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Article

Product Spacing of Stress–Strength under Progressive Hybrid Censored for Exponentiated-Gumbel Distribution

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Abstract: Maximum product spacing for stress–strength model based on progressive Type-II hybrid censored samples with different cases has been obtained. This paper deals with estimation of the stress strength reliability model $R = P(Y < X)$ when the stress and strength are two independent exponentiated Gumbel distribution random variables with different shape parameters but having the same scale parameter. The stress–strength reliability model is estimated under progressive Type-II hybrid censoring samples. Two progressive Type-II hybrid censoring schemes were used, Case I: A sample size of stress is the equal sample size of strength, and same time of hybrid censoring, the product of spacing function under progressive Type-II hybrid censoring schemes. Case II: The sample size of stress is a different sample size of strength, in which the life-testing experiment with a progressive censoring scheme is terminated at a random time $T \in (0, \infty)$. The maximum likelihood estimation and maximum product spacing estimation methods under progressive Type-II hybrid censored samples for the stress strength model have been discussed. A comparison study with classical methods as the maximum likelihood estimation method is discussed. Furthermore, to compare the performance of various cases, Markov chain Monte Carlo simulation is conducted by using iterative procedures as Newton Raphson or conjugate-gradient procedures. Finally, two real datasets are analyzed for illustrative purposes, first data for the breaking strengths of jute fiber, and the second data for the waiting times before the service of the customers of two banks.

Keywords: Exponentiated Gumbel distribution; stress–strength model; progressive Type-II hybrid censoring; maximum product spacing; maximum likelihood

1 Introduction

The stress–strength reliability $R = P(Y < X)$ model is an important application in reliability theory. This model is used in many applications of physics and engineering such as strength failure and system collapse. In electrical and electronic systems R arise as a measure of system performance. Some Authors had used R as a general measure of the difference between two populations. Reference [1] used R as the inequality measure between income distributions. Reference [2] used it to express the evaluation of the area under the receiver operating characteristic (ROC) curve for diagnostic tests with continuous outcomes. For further details and Applications of R, see [3].

Statistical inference about the reliability model has received great attention in the context of reliability. For $P(Y < X)$, X is the strength of a system which is subjected to stress Y. The system fails when stress exceeds strength. Therefore, the stress–strength parameter R measures system reliability. Many authors have used different statistical inference methods to estimate R when samples drawn from the model are based on simple random samples (SRS). However, in recent years, statistical inferences about R model based on the Ranked set sample designs (RSS) have been considered by several researches. For example [4] considered estimation of the stress strength reliability model when the stress and strength are independent exponentiated pareto variables and the samples are drawn using median and ranked set sampling methods.

Other researchers considered censored data when estimating R. Reference [5] discussed estimation of the reliability model for exponential populations using order statistics. Reference [6] proposed three estimators when X and Y are independent one-parameter exponential random variables. the case when stress and strength variables are independent Burr Type-XII distribution was investigated by Reference [7] when samples drawn using several modifications of ranked set sampling designs (RSS). Furthermore [8] discussed the estimation of the reliability model when X and Y independent Lindley populations.

The estimation of R in exponential distributions under censored data has been investigated by Reference [9], and the stress–strength reliability of Weibull and inverse Weibull distributions has been studied under progressively censored data by [10,11]. Reference [12] carried out the estimation of the stress–strength reliability $R = P(Y < X)$ based on progressively Type-II censored samples when X and Y were two independent two parameter bathtub-shaped lifetime distributions.

Many authors have discussed inference under progressive Type-II hybrid censoring using different lifetime distributions. Reference [13] presented the analysis of the Type-II progressively hybrid censored data of the Weibull distribution. Reference [14] discussed the maximum likelihood estimators and approximate maximum likelihood estimators of the parameters of the Weibull distribution with two different progressively hybrid censoring schemes. Reference [15] discussed the estimation and prediction problems for the Burr Type-III distribution under progressive Type-II hybrid censored data. Reference [16] discussed parameter estimation for the generalized Rayleigh distribution under the adaptive Type-II progressive censoring schemes by using maximum product spacing method. Reference [17] discussed statistical inference for the Gompertz distribution based on generalized progressively hybrid censored data. Reference [18] discussed adaptive Type-II progressive censoring schemes of maximum product spacing for Weibull parameters. Reference [19] discussed classical and Bayesian inferences for the generalized DUS exponential distribution under Type-I progressive hybrid censored data.

Reference [20] introduced progressive Type-II hybrid censoring based on the maximum product spacing method for Power Lomax distribution. Reference [21] obtained inference for the stress strength reliability when X and Y are two independent Weibull distributions under progressively Type-II censored samples. Reference [22] obtained step–stress model with Type-II hybrid censored data from the Kumaraswamy Weibull distribution. Reference [23] considered the reliability analysis problem of a constant-stress life test model based on progressively Type-I hybrid censored data from Weibull distribution. Reference [24] discussed classical and Bayesian estimation procedures for stress–strength reliability parameter for Lomax distribution based on Type-II hybrid censored. Reference [25] discussed point and interval estimate of the stress–strength parameter, from both MLE and Bayesian under the Type-II hybrid progressive censoring scheme.

Based on the observed sample $x_{1:m:n} < \ldots < x_{m:m:n}$ from a progressive Type-II hybrid censoring scheme, the MPS under progressive Type-II hybrid censoring scheme will be introduced depending on [26–29,16].

The two cases of the Type-II progressive hybrid censoring scheme are cases I ($X_{1:m:n} < \ldots < X_{m:m:n} < T$) and case II ($X_{h:m:n} < T < X_{h+1:m:n}$). If $X_{h:m:n} < T < X_{h+1:m:n}$, the progressive censoring sample $\{X_{1:m:n}, \ldots, X_{h:m:n}\}$, is described by [20]. Eq. (1) is referred as MPS under Type-II progressive hybrid censoring scheme in general form as follows:

$$S(\Psi) = C_2 \prod_{i=1}^{h} \left[D_{i:m:n} (1 - F(x_{i:m:n}; \Psi))^{R_i} \right] (1 - F(T; \Psi))^{R_h^*}, \tag{1}$$

where $R_h^* = n - h - \sum_{i=1}^{h} R_i$, C_2 is constant not depend on parameters.

In this paper, estimation of the traditional stress–strength model $R = P(Y < X)$ under progressive Type-II hybrid censoring schemes when X and Y are exponentiated Gumble (EG) random variables with cumulative distribution (cdf), probability density function (pdf) and quantile function respectively is investigated.

$$F(x) = \left[\exp\left(-e^{-\left(\frac{x}{\sigma}\right)} \right) \right]^{\alpha}, \qquad \alpha > 0,\ \beta > 0, -\infty < x < \infty, \tag{2}$$

$$f(x) = \frac{\alpha}{\sigma} e^{-\left(\frac{x}{\sigma}\right)} \left[\exp\left(-e^{-\left(\frac{x}{\sigma}\right)} \right) \right]^{\alpha - 1}, \qquad \alpha > 0, \sigma > 0, -\infty < x < \infty, \tag{3}$$

and

$$Q(u) = \frac{-\ln[-\alpha^{-1} \ln u]}{\beta}. \tag{4}$$

Maximum product of spacing (MPS) and maximum likelihood (MLE) estimation methods are used to estimate R and estimator's performances and efficiencies are investigated through a Monte Carlo simulation study and a real data application will be used for illustrative purposes. Finally, the paper is concluded.

2 Stress Strength Parameter

Let $X \sim EG(\alpha, \sigma)$ and $Y \sim EG(\beta, \sigma)$ be two independent random variables with the same scale parameter σ and $R = P(Y < X)$ is the stress–strength reliability model, then:

$$R = P(Y < X) = \int_{-\infty}^{\infty} \int_{-\infty}^{x} \frac{\alpha}{\sigma} e^{-\left(\frac{x}{\sigma}\right)} \left[\exp\left(-e^{-\left(\frac{x}{\sigma}\right)} \right) \right]^{\alpha-1} \times \frac{\beta}{\sigma} e^{-\left(\frac{y}{\sigma}\right)} \left[\exp\left(-e^{-\left(\frac{y}{\sigma}\right)} \right) \right]^{\beta-1} dy\, dx = \frac{\alpha}{\alpha + \beta}. \tag{5}$$

$$\hat{R} = \frac{\hat{\alpha}}{\hat{\alpha} + \hat{\beta}}. \tag{6}$$

The MLE and MPS estimators $\hat{\alpha}$ and $\hat{\beta}$ of the shape parameters α and β will be obtained to get the estimated model parameters as well as the stress–strength system reliability in case of progressive hybrid censored samples.

3 Maximum Likelihood

If $X_{h:m:n} < T < X_{h+1:m:n}$, the progressive censoring sample $\{X_{1:m:n}, \ldots, X_{h:m:n}\}$, and if $Y_{h:\mathcal{M}:\aleph} < T < Y_{w+1:\mathcal{M}:\aleph}$, the progressive censoring sample $\{Y_{1:\mathcal{M}:\aleph}, \ldots, Y_{h:\mathcal{M}:\aleph}\}$ is described. According [30], the general likelihood function under progressive Type-II hybrid censoring schemes for stress–strength model can be written as:

$$L(\alpha,\beta,\sigma) = \prod_{i=1}^{h}\left[f(x_{i:m:n};\alpha,\sigma)(1-F(x_{i:m:n};\alpha,\sigma))^{R_i}\right](1-F(T;\alpha,\sigma))^{R_h^*}$$
$$\prod_{i=1}^{w}\left[f(y_{i:\mathcal{M}:\aleph};\beta,\sigma)(1-F(y_{i:\mathcal{M}:\aleph};\beta,\sigma))^{\Re_i}\right](1-F(T;\beta,\sigma))^{\Re_w^*}. \tag{7}$$

In case of stress and strength sample sizes are equal, and same time of hybrid censoring, the likelihood function of EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model is:

$$L(\alpha,\beta,\sigma) = \prod_{i=1}^{h} f(x_{i:m:n};\alpha,\sigma)(1-F(x_{i:m:n};\alpha,\sigma))^{R_i} f(y_{i:\mathcal{M}:\aleph};\beta,\sigma)$$
$$(1-F(y_{i:\mathcal{M}:\aleph};\beta,\sigma))^{R_i}(1-F(T;\alpha,\sigma))^{R_h^*}(1-F(T;\beta,\sigma))^{R_h^*}. \tag{8}$$

