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Article

Model of Fractional Heat Conduction in a Thermoelastic Thin Slim Strip under Thermal Shock and Temperature-Dependent Thermal Conductivity

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Abstract: The present paper paper, we estimate the theory of thermoelasticity a thin slim strip under the variable thermal conductivity in the fractional-order form is solved. Thermal stress theory considering the equation of heat conduction based on the time-fractional derivative of Caputo of order α is applied to obtain a solution. We assumed that the strip surface is to be free from traction and impacted by a thermal shock. The transform of Laplace (LT) and numerical inversion techniques of Laplace were considered for solving the governing basic equations. The inverse of the LT was applied in a numerical manner considering the Fourier expansion technique. The numerical results for the physical variables were calculated numerically and displayed via graphs. The parameter of fractional order effect and variation of thermal conductivity on the displacement, stress, and temperature were investigated and compared with the results of previous studies. The results indicated the strong effect of the external parameters, especially the time-fractional derivative parameter on a thermoelastic thin slim strip phenomenon.

Keywords: Non-Fourier heat conduction; thermoelasticity; fractional derivative; variable thermal conductivity

1 Introduction

Recently, more attention has been given to the uncoupled classical thermoelastic theory, which predicts that two phenomena have a confliction and do not agree with the physical laboratory results. While the heat conduction equation is the first phenomenon without any elastic terms, the second is the prediction of infinite propagation speed for heatwaves due to the thermal signals in the equation of heat in a parabolic type. Biot [1] is the prior who presented the coupled thermoelasticity theory between the motion equation and equation of heat to overcome and release the confliction of the first shortcoming. However, the equation of heat for the theory of coupled thermoelasticity in a parabolic form. Thus, both theories have shared and addressed the second shortcoming.

During the last five decades, more different theories developed the thermoelasticity theory to the generalized theory of thermoelasticity. Lord et al. [2] who is the first introduced the theory by the heat conduction equation based on the modified law of Fourier's considering one relaxation time that transforms the equation of heat conduction in a parabolic form to hyperbolic form the equation of heat conduction. Green et al. (GL) [3] who modified the equation of energy and the relation of Duhamel-Neumann, taking into account two thermal relaxation times. The third was by Green et al. (GN) [4] who introduced a generalized theory of thermoelasticity. They included the 'displacement gradient-thermal' within the constitutive variables independence but without dissipation accommodate of the thermal energy.

The material characteristics at high temperatures, such as Poisson's ratio, the elasticity modulus, the coefficient of thermal expansion, and the thermal conductivity are not constants any more [5]. Recently and because of scientific and technological development, the need to understand the actual actions of the material features has become actual [6]. Budaev et al. [7] discussed the dependence temperature of shear elasticity for some liquids which indicated that the increase of temperature depends on the decrease of shear modulus and is explainable by increasing the fluctuation free volume. Rishin et al. [8] employed the method of dynamic resonance to define the dependence temperature of the elasticity modulus of some materials that have plasma-sprayed. The increasing results of test temperature in a monotonic decrease in elasticity modulus. Honig et al. [9] discussed a method for the numerical inversion of the Laplace transform.

The equations of fractional derivatives and fractional differential were applied to obtain solutions to some problems in viscoelasticity, fluid mechanics, physics, engineering, biology, signal processing, mechanical engineering, systems identification, control theory, electrical engineering, finance, and fractional dynamics [10]. It describes anomalous diffusion (subdiffusion, superdiffusion, diffusion of non-Gaussian) that does not conform to the classical law of Fick in [11,12]. Numerically, the experiments show that in various one-dimensional systems that have total momentum conservation, the thermal conduction does not form in the Fourier law. Moreover, the thermal conductivity matches the size of the system in [13].

