer applications in peripheral vascular disease using FHD principle through two inclined permanent magnets in a channel. Jimoh et al. [19] studied third-grade fluid in hematocrit with slip velocity. Dutta et al. [20] have developed an analytical study of heat propagation in biological tissues for constant and variable heat flux at the skin surface with hypothermia treatment. Fu et al. [21] reviewed the heat transfer modelling in thermoregulatory responses in the human body. Kengne et al. [22] discussed the bioheat transfer in the spherical biological tissues. Zhang et al. [23] discussed the heat transfer in LN2 cryoprobe systems and obtained effective results. David et al. [24] used the heat transfer in the warming of simulated blood by the generation of electronic components. Zainol et al. [25] investigated the heat transfer model for the prediction of human skin temperature using the bioheat equation.

The consideration of Two-phase flow is due to the presence of numerous interfaces separating two immiscible phases. The blood flow through a tiny tube at a very low shear is responsible for the two-phase flow surrounded by a cell-depleted peripheral layer. Different types of particles have been considered as the second phase in blood flow, but the most recommended and suitable particles are magnetic particles. The magnetic particles in blood have a vital role in numerous medical applications [26,27]. In drug delivery, a specific number of magnetic particles are used to transport the maximum number of a drug to the area of its choice. Due to the mentioned applications of magnetic particles, several researchers used the two-phase blood flow along with magnetic particles. Verma et al. [28] described a dual-phase blood flow model in thin pipes with the fundamental core of deferred erythrocytes and cell unrestricted film and found the results for the nonlinear problem numerically. The thermal and mass concentration effect of the multiphase blood model in a stenosed artery has been investigated by Tripathi et al. [29]. An analytical approach has been used for the results to comprehend the comportment of blood flow rate, wall shear stress and flow resistivity. Arribas et al. [30] created a reliable two-phase RBC model for the blood vessel and calculated the viscosity, phase dispersals and volume fractions using the depletion theory. They have associated their results with numerical as well as experimental study and found extraordinary conclusions. A two-phase model of blood with mild stenosis magnetic field and thermal effects has been explored by Ponalagusamy et al. [31]. They have concluded that thermal and shear stress slow down with increase in the levels of the plasma layer thickness and they are very effective for the diseased arterial treatment. Ali et al. [32] examined the two-phase dusty fluid with heat transfer in a fluctuating plate, and found that by enhancing the number of embedded particles, the dusty fluid velocity increases.

Due to multidimensional features, the non-integer order calculus is attracting the attention of scientists and researchers [33,34]. Fractional calculus is an important and fruitful tool for describing many systems including memory effects. In the preceding few decades, fractional calculus is used for many purposes in various fields, such as electrochemistry, transportation of water in ground level, electromagnetism, elasticity, diffusion and in conduction of heat process [35]. In 2015, Caputo et al. [36] worked together in the field of non-integer order calculus and presented a new expression for the non-integer order derivative with the non-singular kernel. So, keeping in sight the importance of CF operators, many researchers used the CF operator in their studies such as in physics, biological mathematics, and many more. Ali et al. [37] examined the magnetic flow of Walter's-B fluid by using the CF non-singular operator. Salah Uddin et al. [38] investigated the CF model of blood flow with Ferro particles in cylindrical coordinates and their results were in agreement with the previously published works. Ali et al. [39,40] studied the fractionalized model of blood flow having magnetic particles in cylindrical coordinates.

There is no attempt found in the literature relevant to Caputo–Fabrizio fractional approach to find the closed-form solution for magnetite particles-based blood flow with thermal concentration. Hence, in the present article, the work of Saqib et al. [41] has been generalized by taking the flow of blood as a Brinkman type fluid with magnetic particles in cylinder. The governing equations for both fluid and particles are modelled and using the Caputo–Fabrizio fractional-order approach, the closed-form solutions have been obtained by using joint Laplace and Hankel transformations. The impact of different embedded parameters on blood and particles velocities have been examined through graphs.

2 Mathematical Modeling

The blood flow is considered in a vertical cylinder having a radius r_0 as represented in Fig. 1.

