Computers, Materials & Continua
DOI:10.32604/cmc.2021.014224
Article

Ordering Cost Depletion in Inventory Policy with Imperfect Products and Backorder Rebate

Sandeep Kumar[1], Teekam Singh[1,2], Kamaleldin Abodayeh[3] and Wasfi Shatanawi[3,4,5,*]

[1]Department of Mathematics, Graphic Era Hill University, Dehradun, 248002, India
[2]Department of Mathematics, Graphic Era Deemed University, Dehradun, 248002, India
[3]Department of Mathematics and General Sciences, Prince Sultan University, Riyadh, 11586, Saudi Arabia
[4]China Medical University Hospital, China Medical University, Taichung, 40402, Taiwan
[5]Department of Mathematics, Hashemite University, Zarqa, 13133, Jordan
[*]Corresponding Author: Wasfi Shatanawi. Email: wshatanawi@psu.edu.sa
Received: 07 September 2020; Accepted: 09 October 2020

Abstract: This study presents an inventory model for imperfect products with depletion in ordering costs and constant lead time where the price discount in the backorder is permitted. The imperfect products are refused or modified or if they reached to the customer, returned and thus some extra costs are experienced. Lately some of the researchers explicitly present on the significant association between size of lot and quality imperfection. In practical situations, the unsatisfied demands increase the period of lead time and decrease the backorders. To control customers' problems and losses, the supplier provides a price discount in backorders during shortages. Also, an order's policies may result in including some imperfect products in arrival lots. A discount on price may be offered by the supplier on the out-of-stock products to manage the backorder problems. The study aims to develop a model with imperfect products by permitting the price discount in backorders, and the cost of ordering is considered a decision variable. First, it is assumed that the demand for lead time is followed by a normal distribution and then stops it and assumed that the first two moments of demand for lead time are known. Further, the minimax distribution method is used to solve this model, and a separate algorithm is designed. In this study, two models are discussed with and without a normally distributed rate of demand. The current study verified with the help of some numerical examples over various model parameters.

Keywords: Inventory; ordering cost; imperfect product; lead time; backorder

1 Introduction and Literature Review

The occurrence of shortages is an essential factor in the study of inventory control. In markets, several products of profitable brands such as branded shoes and garments may result in a condition in which the customer may like to wait for backorders even during shortages. Asides the branded product, the image of the supplier and showroom attract the customers for backorder. To enhance customer loyalty, the showroom owners upgrade their customer services, provide some gifts to the customers, and maintain the quality of products. But these activities are not free; naturally, some extra costs are there. There should be a policy to lower the cost of shortages of the annual total required cost and lost sales. A vendor must manage an optimal lead time length to find a required rate of back-ordering so that the related inventory cost is minimized. In recent years, researchers have used these characteristics and modified conventional inventory models to include the execution of lead time concepts.

The conventional inventory theory does not consider the time-importance of money for defective products during production. Generally, during production, the maximum products will be imperfect, and due to the cost of opportunity, the time value of money will be there. With this thought, an inventory model for imperfect products must be elaborated to consider into account the value of time of money. Buzacott [1] presented an EOQ model in the case of inflation. Many researchers [2–4] followed the model of Buzacott [1] and introduced models by including the different rates of inflations for all costa, time-value of money, shortages, finite replenishment, etc. Goyal [5] studied complete literature for previous perishable inventory models. These surveys let out that decaying inventory models have attention. Park [6] investigates an economic order quantity as purchasing credit. Vrat et al. [7] presented that the consumption rate of goods is dependent on the size of stock at the initial cycle time. To study the impact of inflation and time value money and inflation for a finite time horizon, Pal [8] added a model with shortages and a linear rate of time-dependent demand. Lio et al. [9] studied a non-deterministic inventory model where the order's quantity is known where the decision variable is lead time. The study of Lio et al. [9] explored by Ben-Daya et al. [10]. Ouyang et al. [11] have taken a systematic survey of where both the review period and lead time are taken as decision variables. Ouyang et al. [12] elaborated on an integrated policy where the lead time is controllable. Several studies have been carried to present few guidelines in different situations with lead time, such as Hoque et al. [13] and Lee [14]. Gallego et al. [15] presented a newsboy problem, which is distributed free.

It can be observed that due to the unsatisfied demands, some customers may opt for the option of the backorder, and some may refuse it. The customers can be attracted for backorders by offering the discount on backorder price. In general, instead of providing a discount on backorder price on stock-out products, it is better to make the customers more prepared to wait for the wanted products. Pan et al. [16] analyzed a desegregated policy with a discount on the backorder price. Lee et al. [17] elaborated on a joint inventory policy with the cost of ordering cost and varying lead time. Salameh et al. [18] analyzed an inspection and joint policy of lot size. Jaber et al. [19] extended the study of Salameh et al. [18] and provided a straightforward approach to find the lot size quantity. Chang et al. [20] also generalized the analysis of Salameh et al. [18]. They supposed that the non-conforming products could be sold at low cost, rejected, or reworked immediately. Hayek et al. [21] presented a model for imperfect products where the production rate is finite, and shortages are allowed. In the recent study of Annadurai et al. [22] considered the product of imperfect quality and presented a policy with set-up cost and varying lead time. Schwaller [23] developed a policy that expands a model by including the hypothesis that a given percentage of imperfect products in an arrived lot and a cost of inspection are needed to find and destroy the imperfect products. Salameh et al. [24] developed a policy for an EOQ model by including the hypothesis that a known percentage of imperfect products is random. It is also included that after the 100% inspection, the imperfect products could be sold at a lower price in a single batch. Chang [25] analyzed an EOQ fuzzy model with an imperfect rate of demand.