The general likelihood function of the EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model is given as:

$$L(\alpha,\beta,\sigma) = \left(\frac{\alpha}{\sigma}\right)^{h} e^{-\sum_{i=1}^{h}\left(\frac{x_{i:m:n}}{\sigma}\right)} \exp\left(-(\alpha-1)\sum_{i=1}^{h} e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\left(1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha}\right)^{R_h^*}$$
$$\left(\frac{\beta}{\sigma}\right)^{w} e^{-\sum_{i=1}^{w}\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)} \exp\left(-(\beta-1)\sum_{i=1}^{w} e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\left(1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta}\right)^{\Re_w^*}$$
$$\prod_{i=1}^{h}\left(1-\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]^{\alpha}\right)^{R_i}\prod_{i=1}^{w}\left(1-\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]^{\beta}\right)^{\Re_i}. \tag{9}$$

According to Eq. (9), the log-likelihood function of the EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model is given as:

$$l(\alpha,\beta,\sigma) = h[\ln(\alpha)-\ln(\sigma)] + w[\ln(\beta)-\ln(\sigma)] - \sum_{i=1}^{h}\left(\frac{x_{i:m:n}}{\sigma}\right) - (\alpha-1)\sum_{i=1}^{h} e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}$$
$$+\sum_{i=1}^{h} R_i \ln\left(1-\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]^{\alpha}\right) - \sum_{i=1}^{w}\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right) - (\beta-1)\sum_{i=1}^{w} e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}$$
$$+\sum_{i=1}^{w}\Re_i \ln\left(1-\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]^{\beta}\right) + \Re_w^* \ln\left(1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta}\right) +$$
$$R_h^* \ln\left(1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha}\right). \tag{10}$$

The MLE of α, β and σ are obtained by simultaneously solving the following normal equations:

$$\frac{\partial l(\alpha,\beta,\sigma)}{\partial\alpha} = \frac{h}{\alpha} - \sum_{i=1}^{h} e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)} - \sum_{i=1}^{h} R_i \frac{\left[\exp\left(-e^{-\left(e^{\frac{x_{i:m:n}}{\sigma}}\right)}\right)\right]^{\alpha} \ln\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]}{1-\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]^{\alpha}} - R_h^* \frac{\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha} \ln\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]}{1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha}}, \tag{11}$$

$$\frac{\partial l(\alpha,\beta,\sigma)}{\partial\beta} = \frac{w}{\beta} - \sum_{i=1}^{w} e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)} - \sum_{i=1}^{w} \Re_i \frac{\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]^{\beta} \ln\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]}{1-\left[\exp\left(-e^{-\left(\frac{y_{i:m:n}}{\sigma}\right)}\right)\right]^{\beta}} - \Re_w^* \frac{\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta} \ln\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]}{1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta}}, \tag{12}$$

and

$$\frac{\partial l(\alpha,\beta,\sigma)}{\partial\sigma} = \frac{-h}{\sigma} - \frac{w}{\sigma} - (\alpha-1)\sum_{i=1}^{h} \frac{x_{i:m:n}}{\sigma^2} e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)} + \alpha \sum_{i=1}^{h} R_i \frac{\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]^{\alpha} e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}}{1-\left[\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)\right]^{\alpha}} \frac{x_{i:m:n}}{\sigma^2} - (\beta-1)\sum_{i=1}^{w} \frac{y_{i:\mathcal{M}:\aleph}}{\sigma^2} e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)} + \beta \sum_{i=1}^{w} \Re_i \frac{y_{i:\mathcal{M}:\aleph}}{\sigma^2} \frac{\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]^{\beta} e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}}{1-\left[\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)\right]^{\beta}} + \beta\Re_w^* \frac{\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta} e^{-\left(\frac{T}{\sigma}\right)} \frac{T}{\sigma^2}}{1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta}} + \alpha R_h^* \frac{\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha} e^{-\left(\frac{T}{\sigma}\right)} \frac{T}{\sigma^2}}{1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\alpha}}. \tag{13}$$

The MLEs $\hat{\alpha}$, $\widehat{\beta}$, $\hat{\sigma}$ of the model parameters are the solution of non-linear Eqs. (11)–(13) after setting them equal to zero. These equations are very difficult to be solved, so iterative procedures are used as Newton Raphson or conjugate-gradient.

4 Maximum Product of Spacing

In case of sample size of stress is equal sample size of strength, and same time of hybrid censoring, the product of spacing function under progressive Type-II hybrid censoring schemes for stress–strength model as follows:

$$g(\alpha,\beta,\sigma) \propto \prod_{i=1}^{h}\left[D_{1\,i:m:n}(1-F(x_{i:m:n};\alpha,\sigma))^{R_i}\right] \prod_{i=1}^{w}\left[D_{2\,i:\mathcal{M}:\aleph}(1-F(y_{i:m:n};\beta,\sigma))^{R_i}\right] \times (1-F(T;\alpha,\sigma))^{R_h^*}(1-F(T;\beta,\sigma))^{R_h^*}, \tag{14}$$

where $R_h^* = n - h - \sum_{i=1}^{h} R_i$ and

$$D_{1\,i:m:n}=\begin{cases}F(x_{1:m:n};\psi)\\ F(x_{i:m:n};\psi)-F\left(x_{(i-1):m:n};\psi\right)\\ 1-F(x_{m:m:n};\psi)\end{cases},D_{2\,i:\mathcal{M}:\aleph}=\begin{cases}F(y_{1:\mathcal{M}:\aleph};\psi)\\ F(y_{i:\mathcal{M}:\aleph};\psi)-F\left(y_{(i-1):\mathcal{M}:\aleph};\psi\right)\\ 1-F(y_{\mathcal{M}:\mathcal{M}:\aleph};\psi)\end{cases},$$

where Ψ is a vector of parameters. The product of spacing function of the EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model is given as:

$$g(\alpha,\beta,\sigma)=(\mathcal{A}_{1:m:n})^{\alpha}[1-(\mathcal{A}_{h:m:n})^{\alpha}](\mathcal{B}_{1:\mathcal{M}:\aleph})^{\beta}\left[1-(\mathcal{B}_{w:\mathcal{M}:\aleph})^{\beta}\right]$$
$$\left(1-\left[\exp\left(-e^{-\left(\frac{\mathrm{T}}{\sigma}\right)}\right)\right]^{\alpha}\right)^{R_h^*}\prod_{i=1}^{h}[(\mathcal{A}_{i:m:n})^{\alpha}-(\mathcal{A}_{i-1:m:n})^{\alpha}][1-(\mathcal{A}_{i:m:n})^{\alpha}]^{R_i}$$
$$\left(1-\left[\exp\left(-e^{-\left(\frac{T}{\sigma}\right)}\right)\right]^{\beta}\right)^{\Re_w^*}\prod_{i=1}^{w}\left((\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}-(\mathcal{B}_{i-1:\mathcal{M}:\aleph})^{\beta}\right)\left(1-(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}\right)^{\Re_i}, \tag{15}$$

where $\mathcal{A}_{i:m:n}=\exp\left(-e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\right)$ and $\mathcal{B}_{i:\mathcal{M}:\aleph}=\exp\left(-e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\right)$. The log- product of spacing function of the EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model is given as:

$$\begin{aligned}ln(g(\alpha,\beta,\sigma))&=-\alpha e^{-\left(\frac{x_{1:m:n}}{\sigma}\right)}+\ln[1-(\mathcal{A}_{h:m:n})^{\alpha}]-\beta e^{-\left(\frac{y_{1:\mathcal{M}:\aleph}}{\sigma}\right)}+\ln\left[1-(\mathcal{B}_{w:\mathcal{M}:\aleph})^{\beta}\right]\\&+R_h^*\ln\left(1-\left[\exp\left(-e^{-\left(\frac{\mathrm{T}}{\sigma}\right)}\right)\right]^{\alpha}\right)+\sum_{i=1}^{h}R_i\ln(1-(\mathcal{A}_{i:m:n})^{\alpha})+\sum_{i=1}^{h}\ln[(\mathcal{A}_{i:m:n})^{\alpha}-(\mathcal{A}_{i-1:m:n})^{\alpha}]\\&+\Re_w^*\ln\left(1-\left[\exp\left(-e^{-\left(\frac{\mathrm{T}}{\sigma}\right)}\right)\right]^{\beta}\right)+\sum_{i=1}^{w}\Re_i\ln\left(1-(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}\right)+\sum_{i=1}^{w}\ln\left[(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}-(\mathcal{B}_{i-1:\mathcal{M}:\aleph})^{\beta}\right],\end{aligned} \tag{16}$$