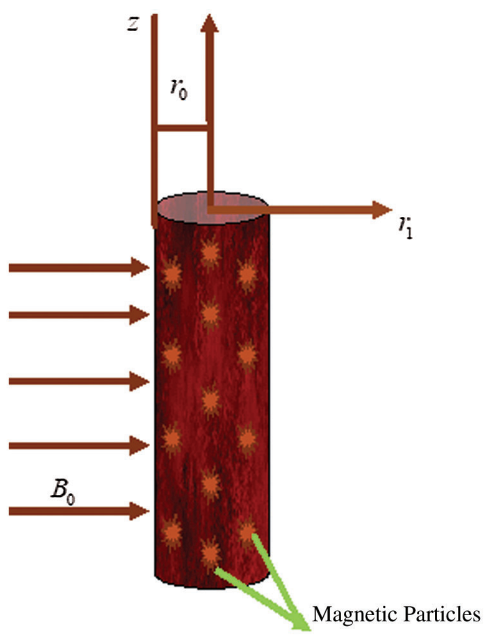

Figure 1: The geometry of the flow

The magnetic particles are equally distributed throughout the blood flow. The cylinder has been considered along the z-axis and r_1-axis is chosen perpendicular to it. The direction of the motion of the blood and magnetic particles are along the z-axis. The biological thermal effect has also been considered and the radiation has been neglected. The induced magnetic field due to a very slight Reynold number has been ignored [42]. For a time $t = 0$, the system is considered to be at rest with ambient temperature T_∞. For $t > 0$ the temperature rises to T_w. The force F_{emag} is described by [43,44]

$$\overrightarrow{F}_{emg} = -\sigma B_0{}^2\, u(r_1, t)\,\overrightarrow{k}, \tag{1}$$

where $\overrightarrow{k}$ denotes the direction along the z-axis and $\overrightarrow{V} = u(r_1, t)\,\overrightarrow{k}$ shows blood velocity.

The unsteady Brinkman-type blood flow in a cylinder is specified by:

$$\rho\left(\frac{\partial u(r_1,t)}{\partial t}+\gamma u(r_1,t)\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u(r_1,t)}{\partial {r_1}^2}+\frac{1}{r_1}\frac{\partial u(r_1,t)}{\partial r_1}\right)+KN\left(u_p(r_1,t)-u(r_1,t)\right) \\ -\sigma_0 B^2{}_0 u(r_1,t)\pm g\beta_T(T-T_\infty), \tag{2}$$

the oscillating pressure gradient [45] is

$$-\frac{\partial p}{\partial z}=P_0+P_1 cos\omega t, \tag{3}$$

where $u(r_1,t)$ is the blood velocity, $u_p(r_1,t)$ is the magnetic particles velocity. The term $KN\left(u_p(r_1,t)-u(r_1,t)\right)$ is the force between the fluid and particle due to relative motion and magnetic particles flow is conducted [46]:

$$m\frac{\partial u_p}{\partial t}=K\left(u(r_1,t)-u_p(r_1,t)\right), \tag{4}$$

The thermal equation is specified by:

$$\frac{\rho c_p}{k}\frac{\partial T_1(r_1,t)}{\partial t}=\frac{\partial^2 T_1(r,t)}{\partial {r_1}^2}+\frac{1}{r_1}\frac{\partial T_1(r_1,t)}{\partial r_1};\ t>0, r_1\in(0,R_0), \tag{5}$$

subjected to the following IBCs

$$\left.\begin{array}{lll} u_1(r_1,0)=0 & , & u_p(r_1,0)=0 \\ u_1(r_0,t)=0 & , & u_p(r_0,t)=0 \\ T_1(r_1,0)=T_\infty & , & T_1(r_0,t)=T_w \\ \left.\frac{\partial u_1}{\partial r_1}\right|_{r_1=0}=0 & & \end{array}\right\}, \tag{6}$$

By incorporating the Non-dimensional variables

$${r_1}^*=\frac{r_1}{r_0}, t^*=\frac{vt}{{r_0}^2}, v=\frac{u_1}{u_0}, v_p=\frac{u_p}{u_0}, \theta=\frac{T_1-T_\infty}{T_w-T_\infty}, {\xi_0}^*=\frac{P_0{r_0}^2}{\mu u_0}, {\xi_1}^*=\frac{P_1{r_0}^2}{\mu u_0}, \tag{7}$$

into Eqs. (2)–(5), then ignoring the * notation, we obtain:

