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Article

Entanglement and Entropy Squeezing for Moving Two Two-Level Atoms Interaction with a Radiation Field

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Abstract: In this paper, we analyzed squeezing in the information entropy, quantum state fidelity, and qubit-qubit entanglement in a time-dependent system. The proposed model consists of two qubits that interact with a two-mode electromagnetic field under the dissipation effect. An analytical solution is calculated by considering the constants for the equations of motion. The effect of the general form of the time-dependent for qubit-field coupling and the dissipation term on the temporal behavior of the qubit-qubit entanglement, quantum state fidelity, entropy, and variance squeezing are examined. It is shown that the intervals of entanglement caused more squeezing for the case of considering the time-dependent parameters. Additionally, the entanglement between the qubits became more substantial for the case of time dependence. Fidelity and negativity rapidly reached the minimum values by increasing the effect of the dissipation parameter. Moreover, the amount of variance squeezing and the amplitude of the oscillations decreased considerably when the time dependence increased, but the fluctuations increased substantially. We show the relation between entropy and variance squeezing in the presence and absence of the dissipation parameter during the interaction period. This result enables new parameters to control the degree of entanglement and squeezing, especially in quantum communication.

Keywords: Entropy squeezing; variance squeezing; qubit-qubit entanglement; moving qubits

1 Introduction

The principles of nonlocal correlation or entanglement mainly appear when two systems interact with each other; one is a pure state, and the other is mixed. Researchers utilize the von Neumann entropy to measure optimally the nonlocal correlation when a system reaches a pure condition. In this case, the density operator takes the form of a separate product state [1,2]. The linear entropy for the two symmetrical 2-level systems that interact with a 2-photon system developed in a squeezed condition is demonstrated in [3]. Additionally, a model consisting of a 2-three-level atom is investigated in [4], where the atom-atom entanglement decreases by increasing the multiplicity of the photons in the absence of a time-dependent coupling effect. A generalized model is considered to study the impact of the linear entropy of two SC-qubits with a linear system, which interacts with a thermal field [5]. Furthermore, a nonlocal correlation between subsystems has been studied. The results demonstrated that the effect of the nonlinear terms is greater than that of the linear terms. Recently, the measurement of the nonlocal correlation has achieved several purposes in the area of quantum information and computation, but the previous research considering the time independence to explain this phenomenon is limited.

The connection between the AES (atomic entropy squeezing) and entanglement has different applications in quantum computing and produces different observable physical phenomena [6–9]. The performance of the nonlocal correlation highly resembles the performance of the absorption coefficient and the GP. The AES for a 2LA coupling to fluctuating electromagnetic fields with a reflecting boundary was studied. The results showed that having the border influences the AES. Additionally, the relationship of the entanglement and FES (field entropy squeezing) of an effective 2-level system in the presence of a Stark Shift FES was studied [10]. The work has been extended to scrutinize the impact of cavity damping on the dynamics of the FES and the entanglement of the dissipation of two-photon JCM for a Kerr-like medium [11]. Recently, the relationship of the AES and entanglement between two two-level atoms and the N-level quantum system has been explored [12]. It was shown that the classical field has a potential role in the evolution of AES and nonlocal correlation. It was found that there is a strong correlation between the spin-orbit interaction and the strength of the AES, which depends on the initial state and the number of squeezed components [13]. Additionally, the relation between the atomic Fisher information and AES of the quantum system for an N-level atom that interacted with a two-level atom was also determined [14]. Experimental results in quantum physics cannot be explained using a closed system (hermitian Hamiltonian). Therefore, the results can be convincingly explained in cases of phenomena observed experimentally in the case of the open system (non-hermitian Hamiltonian) [15]. The non-hermitian generalization of Hamiltonian (NHH) can be used as a paradigm to define an open quantum system [15]. Then, we get the complex eigenvalues of energy. The aforementioned NHHs are valid as a rough and apparent description of an open quantum system, e.g., radioactive decay processes [16]. Therefore, this article explores the relation between the AES and linear entropy as a quantifier of the entanglement and purity of two qubits interacting with a two-mode electromagnetic field.

The contents of this article are arranged as follows: We present the general solution based on solving the differential equations which result from the Schrödinger equation in Section 2. The numerical results for the entropy and variance squeezing in Section 3, and the state fidelity and qubit-qubit entanglement phenomena will be discussed in Section 4. Finally, the results are presented in Section 5.