In the modern era, all production firms try to make perfect quality products, but it is not possible to make all products of excellent quality for various reasons. Generally, it is considered that all products are of the best quality, but practically it can be observed that imperfect products being manufactured due to lousy manufacturing methods. The imperfect products must be destroyed, reworked, or refunded by the customers. Paknejad et al. [26] elaborated on a value-compromised stock inventory policy with non-deterministic demand. Sarkar et al. [27] studied an imperfect manufacturing system for imperfect products in an inventory model with a decreased selling price. Chung et al. [28] analyzed that retailers may pay some amount for other stores for business requirements. Many researchers presented inventory models with perfect and imperfect quality separately. Papachristos et al. [29] analyzed EOQ models for imperfect quality products. Eroglu et al. [30] elaborated on an EOQ model for imperfect products and shortages. Wee et al. [31] presented an optimal policy for the products with bad quality and back-ordering. Chang et al. [20] proposed the solutions for an optimal inventory model for imperfect quality products and shortages. Chang [25] analyzed a fuzzy EOQ model for lousy quality items. Khan et al. [32] provided a review of the extended EOQ model for lousy quality items. Hayek et al. [21] presented a lot of size production policy to repair awful quality products. Goyal et al. [33] and Chan et al. [34] also showed their studies for imperfect products. Sarkar et al. [27] analyzed a policy for imperfect products with non-deterministic demand. Lee et al. [35] presented a policy of a model for imperfect products where the quantity of order and lead time are taken as decision variables. Skouri et al. [36] discussed the supply quality effects on costs. In this study, they provided an alternative approach where whole supplied batches may be of low standard and so refused.

In present work, an inventory policy for imperfect products where the discount in backorder price to the customer is allowed and considered as a decision variable. Here two models are discussed first with demand (normally distributed) and second with demand (generally distributed). We proposed a computational algorithm to find the required optimal results. This paper's blueprint is as follows: In Section 2, notations and assumptions are described, which are used throughout the study. In Section 3, an integrated inventory policy is presented for imperfect products with depletion in ordering cost and constant lead time where the price discount in the backorder is permitted. In Section 3, both the normally distributed model and the non-distributed model are discussed. In Section 4, the numerical verification of the study is provided. Finally, conclusions, suggestions, and future scope of the study are presented in Section 5.

2 Notations and Assumptions

2.1 Notations

The following notations are used in the present study.

B: Demand (annual)

W: Quantity of ordering

C_0: Actual ordering cost without any investment

C: Per order ordering cost, $0 < C < C_0$

M: Quantity of ordering

τ: Length of lead time

α: Per unit discount in backorder price

β: Per unit insignificant profit

γ: Per unit cost of inspection

H: Per year per unit non imperfect cost of holding.

H': Per year per unit imperfect cost of holding.

μ: Backordered fraction of demand in stock-out period duration, $0 \leq \mu < 1$

μ_0: Upper bound (for the ratio of backorder

s: Imperfect rate (per order lot), a random variable, $0 \leq s < 1$

$g(s)$: PDF (probability density function) for s

θ: Opportunity capital cost (fractional annual)

$E(.)$: Mathematics expectation

z_m: $z_m = \max\{0, z\}$

Z: The lead time demand, has PDF f_Z with mean $B\tau$ and S.D $\sigma\sqrt{\tau}$

Π: The class of the CDF (cumulative density function) f_Z with mean DL and S.D $\sigma\sqrt{\tau}$

2.2 Assumptions

The following assumptions are used in the present study.

1. Replenishment is allowed when inventory level goes to the point of re-order.
2. If discount in price is larger than the minor gain i.e., $\alpha > \beta$ then the provider may not be ready to offer the discount in backorder price.
3. Inspection is without error.
4. An arrival lot may have some imperfect products. Suppose that the counting of imperfect products in an arrival lot of size W is considered as a binomial random variables of parameters s ($0 \leq s < 1$) and W. All products of an arrival are checked, and imperfect products are returned back.
5. The lead time (τ) contains n components which are not mutually dependent. Assume that u_i, v_i and w_i are minimum duration, normal duration and per unit time crashing cost for i^{th} component respectively. Where $w_1 \leq w_2 \leq w_3, \ldots, \leq w_n$.
6. The point of re-order (R) is given as

R = awaited demand during lead time + stock of safety

i.e., $R = B\tau + j\sigma\sqrt{\tau}$, here j is the factor of safety.

7. Assume that τ_i is the lead time length for component i. Then τ_i can be presented as follows

$$\tau_i = \sum_{j=1}^{n} v_j - \sum_{j=1}^{i} (v_j - u_j), \text{where } (i = 1, 2, 3 \ldots n)$$

Here the per cycle cost of lead time crashing (U) is as follows

$$U = w_j(\tau_{i-1} - \tau) + \sum_{j=1}^{i-1} w_j(v_j - u_j), \tau_i \leq \tau \leq \tau_{i-1}$$

8. During the period of stock-out, μ (ratio of backordering) is a variable and directly proportional to α (per unit discount in backorder price, provided by supplier). So $\mu = \mu_0\alpha/\beta$, where $0 \leq \mu_0 < 1$ and $0 \leq \alpha \leq \beta$.
9. If the size of arrival lot is W with an imperfect rate s, then all products are identified and separated the imperfect products from the arrival lot W. So, the actual ordered quantity W is decreased by a quantity $W(1 - s)$.

3 Mathematical Formulation of the Model

It is assumed that the lead time demand, i.e., Z has the probability distribution function $f_Z(z)$ with mean $B\tau$ and S.D $\sigma\sqrt{\tau}$. The per cycle expected counting of backorders is $\mu E(Z - R)^+$ where $E(Z - R)^+$ is the shortage against the expected demand after the cycle completion and $(B/W)E(Z - R)^+[\alpha\mu + \beta(1 - \mu)]$ is the annual cost of stock-out. The level of net inventory (expected) before the arrival of lot of order is

$[R - B\tau + (1-\mu)E(Z-R)^+]$. The total expected cost (annual) will consist of imperfect holding, non-imperfect holding cost, ordering cost, and inspection cost, the cost of lead time crashing and cost of stock-out. The combined inventory function of cost (TIC) is given as below

$$\begin{aligned} TIC(W,R,\mu,\tau) = {} & \frac{B}{W\{1-E(s)\}}\left[C + U + E(Z-R)^+\{\alpha\mu + \beta(1-\mu)\} + \gamma W\right] \\ & + \frac{H}{2}\left[W\{1-E(s)\} + \frac{W(E(s^2) - E^2(s))}{1-E(s)} + \frac{E[s(1-s)]}{1-E(s)}\right] \\ & + H\left[R - B\tau + (1-\mu)E(Z-R)^+\right] + H'(W-1)\frac{E[s(1-s)]}{1-E(s)} \end{aligned} \tag{1}$$

Here $B/W\{1-E(s)\}$ is the number of expected orders annually.