$$\frac{\partial v(r_1,t)}{\partial t}+\gamma_1 v(r_1,t)=(\xi_0+\xi_1\cos\omega t)+\left(\frac{\partial^2 v(r_1,t)}{\partial {r_1}^2}+\frac{1}{r_1}\frac{\partial v(r_1,t)}{\partial r_1}\right)+P_c\left(v_p(r_1,t)-v(r_1,t)\right) \\ -Mv(r_1,t)\pm Gr\theta(r_1,t), \tag{8}$$

$$\frac{\partial v_p(r_1,t)}{\partial t}=\lambda_m\left(v(r_1,t)-v_p(r_1,t)\right), \tag{9}$$

$$\frac{\partial\theta(r_1,t)}{\partial t}=\frac{1}{\Pr}\left(\frac{\partial^2\theta(r_1,t)}{\partial {r_1}^2}+\frac{1}{r_1}\frac{\partial\theta(r_1,t)}{\partial r_1}\right), \tag{10}$$

$$\left.\begin{array}{lll} v(r_1,0)=0 & , & v_1(r_1,0)=0 \\ v(1,\ t)=0 & , & v_p(1,\ t)=0 \\ \theta(r_1,0)=0 & , & \theta(1,\ t)=1 \end{array}\right\}, \tag{11}$$

For a generalized fractional model, the newly developed CF time-fractional derivative has been used to covert the linear model to the fractional model, therefore Eqs. (8)–(10) reduces to:

$$
\begin{aligned}
{}^{AB}\wp_t{}^{\alpha} v(r_1,t) =& (\xi_0 + \xi_1 \cos\omega t) + \left(\frac{\partial^2 v(r_1,t)}{\partial {r_1}^2} + \frac{1}{r_1}\frac{\partial v(r_1,t)}{\partial r_1}\right) + P_c\left(v_p(r_1,t) - v(r_1,t)\right) - Mv(r_1,t) \\
& - \gamma_1 v(r_1,t) \pm Gr\theta(r_1,t),
\end{aligned} \tag{12}
$$

$$
{}^{AB}\wp_t{}^{\alpha} v_p(r_1,t) = \lambda_m\left(v(r_1,t) - v_p(r_1,t)\right), \tag{13}
$$

$$
{}^{AB}\wp_t{}^{\alpha} \theta(r_1,t) = \frac{1}{\Pr}\left(\frac{\partial^2 \theta(r_1,t)}{\partial {r_1}^2} + \frac{1}{r_1}\frac{\partial \theta(r_1,t)}{\partial r_1}\right), \tag{14}
$$

where $\wp_t{}^{\alpha} h(\varepsilon,t) = \frac{1}{1-\alpha}\int_0^{\tau} e^{\left(\frac{-\alpha(\tau-t)}{1-\alpha}\right)} h^{/}(\varepsilon,\tau)dt;\ 0 < \alpha < 1$ is the CF operator is [36]:

3 Solution of the Problem

For the solution of Eqs. (14)–(16) the non-dimensional IBC's from Eq. (11) and the Laplace and Hankel transformations are utilized.

3.1 Energy Equation Solution

Applying the joint L&H transforms using Eqs. (11)–(14), we get

$$
\overline{\theta_H}(r_{1n},q) = \frac{J_1(r_{1n})}{r_{1n}q} + r_{1n}J_1(r_{1n})\Pr_{5n}\left(\frac{\Pr_{7n}}{q} + \frac{\Pr_{8n}{r_n}^2}{(q+a_{2n})}\right) \tag{15}
$$

where

$$
\Pr_{8n} = \frac{a_{2n} - \Pr_{6n}}{a_{2n}}, \Pr_{7n} = \frac{\Pr_{6n}}{a_{2n}{r_{1n}}^2}, \Pr_{6n} = \frac{\Pr_{4n}}{\Pr_{3n}}, \Pr_{5n} = \frac{\Pr_{3n}}{\Pr_{2n}}, \Pr_{4n} = {r_{1n}}^2 a_1 - \Pr_{2n}a_{2n}, \Pr_{3n} = {r_{1n}}^2 - \Pr_{2n},
$$

$$
\Pr_{2n} = \Pr .\Pr_{1n}, \Pr_{1n} = \frac{a_{1n}}{\Pr}, a_{2n} = \frac{a_1 {r_n}^2}{a_{1n}}, a_{1n} = a_0 \Pr + {r_{1n}}^2, a_1 = a_0\alpha, a_0 = \frac{1}{1-\alpha}
$$

and $\overline{\theta_H}(r_n,\ q)$ is the Hankel Transform of $\overline{\theta}(r,\ q)$ [39,40].