The MPS of α, β and σ are obtained by simultaneously solving the following normal equations:

$$\begin{aligned}\frac{\partial ln(g(\alpha,\beta,\sigma))}{\partial\alpha}=&-e^{-\left(\frac{x_{1:m:n}}{\sigma}\right)}-\frac{(\mathcal{A}_{h:m:n})^{\alpha}\ln(\mathcal{A}_{h:m:n})}{1-(\mathcal{A}_{h:m:n})^{\alpha}}-R_h^*\frac{(\mathcal{A}_T)^{\alpha}\ln(\mathcal{A}_T)}{1-(\mathcal{A}_T)^{\alpha}}-\sum_{i=1}^{h}R_i\frac{(\mathcal{A}_{i:m:n})^{\alpha}\ln(\mathcal{A}_{i:m:n})}{1-(\mathcal{A}_{i:m:n})^{\alpha}}\\&+\sum_{i=1}^{h}\frac{(\mathcal{A}_{i:m:n})^{\alpha}\ln(\mathcal{A}_{i:m:n})-(\mathcal{A}_{i-1:m:n})^{\alpha}\ln(\mathcal{A}_{i-1:m:n})}{(\mathcal{A}_{i:m:n})^{\alpha}-(\mathcal{A}_{i-1:m:n})^{\alpha}},\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial ln(g(\alpha,\beta,\sigma))}{\partial\beta}=&-e^{-\left(\frac{y_{1:m:n}}{\sigma}\right)}-\frac{(\mathcal{B}_{h:m:n})^{\beta}\ln(\mathcal{B}_{h:m:n})}{1-(\mathcal{B}_{h:m:n})^{\beta}}-\Re_w^*\frac{(\mathcal{B}_T)^{\beta}\ln(\mathcal{B}_T)}{1-(\mathcal{B}_T)^{\beta}}-\sum_{i=1}^{w}\Re_i\frac{(\mathcal{B}_{i:m:n})^{\beta}\ln(\mathcal{B}_{i:m:n})}{1-(\mathcal{B}_{i:m:n})^{\beta}}\\&+\sum_{i=1}^{h}\frac{(\mathcal{B}_{i:m:n})^{\beta}\ln(\mathcal{B}_{i:m:n})-(\mathcal{B}_{i-1:m:n})^{\beta}\ln(\mathcal{B}_{i-1:m:n})}{(\mathcal{B}_{i:m:n})^{\beta}-(\mathcal{B}_{i-1:m:n})^{\beta}},\end{aligned} \tag{18}$$

and

$$\frac{\partial ln(g(\alpha,\beta,\sigma))}{\partial\sigma} = -\alpha\frac{x_{1:m:n}}{\sigma^2}e^{-\left(\frac{x_{1:m:n}}{\sigma}\right)} - \frac{\alpha(\mathcal{A}_{h:m:n})^{\alpha}\mathfrak{A}_{h:m:n}}{1-(\mathcal{A}_{h:m:n})^{\alpha}} - \beta\frac{y_{1:m:n}}{\sigma^2}e^{-\left(\frac{y_{1:m:n}}{\sigma}\right)} - \frac{\beta(\mathcal{B}_{h:m:n})^{\beta}\mathfrak{C}_{h:m:n}}{1-(\mathcal{B}_{h:m:n})^{\beta}}$$
$$- R_h^*\frac{\alpha(\mathcal{A}_T)^{\alpha}\mathfrak{A}_T}{1-(\mathcal{A}_T)^{\alpha}} - \sum_{i=1}^{h}R_i\frac{\alpha(\mathcal{A}_{i:m:n})^{\alpha}\mathfrak{A}_{i:m:n}}{1-(\mathcal{A}_{i:m:n})^{\alpha}} + \alpha\sum_{i=2}^{h}\frac{(\mathcal{A}_{i:m:n})^{\alpha}\mathfrak{A}_{i:m:n}-(\mathcal{A}_{i-1:m:n})^{\alpha}\mathfrak{A}_{i-1:m:n}}{(\mathcal{A}_{i:m:n})^{\alpha}-(\mathcal{A}_{i-1:m:n})^{\alpha}} - R_h^*\frac{\beta(\mathcal{B}_T)^{\beta}\mathfrak{C}_T}{1-(\mathcal{B}_T)^{\beta}} \quad (19)$$
$$+ \sum_{i=1}^{w}\Re_i\frac{\beta(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}\mathfrak{C}_{i:\mathcal{M}:\aleph}}{1-(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}} + \beta\sum_{i=2}^{h}\frac{(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}\mathfrak{C}_{i:\mathcal{M}:\aleph}-(\mathcal{B}_{i-1:\mathcal{M}:\aleph})^{\beta}\mathfrak{C}_{i-1:\mathcal{M}:\aleph}}{(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}-(\mathcal{B}_{i-1:\mathcal{M}:\aleph})^{\beta}}\ln\left[(\mathcal{B}_{i:\mathcal{M}:\aleph})^{\beta}-(\mathcal{B}_{i-1:\mathcal{M}:\aleph})^{\beta}\right],$$

where $\frac{d\mathcal{A}_{i:m:n}}{d\sigma} = \mathfrak{A}_{i:m:n} = -e^{-\left(\frac{x_{i:m:n}}{\sigma}\right)}\frac{x_{i:m:n}}{\sigma^2}$, $\frac{d\mathcal{B}_{i:\mathcal{M}:\aleph}}{d\sigma} = \mathfrak{C}_{i:\mathcal{M}:\aleph} = -e^{-\left(\frac{y_{i:\mathcal{M}:\aleph}}{\sigma}\right)}\frac{y_{i:\mathcal{M}:\aleph}}{\sigma^2}$, and

$\frac{d\mathcal{A}_T}{d\sigma} = \mathfrak{A}_T = -e^{-\left(\frac{T}{\sigma}\right)}\frac{T}{\sigma^2}$.

Again, The MPS $\hat{\alpha}, \hat{\beta}, \hat{\sigma}$ of the model parameters are the solution of those non-linear Eqs. (17)–(19) after setting them equal zero. These equations are very difficult to be solved, so iterative procedures are used as Newton Raphson or conjugate-gradient.

5 Simulation Study

In this section, a Monte-Carlo simulation is done to estimate the parameters of EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model for MLE and MPS methods using R language is described as follows:

Step 1: Generate 10000 random samples of size 30, 50 and 100 from the EG distribution under progressive Type-II hybrid censoring schemes for stress–strength model.

Step 2: Using the quantile $x_i = \frac{-\ln[-\alpha^{-1}\ln u_i]}{\beta}; 0 < u_i < 1$, where x are distributed as EG for different parameters (α, β, σ), Three sets of parameters values are selected as are $(\alpha,\beta,\sigma) = (1.75,\ 2,\ 1.5)$, $(\alpha,\beta,\sigma) = (0.75,\ 2,\ 1.5)$ and is $(\alpha,\beta,\sigma) = (0.75,\ 0.5,\ 1.5)$.

Step 3: In progressive Type-II hybrid censoring schemes for stress–strength model, the effective of sample sizes (failure items) m are selected based on two levels of censoring for all sample size. Selected T are 1.5 and 5 and sets of different samples schemes.

- Scheme 1: $R^{(1)} = R_2 = \ldots = R_{m-1} = 0,\ and\ R_m = n - m$. It is Type-II scheme
- Scheme 2: $R^{(2)} = n - m\ and\ R_2 = R_3 = \ldots = R_{m-1} = 0$.
- Case 1: Sample size of stress is equal sample size of strength, and same time of hybrid censoring.
- Case 2: Sample size of stress is different sample size of strength, and same time of hybrid censoring.

Step 4: The MLE and MPS of the model parameters are obtained by solving the non-linear equations based on progressive Type-II hybrid censoring schemes for stress–strength model.

Step 5: The Bias and mean square errors (MSE) of the parameters are obtained as measures of efficiency.

Step 6: The numerical results of parameters estimation of EG distribution under different censoring schemes are listed in Tabs. 1 and 3.

Table 1: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 1: 1: $(\alpha, \beta, \sigma) = (1.75,\ 2,\ 1.5)$

T				1.5				5			
$n = \aleph$	$m = \mathcal{M}$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
30	**20**	**I**	$\hat{\alpha}$	0.4398	0.2733	0.3418	0.1999	0.2791	0.1201	0.2247	0.0974
			$\hat{\beta}$	0.3479	0.2156	0.2418	0.1562	0.1434	0.0737	0.0899	0.0660
			$\hat{\sigma}$	−0.3591	0.1377	−0.3526	0.1388	0.1011	0.0949	0.0376	0.0770
			$\hat{R}$	0.0161	0.0024	0.0162	0.0024	0.0199	0.0019	0.0194	0.0019
		II	$\hat{\alpha}$	0.1549	0.2389	0.0460	0.1699	0.1516	0.2245	0.0384	0.1609
			$\hat{\beta}$	0.2174	0.4048	0.0579	0.2681	0.2017	0.3504	0.0468	0.2520
			$\hat{\sigma}$	−0.8418	0.7180	−0.7959	0.6427	−0.8322	0.7018	−0.7878	0.6290
			$\hat{R}$	−0.0032	0.0074	0.0009	0.0057	−0.0023	0.0068	0.0011	0.0055
	25	**I**	$\hat{\alpha}$	0.2169	0.1287	0.1278	0.1004	0.1179	0.0646	0.0601	0.0601
			$\hat{\beta}$	0.1863	0.1391	0.0851	0.1133	0.0360	0.0671	−0.0218	0.0721
			$\hat{\sigma}$	−0.5977	0.3639	−0.5936	0.3617	−0.3678	0.1594	−0.4088	0.1881
			$\hat{R}$	0.0073	0.0027	0.0076	0.0027	0.0121	0.0021	0.0115	0.0021
		II	$\hat{\alpha}$	0.0805	0.1295	−0.0208	0.1112	0.0760	0.1244	−0.0269	0.1066
			$\hat{\beta}$	0.1188	0.1979	−0.0117	0.1671	0.1077	0.1874	−0.0231	0.1581
			$\hat{\sigma}$	−0.8101	0.6631	−0.7807	0.6175	−0.8019	0.6494	−0.7746	0.6064
			$\hat{R}$	−0.0021	0.0046	−0.0004	0.0041	−0.0014	0.0044	0.0001	0.0040
50	**35**	**I**	$\hat{\alpha}$	0.3693	0.1848	0.3294	0.1649	0.1932	0.0610	0.1930	0.0670
			$\hat{\beta}$	0.2979	0.1480	0.2613	0.1383	0.0586	0.0321	0.0683	0.0406
			$\hat{\sigma}$	−0.4029	0.1672	−0.4014	0.1520	0.1432	0.0802	0.0155	0.0494
			$\hat{R}$	0.0133	0.0016	0.0126	0.0016	0.0190	0.0012	0.0178	0.0012
		II	$\hat{\alpha}$	0.0805	0.1051	0.0074	0.0906	0.0779	0.1013	0.0039	0.0883
			$\hat{\beta}$	0.1261	0.1624	0.0340	0.1380	0.1188	0.1520	0.0264	0.1318
			$\hat{\sigma}$	−0.8215	0.6796	−0.8083	0.6583	−0.8133	0.6658	−0.8026	0.6483
			$\hat{R}$	−0.0029	0.0040	−0.0023	0.0036	−0.0028	0.0037	−0.0019	0.0036
	45	**I**	$\hat{\alpha}$	0.1260	0.0627	0.0765	0.0567	0.0496	0.0338	0.0257	0.0373
			$\hat{\beta}$	0.0920	0.0728	0.0427	0.0742	−0.0304	0.0412	−0.0454	0.0507
			$\hat{\sigma}$	−0.6787	0.4640	−0.6980	0.4912	−0.5034	0.2628	−0.5621	0.3235
			$\hat{R}$	0.0065	0.0019	0.0058	0.0020	0.0110	0.00146	0.0096	0.0015
		II	$\hat{\alpha}$	0.0362	0.0642	−0.0251	0.0636	0.0314	0.0623	−0.0290	0.0621
			$\hat{\beta}$	0.0297	0.0850	−0.0264	0.0848	0.0201	0.0813	−0.0205	0.0805
			$\hat{\sigma}$	−0.8014	0.6460	−0.8009	0.6453	−0.7923	0.6314	−0.7895	0.6303
			$\hat{R}$	0.0018	0.0026	0.0021	0.0025	0.0023	0.00253	0.0024	0.0025

Table 1 (continued).