In this study the ordering cost C is considered as a decision variable. The sum of capital investment cost and all inventory costs can be minimized by optimizing over W, C, R, μ and τ with the constraint $0 < C \le C_0$, where C_0 is the actual cost of ordering. Now, the capital investment of supplier is $\theta I(C)$, where θ is the per year opportunity capital cost (fractional annual) and obeys the logarithmic investment function defined as $I(C) = m\ln(C_0/C)$, $0 < C \le C_0$, where $1/m$ is the fraction of decrement in C against the increment in investment (per dollar). Thus from Eq. (1)

$$\begin{aligned} TIC(W,C,R,\mu,\tau) = {} & m\ln(C_0/C) + \frac{B}{W\{1-E(s)\}}\left[C + U + E(Z-R)^+\{\alpha\mu + \beta(1-\mu)\} + \gamma W\right] \\ & + \frac{H}{2}\left[W\{1-E(s)\} + \frac{W(E(s^2) - E^2(s))}{1-E(s)} + \frac{E[s(1-s)]}{1-E(s)}\right] \\ & + H\left[R - B\tau + (1-\mu)E(Z-R)^+\right] + H'(W-1)\frac{E[s(1-s)]}{1-E(s)} \end{aligned} \tag{2}$$

Furthermore during the period of stock-out, the ratio of backorder (μ) is a variable and directly varies to the price discount in backorder (α) provided by the supplier (per unit). Thus $\mu = \mu_0\alpha/\beta$, where $0 \le \mu_0 < 1$ and $0 \le \alpha \le \beta$. So the per unit price discount of backorder (α) is considered as a decision inconstant in place of ratio of backorder (μ). So Eq. (2) will become

$$\begin{aligned} TIC(W,C,R,\mu,\tau) = {} & m\ln(C_0/C) + \frac{B}{W\{1-E(s)\}}\left[C + U + E(Z-R)^+\left\{\frac{\mu_0\alpha^2}{\beta} + \beta - \mu_0\alpha\right\} + \gamma W\right] \\ & + \frac{H}{2}\left[W\{1-E(s)\} + \frac{W(E(s^2) - E^2(s))}{1-E(s)} + \frac{E[s(1-s)]}{1-E(s)}\right] \\ & + H\left[R - B\tau + (1-\frac{\mu_0\alpha}{\beta})E(Z-R)^+\right] + H'(W-1)\frac{E[s(1-s)]}{1-E(s)} \end{aligned} \tag{3}$$

3.1 Normal Distribution Model

It is assumed that the demand of lead time Z abides normal distribution with p.d.f $f_Z(z)$, mean DL and S.D $\sigma\sqrt{\tau}$. We have $R = B\tau + j\sigma\sqrt{\tau}$, here j is safety factor and the shortage quantity (expected) at the completion of cycle is given as $E(Z-R)^+ = \int_R^\infty (z-R)^+ f_Z(z)dz = \sigma\sqrt{\tau}\phi(j) > 0$, where $\phi(j) = \phi_1(j) - j\{1-\phi_2(j)\}$. Here $\phi_1(j)$ is standard normal p.d.f and $\phi_2(j)$ is cumulative distribution function. Hence Eq. (3) can be expressed as

$$TIC_N(W,C,j,\alpha,\tau) = m\ln(C_0/C) + \frac{B}{W\{1-E(s)\}}\left[C+U+\sigma\sqrt{\tau}\phi(j)\left\{\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right\}+\gamma W\right]$$
$$+\frac{H}{2}\left[W\{1-E(s)\}+\frac{W(E(s^2)-E^2(s))}{1-E(s)}+\frac{E[s(1-s)]}{1-E(s)}\right]$$
$$+H\left[R-B\tau+(1-\frac{\mu_0\alpha}{\beta})\sigma\sqrt{\tau}\phi(j)\right]+H'(W-1)\frac{E[s(1-s)]}{1-E(s)} \quad (4)$$

For the solution of Eq. (4) we differentiae $TIC_N(W,C,j,\alpha,\tau)$ partially with respect to W, C, j, α and τ respectively. We have

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial W} = -\frac{B[C+U+(\mu_0\alpha^2/\beta+\beta-\mu_0\alpha)\sigma\sqrt{\tau}\phi(j)]}{W^2\{1-E(s)\}} \quad (5)$$

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial C} = -\frac{\theta m}{C}+\frac{B}{W\{1-E(s)\}} \quad (6)$$

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial j} = H\sigma\sqrt{\tau}+\left[H\left(1-\frac{\mu_0\alpha}{\beta}\right)+\frac{B}{W(1-E(s))}\left(\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right)\right]\times\sigma\sqrt{\tau}P_x(j) \quad (7)$$

where $P_x(j)=P(x\geq j)$, x is standard normal variable.

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial \alpha} = \left[\frac{B}{W(1-E(s))}\left(\frac{2\mu_0\alpha}{\beta}-\mu_0-\frac{H\mu_0}{\beta}\right)\right]\sigma\sqrt{\tau}\phi(j) \quad (8)$$

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial \tau} = \frac{B}{2W(1-E(s))}\left(\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right)\frac{\sigma\phi(j)}{\sqrt{\tau}}$$
$$+\frac{H\sigma}{2\sqrt{\tau}}\left[j+(1-\frac{\mu_0\alpha}{\beta})\phi(j)\right]-\frac{B}{W(1-E(s))} \quad (9)$$

By checking the sufficient conditions, for fixed (W,C,j,α), $TIC_N(W,C,j,\alpha,\tau)$ is concave for $\tau_i\leq\tau\leq\tau_{i-1}$ as

$$\frac{\partial TIC_N(W,C,j,\alpha,\tau)}{\partial \tau} = -\frac{B}{4W(1-E(s))}\left(\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right)\frac{\sigma\phi(j)}{\tau^{3/2}}$$
$$+\frac{H\sigma}{4\tau^{3/2}}\left[j+(1-\frac{\mu_0\alpha}{\beta})\phi(j)\right]<0 \quad (10)$$