Podlubny [14] introduced an important overview of different fractional calculus applications in science, engineering, and technology. Ross [15] and Miller et al. [16] introduced a historical brief of the progress of fractional order calculus. Youssef et al. [17] discussed and made a new model for generalized thermoelasticity theory in fractional order. Sherief et al. [18] presented a fractional calculus-based thermoelasticity theory. Povstenko [19] studied axisymmetric stresses from pulses instantaneous and diffusion sources in an infinite space in a diffusion of time-fractional equation in a two-dimensional. Povstenko [20] applied Caputo time-fractional derivative technique on the thermal stresses theory based on the equation of thermal conduction to explore thermal stresses in a circular cylindrical hole an infinite body. Allam et al. [21] examined the interactions between the electromagneto-thermoelastic in a body with a spherical cavity an infinite perfectly conducting considering the GN model. The elasticity modulus is considered a linear function of temperature. Abouelregal [22] investigated the generalized thermo-piezoelectric semi-infinite under the assumption of the fractional-order with temperature-dependent and ramp-type heating. Structural continuous dependence in micropolar porous bodies is discussed in [23]. Study of heat and mass transfer in the Eyring–Powell model of peristaltically fluid propagating through a rectangular compliant channel has been discussed by Riaz et al. [24]. Some new related works have been discussed in [25–28].

In this work, we studied the thermoelasticity theory of a thin slim strip under the variable thermal conductivity in the fractional-order form is solved. Thermal stress theory considering the equation of heat conduction based on the time-fractional derivative of Caputo of order α is applied to obtain a solution. We assumed that the strip surface is to be free from traction and impacted by a thermal shock. The transform of Laplace (LT) and numerical inversion techniques of Laplace were considered for solving the governing basic equations. The inverse of the LT was applied in a numerical manner considering the Fourier expansion technique. The numerical results for the physical variables were calculated numerically and displayed via graphs. The parameter of fractional order effect and variation of thermal conductivity on the displacement, stress, and temperature were investigated and compared with the results of previous studies. The results indicated the strong effect of the external parameters, especially the time-fractional derivative parameter on a thermoelastic thin slim strip phenomenon. The obtained results are deduced to special case if thermal conductivity and thermal shock neglect.

2 The Governing Equations

Considering an isotropic homogeneous thermoelastic thin slim strip, the generalized thermoelastic governing differential equations in the fractional-order form [29] consist of:

(i) The motion equations, if the body forces were neglected

$$\rho \ddot{u}_i = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj} - \gamma T_i \tag{1}$$

(ii) The constitutive (stress-strain) equations

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma T \delta_{ij} \tag{2}$$

where λ and μ are the Lamé parameters, σ_{ij} is the tensor of stress, $\gamma = \alpha_t (3\lambda + 2\mu)$, α_t is the thermal expansion coefficient, and ρ is the medium density, e_{ij} are the elements of the strain tensor, T_0 is the temperature reference, T is the temperature, and u_i are the elements of the displacement vector.

(iii) Assuming series of Taylor of time-fractional order α is the new fractional form in the present paper [21] presented the time-nonlocal dependence between the the heat flux vector and temperature gradient discussed concerning fractional integrals and derivatives as follows:

$$q_i + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} q_i = -K T_{,i} \tag{3}$$

where K is the heat conductivity, q_i is the vector of thermal flux, $\frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional derivative of Caputo, and $\alpha (0 < \alpha \leq 1)$ is the fractional-order parameter.

(iv) The equation of heat conduction as time-fractional form takes the form [29]

$$\left(K T_{,i} \right)_{,i} = \left(\delta + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \left(\rho C_E \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial e_{kk}}{\partial t} \right) \tag{4}$$

where C_E is the unit mass specific heat.

Eq. (4) is the fractional derivatives of generalized energy equation considering the relaxation time τ_0. Some of the theories of the law of thermal conduction follow different values of α and τ_0 as limit cases. The theories of coupled or generalized thermoelasticity with one relaxation time and the generalized theory of thermoelasticity without energy dissipation (TWOED) adopt restricted cases according to the value of δ, τ_0 and α.

The temperature Eq. (4), $\alpha \to 0$ and $\delta = 1$ tends to:

$$\left(K T_{,i} \right)_{,i} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho C_E T + \gamma T_0 e) \tag{5}$$

this is the generalized theory that has a thermal relaxation time.

In the restricted case, when $\alpha \rightarrow 0, \ \ \tau_0 = 1$ and $\delta = 0$, the heat conduction Eq. (4), tends to

$$\left(KT_{,i}\right)_{,i} = \rho C_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 e}{\partial t^2} \tag{6}$$

This is the GN generalized theory without energy dissipation.