T				1.5				5			
$n=\aleph$	$m=\mathcal{M}$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
100	**65**	**I**	$\hat{\alpha}$	0.4347	0.2101	0.4447	0.2248	0.2121	0.0550	0.2528	0.0773
			$\hat{\beta}$	0.3341	0.1381	0.3563	0.1618	0.0441	0.0149	0.0969	0.0268
			$\hat{\sigma}$	−0.3161	0.1024	−0.3935	0.1583	0.5179	0.3431	0.3130	0.1598
			$\hat{R}$	0.0169	0.0009	0.0157	0.0009	0.0232	0.0009	0.0219	0.0009
		II	$\hat{\alpha}$	0.0603	0.0510	0.0153	0.0485	0.0576	0.0492	0.0135	0.0474
			$\hat{\beta}$	0.0671	0.0727	0.0188	0.0722	0.0602	0.0700	0.0145	0.0701
			$\hat{\sigma}$	−0.8107	0.6598	−0.8093	0.6474	−0.8018	0.6452	−0.8141	0.6465
			$\hat{R}$	0.0006	0.0019	0.0003	0.0019	0.0011	0.0018	0.0006	0.0019
	85	**I**	$\hat{\alpha}$	0.1579	0.0470	0.1494	0.0493	0.0324	0.0123	0.0579	0.0186
			$\hat{\beta}$	0.1154	0.0414	0.1189	0.0493	−0.0747	0.0206	−0.0305	0.0211
			$\hat{\sigma}$	−0.6126	0.3770	−0.6624	0.4407	−0.2800	0.0901	−0.4039	0.1715
			$\hat{R}$	0.0077	0.0009	0.0062	0.0009	0.0142	0.00068	0.0120	0.0007
		II	$\hat{\alpha}$	0.0037	0.0307	−0.0298	0.0338	−0.0004	0.0297	−0.0319	0.0333
			$\hat{\beta}$	0.0038	0.0422	−0.0279	0.0479	−0.0039	0.0409	−0.0318	0.0471
			$\hat{\sigma}$	−0.8002	0.6422	−0.8171	0.6695	−0.7917	0.6286	−0.8130	0.6626
			$\hat{R}$	0.0003	0.0013	−0.0005	0.0014	0.0007	0.00127	−0.0003	0.0013

Table 2: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 1: $(\alpha, \beta, \sigma) = (0.75,\ 2,\ 1.5)$

T				1.5				5			
$n=\aleph$	$m=\mathcal{M}$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
30	**20**	**I**	$\hat{\alpha}$	0.5521	0.3903	0.5589	0.3396	0.6403	0.4553	0.6028	0.3872
			$\hat{\beta}$	0.4928	0.4585	0.1854	0.1165	0.2676	0.1720	0.0893	0.0662
			$\hat{\sigma}$	−0.4946	0.2764	−0.2743	0.0914	−0.0872	0.1105	0.0394	0.0793
			$\hat{R}$	0.0719	0.0110	0.1029	0.0124	0.1082	0.0152	0.1211	0.0164
		II	$\hat{\alpha}$	−0.5507	0.4297	0.0449	0.0360	−0.5004	0.3893	0.0505	0.0367
			$\hat{\beta}$	0.6294	1.2159	0.0562	0.2658	0.5809	1.5331	0.0468	0.2520
			$\hat{\sigma}$	−1.2806	1.6617	−0.7956	0.6417	−1.2593	1.6134	−0.7877	0.6288
			$\hat{R}$	−0.2007	0.0553	0.0109	0.0042	−0.1814	0.0495	0.0130	0.0043

(Continued)

Table 2 (continued).

T				1.5				5			
$n = \aleph$	$m = \mathcal{M}$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
	25	**I**	$\hat{\alpha}$	−0.0568	0.2563	0.2715	0.0960	0.2285	0.1801	0.3195	0.1225
			$\hat{\beta}$	0.4058	0.3171	0.0559	0.0988	0.2877	0.3478	−0.0222	0.0722
			$\hat{\sigma}$	−0.3962	0.1500	−0.2056	0.0321	−0.6409	0.4903	−0.4081	0.1879
			$\hat{R}$	−0.0656	0.0257	0.0610	0.0058	0.0273	0.0125	0.07957	0.0083
		II	$\hat{\alpha}$	−0.5644	0.4244	0.0304	0.0248	−0.5172	0.3898	0.0345	0.0244
			$\hat{\beta}$	0.5551	0.8018	−0.0122	0.1675	0.5652	0.9881	−0.0233	0.1581
			$\hat{\sigma}$	−1.2768	1.6507	−0.7805	0.6166	−1.2539	1.5989	−0.7745	0.6063
			$\hat{R}$	−0.2041	0.0550	0.0128	0.0034	−0.1891	0.0497	0.01481	0.0033
50	**35**	**I**	$\hat{\alpha}$	−0.3745	0.3270	0.5081	0.2753	0.6044	0.3862	0.5785	0.3491
			$\hat{\beta}$	0.4682	0.9387	0.2014	0.0969	0.2925	0.3831	−0.0460	0.0507
			$\hat{\sigma}$	−1.1500	1.3729	−0.3655	0.1401	−0.8386	0.7723	−0.5613	0.3228
			$\hat{R}$	0.0643	0.0080	0.0917	0.0096	0.1128	0.0144	0.1190	0.0152
		II	$\hat{\alpha}$	−0.6077	0.4387	0.0220	0.0198	−0.5689	0.4118	0.0262	0.0197
			$\hat{\beta}$	0.5378	0.7514	0.0325	0.1363	0.5182	0.7249	0.0264	0.1318
			$\hat{\sigma}$	−1.2905	1.6788	−0.8076	0.6569	−1.2728	1.6384	−0.8025	0.6482
			$\hat{R}$	−0.2171	0.0560	0.0055	0.0028	−0.2051	0.0524	0.0071	0.0028
	45	**I**	$\hat{\alpha}$	0.4845	0.2880	0.1646	0.0400	0.0172	0.1509	0.2181	0.0594
			$\hat{\beta}$	0.4526	0.4279	0.0252	0.0674	0.1631	0.0736	0.0675	0.0408
			$\hat{\sigma}$	−0.5419	0.3117	−0.6779	0.4634	−0.0433	0.0711	0.0181	0.0520
			$\hat{R}$	−0.1498	0.0407	0.0398	0.0032	−0.0242	0.0160	0.05959	0.0049
		II	$\hat{\alpha}$	−0.6402	0.4539	0.0150	0.0139	−0.5959	0.4212	0.0190	0.0138
			$\hat{\beta}$	0.5014	0.7129	−0.0425	0.0872	0.4475	0.6278	−0.0493	0.0852
			$\hat{\sigma}$	−0.9888	1.1696	−0.8005	0.6446	−0.9278	1.0649	−0.7948	0.6352
			$\hat{R}$	−0.2254	0.0577	0.0102	0.0021	−0.2099	0.0536	0.01193	0.0021
100	**65**	**I**	$\hat{\alpha}$	0.6211	0.3981	0.6178	0.3903	0.7270	0.5354	0.7083	0.5083
			$\hat{\beta}$	0.4112	0.2036	0.2844	0.1085	0.1166	0.0324	0.0957	0.0269
			$\hat{\sigma}$	−0.4075	0.1694	−0.3011	0.0939	0.3418	0.1973	0.3176	0.1672
			$\hat{R}$	0.0901	0.0089	0.1021	0.0110	0.1386	0.0198	0.1379	0.0195
		II	$\hat{\alpha}$	−0.6659	0.4660	0.0179	0.0105	−0.6402	0.4482	0.0217	0.0105
			$\hat{\beta}$	0.4848	0.6018	0.0181	0.0725	0.4635	0.6451	0.0144	0.0712
			$\hat{\sigma}$	−1.3026	1.7016	−0.8184	0.6722	−1.2915	1.6766	−0.8140	0.6648
			$\hat{R}$	−0.2363	0.0594	0.0045	0.0015	−0.2260	0.0569	0.0059	0.0015

Table 2 (continued).