So, for fixed (W,C,j,α), the total expected minimum cost (annual) will exist at the last of the interval $[\tau_i,\tau_{i-1}]$. Now, by putting Eq. (6) equal to zero and then solving for C, we have

$$C=\frac{\theta mW(1-E(s))}{B} \quad (11)$$

Now, putting Eq. (6) equal to zero and then solving for α, we have

$$\alpha=\frac{HW(1-E(s))}{2B}+\frac{\beta}{2}c \quad (12)$$

Putting the value of α from Eq. (12) in Eq. (5) and then putting it equal to zero, have

$$W = \sqrt{\frac{2B[C + U\{\beta(4-\beta_0)/4\}\sigma\sqrt{L}\phi(j)}{HS(E_s)[1-(H\mu_0/2D\beta)\sigma\sqrt{L}\phi(j)]}} \tag{13}$$

where $S(E_s) == 1 - 2E(s) + E(s^2) + (2H'/H)E(1 - E(s))$

Now, again putting the value of α from Eq. (12) in Eq. (7) and then setting it equal to zero, we have

$$P_x(j) = 4H\beta BW(1-E(s))\{(4-\mu_0)\beta^2B^2 + (4-2\mu_0)H\beta BW(1-E(s))\} - \frac{\mu_0 H^2 W^2}{(1-E(s))} \tag{14}$$

To find the values of (W, C, j, α), we solve the Eqs. (11) to (14) for the interval $[\tau_i, \tau_{i-1}]$ and denote these values by $(W^*, C^*, j^*, \alpha^*)$ respectively. The following postulation claims that, for fixed interval $[\tau_i, \tau_{i-1}]$ the point $(W^*, C^*, j^*, \alpha^*)$ is the optimal solution for the minimum expected annual cost.

Postulation 1. The Hessian matrix for $TIC_N(W, C, j, \alpha, \tau)$ in interval $[\tau_i, \tau_{i-1}]$ is positive define at the point $(W^*, C^*, j^*, \alpha^*)$.

The effects of C on the total annual cost (expected) can be examined by finding the partial derivative of second order of $TIC_N(W, C, j, \alpha, \tau)$ with respect to C and obtaining $\frac{\partial^2 TIC_N(W, C, j, \alpha, \tau)}{\partial C^2} = \theta m/C^2 > 0$. Thus $TIC_N(W, C, j, \alpha, \tau)$ is convex in C for fixed $[\tau_i, \tau_{i-1}]$ and (W, C, j, α). Since it is not easy to find the solution for(W, C, j, α, τ), so the following algorithm is used to find the solution

Algorithm 1

Step 1. For every τ_i (i = 0,1, 2, 3…n) perform the following sub steps

(i) Start with $j_{i1} = 0$ and $C_{i1} = C_0$

(ii) Evaluate W_i by substituting $\phi(j_{i1})$ and C_{i1}

(iii) With this value of W_i find C_{i2} by Eq. (11), α_i by Eq. (12) and $P_x(j_{i2})$ by Eq. (14)

(iv) Determine j_{i2} and then $\phi(j_{i2})$

(v) Repeat sub steps (ii) to (iv) until the values of W_i, C_i, α_i and j_i are unchanged.

Step 2. Compare C_i with C_0 and α_i with α_0, there will be two cases

(i) If $C_i \le C_0$ and $\alpha_i \le \alpha_0$ then C_i and α_i are feasible. Go to Step 3.

(ii) If $C_i > C_0$ and $\alpha_i > \alpha_0$ then C_i and α_i are infeasible. For a particular τ_i, let $C_i = C_0$ and $\alpha_i = \alpha_0$ and find the values of (W_i, j_i, α_i) by iterative methods with the help of Eqs. (12), (13) and (14). Go to Step 3.

Step 3. For every $(W_i, C_i, j_i, \alpha_i, \tau_i)$ determine the corresponding total expected cost (annual) by $TIC_N(W_i, C_i, j_i, \alpha_i, \tau_i)$ using Eq. (4).

Step 4. $M = \min\{TIC_N(W_i, C_i, j_i, \alpha_i, \tau_i) : i = 0, 1, 2, 3, \ldots.., n\}$ and put $TIC_N(W^*, C^*, j^*, \alpha^*, \tau_i{}^*) = M$. Then $(W^*, C^*, j^*, \alpha^*, \tau_i{}^*)$ is the final solution and the optimal reordering point $R^* = B\tau^* + J^*\sigma\sqrt{\tau^*}$ can be determined.

3.2 Model without Distribution

In real situations the distribution of probability for the demand of lead time is restricted. A good manager can predict the variance and mean value of the demand of lead time. The actual distribution of probability may not be known. Due to the absence of required things the anticipated shortage $E(Z-R)^+$ is not known and so (W, C, j, α, τ) cannot be determined. To defeat this situation, assumption of normal distribution is moderated and assumed that the demand of lead time Z considered with first two moments. For instance the p.d.f f_Z of Z comes from the class Π with mean DL and S.D $\sigma\sqrt{\tau}$. Thus the procedure of minmax distribution-free is used to resolve this problem i.e., to determine the best p.d.f f_Z in Π for every (W, C, j, α, τ) and the total expected cost can be minimized.