Eq. (4) in the thermoelasticity coupled theory in the limiting case $\alpha \rightarrow 0, \ \ \delta = 1$ and $\tau_0 \rightarrow 0$ as

$$\left(KT_{,i}\right)_{,i} = \rho c_E \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial e}{\partial t}. \tag{7}$$

Variations in mechanical properties due to an imposed temperature field are not the only ones that accompany heating. Similar variations are observed in the thermal properties characterized by such coefficients as the thermal linear expansion coefficients of α_t, conductivity of thermal K and others. An acceptable approximation in limited temperature interval is obtained by considering the thermal conductivity to depend linearly on the change of temperature.

The function of thermal conductivity formed as a linear function of temperature is given as [30]

$$K = K(T) = K_0(1 + K_1 T), \qquad \frac{K}{k} = \rho C_E \tag{8}$$

where $K(T)$ is thermal conductivity as temperature-dependent, K_1 is the thermal conductivity variation (usually negative experimental coefficient), K_0 denotes the thermal conductivity at $T = T_0$, k is the diffusivity, and t is the time.

Using Eq. (4) with Eq. (8), we get

$$\left(KT_{,i}\right)_{,i} = \left(\delta + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{K}{k}\frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial^2 e}{\partial t}\right) \tag{9}$$

Using the mapping

$$\Theta = \frac{1}{K_0}\int_0^T K(\tau)d\tau \tag{10}$$

where Θ is the mapping function.

Differentiating with respect to the coordinates, Eq. (10) tends to

$$\Theta_{,i} = (1 + K_1 T)T_{,i} \tag{11}$$

Redifferentiating concerning the coordinates axis, we obtain

$$\Theta_{,ii} = \left[(1 + K_1 T)T_{,i}\right]_{,i} \tag{12}$$

Similarly, by differentiating with respect to time, the mapping is

$$\dot{\Theta} = (1 + K_1 T)\dot{T} \tag{13}$$

From Eqs. (12) and (13), Eq. (9) is given as

$$\Theta_{,ii} = \frac{\partial}{\partial t}\left(\delta + \frac{\tau_0^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\frac{\Theta}{k} + \frac{\gamma T_0}{K_0}e\right) \tag{14}$$

From Eqs. (10) and (11), we have

$$\Theta = T + \frac{K_1}{2} T^2 \tag{15}$$

Substituting from Eq. (11) into Eq. (1), we obtain

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \frac{\gamma}{(1 + K_1 T)} \Theta_{,i} \tag{16}$$

For the linearity governing partial differential equations, considering the condition $\frac{|T - T_0|}{T_0} << 1$, give the approximating function of the thermal conductivity $K(T)$.

Then, Eq. (16) takes the form

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \gamma \Theta_{,i} \tag{17}$$

Using Eq. (15) and neglecting the small values of temperature, the constitutive relation reduces to

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma \Theta \delta_{ij} \tag{18}$$

3 Formulation of the Problem

Taking into account a thin rod semi-infinite with the half-space region $x \geq 0$, the $x-$axis perpendicular to the layer, parallel to oyz plane, the one-dimension displacement vector is given as

$$u_x = u(x, t), \qquad u_y = u_z = 0$$

The strain components are

$$e = e_{xx} = \frac{\partial u}{\partial x}$$

The heat equation is

$$\frac{\partial^2 \Theta}{\partial x^2} = \frac{\partial}{\partial t} \left(\delta + \frac{\tau_0^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \left(\frac{1}{k} \Theta + \frac{\gamma T_0}{K_0} e \right) \tag{19}$$

The equation of motion is

$$\frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial \Theta}{\partial x} \tag{20}$$

The constitutive relation takes the form

$$\sigma_{xx} = \sigma = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \Theta \tag{21}$$

4 Boundary Conditions

Considering a half-space $x \geq 0$ that primarily rests and has an initial temperature T_0 with zero velocity of temperature, the original conditions are

$$u(x,0)=\frac{\partial u(x,0)}{\partial t}=0$$

$$T(x,0)=\frac{\partial T(x,0)}{\partial t}=0 \tag{22}$$

We assume that when $x=0$, the surface becomes free from stresses and is put in sudden heating. The boundary conditions take the form

- The thermal boundary conditions:

$$T=T_0H(t),\quad \text{for}\quad x=0 \tag{23}$$

$$\Theta=\upsilon_0H(t),\quad \text{for}\quad x=0 \tag{24}$$

where $H(t)$ is function of Heaviside unit step

$$\upsilon_0=T_0\left(1+\frac{K_1}{2}T_0\right)$$

- The boundary conditions concerning mechanical stress, displacement, and temperature are

$$\sigma_{xx}=0,\qquad \text{for } x=0 \tag{25}$$

- and

$$\{u(x,t),\quad T(x,t),\ \Theta(x,t)\}\rightarrow 0,\qquad \text{as}\qquad x\rightarrow\infty,\qquad t>0.$$