$n = \aleph$	$m = \mathcal{M}$	Scheme		T = 1.5 MLE Bias	MSE	MPS Bias	MSE	T = 5 MLE Bias	MSE	MPS Bias	MSE
	85	**I**	$\hat{\alpha}$	−0.1330	0.2047	0.2394	0.0643	0.3079	0.1111	0.3268	0.1130
			$\hat{\beta}$	0.4362	0.4231	0.0923	0.0399	0.0724	0.0402	−0.0314	0.0213
			$\hat{\sigma}$	−0.9935	1.0431	−0.6312	0.4002	−0.5009	0.2705	−0.4021	0.1705
			$\hat{R}$	−0.0802	0.0237	0.0489	0.0031	0.0654	0.0059	0.08122	0.0072
		II	$\hat{\alpha}$	−0.6082	0.4174	0.0002	0.0070	−0.5666	0.4064	0.0035	0.0070
			$\hat{\beta}$	0.4509	0.5741	−0.0282	0.0477	0.5042	0.5865	−0.0320	0.0471
			$\hat{\sigma}$	−1.2305	1.4705	−0.8171	0.6695	−1.0297	0.2088	−0.8130	0.1625
			$\hat{R}$	−0.2423	0.0161	0.0039	0.0010	−0.2373	0.0593	0.00516	0.0010

Table 3: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 1: $(\alpha, \beta, \sigma) = (1.75,\ 0.5,\ 1.5)$

$n = \aleph$	$m = \mathcal{M}$	Scheme		T = 1.5 MLE Bias	MSE	MPS Bias	MSE	T = 5 MLE Bias	MSE	MPS Bias	MSE
30	**20**	**I**	$\hat{\alpha}$	0.1182	1.1602	0.5704	0.3491	0.5640	0.6757	0.6027	0.3871
			$\hat{\beta}$	−0.0056	0.2778	0.5788	0.3527	0.3015	0.3690	0.6266	0.4136
			$\hat{\sigma}$	−1.0480	1.2118	−0.5116	0.2702	−0.6438	0.7024	0.0419	0.0827
			$\hat{R}$	0.0438	0.1288	−0.0498	0.0041	0.0259	0.0494	−0.0542	0.0045
		II	$\hat{\alpha}$	0.0649	1.5527	0.0460	0.0361	0.0710	1.5253	0.0506	0.0367
			$\hat{\beta}$	−0.2497	0.2374	0.0465	0.0233	−0.2415	0.2356	0.0518	0.0239
			$\hat{\sigma}$	−1.3006	1.6924	−0.7942	0.6389	−1.3000	1.6911	−0.7875	0.6286
			$\hat{R}$	0.0034	0.1197	−0.0072	0.0056	0.0078	0.1215	−0.0081	0.0055
	25	**I**	$\hat{\alpha}$	0.4936	0.7761	0.2872	0.1032	0.1785	0.9562	0.3196	0.1226
			$\hat{\beta}$	−0.2174	0.2729	0.2949	0.1010	−0.2214	0.2505	0.3379	0.1298
			$\hat{\sigma}$	−1.2936	1.1674	−0.1660	0.0457	−1.2572	1.6055	−0.4073	0.1878
			$\hat{R}$	0.1021	0.0824	−0.0340	0.0034	0.0798	0.1198	−0.03916	0.0035
		II	$\hat{\alpha}$	−0.0710	1.3221	0.0309	0.0244	−0.0526	1.3410	0.0346	0.0244
			$\hat{\beta}$	−0.2262	0.2328	0.0425	0.0156	−0.2503	0.2276	0.0468	0.0160
			$\hat{\sigma}$	−1.2946	1.6768	−0.7802	0.6153	−1.2944	1.6763	−0.7744	0.6061
			$\hat{R}$	−0.0234	0.1192	−0.0103	0.0040	−0.0085	0.1132	−0.0110	0.0040

(Continued)

Table 3 (continued).

$n = \aleph$	T			1.5				5			
	$m = \mathcal{M}$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
50	**35**	**I**	$\hat{\alpha}$	−0.0116	0.0897	−0.0044	0.0024	−0.0143	0.0931	−0.0051	0.0024
			$\hat{\beta}$	0.2814	0.6780	0.5262	0.2918	−0.0119	0.9637	0.5785	0.3491
			$\hat{\sigma}$	−0.2239	0.2091	0.5351	0.2963	0.3093	0.3099	0.6113	0.3857
			$\hat{R}$	−1.1861	1.4575	−0.2584	0.0732	−1.2806	1.6409	−0.5603	0.3218
		II	$\hat{\alpha}$	0.0395	0.0906	−0.0220	0.0021	0.0248	0.0370	−0.0283	0.0022
			$\hat{\beta}$	−0.1454	1.1921	0.0229	0.0195	−0.1040	1.2859	0.0263	0.0197
			$\hat{\sigma}$	−0.2713	0.2145	0.0304	0.0115	−0.2788	0.2155	0.0341	0.0118
			$\hat{R}$	−1.2878	1.6591	−0.8068	0.6551	−1.2878	1.6593	−0.8024	0.6480
	45	**I**	$\hat{\alpha}$	0.1374	0.0912	−0.0481	0.0034	0.0432	0.1114	−0.0554	0.0040
			$\hat{\beta}$	−0.0760	0.5924	0.1798	0.0446	0.5292	0.4926	0.2183	0.0596
			$\hat{\sigma}$	−0.3051	0.1863	0.1778	0.0389	−0.2478	0.2460	0.2254	0.0591
			$\hat{R}$	−1.2813	1.0642	−0.6473	0.4225	−0.5883	0.5865	0.02115	0.0547
		II	$\hat{\alpha}$	−0.0229	0.1005	−0.0077	0.0035	−0.0054	0.0985	−0.0083	0.0034
			$\hat{\beta}$	−0.2177	1.0696	0.0156	0.0138	−0.2251	1.0534	0.0191	0.0138
			$\hat{\sigma}$	−0.2847	0.2070	0.0189	0.0077	−0.2838	0.2053	0.0228	0.0078
			$\hat{R}$	−1.2807	1.6409	−0.7998	0.6431	−1.2806	1.6407	−0.79471	0.6350
100	**65**	**I**	$\hat{\alpha}$	−0.1937	0.6361	0.6345	0.4099	0.7123	0.5288	0.7081	0.5080
			$\hat{\beta}$	0.1087	0.2308	0.6365	0.4102	0.6951	0.5206	0.7489	0.5677
			$\hat{\sigma}$	−1.2688	1.6105	−0.5831	0.3417	−0.0078	0.1242	0.3256	0.1801
			$\hat{R}$	−0.1909	0.1935	−0.0509	0.0030	−0.0462	0.0054	−0.0613	0.0042
		II	$\hat{\alpha}$	−0.2710	0.9207	0.0184	0.0105	−0.3292	0.8551	0.0218	0.0105
			$\hat{\beta}$	−0.3136	0.1968	0.0178	0.0060	−0.3142	0.1971	0.0213	0.0060
			$\hat{\sigma}$	−1.2726	1.6202	−0.8179	0.6712	−1.2727	1.6204	−0.8139	0.6647
			$\hat{R}$	0.0037	0.0776	−0.0029	0.0018	−0.0042	0.0749	−0.0034	0.0018
	85	**I**	$\hat{\alpha}$	0.4923	0.5208	0.2604	0.0743	0.0692	0.6417	0.3272	0.1133
			$\hat{\beta}$	−0.3351	0.2152	0.2574	0.0704	−0.3633	0.1881	0.3448	0.1239
			$\hat{\sigma}$	−1.2688	1.6105	−0.5831	0.3417	−1.2581	1.5895	−0.3993	0.1691
			$\hat{R}$	0.0830	0.0491	−0.1648	0.0302	0.1465	0.0844	−0.03948	0.0021
		II	$\hat{\alpha}$	−0.3940	0.7292	0.0010	0.0070	−0.4171	0.6830	0.0036	0.0070
			$\hat{\beta}$	−0.3152	0.1910	0.0088	0.0043	−0.3165	0.1908	0.0115	0.0043
			$\hat{\sigma}$	−1.2686	1.6098	−0.8162	0.6678	−1.2686	1.6099	−0.8128	0.6623
			$\hat{R}$	−0.0018	0.0689	−0.0039	0.0013	−0.0083	0.0684	−0.00441	0.0013

Table 4: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 2: $(\alpha, \beta, \sigma) = (0.75,\ 0.5,\ 1.5)$

T				1.5				5			
$(n,\ \aleph)$	$(m,\ \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 35)	**(20, 25)**	**I**	$\hat{\alpha}$	0.4905	0.9425	0.5656	0.3456	0.5852	0.7404	0.6041	0.3904
			$\hat{\beta}$	−0.2025	0.2234	0.5119	0.2757	0.1626	0.2936	0.5670	0.3377
			$\hat{\sigma}$	−1.1768	1.4461	−0.2306	0.0666	−0.7515	0.8278	−0.0119	0.0672
			$\hat{R}$	0.1583	0.0984	−0.0354	0.0029	0.0725	0.0548	−0.0410	0.0032
	(20, 25)	**II**	$\hat{\alpha}$	0.2757	2.0458	0.0544	0.0380	0.2727	2.0314	0.0588	0.0381
			$\hat{\beta}$	−0.2892	0.2188	0.0372	0.0161	−0.2991	0.2145	0.0419	0.0163
			$\hat{\sigma}$	−1.2962	1.6810	−0.7957	0.6404	−1.2962	1.6810	−0.7899	0.6308
			$\hat{R}$	0.0505	0.1165	−0.0027	0.0050	0.0436	0.1129	−0.0035	0.0049
	(25, 30)	**I**	$\hat{\alpha}$	0.2325	1.3462	0.2852	0.1039	0.2421	0.9515	0.3183	0.1234
			$\hat{\beta}$	−0.2548	0.2476	0.2653	0.0823	−0.2784	0.2330	0.3090	0.1089
			$\hat{\sigma}$	−1.2896	1.6639	−0.5358	0.2941	−1.2694	1.6247	−0.4348	0.2067
			$\hat{R}$	0.0758	0.1252	−0.0259	0.0030	0.1103	0.1174	−0.0314	0.0030
		II	$\hat{\alpha}$	0.0900	1.6671	0.0320	0.0261	0.1329	1.7392	0.0352	0.0261
			$\hat{\beta}$	−0.2746	0.2190	0.0364	0.0138	−0.2741	0.2166	0.0402	0.0138
			$\hat{\sigma}$	−1.2911	1.6677	−0.7863	0.6241	−1.2912	1.6680	−0.7814	0.6161
			$\hat{R}$	0.0140	0.1112	−0.0082	0.0044	0.0195	0.1143	−0.0089	0.0043
(50, 60)	**(35,40)**	**I**	$\hat{\alpha}$	0.3633	0.7463	0.5331	0.2976	0.3633	0.7063	0.5331	0.2976
			$\hat{\beta}$	−0.1805	0.2116	0.5891	0.3555	−0.1805	0.2116	0.5891	0.3555
			$\hat{\sigma}$	−1.1424	1.3732	−0.2158	0.0530	−0.5142	1.3732	−0.2158	0.0530
			$\hat{R}$	0.1341	0.0843	−0.0594	0.0044	0.1341	0.0843	−0.0594	0.0044
		II	$\hat{\alpha}$	−0.0139	1.4259	0.0283	0.0211	−0.0275	1.4465	0.0322	0.0212
			$\hat{\beta}$	−0.3029	0.2069	0.0260	0.0101	−0.3009	0.2050	0.0303	0.0103
			$\hat{\sigma}$	−1.2824	1.6453	−0.8066	0.6547	−1.2824	1.6453	−0.8016	0.6467
			$\hat{R}$	0.0209	0.0951	−0.0045	0.0031	0.0137	0.0928	−0.0052	0.0031
	(45, 50)	**I**	$\hat{\alpha}$	−0.0767	0.9379	0.2092	0.0564	−0.0767	0.9379	0.2092	0.0564
			$\hat{\beta}$	−0.3067	0.2281	0.2674	0.0789	−0.3067	0.2281	0.2674	0.0789
			$\hat{\sigma}$	−1.2790	1.6365	−0.5835	0.3441	−1.2790	1.6365	−0.5835	0.3441
			$\hat{R}$	0.0585	0.0934	−0.0449	0.0034	0.0585	0.0934	−0.0449	0.0034
		II	$\hat{\alpha}$	−0.1432	1.1865	0.0141	0.0157	−0.1151	1.2613	0.0180	0.0156
			$\hat{\beta}$	−0.3149	0.2052	0.0233	0.0076	−0.3193	0.1993	0.0276	0.0077
			$\hat{\sigma}$	−1.2786	1.6355	−0.8016	0.6459	−1.2786	1.6354	−0.7962	0.6371
			$\hat{R}$	0.0244	0.0862	−0.0074	0.0024	0.0239	0.0862	−0.0082	0.0024