2 Analytical Solution

A time-dependent parameter and the atomic dissipation effect are added in our proposed model. Therefore, the Hamiltonian of the system takes the following form:

$$\frac{\hat{H}}{\hbar}=\frac{1}{2}\sum_{j=1}^{2}\left(-i\gamma\hat{\sigma}_{+}^{(j)}\hat{\sigma}_{-}^{(j)}+\Omega_j(t)\hat{\sigma}_{z}^{(j)}+2\omega_j(t)\,\hat{a}_j^{+}\hat{a}_j\right)+\sum_{j=1}^{2}g(t)\left(\hat{a}_1\hat{a}_2\,\hat{\sigma}_{+}^{(j)}+\hat{a}_1^{+}\hat{a}_2^{+}\,\hat{\sigma}_{-}^{(j)}\right), \tag{1}$$

The operators $\hat{\sigma}_{+}^{(j)}(\hat{\sigma}_{-}^{(j)})$ and $\hat{\sigma}_{z}^{(j)}(j=1,2)$ represent the Pauli matrices and fulfill the relationships of commutation $[\hat{\sigma}_{z}^{(j)},\hat{\sigma}_{\pm}^{(j)}]=\pm 2\hat{\sigma}_{\pm}^{(j)}$ and $[\hat{\sigma}_{+}^{(i)},\hat{\sigma}_{-}^{(j)}]=\hat{\sigma}_{z}^{(j)}\delta_{ij}$. While $\hat{a}_j^{+}$ and $\hat{a}_j$ represent the operators of creation and annihilation for the cavity mode. $[\hat{X},\hat{X}^{+}]=\hat{I}$, $X=\hat{a}_1$ and $\hat{a}_2$, $\omega_{1,2}$ and Ω_j, $j=1,2$ are the frequencies of the two modes and the atoms, respectively, while $g(t)$ is the time-dependent coupling between the field and the atoms and γ is the atomic corresponding decay rate. The behavior of the Hamiltonian (1) can be explained by calculating the wave function and solving the differential equations which are obtained from the Schrödinger formula. However, the dynamical operators can be calculated by the Heisenberg relationship. Therefore, the statistical results can be analyzed by employing these operators. The dynamical operator can be written as:

$$\begin{aligned} i\frac{d\hat{a}_1^{+}\hat{a}_1}{dt} &= g(t)\sum_{j=1}^{2}\hat{a}_1\hat{a}_2\hat{\sigma}_{+}^{(j)} - \hat{a}_1^{+}\hat{a}_2^{+}\ \hat{\sigma}_{-}^{(j)}, \\ i\frac{d\hat{a}_2^{+}\hat{a}_2}{dt} &= g(t)\sum_{j=1}^{2}\hat{a}_1\hat{a}_2\hat{\sigma}_{+}^{(j)} - \hat{a}_1^{+}\hat{a}_2^{+}\ \hat{\sigma}_{-}^{(j)} \\ -i\frac{d\ \hat{\sigma}_z^{(j)}}{dt} &= 2\ g(t)\left(\hat{a}_1\hat{a}_2\ \hat{\sigma}_{+}^{(j)} - \hat{a}_1^{+}\hat{a}_2^{+}\ \hat{\sigma}_{-}^{(j)}\right),\ j = 1,\, 2 \end{aligned} \tag{2}$$

Therefore, the constants of motion are given by:

$$\hat{N}_j = \hat{a}_j^{+}\hat{a}_j + \frac{\hat{\sigma}_z^{(1)} + \hat{\sigma}_z^{(2)}}{2},\ j = 1,\, 2 \tag{3}$$

By applying Eq. (3) to the Hamiltonian system (1), we can obtain:

$$\frac{\hat{H}}{\hbar} = \omega_a(t)\hat{N}_1 + \omega_b(t)\hat{N}_2 + \hat{R} - \frac{i\gamma}{4}\hat{I}, \tag{4}$$

where, $\hat{I}$ is the identity operator and $\hat{R}$ is given by:

$$\hat{R} = \sum_{j=1}^{2}\frac{\delta_j(t)}{2}\ \hat{\sigma}_z^{(j)} + \sum_{j=1}^{2}g(t)\left(\hat{a}_1\hat{a}_2\ \hat{\sigma}_{+}^{(j)} + \hat{a}_1^{?}\hat{a}_2^{?}\ \hat{\sigma}_{-}^{(j)}\right), \tag{5}$$

where δ_1 and δ_2 are the detuning parameters defined by:

$$\delta_j(t) = \Omega_j(t) - \frac{i\gamma}{2} - \omega_1(t) - \omega_2(t). \tag{6}$$

Here, we consider that $\Omega_j(t) >> \gamma$ [17]. We assume that the primary conditions of the atoms and the field are:

$$|\Theta(0)\rangle = |e,\, e\rangle \otimes |b\rangle, \tag{7}$$

where $|e,\, e\rangle$ represents an excited state and $|b\rangle$ is the pair coherent state [18], which is given by:

$$|b\rangle = \sum_{m=0}^{\infty}Q_m\ |m\rangle,\ \ Q_m\ = M\frac{b^m}{\sqrt{m!(m+q)!}},\ \ M^{-2} = \sum_{m=0}^{\infty}\frac{|b|^{2m}}{m!(m+q)!} \tag{8}$$