5 The Solution of the Problem

To simplify the physical quantities, we put them in the following non-dimensional forms

$$x'=\frac{c_1}{k}x,\qquad u'=\frac{c_1}{k}u,\qquad t'=\frac{c_1^2}{k}t$$

$$\sigma'_{xx}=\frac{\sigma_{xx}}{\rho c_1^2},\qquad \tau'_0=\frac{c_1^2}{k}\tau_0, \tag{26}$$

$$\Theta'=\frac{\gamma}{\rho c_1^2}\Theta,\qquad c_1^2=\frac{(\lambda+2\mu)}{\rho}$$

From Eq. (26), the governing equations (eliminating the primes for convenience) take the forms

$$\sigma=\frac{\partial u}{\partial x}-\Theta=e-\Theta \tag{27}$$

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}-\frac{\partial\Theta}{\partial x}=De-D\Theta=D\sigma \tag{28}$$

$$\frac{\partial^2 e}{\partial t^2}=D^2e-D^2\Theta=D^2\sigma \tag{29}$$

$$D^2\Theta=\frac{\partial}{\partial t}\left(\delta+\frac{\tau_0^\alpha}{\alpha!}\frac{\partial}{\partial t}\right)(\Theta+\varepsilon e) \tag{30}$$

where $D=\partial/\partial x$ and $\varepsilon=\gamma^2T_0k/\left(\rho c_1^2K_0\right)$

6 The Solution in the Laplace Transform Domain

If we apply the following LT

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt$$

Applying it in Eqs. (27)–(30), and using the initial conditions in Eq. (22), we get

$$\bar{\sigma} = \bar{e} - \bar{\Theta} \tag{31}$$

$$D^2\bar{\sigma} = s^2\bar{e} \tag{32}$$

$$\left(D^2 - g\right)\bar{\Theta} = g\varepsilon\bar{e}, \qquad g = s\left(\delta + \frac{\tau_0^\alpha}{\alpha!}s^\alpha\right) \tag{33}$$

The boundary conditions in Eqs. (24) and (25) and the regularity condition in the LT domain take the forms

$$\bar{\Theta} = \frac{\upsilon_0}{s} \quad \text{for} \quad x = 0 \tag{34}$$

$$\overline{\sigma} = 0 \quad \text{for} \quad x = 0 \tag{35}$$

$$\{u(x,s), \quad T(x,s)\} \to 0, \quad \text{as} \quad x \to \infty$$

By eliminating $\bar{e}$, we get

$$\left(D^2 - s^2\right)\bar{\sigma} = s^2\bar{\Theta} \tag{36}$$

$$\left(D^2 - g(1+\varepsilon)\right)\bar{\Theta} = g\varepsilon\bar{\sigma} \tag{37}$$

By eliminating $\bar{\sigma}$ between Eqs. (36) and (37), we get

$$D^4 - \left(s^2 + g(1+\varepsilon)\right)D^2 + g\,\varepsilon\,s^2]\bar{\Theta} = 0 \tag{38}$$

Also, we can show that $\bar{\sigma}$ satisfies

$$[D^4 - LD^2 + M]\bar{\sigma} = 0 \tag{39}$$

where

$$L = \left(s^2 + g(1+\varepsilon)\right), \qquad M = \varepsilon s^2 g$$

The solution of Eqs. (38) and (39) takes the form

$$\bar{\sigma} = A_1 s^2 \exp(-m_1 x) + A_2 s^2 \exp(-m_2 x) \tag{40}$$

$$\bar{\Theta} = A_1\left(m_1^2 - s^2\right)\exp(m_1 x) + A_2\left(m_2^2 - s^2\right)\exp(m_2 x) \tag{41}$$

where the parameters m_1 and m_2 satisfy the equation

$$m^4 - Lm^2 + M = 0$$

We can get the displacement using Eq. (28), such that

$$\bar{u} = \frac{1}{s^2} D\bar{\sigma}$$

Thus, we obtain

$$\bar{u} = A_1 m_1 \exp(m_1 x) + A_2 m_2 \exp(m_2 x) \tag{42}$$

The temperature increment $\bar{T}$ is given by providing a solution to (15) to give