(Continued)

Table 4 (continued).

T				1.5				5			
$(n, \aleph)$	$(m, \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(100, 120)	**(60, 85)**	**I**	$\hat{\alpha}$	0.4445	0.6167	0.6276	0.4013	0.7045	0.5193	0.7048	0.5036
			$\hat{\beta}$	−0.0858	0.1979	0.6123	0.3787	0.6746	0.4838	0.7313	0.5399
			$\hat{\sigma}$	−1.0549	1.2013	−0.1844	0.0364	−0.0249	0.1076	0.3198	0.1643
			$\hat{R}$	0.1287	0.0714	−0.0469	0.0026	−0.0449	0.0044	−0.0584	0.0038
		II	$\hat{\alpha}$	−0.1440	1.0831	0.0195	0.0106	−0.1257	1.1744	0.0227	0.0107
			$\hat{\beta}$	−0.3313	0.1931	0.0145	0.0046	−0.3395	0.1919	0.0178	0.0047
			$\hat{\sigma}$	−1.2683	1.6090	−0.8179	0.6710	−1.2684	1.6093	−0.8141	0.6648
			$\hat{R}$	0.0386	0.0739	−0.0014	0.0016	0.0412	0.0720	−0.0019	0.0015
	(85, 100)	**I**	$\hat{\alpha}$	−0.0498	0.8044	0.2730	0.0813	0.1037	0.6593	0.3404	0.1220
			$\hat{\beta}$	−0.3643	0.2038	0.2816	0.0831	−0.3725	0.1854	0.3749	0.1455
			$\hat{\sigma}$	−1.2651	1.6008	−0.5655	0.3214	−1.2513	1.5753	−0.3651	0.1442
			$\hat{R}$	0.1197	0.0758	−0.0333	0.0018	0.1591	0.0836	−0.0451	0.0026
		II	$\hat{\alpha}$	−0.1752	1.0601	0.0050	0.0077	−0.1713	1.0789	0.0074	0.0076
			$\hat{\beta}$	−0.3342	0.1940	0.0098	0.0035	−0.3336	0.1931	0.0123	0.0036
			$\hat{\sigma}$	−1.2649	1.6005	−0.8157	0.6669	−1.2648	1.6002	−0.8126	0.6618
			$\hat{R}$	0.0366	0.0705	−0.0036	0.0013	0.0362	0.0720	−0.0040	0.0013

Table 5: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 2: $(\alpha, \beta, \sigma) = (0.75,\ 2,\ 1.5)$

T				1.5				5			
$(n, \aleph)$	$(m, \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 35)	**(20, 25)**	**I**	$\hat{\alpha}$	0.5567	0.3814	0.5518	0.3339	0.6476	0.4595	0.6041	0.3904
			$\hat{\beta}$	0.3816	0.3420	0.1274	0.0828	0.1585	0.1036	0.0198	0.0457
			$\hat{\sigma}$	−0.5131	0.2860	−0.3287	0.1212	−0.1067	0.0943	−0.0148	0.0640
			$\hat{R}$	0.0828	0.0116	0.1075	0.0134	0.1211	0.0176	0.1290	0.0184
	(20, 25)	**II**	$\hat{\alpha}$	−0.4958	0.4034	0.0535	0.0387	−0.4421	0.3629	0.0587	0.0381
			$\hat{\beta}$	0.4988	0.7578	0.0273	0.1792	0.4670	0.6921	0.0174	0.1692
			$\hat{\sigma}$	−1.2527	1.5992	−0.7961	0.6419	−1.2289	1.5449	−0.7900	0.6310
			$\hat{R}$	−0.1800	0.0515	0.0143	0.0043	−0.1626	0.0456	0.0164	0.0042

Table 5 (continued).

T				1.5				5			
$(n,\ \aleph)$	$(m,\ \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
	(25, 30)	**I**	$\hat{\alpha}$	−0.0145	0.2432	0.2684	0.0964	0.2543	0.1744	0.3182	0.1233
			$\hat{\beta}$	0.4652	0.7792	0.0253	0.0816	0.2079	0.2596	−0.0522	0.0620
			$\hat{\sigma}$	−0.9399	0.9571	−0.5826	0.3468	−0.6305	0.4613	−0.4361	0.2072
			$\hat{R}$	−0.0494	0.0238	0.0631	0.0062	0.0397	0.0118	0.0822	0.0087
		II	$\hat{\alpha}$	−0.5318	0.4079	0.0309	0.0264	−0.4799	0.3709	0.0351	0.0261
			$\hat{\beta}$	0.5173	1.0547	−0.0178	0.1428	0.4905	0.9445	−0.0270	0.1370
			$\hat{\sigma}$	−1.2620	1.6174	−0.7878	0.6270	−1.2343	1.5551	−0.7816	0.6162
			$\hat{R}$	−0.1900	0.0529	0.0128	0.0035	−0.1742	0.0476	0.0148	0.0034
(50, 60)	**(35, 40)**	**I**	$\hat{\alpha}$	0.5120	0.2919	0.5118	0.2780	0.6137	0.3929	0.5841	0.3538
			$\hat{\beta}$	0.4560	0.2820	0.2656	0.1175	0.1909	0.0726	0.1197	0.0437
			$\hat{\sigma}$	−0.4942	0.2534	−0.3527	0.1308	0.0590	0.0791	0.0901	0.0673
			$\hat{R}$	0.0671	0.0065	0.0855	0.0083	0.1114	0.0137	0.1139	0.0139
		II	$\hat{\alpha}$	−0.5751	0.4227	0.0278	0.0214	−0.5321	0.3927	0.0321	0.0212
			$\hat{\beta}$	0.4874	0.6643	0.0131	0.1144	0.4303	0.6100	0.0069	0.1107
			$\hat{\sigma}$	−1.2748	1.6437	−0.8069	0.6556	−1.2538	1.5962	−0.8017	0.6468
			$\hat{R}$	−0.2039	0.0549	0.0081	0.0026	−0.1896	0.0497	0.0098	0.0025
	(45, 50)	**I**	$\hat{\alpha}$	−0.2049	0.2464	0.1838	0.0476	0.1858	0.1083	0.2514	0.0751
			$\hat{\beta}$	0.5429	0.8201	0.1342	0.0781	0.2326	0.1607	0.0356	0.0420
			$\hat{\sigma}$	−1.0368	1.1395	−0.6436	0.4180	−0.6210	0.4374	−0.4594	0.2232
			$\hat{R}$	−0.1046	0.0292	0.0326	0.0024	0.0217	0.0077	0.0577	0.0045
		II	$\hat{\alpha}$	−0.6046	0.4366	0.0140	0.0159	−0.5654	0.4103	0.0179	0.0156
			$\hat{\beta}$	0.4553	0.6189	−0.0213	0.0785	0.4593	0.6791	−0.0278	0.0760
			$\hat{\sigma}$	−1.2802	1.6540	−0.8017	0.6463	−1.2593	1.6073	−0.7963	0.6373
			$\hat{R}$	−0.2155	0.0558	0.0073	0.0020	−0.2029	0.0525	0.0090	0.0020
(100, 120)	**(60, 85)**	**I**	$\hat{\alpha}$	0.7483	0.5685	0.7306	0.5407	0.8241	0.6876	0.8032	0.6539
			$\hat{\beta}$	1.3329	1.8357	1.1585	1.3886	−0.0339	0.0135	−0.0343	0.0136
			$\hat{\sigma}$	−0.0661	0.0088	0.0481	0.0083	0.3596	0.1981	0.2990	0.1449
			$\hat{R}$	0.0380	0.0019	0.0469	0.0026	0.1720	0.0302	0.1687	0.0290
		II	$\hat{\alpha}$	−0.6789	0.4756	0.0149	0.0120	−0.5515	0.3935	0.0286	0.0121
			$\hat{\beta}$	0.9752	1.6817	0.2216	0.2035	0.3955	0.5150	0.0011	0.0464
			$\hat{\sigma}$	−1.3120	1.7241	−0.8292	0.6908	−1.2437	1.5680	−0.8150	0.6662
			$\hat{R}$	−0.2448	0.0624	−0.0131	0.0021	−0.1986	0.0497	0.0081	0.0014

(Continued)

Table 5 (continued).