Applying inverse L&H transformations to Eq. (15), and by using Lorenzo and Hartley's' and Robotnov and Hartley's' functions, respectively [43], yields:

$$
\theta(r_1,t) = 1 + 2\sum_{n=1}^{\infty}\frac{J_0(r_1 r_{1n})r_{1n}}{J_1(r_{1n})}(\Pr_{7n} + \exp(-a_{2n},t)\Pr_{8n}). \tag{16}
$$

3.1.1 Heat Transfer Rate (Nu)

The Non-dimensional Nusselt number is given by

$$
\mathrm{Nu} = -\left(\frac{\partial\theta(r_1,\ t)}{\partial r_1}\right)_{r_1=1}. \tag{17}
$$

3.2 Solution of the Blood and Particle Velocities

To obtain the solution for the blood velocity and Magnetic particles velocity, the Laplace and Hankel transforms have been applied on Eq. (15) using the corresponding transformed boundary conditions by letting $\lambda_{m1} = 1 + \lambda_m a_0$, $\lambda_{m2} = \frac{a_1}{\lambda_{m1}}$ and we get:

$$\overline{v_{pH}}(r_{1n},q)=\overline{v_H}(r_{1n},q)\left(\frac{q+a_1}{\lambda_{m1}(q+\lambda_{m2})}\right). \tag{18}$$

Now for the blood velocity, Eq. (18) has been incorporated into Eq. (12) using the corresponding transformations and boundary condition $\overline{v}(1,\,q)=0$, which yields:

$$\begin{aligned}&\left(\frac{a_0q}{q+a_1}+\gamma_1+{r_{1n}}^2-P_c\frac{q+a_1}{\lambda_{m1}(q+\lambda_{m2})}+P_c+M\right)\overline{v_H}(r_{1n},q)\\&=\left(\left(\frac{\xi_0}{q}+\frac{\xi_1q}{q^2+\omega^2}\right)\pm Gr\overline{\theta}_H(r_{1n},q)\right)\left(\frac{J_1(r_{1n})}{r_{1n}}\right)\end{aligned} \tag{19}$$

The simplified form of Eq. (19) is

$$\overline{v_H}(r_{1n},q)=\left(\left(\frac{\xi_0}{q}+\frac{\xi_1q}{q^2+\omega^2}\right)\pm Gr\overline{\theta}_H(r_{1n},q)\right)\left(\frac{J_1(r_{1n})}{r_{1n}}\right)\left(\frac{\lambda_{m1}(q+\lambda_{m2})(q+a_1)}{q^2X_{1n}+qX_{2n}+X_{3n}}\right) \tag{20}$$

After further simplification the Eq. (20) will take the following form as

$$\overline{v_H}(r_{1n},q)=\left(\left(\frac{\xi_0}{q}+\frac{\xi_1q}{q^2+\omega^2}\right)\pm Gr\overline{\theta}_H(r_{1n},q)\right)\left(\frac{J_1(r_{1n})}{r_{1n}}\right)\left(\frac{b_{3n}X_{9n}}{(q+b_{1n})}-\frac{b_{3n}X_{8n}}{(q+b_{2n})}\right) \tag{21}$$

where

$$X_{0n}=\left(\gamma_1+{r_{1n}}^2+P_c+M\right)\lambda_{m1},\quad X_{1n}=\lambda_{m3}+X_{0n}-Pc,\quad X_{2n}=\lambda_{m3}\lambda_{m2}+X_{0n}\lambda_{m2}a_1-P_c{a_1}^2,$$

$$X_{4n}=\frac{\lambda_{m1}}{X_{1n}},\quad X_{5n}=\frac{X_{2n}}{X_{1n}},\quad X_{6n}=\frac{X_{3n}}{X_{1n}},\quad {X^2}_{7n}={X^2}_{5n}-4X_{6n},\quad b_{1n}=\frac{X_{5n}+X_{7n}}{2},\quad b_{2n}=\frac{X_{5n}-X_{7n}}{2},$$

$$X_{8n}=(X_{4n}b_{2n}-a_1X_{4n})\lambda_{m2}+X_{4n}b_{2n}a_1,\quad X_{9n}=X_{4n}(\lambda_{m2}+a_1)-\lambda_{m2}a_1X_{4n},\quad b_{3n}=\frac{1}{b_{1n}-b_{2n}}$$