$$\bar{T} = \frac{-1 + \sqrt{1 + 2K_1\bar{\Theta}}}{K_1}. \tag{43}$$

We utilize the problem's boundary conditions to evaluate the A_1 and A_2 parameters. Eqs. (34) and (35) with Eqs. (40) and (41), they immediately give

$$A_1 + A_2 = 0 \tag{44}$$

$$A_1\left(m_1^2 - s^2\right) + A_2\left(m_2^2 - s^2\right)\exp(m_2 x) = \frac{\bar{\upsilon}_0}{s} \tag{45}$$

The solution of the former system of the linear equations provides the parameters A_1 and A_2 in the form

$$A_1 = -\frac{\bar{\upsilon}_0}{s\left(m_2^2 - m_1^2\right)}, \qquad A_1 = \frac{\bar{\upsilon}_0}{s\left(m_2^2 - m_1^2\right)} \tag{46}$$

Hence,

$$\bar{\sigma} = -\frac{\bar{\upsilon}_0 s}{\left(m_2^2 - m_1^2\right)}\exp(-m_1 x) + \frac{\bar{\upsilon}_0 s}{\left(m_2^2 - m_1^2\right)}\exp(-m_2 x) \tag{47}$$

$$\bar{\Theta} = -\frac{\bar{\upsilon}_0\left(m_1^2 - s^2\right)}{s\left(m_2^2 - m_1^2\right)}\exp(m_1 x) + \frac{\bar{\upsilon}_0\left(m_2^2 - s^2\right)}{s\left(m_2^2 - m_1^2\right)}\exp(m_2 x) \tag{48}$$

$$\bar{u} = -\frac{\bar{\upsilon}_0 m_1}{s\left(m_2^2 - m_1^2\right)}\exp(m_1 x) + \frac{\bar{\upsilon}_0 m_2}{s\left(m_2^2 - m_1^2\right)}\exp(m_2 x) \tag{49}$$

Accordingly, the problem is solved in the transformed domain completely.

7 Inversion of the Laplace Transform

It is too difficult to obtain the analytical inverse the LT of the intricate solutions to the temperature, displacement, stress, and strain in the LT domain. The method of numerical inversion is outlined to solve the problem in the physical domain. Durbin [31] obtained the approximation formula

$$f(t) = \frac{2e^{st}}{t_1}\left(-\frac{1}{2}Re[F(s)] + Re\sum_{n=0}^{N}\left[\begin{array}{l} Re\left(F\left(s + \frac{2in\pi}{t_1}\right)\right)\cos\left(\frac{2n\pi}{t_1}\right) \\ -Im\left(F\left(s + \frac{2in\pi}{t_1}\right)\right)\sin\left(\frac{2n\pi}{t_1}\right)\end{array}\right]\right) \tag{50}$$

It is worth noting that choosing the free parameters N and st_1 is significant for the accurate results and applying the method's Korrecktur and the methods of convergence acceleration. The values of the parameters in Eq. (50) are defined as $t_1 = 20$, $s = 0.25$, and $N = 1000$.

8 Numerical Results

To calculate the analytical procedure, we take into account a numerical physical example. The findings depict the variations of the non-dimensional values of temperature, displacement, and thermal stresses. Thus, we consider the following values material constants (Copper material and the type 316) as shown in Tab. 1.

Table 1: The constants of the material [32]

Parameter	Value	Parameter	Value
C_E	$383.1\ \mathrm{J\,Kg^{-1}\,K^{-1}}$	μ	$0.497425\ \lambda$
ρ	$8954\ \mathrm{Kg\,m^{-3}}$	β	-2654.53
α_t	$1.78\ \times 10^{-4}\ \mathrm{K^{-1}}$	K_0	$386\ \mathrm{W\,m^{-1}\,K^{-1}\,s^{-1}}$
T_0	$293\ \mathrm{K}$	ϵ	0.0150
K_1	-0.1		

The computations for the results obtained are carried out for the time $t = 0.15$ to obtain the displacement u, temperature T, and stress σ_{xx}. They were conducted for several $x(0 \le x \le 5)$, for different values of the parameter of fractional-order α with a wide range of $(0 < \alpha \le 1)$ that contains both cases of conductivity; $(0 < \alpha < 1)$ for low conductivity and $\alpha = 1$ for normal conductivity. Here, the numerical results are displayed graphically in Figs. 1–3.