The general solution $|\Theta(t)\rangle$ for $t > 0$ takes the form,

$$|\Theta(t)\rangle = |e,\, e\rangle|A\rangle + |e,\, g\rangle|B\rangle + |g,\, e\rangle|C\rangle + |g,\, g\rangle|D\rangle \tag{9}$$

and

$$
\begin{aligned}
|A\rangle &= \sum_{m=0}^{\infty} y_1(m,\, t)|m+q,\, m\rangle \\
|B\rangle &= \sum_{m=0}^{\infty} y_2(m,\, t)|m+q+1,\, m\rangle \\
|C\rangle &= \sum_{m=0}^{\infty} y_3(m,\, t)|m+q+1,\, m\rangle \\
|D\rangle &= \sum_{m=0}^{\infty} y_4(m,\, t)|m+q+2,\, m\rangle
\end{aligned}
$$

where $y_j(m,\, t)$ are the solution of the following system of differential equations,

$$
\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{-\gamma}{4} & -i\,u_1(m) & -i\,u_1(m) & 0 \\ -i\,u_1(m) & -i\,\delta_1 - \frac{\gamma}{4} & 0 & -i\,u_2(m) \\ -i\,u_1(m) & 0 & i\,\delta_1 - \frac{\gamma}{4} & -i\,u_2(m) \\ 0 & -i\,u_2(m) & -i\,u_2(m) & \frac{-\gamma}{4} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \tag{10}
$$

where $u_j(m) = g(t)\sqrt{(m+q+j)(m+j)}\; j = 1, 2$

By specifying $\omega_a(t) = \omega_0(\alpha t + t^2\beta + \phi)$, $\omega_b(t) = \omega_f(\alpha t + t^2\beta + \phi)$, $g(t) = g(\alpha t + t^2\beta + \phi)$, the time dependence of the coefficients $y_j(m,\, t), j = 1,\, 2,\, 3,\, 4$ takes the following form:

$$
\begin{aligned}
y_1(m,\, t) =& Q_m \exp(\frac{-\gamma t}{4})\left[\frac{(\mu_1(m))^2 - 2u_1^2(m)(1-\cos\mu_1(m)\varepsilon(t))}{(\mu_1(m))^2}\right], \\
y_2(m,\, t) =& -Q_m \exp(\frac{-\gamma t}{4})\left[\frac{\delta u_1(m)(1-\cos\mu_1(m)\varepsilon(t))}{(\mu_1(m))^2} + i\frac{u_1(m)\sin\mu_1(m)\varepsilon(t)}{\mu_1(m)}\right], \\
y_3(m,\, t) =& -\overline{y_2}(m,\, t), \\
y_4(m,\, t) =& Q_m \exp(\frac{-\gamma t}{4})\frac{2u_1(n)u_2(m)(\cos\mu_1(m)\varepsilon(t)-1)}{(\mu_1(m))^2},
\end{aligned} \tag{11}
$$

where

$$
\begin{aligned}
\mu_j(n) =& \sqrt{|\delta|^2 + (3-j)\mu_3^2(n)},\ j = 1,\, 2 \\
\mu_3(n) =& \sqrt{(u_1(m))^2 + (u_2(m))^2},\ \varepsilon(t) = \frac{\alpha t^2}{2} + \frac{\beta t^3}{3} + \phi t
\end{aligned} \tag{12}
$$

Next, the density matrix will be calculated according to various statistical quantities, and therefore, the physical phenomena can be explained. For the case where two atoms are identical and the trace is taken, we have:

$$
\hat{\rho}_A(t) = Tr_F|\Theta(t)\rangle\langle\Theta(t)|, \tag{13}
$$

$$
\begin{aligned}
\hat{\rho}_{A(j)} =& Tr_{A(i)}\hat{\rho}_{atoms}(t), \quad i, j = 1,\, 2 \\
\hat{\rho}_{(i)} =& \rho_{11}|+\rangle\langle+| + \rho_{12}|+\rangle\langle-| + \rho_{21}|-\rangle\langle+| + \rho_{22}|-\rangle\langle-|,
\end{aligned} \tag{14}
$$

Section 3 discusses the dynamical behavior of the entropy and variance squeezing based on the single-atom density matrix (14).