In component form Eq. (21) is expressed as:

$$\overline{v_H}(r_{1n},q)=\left(\overline{F_{1H}}(q)+\overline{F_{2H}}(q)\right) \tag{22}$$

where

$$\overline{F_{1H}}(q)=\left(\xi_0\left(\frac{b_{4n}q^{-1}}{q+b_{1n}}-\frac{b_{5n}q^{-1}}{q+b_{2n}}\right)+\xi_1\left(\frac{b_{4n}}{(q+b_{1n})}-\frac{b_{5n}}{q+b_{2n}}\right)\frac{1}{(q^2+\omega^2)}\right)\frac{J_1(r_{1n})}{r_{1n}}$$

and

$$\begin{aligned}\overline{F_{2H}}(q)&=\left(\frac{b_{4n}q^{-1}}{q+b_{1n}}-\frac{b_{5n}q^{-1}}{q+b_{2n}}\right)\frac{J_1(r_{1n})}{r_{1n}}\\&+r_{1n}J_1(r_{1n})\left(\frac{b_{6n}q^{-1}}{(q+b_{1n})}-\frac{b_{7n}q^{-1}}{q+b_{2n}}-\frac{b_{10n}}{q+a_{2n}}+\frac{b_{10n}}{q-b_{1n}}+\frac{b_{11n}}{q+a_{2n}}+\frac{b_{10n}}{q-b_{1n}}\right)\end{aligned}$$

$$b_{4n}=b_{3n}X_{9n},\quad b_{5n}=b_{3n}X_{8n},\quad b_{6n}=\mathrm{Pr}_{5n}\mathrm{Pr}_{7n}b_{4n},\quad b_{7n}=\mathrm{Pr}_{5n}\mathrm{Pr}_{7n}b_{5n},\quad b_{8n}=\mathrm{Pr}_{8n}b_{4n},\quad b_{9n}=\mathrm{Pr}_{8n}b_{5n},$$

$$b_{10n}=\frac{b_{8n}}{b_{1n}-a_{2n}},\quad b_{11n}=\frac{b_{9n}}{b_{1n}-a_{2n}}.$$

By applying inverse Laplace transform to Eq. (22), by using the Lorenzo and Hartley's' respectively [46],

$$L^{-1}\left(\frac{\varsigma^{-\upsilon}}{\varsigma^{\beta}+\varphi}\right)=\Re_{\beta,\upsilon}(-\varphi,\tau),$$

we get

$$v_H(r_{1n},t)=F_{1H}(t)+F_{2H}(t) \tag{23}$$

where

$$F_{1H}(t)=\left(\xi_0\left(b_{4n}\Re_{(1,1)}(-b_{1n},t)-b_{5n}\Re_{(1,1)}(-b_{2n},t)\right)+\xi_1\begin{pmatrix}b_{4n}\exp(-b_{1n},t)*\cos(\omega t)-\\ b_{5n}\exp(-b_{2n},t)*\cos(\omega t)\end{pmatrix}\right)\frac{J_1(r_{1n})}{r_{1n}}$$

$$F_{2H}(t)=\left(b_{4n}\Re_{(1,1)}(-b_{1n},t)-b_{4n}\Re_{(1,1)}(-b_{2n},t)\right)\frac{J_1(r_{1n})}{r_{1n}}$$
$$+\,r_{1n}J_1(r_{1n})\begin{pmatrix}b_{6n}\Re_{(1,1)}(-b_{1n},t)-b_{7n}\Re_{(1,1)}(-b_{2n},t)\\ -b_{10n}\exp(-a_{2n},t)+b_{10n}\exp(b_{1n},t)\\ +b_{11n}\exp(-a_{2n},t)+b_{10n}\exp(b_{1n},t)\end{pmatrix}$$