To find Min-Max $TIC(W, C, j, \alpha, \tau)$, the following preposition is required which was presented by Gallego et al. [6]

$$E(Z-R)^{+} \leq \frac{1}{2}\left\{\sqrt{\sigma^2\tau + (R - B\tau)} - (R - B\tau)\right\} \tag{15}$$

Substituting $R = B\tau + j\sigma\sqrt{\tau}$ in (15), we have

$$E(Z-R)^{+} \leq \frac{1}{2}\sigma\sqrt{\tau}\left(\sqrt{1+j^2} - j\right) \tag{16}$$

Now using Eq. (3) and inequality (16) and taking the factor of safety j as a decision v factor in place of R, the function of can be presented as

$$\begin{aligned} TIC_U(W, C, R, \mu, \tau) &= m\ln(C_0/C) + \frac{B}{W\{1-E(s)\}}[C + U + \gamma W] \\ &+ \frac{H}{2}\left[2j\sigma\sqrt{\tau} + W\{1-E(s)\} + \frac{W(E(s^2) - E^2(s))}{1-E(s)} + \frac{E[s(1-s)]}{1-E(s)}\right] \\ &+ \frac{1}{2}\left[\frac{B}{W\{1-E(s)\}}\left(\frac{\mu_0\alpha^2}{\beta} + \beta - \mu_0\alpha\right) + H\left(1 - \frac{\mu_0\alpha}{\beta}\right)\right]\sigma\sqrt{\tau}\left(\sqrt{1+j^2} - j\right) \end{aligned} \tag{17}$$

Here $TIC_U(W, C, R, \mu, \tau)$ is the t final expected cost for distribution-free and it is the supremum of (W, C, R, μ, τ). Now we differentiate Eq. (17) with respect to W, C, R, μ and τ respectively in the interval $[\tau_i, \tau_{i-1}]$, we have

$$\begin{aligned} \frac{\partial TIC_U(W, C, R, \mu, \tau)}{\partial W} &= \left[-B\left\{C + U + \frac{1}{2}\left(\frac{\mu_0\alpha^2}{\beta} + \beta - \mu_0\alpha\right)\sigma\sqrt{\tau}\left(\sqrt{1+j^2} - j\right)\right\}\frac{W^2}{1-E(s)}\right] \\ &+ \frac{H}{2}\left[\{1-E(s)\} + \frac{W(E(s^2) - E^2(s))}{1-E(s)} + H'\frac{E\{s(1-s)\}}{1-E(s)}\right] \end{aligned} \tag{18}$$

$$\frac{\partial TIC_U(W, C, R, \mu, \tau)}{\partial C} = -\frac{\theta m}{C} + \frac{B}{W\{1-E(s)\}} \tag{19}$$

$$\begin{aligned} \frac{\partial TIC_U(W, C, R, \mu, \tau)}{\partial j} &= H\sigma\sqrt{\tau} - \frac{1}{2}\left[H\left(1 - \frac{\mu_0\alpha}{\beta}\right) + \frac{B}{W\{1-E(s)\}}\left(\frac{\mu_0\alpha^2}{\beta} + \beta - \mu_0\alpha\right)\right] \\ &\times \sqrt{\tau}\sigma\left(1 - \frac{j}{\sqrt{1+j^2}}\right) \end{aligned} \tag{20}$$

$$\frac{\partial TIC_U(W, C, R, \mu, \tau)}{\partial \alpha} = \left[\frac{B}{W\{1-E(s)\}}\left(\frac{2\mu_0\alpha}{\beta} - \mu_0\right) - \frac{H\mu_0}{\beta}\right]\sigma\sqrt{\tau}\left(\sqrt{1+j^2} - j\right) \tag{21}$$

$$\frac{\partial TIC_U(W,C,R,\mu,\tau)}{\partial\tau} = \frac{1}{4}\left[H\left(1-\frac{\mu_0\alpha}{\beta}\right)+\frac{B}{W\{1-E(s)\}}\left(\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right)\right] \times\frac{\sigma\left(\sqrt{1+j^2}-j\right)}{\sqrt{\tau}}+\frac{Hj\sigma}{2\sqrt{\tau}}-\frac{BCi}{W\{1-E(s)\}} \tag{22}$$

By checking the second order partial derivative, the sufficient conditions, for fixed (W, C, R, μ), $TIC_U(W, C, R, \mu, \tau)$ is concave in the interval $[\tau_i, \tau_{i-1}]$ as

$$\frac{\partial^2 TIC_U(W,C,R,\mu,\tau)}{\partial\tau^2} = -\frac{1}{8}\left[H\left(1-\frac{\mu_0\alpha}{\beta}\right)+\frac{B}{W\{1-E(s)\}}\left(\frac{\mu_0\alpha^2}{\beta}+\beta-\mu_0\alpha\right)\right] \times\frac{\sigma\left(\sqrt{1+j^2}-j\right)}{\tau^{3/2}}-\frac{Hj\sigma}{4\tau^{3/2}}<0 \tag{23}$$

So, for fixed (W, C, j, α), the total expected minimum cost (annual) will exist at the last of the interval $[\tau_i, \tau_{i-1}]$. Now, by putting Eq. (19) equal to zero and then solving for C, we have

$$C = \frac{\theta m W\{1-E(s)\}}{B} \tag{24}$$

Similarly solving for α by putting Eq. (21) to zero, we have

$$C = \frac{HW\{1-E(s)\}}{2B}+\frac{\beta}{2} \tag{25}$$

Again putting Eq. (25) into Eq. (18) and then putting it to zero, we have

$$W = \sqrt{\frac{2B[C+U\{\beta(4-\beta_0)/8\}\sigma\sqrt{\tau}(\sqrt{1+j^2}-j)]}{HS(E_s)[1-(H\mu_0/2B\beta)\sigma\sqrt{\tau}(\sqrt{1+j^2}-j)]}} \tag{26}$$

where $S(E_s) == 1-2E(s)+E(s^2)+(2H'/H)E\{s(1-E(s))\}$

Now, putting Eq. (25) into Eq. (20) and then putting it to zero, we have

$$1-\frac{j}{\sqrt{j^2+1}} = 8H\beta BW\{1-E(s)\}\left[(4-\mu_0)\beta^2B^2+(4-2\mu_0)H\beta BW\{1-E(s)\}\right] - \mu_0H^2W^2\{1-E(s)\}^{-1} \tag{27}$$

To find the values of (W, C, j, α), we solve the Eqs. (24) to (27) for the interval $[\tau_i, \tau_{i-1}]$ and denote these values by respectively (we represent these terms by $(W^*, C^*, j^*, \alpha^*)$). The following postulation claims that, for fixed interval $[\tau_i, \tau_{i-1}]$, the point $(W^*, C^*, j^*, \alpha^*)$ is the optimal solution for the minimum expected annual cost.

Postulation 3. The Hessian matrix for $TIC_U(W, C, j, \alpha, \tau)$ in interval $[\tau_i, \tau_{i-1}]$ is positive define at the point $(W^*, C^*, j^*, \alpha^*)$.