3 Entropy and Variance Squeezing

The uncertainty principle, which was first introduced by Heisenberg, shows the limits of the error in the conventional measurements of non-commutating operators for measuring quantum states [19–23]. In general, the uncertainty principle for any two Hermitian operators $\hat{A}$ and $\hat{B}$ yields the relationship $[\hat{A}, \hat{B}] = i\hat{C}$. Therefore, the Heisenberg uncertainty inequality is given by:

$$\langle(\Delta\hat{A})^2\rangle\langle(\Delta\hat{B})^2\rangle \geq \frac{1}{4}|\langle\hat{C}\rangle|^2, \tag{15}$$

where $\langle(\Delta\hat{A})^2\rangle = (\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2)$. As an important application for a Pauli operator $\hat{S}_x$, $\hat{S}_y$ and $\hat{S}_z$, which describe the way a two-level atom interacts with the electromagnetic field, such that $[\hat{\sigma}_x, \hat{\sigma}_y] = i\hat{\sigma}_z$, can be used to define the uncertainty as $\Delta\hat{\sigma}_x\Delta\hat{\sigma}_y \geq \frac{1}{2}|\langle\hat{\sigma}_z\rangle|$. The variance squeezing (VS) for component $\Delta\hat{\sigma}_\alpha$ is squeezed if $\hat{\sigma}_\alpha$ fulfills the requirement of Eq. (15),

$$\begin{aligned} V(\hat{\sigma}_\alpha) &= \left(\Delta\hat{\sigma}_\alpha - \sqrt{\left|\frac{\langle\hat{\sigma}_z\rangle}{2}\right|}\right) < 0, \\ \Delta\hat{\sigma}_\alpha &= \sqrt{\langle\hat{\sigma}_\alpha\rangle^2 - \langle\hat{\sigma}_\alpha^2\rangle}, \qquad \alpha = x \ \ or \ \ y. \end{aligned} \tag{16}$$

Using the next formula, the AES can be written as:

$$E(\hat{\sigma}_\alpha) = \delta H(\hat{\sigma}_\alpha) - \frac{2}{\sqrt{\delta H(\hat{\sigma}_z)}} < 0, \ \alpha = x \ or \ y \tag{17}$$

where $H(\hat{\sigma}_\alpha)$ is the atomic operators' Shannon information entropies $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$.

By applying the previous condition, the behavior of the ES and VS related to the uncertainty principle can be examined.

When $\delta = \gamma = 0$ and $\alpha = \beta = 0$, $\phi = 1$, it is obvious that the AES becomes feasible for $E(\hat{\sigma}_x)$ several times (at regions of collapses for the atomic population inversion [9]) and does not occur when $E(\hat{\sigma}_y)$. It is noteworthy that the collapse periods have a direct relationship to the phenomenon of maximally entangled between parts of the system. These periods have applications in quantum computing [24] and quantum algorithm [25]. In addition, higher values of squeezing are found at the center of the collapse points, as seen in Fig. 1a. When considering the effect of the linear time dependence $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0$, $\phi = 1$, the intervals of squeezing for the function $E(\hat{\sigma}_x)$ decrease to small regions, and more oscillations of the AES function are built-up (see Fig. 1b). Fig. 1c shows that the maximum values of squeezing increase, and more fluctuations in the squeezing function occur by employing the parameter β. Additionally, the fluctuations of the oscillations increase substantially by adapting $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0.5$, $\phi = 1$. For the off-resonance case $\delta = 4g$ and $\gamma = 0$, $\alpha = 1$, $\beta = 0.5$, $\phi = 1$, the squeezing in the previous cases deteriorates after adding the detuning term in the system. The squeezing phenomenon exists in a few regions, as shown in Fig. 1d. After inserting the dissipation term into the interaction cavity, the squeezing periods appear after the start of the interaction and decrease quickly until disappearing, as shown in Fig. 5a. Therefore, the entropy squeezing is affected by changing both the detuning and the dissipation parameters. Generally, the squeezing disappears.

Fig. 2a shows the VS when $\delta = \gamma = 0$ and $\alpha = \beta = 0$, $\phi = 1$. It is shown that the squeezing occurs in many intervals, but the regions of the VS are less than those of the ES. On the contrary, when considering the time dependence $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0$, $\phi = 1$, the squeezing increases compared with the beforehand case and exists in many intervals (where the higher value of squeezing occurs at -0.08) as shown in Fig. 2b. When $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0.5$, $\phi = 1$, the squeezing decreases, and the maximum values reduce to

-0.06. Additionally, the fluctuations increase, with a slight shift in $V(\hat{\sigma}_x)$ after the onset of the considered time, as shown in Fig. 2c. Finally, the amount of squeezing and the amplitude of the oscillations decrease considerably when the interaction time increases, but the fluctuations increase substantially, as shown in Fig. 2d. When we take into account the dissipation term, the squeezing periods are only fully realized in one period at the start of the interaction. The maximum values of the $V(\hat{\sigma}_x)$ and $V(\hat{\sigma}_y)$ functions increase, and the amplitude of the vibrations decreases rapidly, as evident in Fig. 5d.