Applying the finite Hankel transform of order zero to Eq. (23), we get

$$v(r_1,t)=2\sum_{n=1}^{\infty}\left(\frac{J_0(r_1r_{1n})}{r_{1n}J_1(r_{1n})}\right)(F_1(t)+F_2(t)) \tag{24}$$

where

$$F_1(t)=\left(\xi_0\left(b_{4n}\Re_{(1,1)}(-b_{1n},t)-b_{5n}\Re_{(1,1)}(-b_{2n},t)\right)+\xi_1\begin{pmatrix}b_{4n}\exp(-b_{1n},t)*\cos(\omega t)-\\ b_{5n}\exp(-b_{2n},t)*\cos(\omega t)\end{pmatrix}\right)$$

$$F_2(t)=\left(b_{4n}\Re_{(1,1)}(-b_{1n},t)-b_{4n}\Re_{(1,1)}(-b_{2n},t)\right)+\begin{pmatrix}b_{6n}\Re_{(1,1)}(-b_{1n},t)-b_{7n}\Re_{(1,1)}(-b_{2n},t)\\ -b_{10n}\exp(-a_{2n},t)+b_{10n}\exp(b_{1n},t)\\ +b_{11n}\exp(-a_{2n},t)+b_{10n}\exp(b_{1n},t)\end{pmatrix}$$

Now for the solution of magnetic particles velocity applying the inverse L&H transformations to Eq. (16), yields:

$$v_p(r_1,t)=2\sum_{n=1}^{\infty}v(r_1,t)*\left(\frac{1}{\lambda_{m1}}\begin{pmatrix}\Re_{(1,-1)}(-\lambda_{m2},t)\\ +\Re_{(1,0)}(-\lambda_{m2},t)\end{pmatrix}\right). \tag{25}$$

From Tab. 1 it can be seen that by growing the fractional parameter an enhancement in (Nu) occurs for time.

Table 1: Time and α variation on Nusselt number

α	t	Nu
0.3	2	2.503
0.5	2	2.684
0.7	2	3.012
0.9	2	3.412
1	2	3.576

4 Graphical Results and Discussion

The considered work aims to study the generalized two-phase blood flow of Brinkman type fluid in a cylindrical tube. The analytical solutions have been attained for energy, velocity as well as for the magnetic particles contained in the blood. Various parameters have been discussed physically on velocities of the blood, particles and temperature. Fig. 1 shows the physical model of the considered problem. Fig. 2 shows the effect of fractional parameter on the temperature profile. It can be concluded from the figure that by using the fractional parameter, we obtained different temperature profiles by keeping the other parameters constant and this effect is called the memory effect, which is impossible in integer-order. In this graph, we obtained dual behaviour of temperature memory for different values of α and the same behaviour has been noticed as reported by Ali et al. [39].

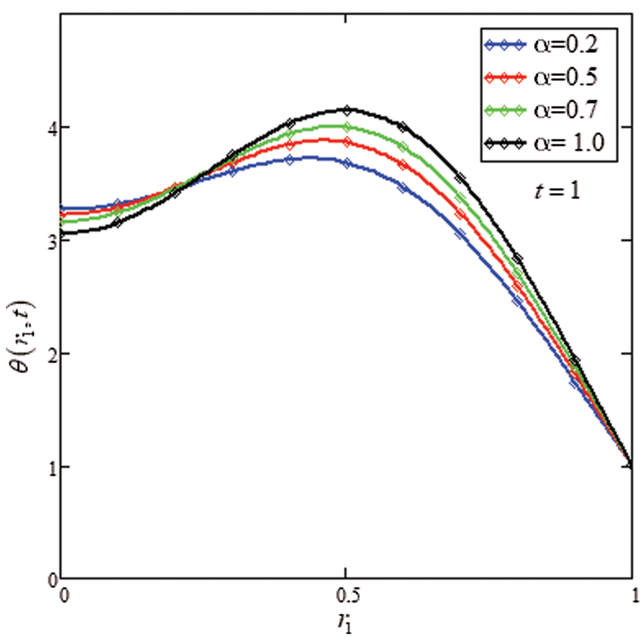

Figure 2: Variation in Temperature for diverse values of α for Pr = 22.64 at $t = 1$

Fig. 3 shows the effect of the fractional parameter α on velocity profiles. The corresponding results for regular blood and particles velocity are compared with the fractional order in a fixed time and with classical order and the fluid and particles memory has been discussed.