The effects of C on the total annual cost (expected) can be examined by finding the partial derivative of second order of $TIC_U(W, C, j, \alpha, \tau)$ with respect to C and obtaining $\frac{\partial^2 TIC_N(W,C,j,\alpha,\tau)}{\partial C^2} = \theta m/C^2 > 0$.

Thus $TIC_U(W, C, j, \alpha, \tau)$ is convex in C for fixed $[\tau_i, \tau_{i-1}]$ and (W, C, j, α). Since it is not easy to find the solution for (W, C, j, α, τ), so the Algorithm 2 is executed to determine the solution of $(W^*, C^*, j^*, \alpha^*, \tau^*)$ represented by $TIC_N(W^*, C^*, j^*, \alpha^*, \tau^*)$.

Algorithm 2

Step 1. For every τ_i (i = 0,1, 2, 3, …, n) perform the following sub steps

(i) Start with $j_{i1} = 0$ and $C_{i1} = C_0$

(ii) Evaluate W_i by substituting C_{i1} in Eq. (26)

(iii) With this value of W_i find C_{i2} by Eq. (24), α_i by Eq. (25) and j_{i2} by Eq. (27)

(iv) Repeat sub steps (ii) to (iii) until the values of W_i, C_i, α_i and j_i are unchanged.

Step 2. Compare C_i with C_0 and α_i with α_0, there will be two cases

(i) If $C_i \leq C_0$ and $\alpha_i \leq \alpha_0$ then C_i and α_i are feasible. Go to Step 3.

(ii) If $C_i > C_0$ and $\alpha_i > \alpha_0$ then C_i and α_i are infeasible. For a particular τ_i, let $C_i = C_0$ and $\alpha_i = \alpha_0$ and find the values of (W_i, j_i, α_i) by iterative methods with the help of Eqs. (26), (27) and (25). Go to Step 3.

Step 3. For every $(W_i, C_i, j_i, \alpha_i, \tau_i)$ determine the corresponding total expected cost (annual) by $TIC_U(W, C, j, \alpha, \tau)$ using Eq. (17).

Step 4. $M = \min\{TIC_U(W_i, C_i, j_i, \alpha_i, \tau_i) : i = 0, 1, 2, 3, \ldots, \text{n}\}$ and put $TIC_U(W^*, C^*, j^*, \alpha^*, \tau^*) = \text{M}$. Then $(W^*, C^*, j^*, \alpha^*, \tau^*)$ is the solution and the reorder point $R^* = B\tau^* + J^*\sigma\sqrt{\tau^*}$ can be obtained.

4 Numerical Verification of the Study

To verify the present study and illustrate the effects of reduction in ordering cost, an inventory product is considered with the same parameter values as in Pan et al. [16]. B = 600 units (every year), H = 20\$/unit/year, C_0 = 200\$/order, H' = 12\$/unit/year, σ = 7 units/week, β = 150\$/unit lost, γ = 1.6\$/unit. For the reduction in ordering cost, we consider θ = 0.1 and m = 5800. There are three components of lead time

Table 1: Data (Lead time)

Component of lead time (i)	Min. duration u_i (in days)	Normal distribution v_i (in days)	Unit cost of crashing w_i (\$ per day)
13	205	65	0.43
23	205	65	1.23
33	165	95	5.03

Example 1. Suppose a normal distribution follows the demand for lead time. The results are presented in Tab. 1 and Tab. 2 for = 0.2, 0.4, 0.6, 0.8 by using the Algorithm 1. Next, an analysis of optimal solutions is presented in Tab. 3 and to observe the impacts of reduction in the cost of ordering and discount in backorder price. The same Tab. 3 includes the results with the fixed cost of ordering and with no discount on the backorder price. The shavings can be observed by comparing both the cases, which range between 18.52% to 21.45%.

Example 2. In this example the data of Example 1 is considered except the condition that distribution of probability of the demand of lead time is not known. Using the Algorithm 2, find the results presented in Tab. 4 and Tab. 5 includes the analyzed optimal values. The comparative results of Tab. 5 reflect that in case of bad distribution of demand of lead time, it is preferable to invest in reduction of ordering cost and permit discount in price of backorder to resolve the problem of ordering during the period of shortages. The shavings can be observed by comparing both the cases, which range between 18.44% to 20%.

Table 2: Solution process for Example 1 (τ_i in weeks)

μ_0	τ_i	U_i	W_i	C_i	R_i	α_i	$TIC_N(W^*, C^*, j^*, \alpha^*, \tau^*)$
0.2	7	0	82	66	161	75.65	3912.46
	5	5.2	95	70	132	75.62	3724.24
	4	21.4	110	78	112	75.54	3622.56
	3	56.2	117	90	96	75.44	3738.24
0.4	7	0	84	67	157	75.72	3876.76
	5	5.2	95	70	128	75.54	3713.12
	4	21.4	110	78	106	75.44	3588.86
	3	56.2	117	90	89	75.26	3782.24
0.6	7	0	84	67	154	75.86	3792.25
	5	5.2	94	70	122	75.42	3624.50
	4	21.4	111	78	104	75.38	3798.25
	3	56.2	117	91	86	75.14	3695.24
0.8	7	0	87	68	134	75.92	3052.12
	5	5.2	94	71	117	75.542	3697.45
	4	21.4	111	79	101	75.14	3682.23
	3	56.2	118	91	82	75.08	3586.22

Table 3: Summarized optimal solution for Example 1 (τ_i in weeks)

Analyzed model							Model (with ordering cost C and without α)				Shavings (%)
μ_0	τ^*	W^*	C^*	R^*	α^*	Cost-1	τ^*	R^*	W^*	Cost-2	
0.2	4	113	81	115	75.64	3687.52	4	134	78	4521.21	18.44
0.4	4	116	81	109	75.52	3624.25	4	134	75	4473.25	19.00
0.6	4	117	81	107	75.25	3546.84	4	135	72	4422.28	19.80
0.8	4	117	82	104	75.21	3493.28	4	135	69	4365.85	20.00

With the help of these two examples, it can be observed that the shavings of total expected cost (annual) are examined by the reduction in ordering cost and discount in price of backorder'. Next, we test the importance of distribution-free concept in comparison of normal distribution. If we use the solution in place of $(W^*, C^*, j^*, \alpha^*, \tau^*)$, then $TIC_N(W^*, C^*, j^*, \alpha^*, \tau^*) - TIC_N(W^*, C^*, j^*, \alpha^*, \tau^*)$ will be the added cost.