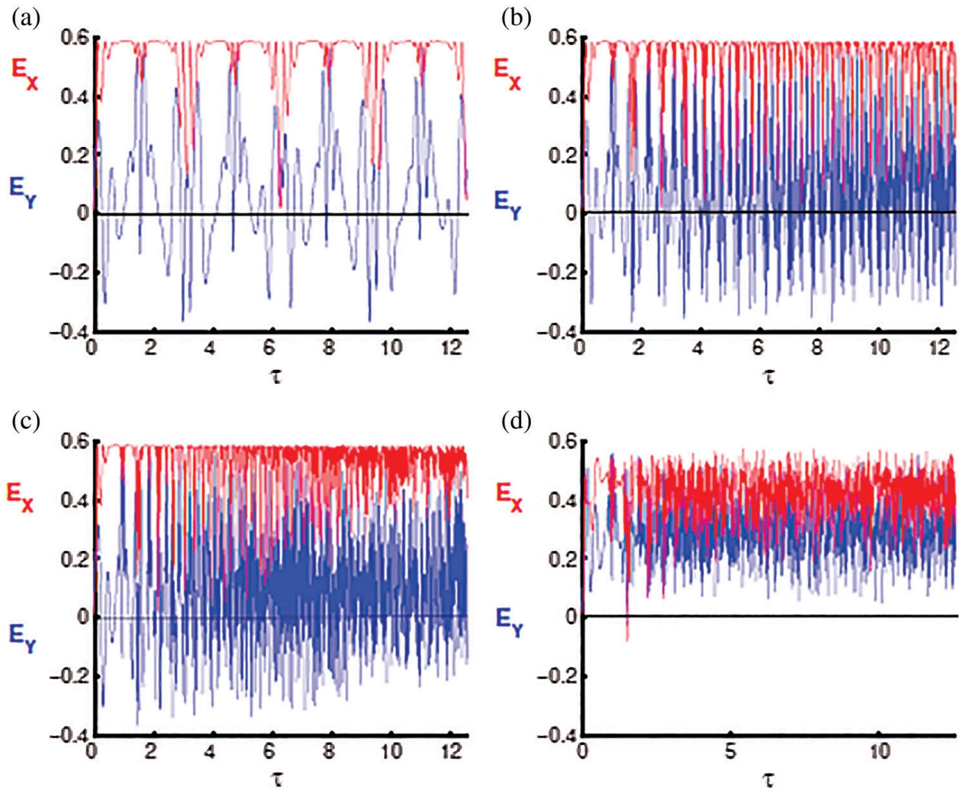

Figure 1: The dynamics of the AES components E_X and E_Y as a function of the scaled time τ, with the atoms primarily in excited states, and the fields are prepared in a pair coherent states with $\gamma = 0$ and fixed parameter $b = 5$. (a) $\delta = 0$, $\alpha = \beta = 0$, $\varphi = 1$, (b) $\delta = 0$, $\alpha = 1$, $\beta = 0$, $\varphi = 1$, (c) $\delta = 0$, $\alpha = 1$, $\beta = 0.5$, $\varphi = 1$, and (d) $\delta = 4$ g, $\alpha = 1$, $\beta = 0.5$, $\varphi = 1$

4 State Fidelity and Qubit-Qubit Entanglement

Next, we will analyze the degree of entanglement by employing fidelity since it is the primary criterion for measuring the entanglement of the system components [26–29]. Recently it has been found that the fidelity can measure the entanglement between parts of a system. It also plays an important role in quantum information in terms of estimating purity periods and partial entanglement [30]. The state fidelity of the present system can be written as

$$\xi(t) = |\langle\Theta(0)|\Theta(t)\rangle|^2 \tag{18}$$

First, the case $\delta = \gamma = 0$ and $\alpha = \beta = 0$, $\phi = 1$ is considered. It is evident that the function $\xi(t)$ varies between 0 and 1, where fidelity starts from the pure state ($\xi(t) = 1$) followed by partial entanglement. Then, the function $\xi(t)$ is fixed (stability) for a period, after which the function $\xi(t)$ has periodically oscillations between the lower and higher values, as shown in Fig. 3a. For the case $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0$ and $\phi = 1$ (velocity case), the oscillations increase, and the periods of the fixed intervals decrease. The results indicate that there are rapid fluctuations after adding the time dependence to the interaction cavity. Therefore, there is a strong entanglement between the parts of the system, as confirmed by Fig. 3b. While the oscillations of the function $\xi(t)$ increase sharply, the maximum values decrease, and the smaller values gradually increase after taking into account the acceleration case $\delta = \gamma = 0$ and $\alpha = 1$, $\beta = 0.5$, and $\phi = 1$, as shown in Fig. 1c. After adding the detuning to the interaction cavity $\delta = 4g$ and $\gamma = 0$, $\alpha = 1$, $\beta = 0.5$ and $\phi = 1$, the function $\xi(t)$ approaches the pure state compared to that of the previous cases, and the lower values increase. Thus, the entanglement decreases as expected from the effect of the detuning parameter, as shown in Fig. 3d. After the dissipation term is inserted into the interaction cavity, the maximum values decrease gradually over time. We also note that the correlation between parts of the system decreases until the function $\xi(t)$ reaches the stability state, as is evident in Fig. 5c.