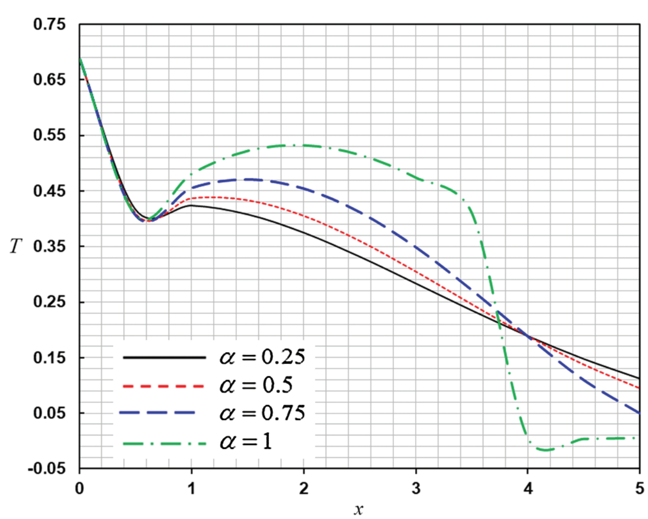

Figure 1: The effect of fractional order parameter on temperature distribution

It should be pointed out that, the increasing value of α decreases the speed of wave propagation of the stress and the temperature, whereas the displacement increases. We have noticed that the α value has a strong effect on all distributions. From these figures, the surface stress equals zero and matches the prescribed boundary condition.

In Figs. 4–6, we presented the stress, temperature, and displacement, respectively with different values of K_1. The thermal conductivity parameter has a significant impact on all fields. Physically, for the variable K, the temperature is a linear function with negative values of K_1, the heat conductivity decreases with the arising of the temperature, and the distance between the particles increases. The wave speed progress of all fields is slower. Thus, the values of all fields of quantities decrease.

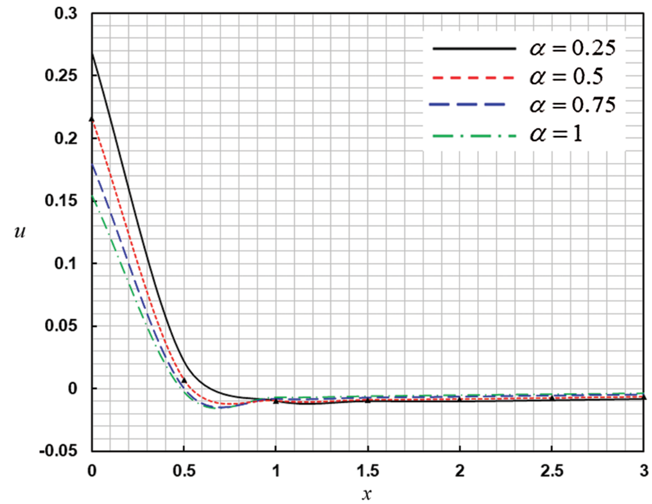

Figure 2: The effect of fractional order parameter on displacement distribution

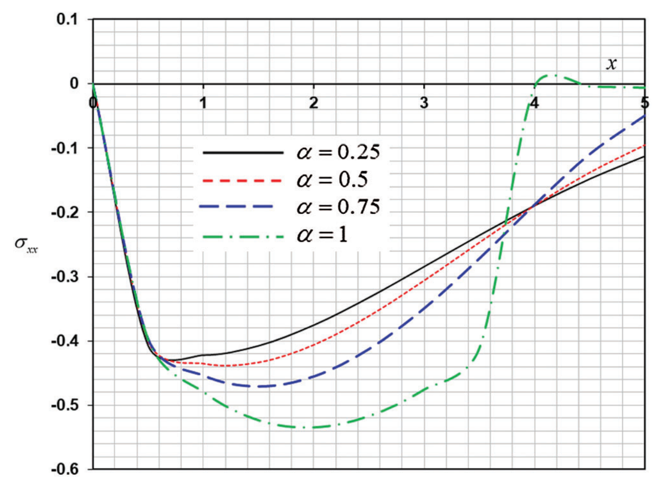

Figure 3: The effect of fractional order parameter on stress distribution

The physical field variables numerical values were calculated and presented by graphs in Figs. 7–9 respect to the axis x to clear the variations nature of the field in the context of different thermoelastic models. The figures illustrate that all variables nearly have the same nature for the (LS), (GN), and (CD) models. Their behaviors are significantly different. The wave propagation with finite speeds is manifested in all figures for theories of (LS) and (GN). It differs for the case for considering the coupled equation of heat conduction (CD) model, in which the mechanical and thermal effects fill the entire space.