T				1.5				5			
$(n, \aleph)$	$(m, \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
	(85, 100)	**I**	$\hat{\alpha}$	0.0124	0.1395	0.2476	0.0686	0.3374	0.1255	0.3398	0.1216
			$\hat{\beta}$	0.4363	0.4036	0.1220	0.0419	0.0590	0.0270	−0.0115	0.0177
			$\hat{\sigma}$	−0.8955	0.8466	−0.6256	0.3932	−0.4293	0.2020	−0.3693	0.1459
			$\hat{R}$	−0.0374	0.0143	0.0475	0.0029	0.0731	0.0065	0.0816	0.0072
		II	$\hat{\alpha}$	−0.6628	0.4606	0.0050	0.0072	−0.6356	0.4423	0.0073	0.0076
			$\hat{\beta}$	0.4382	0.5836	−0.0269	0.0391	0.4014	0.6756	−0.0298	0.0385
			$\hat{\sigma}$	−1.2971	1.6871	−0.8174	0.6698	−1.2826	1.6542	−0.8127	0.6619
			$\hat{R}$	−0.2336	0.0583	0.0048	0.0010	−0.2241	0.0556	0.0057	0.0010

Table 6: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes, Case 2: $(\alpha, \beta, \sigma) = (1.75,\ 2,\ 1.5)$

T				1.5				5			
$(n, \aleph)$	$(m, \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(30, 35)	**(20, 25)**	**I**	$\hat{\alpha}$	0.4658	0.3047	0.3713	0.2290	0.3017	0.1406	0.2524	0.1185
			$\hat{\beta}$	0.2624	0.1431	0.1816	0.1130	0.0525	0.0427	0.0202	0.0456
			$\hat{\sigma}$	−0.3973	0.1657	−0.4035	0.1749	0.0615	0.0753	−0.0164	0.0624
			$\hat{R}$	0.0278	0.0029	0.0260	0.0028	0.0330	0.0026	0.0308	0.0025
	(20,25)	**II**	$\hat{\alpha}$	0.1761	0.2422	0.0638	0.1799	0.1739	0.2367	0.0591	0.1735
			$\hat{\beta}$	0.1580	0.2376	0.0291	0.1805	0.1497	0.2229	0.0174	0.1692
			$\hat{\sigma}$	−0.8309	0.6985	−0.7973	0.6441	−0.8230	0.6847	−0.7901	0.6311
			$\hat{R}$	0.0041	0.0064	0.0049	0.0053	0.0048	0.0062	0.0056	0.0052
	(25, 30)	**I**	$\hat{\alpha}$	0.2383	0.1499	0.1493	0.1179	0.1372	0.0767	0.0805	0.0704
			$\hat{\beta}$	0.1339	0.1028	0.0508	0.0910	−0.0090	0.0531	−0.0520	0.0620
			$\hat{\sigma}$	−0.6109	0.3791	−0.6135	0.3842	−0.3892	0.1718	−0.4364	0.2074
			$\hat{R}$	0.0153	0.0030	0.0137	0.0029	0.0197	0.0023	0.0175	0.0024
		II	$\hat{\alpha}$	0.0935	0.1477	−0.0070	0.1254	0.0881	0.1413	−0.0136	0.1200
			$\hat{\beta}$	0.0971	0.1655	−0.0170	0.1432	0.0870	0.1574	−0.0265	0.1379
			$\hat{\sigma}$	−0.8092	0.6609	−0.7875	0.6269	−0.8014	0.6478	−0.7816	0.6164
			$\hat{R}$	0.0012	0.0049	0.0012	0.0045	0.0016	0.0047	0.0014	0.0044

Table 6 (continued).

T				1.5				5			
$(n, \aleph)$	$(m, \mathcal{M})$	Scheme		MLE		MPS		MLE		MPS	
				Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
(50, 60)	**(35, 40)**	**I**	$\hat{\alpha}$	0.3548	0.1687	0.3197	0.1527	0.1659	0.0494	0.1692	0.0555
			$\hat{\beta}$	0.3428	0.1639	0.3190	0.1576	0.0999	0.0330	0.1205	0.0437
			$\hat{\sigma}$	−0.3744	0.1448	−0.4197	0.1827	0.2294	0.1220	0.0877	0.0647
			$\hat{R}$	0.0065	0.0012	0.0049	0.0012	0.0104	0.0008	0.0084	0.0008
		II	$\hat{\alpha}$	0.0916	0.1110	0.0186	0.0987	0.0887	0.1080	0.0154	0.0962
			$\hat{\beta}$	0.0941	0.1263	0.0133	0.1145	0.0870	0.1212	0.0070	0.1107
			$\hat{\sigma}$	−0.8167	0.6717	−0.8072	0.6563	−0.8090	0.6589	−0.8018	0.6470
			$\hat{R}$	0.0013	0.0034	0.0010	0.0033	0.0017	0.0033	0.0014	0.0032
	(45, 50)	**I**	$\hat{\alpha}$	0.1045	0.0551	0.0625	0.0537	−0.0066	0.0263	−0.0171	0.0323
			$\hat{\beta}$	0.1841	0.0890	0.1496	0.0865	0.0293	0.0335	0.0361	0.0420
			$\hat{\sigma}$	−0.6313	0.4021	−0.6601	0.4400	−0.3762	0.1571	−0.4603	0.2238
			$\hat{R}$	−0.0074	0.0017	−0.0091	0.0018	−0.0045	0.0011	−0.0068	0.0012
		II	$\hat{\alpha}$	0.0339	0.0676	−0.0269	0.0673	0.0292	0.0653	−0.0306	0.0660
			$\hat{\beta}$	0.0435	0.0782	−0.0206	0.0790	0.0340	0.0741	−0.0276	0.0761
			$\hat{\sigma}$	−0.7996	0.6431	−0.8018	0.6466	−0.7909	0.6292	−0.7964	0.6375
			$\hat{R}$	−0.0006	0.0026	−0.0012	0.0025	−0.0001	0.0025	−0.0009	0.0025
(100, 120)	**(60, 85)**	**I**	$\hat{\alpha}$	0.5631	0.3438	0.5761	0.3661	0.3400	0.1270	0.3877	0.1656
			$\hat{\beta}$	0.1989	0.0563	0.2271	0.0741	−0.0871	0.0166	−0.0334	0.0132
			$\hat{\sigma}$	−0.3261	0.1086	−0.4074	0.1691	0.5073	0.3218	0.2953	0.1396
			$\hat{R}$	0.0458	0.0026	0.0440	0.0026	0.0554	0.0034	0.0541	0.0033
		II	$\hat{\alpha}$	0.0799	0.0599	0.0319	0.0560	0.0774	0.0578	0.0306	0.0549
			$\hat{\beta}$	0.0399	0.0447	0.0055	0.0478	0.0325	0.0427	0.0011	0.0464
			$\hat{\sigma}$	−0.8051	0.6505	−0.8201	0.6750	−0.7963	0.6363	−0.8151	0.6663
			$\hat{R}$	0.0057	0.0017	0.0033	0.0018	0.0063	0.0017	0.0037	0.0018
	(85, 100)	**I**	$\hat{\alpha}$	0.1608	0.0484	0.1557	0.0517	0.0229	0.0121	0.0529	0.0183
			$\hat{\beta}$	0.1342	0.0422	0.1472	0.0521	−0.0642	0.0169	−0.0108	0.0176
			$\hat{\sigma}$	−0.5972	0.3586	−0.6526	0.4279	−0.2350	0.0682	−0.3708	0.1466
			$\hat{R}$	0.0057	0.0008	0.0035	0.0009	0.0114	0.0006	0.0088	0.0006
		II	$\hat{\alpha}$	0.0099	0.0321	−0.0227	0.0351	0.0061	0.0310	−0.0246	0.0345
			$\hat{\beta}$	−0.0014	0.0341	−0.0263	0.0389	−0.0088	0.0333	−0.0296	0.0385
			$\hat{\sigma}$	−0.7953	0.6344	−0.8158	0.6673	−0.7877	0.6223	−0.8127	0.6620
			$\hat{R}$	0.0015	0.0013	2.2E-07	0.0013	0.0019	0.0012	0.0002	0.0013

The simulation study showed that the bias and MSE of all estimators for different cases decrease when sample size of stress or/and strength increases. Furthermore, model efficiency increases when the effective sample size of the censored scheme increases. In this study, we noted that scheme I of the progressive Type-II hybrid censoring was found to be superior to scheme II. Moreover, the results showed that efficiency of the MPS estimators are over MLE's which means that MPS estimation method is good alternative to MLE method.

6 Applications

We discuss a stress–strength reliability of EG distribution using real data set to illustrate estimation methods of EG distribution based on stress–strength reliability model provides significant improvements over.

Data Set 1: The real data sets of the waiting times before service of the customers of two banks A and B, respectively have been used. These data sets have been discussed by Reference [31] for estimating the stress–strength reliability in case of the Generalized Lindley distribution.

Data of Bank A: 0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2, 6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6, 13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5.

Data of Bank B: 0.1, 0.2, 0.3, 0.7, 0.9, 1.1, 1.2, 1.8, 1.9, 2.0, 2.2, 2.3, 2.3, 2.3, 2.5, 2.6, 2.7, 2.7, 2.9, 3.1, 3.1, 3.2, 3.4, 3.4, 3.5, 3.9, 4.0, 4.2, 4.5, 4.7, 5.3, 5.6, 5.6, 6.2, 6.3, 6.6, 6.8, 7.3, 7.5, 7.7, 7.7, 8.0, 8.0, 8.5, 8.5, 8.7, 9.5, 10.7, 10.9, 11.0, 12.1, 12.3, 12.8, 12.9, 13.2, 13.7, 14.5, 16.0, 16.5, 28.0.

Fig. 1 Shows plots of the fitted pdf, cdf and p-p plot of the EG distribution for these data and the results of MLE estimates of R along with the value of standard error, Kolmogorov–Smirnov and the p-value are confirmed in Tab. 7, while Tab. 8 provides the MLE estimates of R for the Bank data based on stress–strength model under different Censoring Schemes.