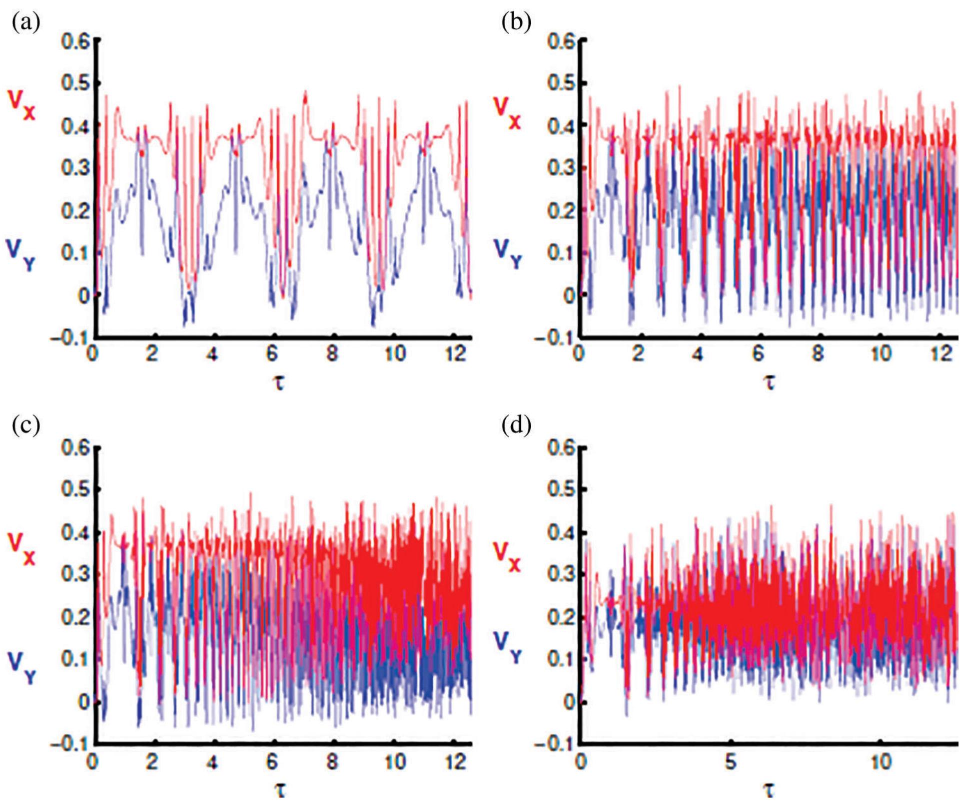

Figure 2: Variance squeezing as a function of the scaled time τ when the other conditions are the same as those in Fig. 1

The quantum entanglement between the subsystems is useful in quantum computing and quantum information processing [31,32]. Using the Peres-Horodecki criterion, qubit-qubit entanglement is attained from the evolution of negativity [33,34]. In the following function

$$\langle i_A, j_B | \rho^{T_A}(t) | k_A, l_B \rangle = \langle k_A, j_B | \rho_{SC_{AB}}(t) | i_A, l_B \rangle, \quad (19)$$

where $\rho^{T_A}(t)$ represents the partial transpose of $\rho_{AB}(t) = tr\{|\Theta(t)\rangle\langle\Theta(t)|\}$ in terms of the first atom A. Therefore, negativity takes the following form [35,36]