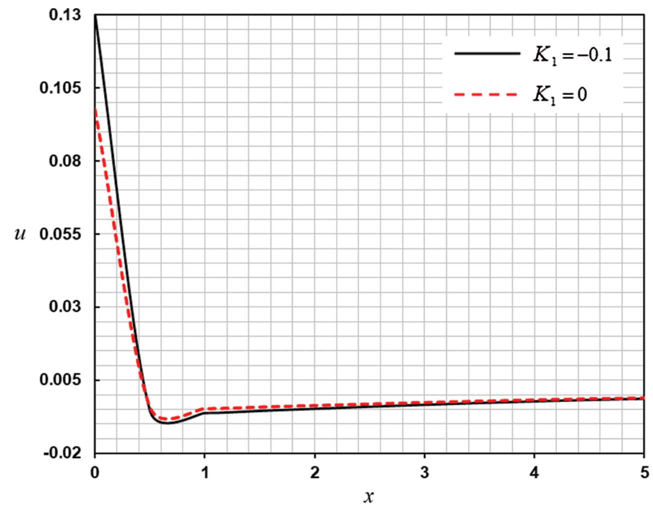

Figure 4: The effect of variation of thermal conductivity on displacement distribution

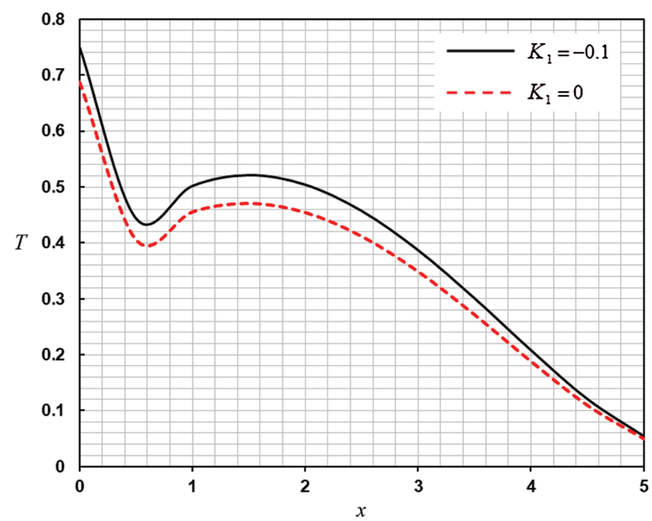

Figure 5: The effect of variation of thermal conductivity on temperature distribution

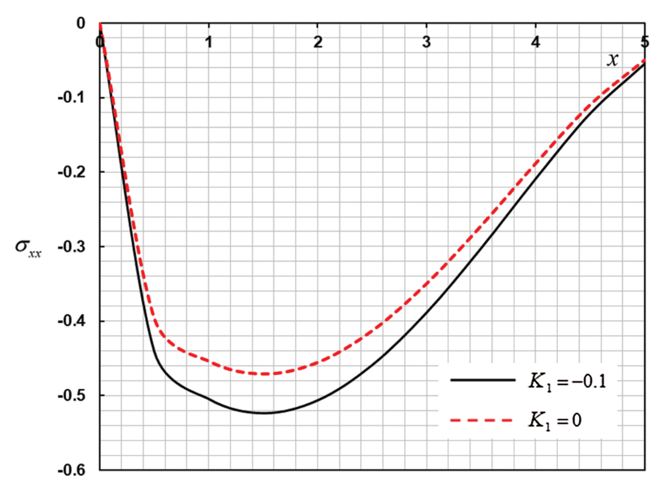

Figure 6: The effect of variation of thermal conductivity on the stress distribution