This is hugest quantity for the information p.d.f f_Z and the amount is assumed as the additional information of expected value (AIEV) and its précis is shown in Tab. 6. In practice, the model (Reduction in ordering cost) with the discount in backorder price is more like the supply chain of real life.

Table 4: Solution process for Example 2 (τ_i in weeks)

μ_0	τ_i	U_i	W_i	C_i	R_i	α_i	$TIC_U(W^*, C^*, j^*, \alpha^*, \tau^*)$
0.2	7	0	82	66	161	75.65	3912.46
	5	5.2	95	70	132	75.62	3724.24
	4	21.4	110	78	112	75.54	3622.56
	3	56.2	117	90	96	75.44	3738.24
0.4	7	0	84	67	157	75.72	3876.76
	5	5.2	95	70	128	75.54	3713.12
	4	21.4	110	78	106	75.44	3588.86
	3	56.2	117	90	89	75.26	3782.24
0.6	7	0	84	67	154	75.86	3792.25
	5	5.2	94	70	122	75.42	3624.50
	4	21.4	111	78	104	75.38	3798.25
	3	56.2	117	91	86	75.14	3695.24
0.8	7	0	87	68	134	75.92	3052.12
	5	5.2	94	71	117	75.542	3697.45
	4	21.4	111	79	101	75.14	3682.23
	3	56.2	118	91	82	75.08	3586.22

Table 5: Summarized optimal solution for Example 2 (L_i in weeks)

The analysed model							Model with ordering cost C and without α				Shavings (%)
μ_0	τ^*	W^*	C^*	R^*	α^*	Cost-1	τ^*	R^*	W^*	Cost-2	
0.2	4	117	120	93	80.64	4786.52	3	184	70	5692.24	15.91
0.4	4	113	114	81	80.72	4454.35	3	174	65	5350.25	16.74
0.6	3	107	100	86	80.85	4226.84	4	160	72	5079.26	16.78
0.8	3	101	91	74	80.91	3915.28	4	152	63	4808.85	18.58

Table 6: Determination of additional information of expected value (AIEV)

μ_0	$TIC_N(W^*, C^*, j^*, \alpha^*, \tau^*)$	$TIC_N(Q^*, C^*, j^*, \alpha^*, L^*)$	AIEV	Cost of penalty
0.2	3961.52	3678.85	282.67	1.046
0.4	3838.54	3618.68	219.86	1.052
0.6	3748.12	3538.78	209.74	1.057
0.8	3676.34	3485.42	190.92	1.042

5 Conclusion and Future Scope

The occurrence of unsatisfied demands increases the period of lead time and decreases the backorders in the same ratio. In the case of probabilistic needs, the shortages cannot be ignored. To solve this type of problem and to cover the losses of customers, a discount in backorders may be offered by the supplier.

Also, our inventory policies may not be good for the arriving lots, which include some imperfect products. This study aims to check the impact of imperfect products on a unified policy by permitting the discount in the price of backorders and considering the cost of ordering as decision factors. First, it is assumed that the demand for lead time is followed by a normal distribution and then stops it and assumed that the first two moments of demand for lead time are known. Further, the minimax distribution method is used to solve this model, and a separate algorithm is designed. Since the situations of markets are regularly changing, so the policies of corporate should be managed suitably. If it is possible to decrease the cost of ordering correctly and the orders fixed well, the per unit time total concern cost will be upgraded. On the other side, if the seller provides a better discount in the backorder to the purchaser, then the service level will be improved, and the total expected cost (annual) will be reduced. This study covers the above literature gaps and is supported by numerical verification.

By the examples, it is observed that a notable quantity of shavings can be attained. It also reflects the suggestions to invest only when there is a chance of improvement. Computed results tell us the significant effects of weak quality of supply on the related cost and parameter sensitivity. This study provides an application in inventory models involving inspections, ordering quantity, and imperfect products. The presented research further is suitable for more general inventory characteristics in various real-life problems.

Funding Statement: The author(s) received no specific funding for this study. The Graphic Era Hill University Dehradun supported the research of the Sandeep Kumar and Teekam Singh. The corresponding and the third authors thank Prince Sultan University for the financial support.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] J. A. Buzacott, "Economic order quantity with inflation," *Operational Research Quarterly*, vol. 26, no. 3, pp. 553–558, 1975.

[2] R. B. Misra, "A note on optimal inventory management under inflation," *Naval Research Logistics Quarterly*, vol. 26, no. 1, pp. 161–165, 1979.

[3] J. K. Dey, S. K. Mondal and M. Matti, "Two shortage inventory problem with dynamic demand and interval valued lead time over finite time horizon under inflation and time-value of money," *European Journal of Operational Research*, vol. 185, no. 1, pp. 170–194, 2008.

[4] F. Jolai, R. T. Moghaddam, M. Rabbani and M. R. Sadoughian, "An economic production lot size model with deteriorating items, stock-dependent demand, inflation, and partial backlogging," *Applied Mathematics and Computation*, vol. 181, no. 1, pp. 380–389, 2006.

[5] S. K. Goyal, "Economic ordering policy for a product with periodic price changes," in *Proc. of Third Int. Symp. on Inventories*, Budapest, Hungry, 1984.

[6] K. S. Park, "Inflationary effect on EOQ under trade-credit financing," *International Journal on Policy and Information*, vol. 10, no. 2, pp. 65–69, 1986.

[7] P. Vrat and G. Padamanabban, "An inventory model under inflation for stock dependent consumption rate items," *Engineering Costs and Production Economics*, vol. 19, no. 1–3, pp. 379–383, 1990.

[8] T. K. Datta and A. K. Pal, "Effects of inflation and time value of money on an inventory model with linear time-dependent demand rate and shortages," *European Journal of Operational Research*, vol. 52, no. 3, pp. 326–333, 1991.