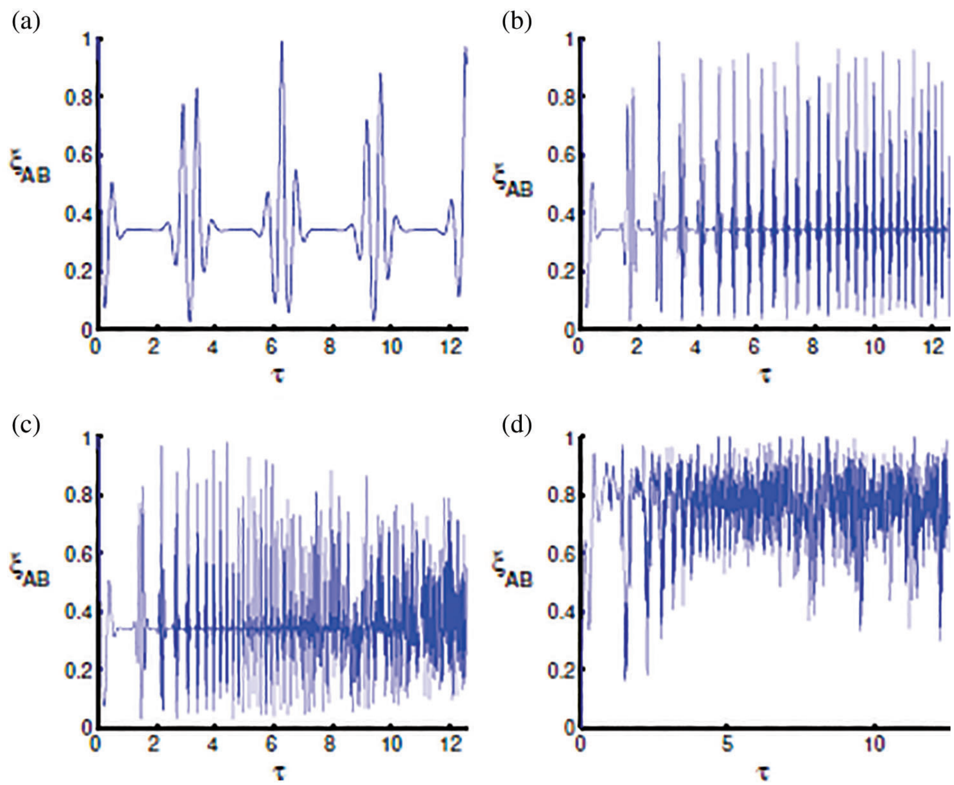

Figure 3: Fidelity as a function of the scaled time τ the other conditions are the same as those in Fig. 1

$$N(\rho) = \max\left\{0, -2\sum_{m} \chi_m(t)\right\}, \tag{20}$$

The sum is taken over each negative eigenvalue of $\rho^{T_A}(t)$. The entanglement of the solid-state system takes place if $N(\rho)$ is positive. The state $N(\rho) = 1$ corresponds to the maximal entanglement, whereas $N_{AB} = 0$ indicates that the atoms A and B are not related.

Using those same conditions of the previous sections, we first exclude the dissipation term, detuning, and time dependence for the velocity and acceleration. It is observed that the negativity oscillates periodically between 0and the maximum value, and the $N(\rho)$ function reaches zero before and after the points $t = \frac{n\pi}{\lambda}$ [5]. The results indicate that negativity reaches zero during the periods when the fidelity is fixed by comparing Figs. 3a and 4a. When adding the time dependence to the interaction cavity, we find that the fluctuations increase, and the oscillations become fast between the maximum and minimum values. This finding indicates that the entanglement between the qubits becomes significant for the case of time dependence (see Fig. 4b). The speed of the oscillations increases sharply, the smallest values increase, and the maximum values decrease after considering the acceleration, as seen in Fig. 4c. After adding the detuning to the interaction, the entanglement between the qubits decreases, and the center of the oscillations becomes approximately 0.25. After taking into account the dissipation term, negativity decreases gradually until it reaches its lowest value after some time as shown in Fig. 5d. In contrast, the way correlations behave can be greatly influenced by the selection of time dependence, especially the acceleration state and the parameter of dissipation term.