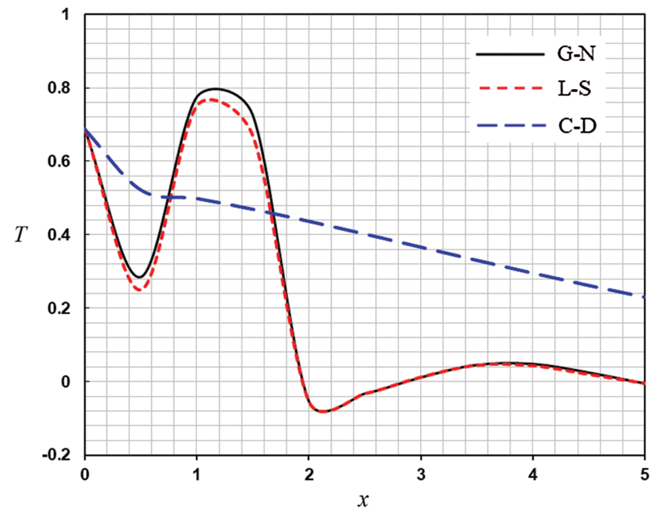

Figure 7: The temperature distribution in different theories of thermoelasticity

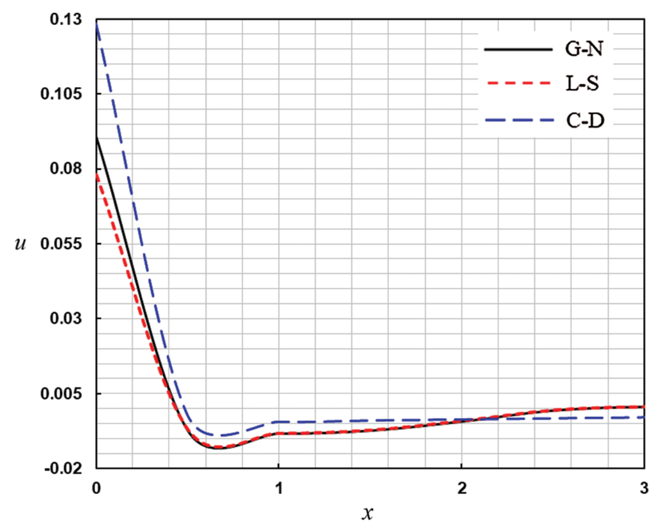

Figure 8: The displacement distribution in different theories of thermoelasticity

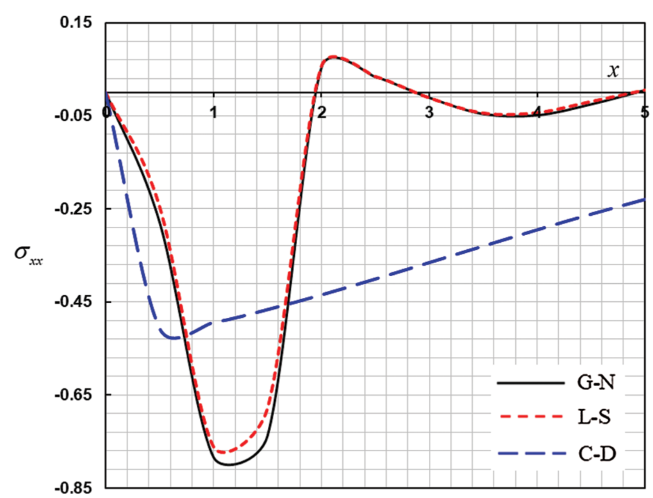

Figure 9: The stress distribution in different theories of thermoelasticity

9 Conclusion

The main observations from these figures are organized as:

- The LT technique is applied to derive the temperature, displacement, and stress due to the mechanical and thermal shock temperatures.
- The parameter α has a strong impact on all the physical quantities.
- Considering the new models applied, we introduced a novel classification for the materials based on their fractional order parameter α- a novel sign of ability.
- The results motivate the investigation of the conducting thermoelastic.
- The graphs illustrate the significant effect of the thermal conductivity on all the quantities fields and in different materials that we take into account in any analysis of heat conduction.
- The field quantities, displacement, temperature, stress, and do not depend only on the state and the space variables t and x but rely on the value of K_1, as well which has a considerable role in developing all quantities.
- The different thermoelasticity theories, i.e., Lord and Shulman, GN, and classical dynamical coupled theories were compared.
- In the generalized thermoelasticity, the wave propagation with a finite speed is evident in all these figures. This is not the case of the theory of coupled thermoelasticity, where the considered function has non-vanishing values for all of x values due to the propagation with an infinite speed of the signal thermal waves.

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