Fig. 4 shows the impact of the Magnetic parameter on blood velocity and particle velocity. From the figures, it has been concluded that for the higher values of M, the flow of blood, as well as magnetic particles, retards. It is physically true that by increasing M, the Lorentz forces increase, which produces resistive forces due to which the flow retards. This effect is very useful in various medical fields such as Magnetic drug targeting and separation techniques by the magnetic field for the cure of different types of diseases and to maintain the normal aspects of the human body.

Fig. 5 shows the impact of the Brinkman type parameter γ_1 on the velocities of blood and particles. It is observed that the velocity decreases with the increasing values of γ_1. This is because γ_1 is the ratio between resistive forces and density. By increasing γ_1, the opposite forces increase, which retards the fluid velocity and is directly related to the blood flow. These results are strongly in agreement with Saqib et al. [41].

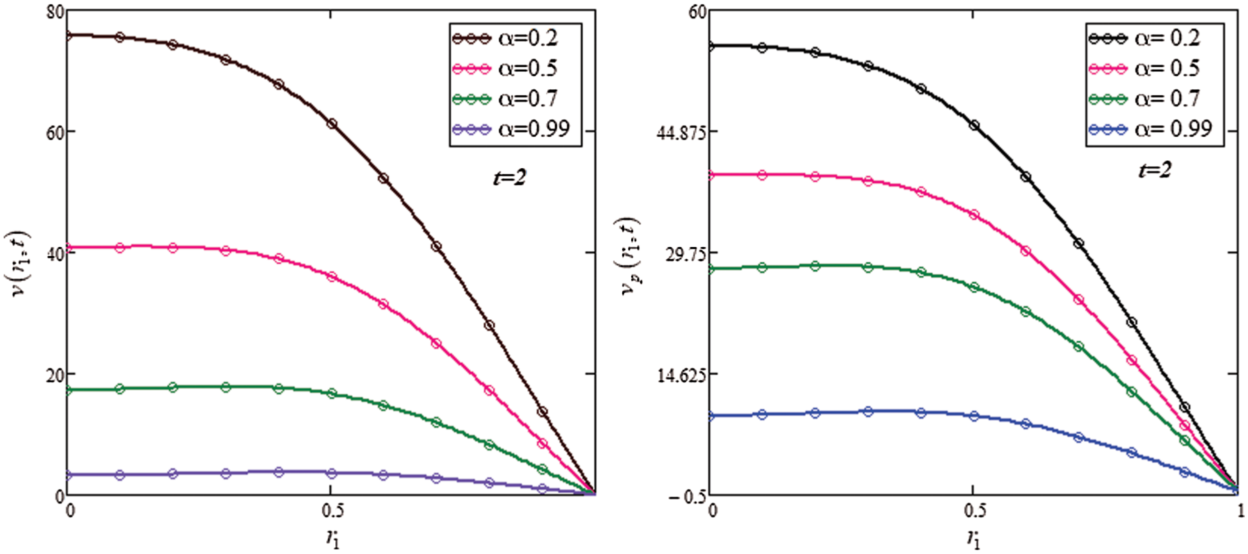

Figure 3: Blood and Particle velocities sketch of α at $t = 2$, $Gr = 3.2 \times 10^2, \Pr = 22.64$