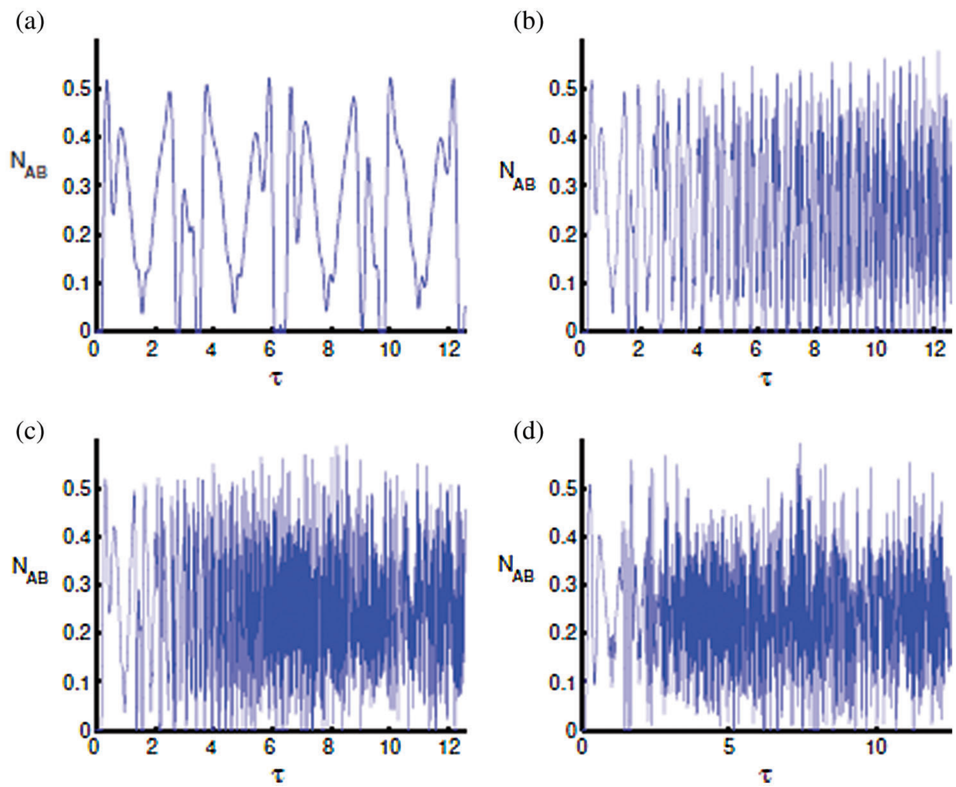

Figure 4: Negativity N(ρ) for the parameters utilized of Fig. 1

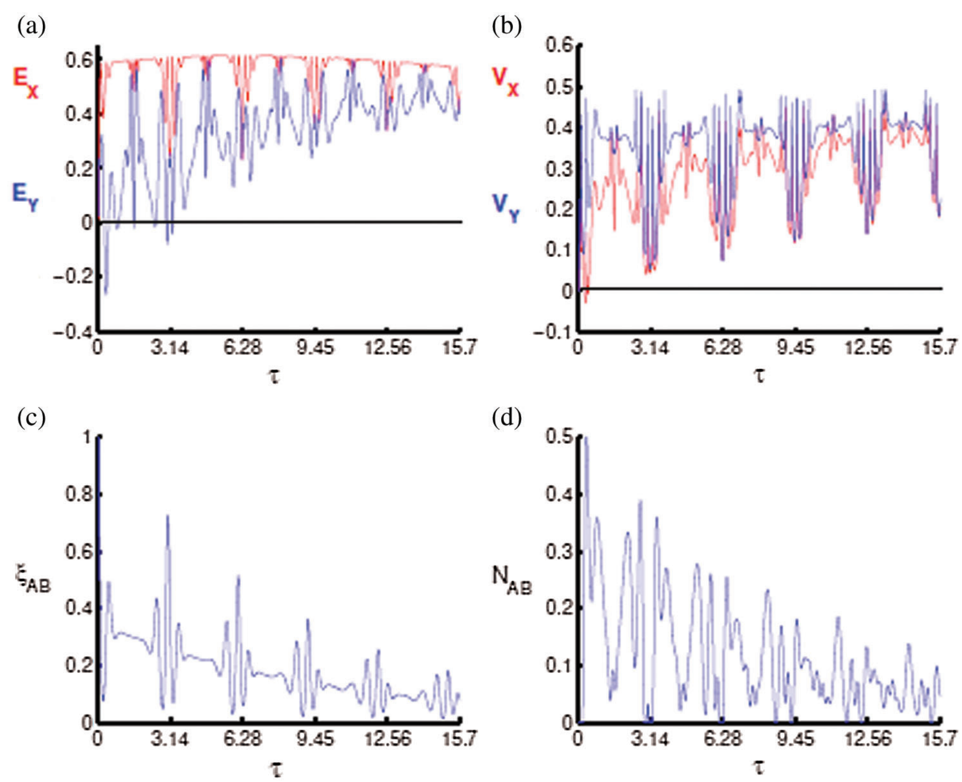

Figure 5: (a) Entropy squeezing, (b) variance squeezing, (c) fidelity and (d) negativity N(ρ) for the parameters $\delta = 0$, $\alpha = \beta = 0$, $\varphi = 1$, $\gamma = 0.01$

5 Conclusion

The effect of time dependence on a system containing two qubits within a cavity consisting of a pair of amplifier-type of electromagnetic fields in the presence of dissipation effect was studied. The constants of motion were calculated, and the general solution was obtained by solving the Schrödinger differential equations. The total density matrix was written via the wave function and was used to calculate and analyze the influence of the time dependence and the dissipation parameter on the entropy and variance squeezing. The results show that there was a superstructure between the atomic state fidelity and negativity based on a comparison of Figs. 3 and 4. Moreover, the degree of entanglement was proportional to the value of the time-dependent parameters. It was found that the degree of entanglement decreased after taking into account the detuning and time-dependent parameters. The dissipation parameter due to the interacting qubits and the electromagnetic field can be controlled, which helps improve and alleviate the correlation between the qubits during the interaction period. The ES and VS were examined. Furthermore, the squeezing phenomena occurred in the quadratures $E(\hat{\sigma}_x)$, $V(\hat{\sigma}_x)$ and rarely occurred in the quadratures $E(\hat{\sigma}_y)$, $V(\hat{\sigma}_y)$. The squeezing periods appeared after the start of the interaction and decreased quickly until disappearing after adding the dissipation term into the interaction cavity.

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