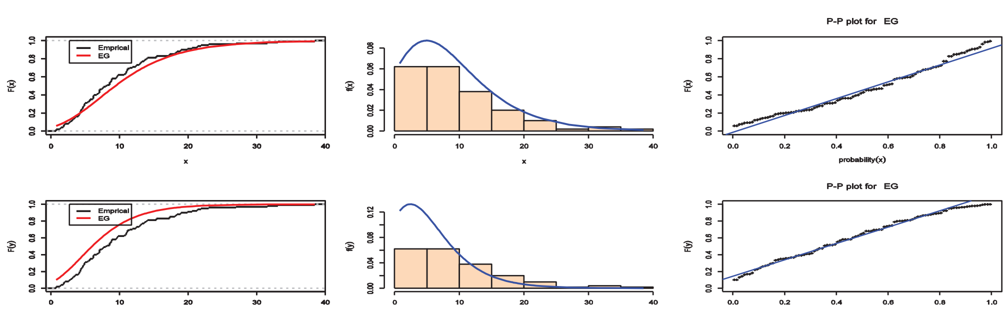

Figure 1: Plots of the fitted pdf, cdf and p-p plot of the EG distribution for banking data

For this data, MPS method can't be used since there are equal observation in the data, so the spacing will be zero and hence the product will also be zero a. Despite the effectiveness of the MPS method, this problem hinders their use in the estimation process (for more information of this method see [16,18,20].

Data Set 2: The analysis of a pair of real data sets is presented for illustrative purposes. These data show the breaking strengths of jute fiber at two different gauge lengths. These two data sets were used by [32] where X is the breaking strength of jute fibre with 10 mm, and Y is the breaking strength of jute fibre with 20 mm. These data sets have been discussed by Reference [33] for estimating the stress–strength reliability under progressive Type-II censoring scheme in case of the exponential distribution.

Table 7: Estimate, stander error, Kolmogorov–Smirnov test and reliability for EG distribution for banking data

	x		***y***	
	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$	$\hat{\sigma}$
Estimate	3.247608	6.101491	2.753723	4.364379
SE	0.434372	0.606014	0.434255	0.548213
KS	0.11009		0.12581	
P-value	0.177		0.2984	
$\hat{R}$	0.541148			

Table 8: MLE of EG distribution based on stress–strength model under different censoring schemes for banking data

	T		3				5			
Scheme	m1, m2		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$
I	**(70, 45)**	Estimate	5.5875	3.3451	3.2171	0.6255	4.8303	3.0050	3.7637	0.6165
		SE	0.9572	0.5631	0.5441		0.7404	0.4597	0.4527	
II	**(70, 45)**	Estimate	4.6829	2.9723	3.9983	0.6117	5.2823	3.3013	3.3704	0.6154
		SE	0.7303	0.4560	0.4504		0.8811	0.5381	0.4258	
	(80, 45)	Estimate	5.7254	3.7200	3.1652	0.6062	5.0996	3.3877	3.5587	0.6009
		SE	1.0509	0.6737	0.5880		0.8384	0.5560	0.4573	
	(90, 50)	Estimate	5.5294	3.6505	3.1239	0.6023	4.8343	3.2397	3.6534	0.5987
		SE	0.9771	0.6491	0.5522		0.7571	0.5183	0.4549	

Breaking strength of jute fibre of gauge length 10 mm are 693.73, 704.66, 323.83, 778.17, 123.06, 637.66, 383.43, 151.48, 108.94, 50.16, 671.49, 183.16, 257.44, 727.23, 291.27, 101.15, 376.42, 163.40, 141.38, 700.74, 262.90, 353.24, 422.11, 43.93,590.48, 212.13, 303.90, 506.60, 530.55, 177.25.

Breaking strength of jute fibre of gauge length 20 mm are 71.46, 419.02, 284.64, 585.57, 456.60, 113.85, 187.85, 688.16, 662.66, 45.58, 578.62, 756.70, 594.29, 166.49, 99.72, 707.36, 765.14, 187.13, 145.96, 350.70, 547.44, 116.99, 375.81, 581.60, 119.86, 48.01, 200.16, 36.75, 244.53, 83.55.

The fitted pdf, cdf and p-p plot of the EG distribution for the breaking strengths of jute fiber are presented in Fig. 2, while the results of MLE estimates of two variables along with the value of standard error, Kolmogorov-Smirnov and the *p*-value are given in Tabs. 9, 10 provide the MLE and MPS estimates of R for the breaking strength of jute fibre.

From these two applications we observe that the standard error (SE) of most estimators in decreases as the sample's sizes increase and that MPS estimators are mostly have lower SE than MLE estimators. Moreover, the progressive Type-II censoring Scheme I provide estimators with lower SE that those estimators under Scheme II. To more applications of progressive Type-II censoring scheme see [34].

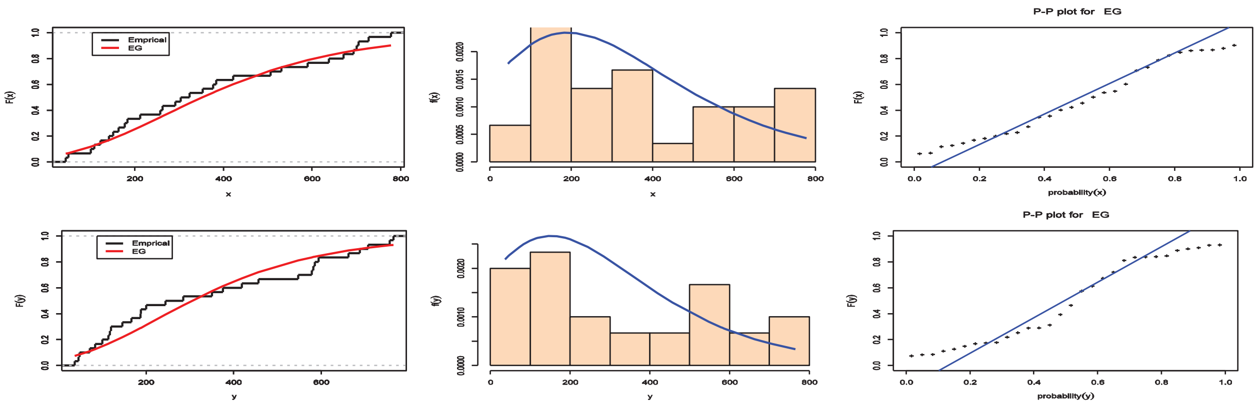

Figure 2: Plots of the fitted pdf, cdf and p-p plot of the EG distribution for fibre data

Table 9: Estimate, stander error, Kolmogorov–Smirnov test and reliability for EG distribution for fibre data

	x		***y***	
	$\hat{\alpha}$	$\hat{\sigma}$	$\hat{\beta}$	$\hat{\sigma}$
Estimate	3.360852	223.3009	3.125314	202.7625
SE	0.806017	38.91472	0.673863	30.69772
KS	0.10567		0.15462	
P-value	0.8565		0.4271	
$\hat{R}$	0.5181569			

Table 10: MLE and MPS of EG distribution based on stress–strength model under different censoring schemes for fibre data

Scheme	T		300				500			
	m1, m2		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$
MLE										
I	**15, 15**	Estimate	4.2545	3.9095	287.9807	0.5211	3.1636	3.1194	532.2289	0.5035
		SE	1.0404	0.9126	60.0047		0.6985	0.6803	113.7408	
	20, 20	Estimate	3.5632	3.6440	269.6446	0.4944	2.8632	3.0159	421.6083	0.4870
		SE	0.8373	0.8478	55.2740		0.6099	0.6453	77.6681	
II	**15, 15**	Estimate	4.8226	3.8121	140.7540	0.5585	3.7001	3.1003	206.7645	0.5441
		SE	1.3558	0.9538	28.2806		0.9164	0.7085	40.8953	
	15, 20	Estimate	4.7091	3.5810	144.6408	0.5680	3.7695	3.0860	200.1826	0.5498
		SE	1.2872	0.8729	27.7617		0.9414	0.6982	37.1523	
	20, 15	Estimate	5.1036	3.7409	140.3197	0.5770	3.5649	2.8435	218.9256	0.5563
		SE	1.4408	0.9266	26.6407		0.8582	0.6258	41.1987	
	20, 20	Estimate	4.4944	3.6070	160.0743	0.5548	3.3911	2.9931	229.7528	0.5312
		SE	1.2007	0.8719	30.2854		0.8033	0.6591	41.0765	

Table 10 (continued).

Scheme	T		300				500			
	m1, m2		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	$\hat{R}$
MPS										
I	**15, 15**	Estimate	3.5548	3.3864	343.0092	0.5121	3.0521	3.0064	523.6670	0.5038
		SE	0.8586	0.7905	54.6717		0.6682	0.6501	111.0147	
	20, 20	Estimate	3.4498	3.5332	264.1144	0.4940	2.8922	3.0367	386.9390	0.4878
		SE	0.8056	0.8173	54.0762		0.6130	0.6453	67.5605	
II	**15, 15**	Estimate	4.7015	3.7409	133.7630	0.5569	3.6866	3.0843	190.5830	0.5445
		SE	1.2655	0.9077	24.5420		0.8952	0.6954	34.9763	
	15, 20	Estimate	5.1007	3.7198	131.3513	0.5783	3.8995	3.1289	179.6775	0.5548
		SE	1.2714	0.8623	22.2539		0.9330	0.6898	30.1102	
	20, 15	Estimate	4.7039	3.6248	133.6804	0.5648	3.6078	2.8958	196.9227	0.5547
		SE	1.2404	0.8605	23.0879		0.8579	0.6327	34.2576	
	20, 20	Estimate	4.5719	3.6593	146.9008	0.5554	3.5125	3.0592	204.7143	0.5345
		SE	1.1792	0.8616	24.8273		0.8007	0.6468	33.2165	

7 Conclusions

In this paper, the MPS method was introduced as an alternative estimation method for the estimation of stress–strength model of EG distribution under progressive Type-II hybrid censoring scheme. Two different schemes of progressive Type-II hybrid censoring were proposed and used to estimate the reliability parameter using MPS and MLE methods. Because the MLE and MPS cannot be obtained in a closed form for EG distribution to estimate parameters, iterative procedures as conjugate-gradient are done by using R program. The MPS method can be used as an alternative method for the MLE method. In the case of EG distribution based on the stress–strength model under the progressive Type-II hybrid censoring scheme, the estimators based on the MPS method are better than the estimators based on the MLE. We can conclude that the MPS method is a good alternative method to the usual MLE method when progressive hybrid censoring schemes are used.

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