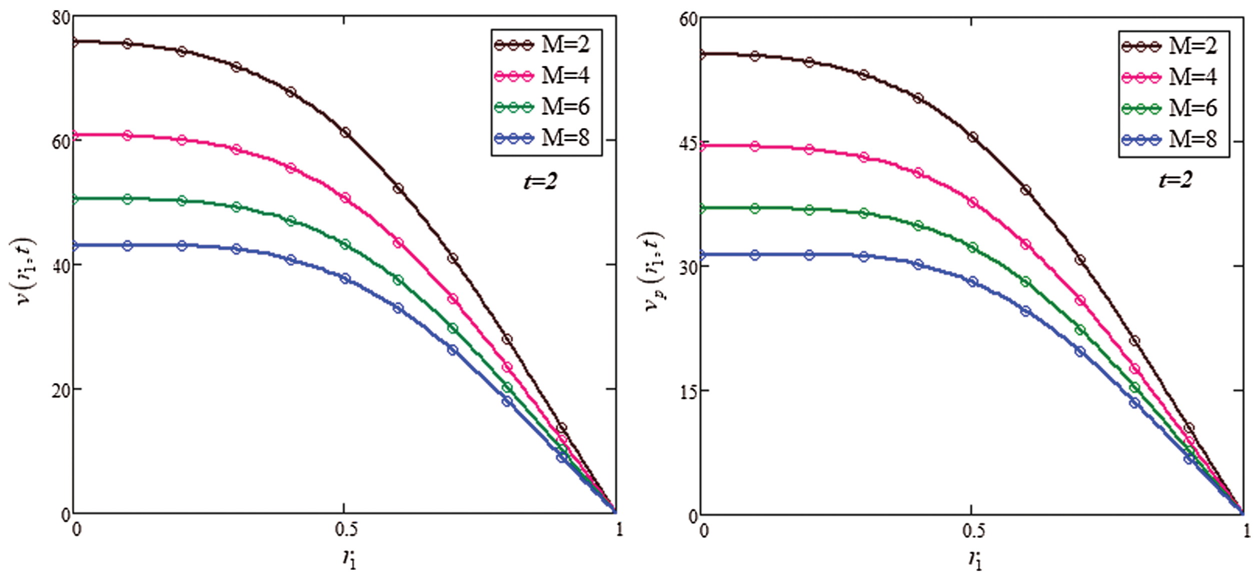

Figure 4: Blood velocity sketch of M at $t = 2$, $Gr = 3.2 \times 10^2, \Pr = 22.64$

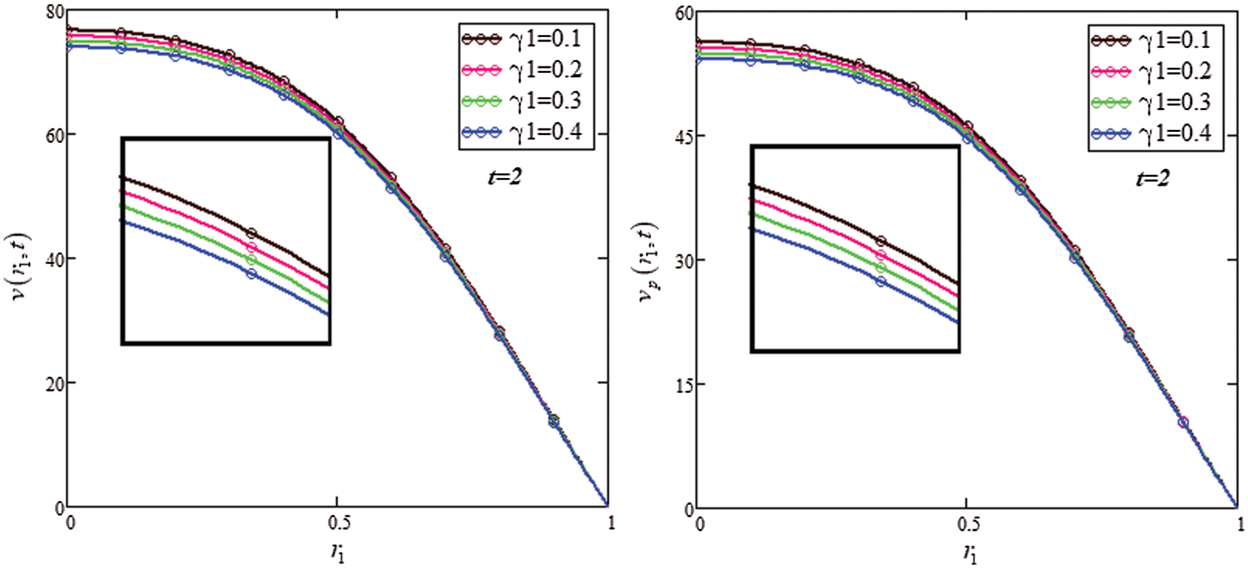

Figure 5: Blood velocity graph of γ_1 at $t = 2$, $Gr = 3.2 \times 10^2$, $\Pr = 22.64$

5 Conclusions

The Caputo–Fabrizio time-fractional derivative has been used. The effect of relative parameters has been shown graphically. Closed-form expressions have been obtained by using the Laplace transform and Hankel transform techniques. Based on the graphical study, it has been concluded that the velocity profile decreases in the response of an external applied magnetic field and Brinkman parameter. This phenomenon might play an important role in Magnetic wounds. Furthermore, by increasing the fractional parameter, the fluid memory becomes thicker.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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