[9] C. J. Liao and C. H. Shyu, "An analytical determination of lead time with normal demand," *International Journal of Operations and Production Management*, vol. 11, no. 9, pp. 72–78, 1991.

[10] M. Ben-Daya and A. Raouf, "Inventory models involving lead time as a decision variable," *Journal of the Operational Research Society*, vol. 45, no. 5, pp. 579–582, 1994.

[11] L. Y. Ouyang and B. R. Chuang, “A periodic review inventory model involving variable lead time with a service level constraint,” *International Journal of System Science*, vol. 31, no. 10, pp. 1209–1215, 2000.

[12] L. Y. Ouyang, K. S. Wu and C. H. Ho, “An integrated vendor-buyer inventory model with quality improvement and lead time reduction,” *International Journal of Production Economics*, vol. 108, no. 1–2, pp. 349–358, 2007.

[13] M. A. Hoque and S. K. Goyal, “An alternative simple solution algorithm of an inventory model with fixed and variable lead time crash costs under unknown demand distribution,” *International Journal of Systems Science*, vol. 40, no. 8, pp. 821–827, 2009.

[14] W. C. Lee, “Inventory model involving controllable backorder rate and variable lead time demand with the mixtures of distribution,” *Applied Mathematics and Computation*, vol. 160, no. 3, pp. 701–717, 2005.

[15] G. Gallego and I. Moon, “The distribution free newsboy problem: Review and extensions,” *Journal of Operational Research Society*, vol. 44, no. 8, pp. 825–834, 1993.

[16] J. C. H. Pan and Y. C. Hsiao, “Integrated inventory models with controllable lead time and backorder discount considerations,” *International Journal of Production Economics*, vol. 94, no. 4, pp. 387–397, 2005.

[17] W. C. Lee, J. W. Wu and C. L. Lei, “Computational algorithm procedure for optimal inventory policy involving ordering cost reduction and back-order discounts when lead time demand is controllable,” *Applied Mathematics and Computation*, vol. 189, no. 1, pp. 186–200, 2007.

[18] M. K. Salameh and M. Y. Jaber, “Economic production quantity model for items with imperfect quality,” *International Journal of Production Economics*, vol. 64, no. 1–3, pp. 59–64, 2000.

[19] M. Y. Jaber, S. K. Goyal and M. Imran, “Economic production quantity model for items with imperfect quality subject to learning effects,” *International Journal of Production Economics*, vol. 115, no. 1, pp. 143–150, 2008.

[20] H. C. Chang and C. H. Ho, “Exact closed form solutions for optimal inventory model for items with imperfect quality and shortage backordering,” *Omega*, vol. 38, no. 3–4, pp. 233–237, 2010.

[21] P. A. Hayek and M. K. Salameh, “Production lot sizing with the reworking of imperfect quality items produced,” *Production Planning & Control*, vol. 12, no. 1, pp. 584–590, 2001.

[22] K. Annadurai and R. Uyhayakumar, “Controlling set-up cost in (Q, r, L) inventory models with defective items,” *Applied Mathematical Modeling*, vol. 34, no. 6, pp. 1418–1427, 2010.

[23] R. L. Schwaller, “EOQ under inspection costs,” *Production and Inventory Management Journal*, vol. 29, no. 3, pp. 22–29, 1988.

[24] M. K. Salmeh and M. Y. Jaber, “Economic production quantity model for items with imperfect quality,” *International Journal of Production Economics*, vol. 64, no. 1–3, pp. 59–64, 2000.

[25] H. C. Chang, “An application of fuzzy sets theory to the EOQ model with imperfect quality items,” *Computer & Operation Research*, vol. 31, no. 12, pp. 2079–2092, 2004.

[26] M. J. Paknejad, F. Nasri and J. F. Affiso, “Defective units in a continuous review system,” *International Journal of Production Research*, vol. 33, no. 10, pp. 2767–2777, 1995.

[27] B. Sarkar, S. S. Sana and K. Chaudhuri, “An economic production quantity model with stochastic demand in an imperfect production system,” *International Journal of Services and Operations Management*, vol. 9, no. 3, pp. 259–283, 2011.

[28] K. J. Chung, C. C. Her and S. D. Lin, “A two-warehouse inventory model with imperfect quality production processes,” *Computers & Industrial Engineering*, vol. 56, no. 1, pp. 193–197, 2009.

[29] S. Papachristos and I. Konstantaras, “Economic ordering quantity models for items with imperfect quality,” *International Journal of Production Economics*, vol. 100, no. 1, pp. 148–154, 2006.

[30] A. Eroglu and G. Ozdemir, “An economic order quantity model with defective items and shortages,” *International Journal of Production Economics*, vol. 106, no. 2, pp. 544–549, 2007.

[31] H. Wee, J. Yu and M. Chen, “Optimal inventory model for items with imperfect quality and shortage backordering,” *Omega*, vol. 35, no. 1, pp. 7–11, 2007.

[32] M. Khan, M. Y. Jaber, A. L. Guiffrida and S. Zolfaghari, “A review of the extensions of a modified EOQ model for imperfect quality iteming,” *International Journal of Production Economics*, vol. 132, no. 1, pp. 1–12, 2011.

[33] S. K. Goyal and L. E. Cardenas-Barron, "Note on economic production quantity model for items with imperfect quality—A practical approach," *International Journal of Production Economics*, vol. 77, no. 1, pp. 85–87, 2002.

[34] W. M. Chan, R. N. Ibrahim and P. B. Lochert, "A new EPQ model: Integrating lower pricing, rework and reject situations," *Production Planning & Control*, vol. 14, no. 7, pp. 588–595, 2003.

[35] W. C. Lee, J. W. Wu, H. H. Tsou and C. L. Lei, "Computational procedure of optimal inventory model involving controllable backorder rate and variable lead time with defective units," *International Journal of Systems Science*, vol. 43, no. 10, pp. 1927–1942, 2012.

[36] K. Skouri, I. Konstantaras, A. G. Lagodimos and S. Papachristos, "An EOQ model with backorders and rejection of defective supply batches," *International Journal of Production Economics*, vol. 155, no. 1, pp. 148–154, 2014.