

Optimal Solution of Fuzzy Transportation Problem Using Octagonal Fuzzy Numbers

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Abstract: In this paper a fuzzy transportation problem under a fuzzy environment is solved using octagonal fuzzy numbers. The transportation problem is significant and has been widely studied in the field of applied mathematics to solve a system of linear equations in many applications in science. Systems of concurrent linear equations play a vital major role in operational research. The main perspective of this research paper is to find out the minimum amount of transportation cost of some supplies through a capacitated network formerly the availability and the demand notes are octagonal fuzzy numbers. Octagonal fuzzy numbers are used and showed a membership function. To illustrate this method, a fuzzy transportation problem is solved by using octagonal fuzzy numbers using the ranking technique. It is shown that it is the best optimal solution and it is demonstrated with a numerical example.

Keywords: Linear equation; transportation problem; fuzzy transportation problem; ranking technique; trapezoidal fuzzy numbers;

1 Introduction

Transportation problems (TPs) encompass possibly the most pro use of interest research. Transportation models consist of full applications within the stock and-provide chain to lowering the well worth calculations which are had been produced for finding out a TP once the price coefficients and consequently the natural market quantities are known explicitly. The commonness of global irregularity and estimation is unavoidable due to sudden situations. There exists rectangular-measure cases in which the really worth coefficients just because the offer-and-request quantities of a TP ought to likewise be dubious due to some factors. To manage unfastened data in selecting alternatives [1], the idea of fluffiness changed into offered. In Ref. [2], a fuzzy set speculation changed into created. In Ref. [3], fuzzy straight fragmentary writing pc programs was brought wherein usage of fuzzy transportation troubles turned into furthermore applicable to transportation fashions and vast global problems addressed. A fuzzy TP is probably one at which the transportation expenses and deliver and-request area devices are fuzzy amounts. Amaliah and Fatichah [4] added an underlying vital possible arrangement (IBFS), that is one of the significant strides in obtaining a really perfect answer for TP. As a characteristic of this examination paintings, a proposed new technique called



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TOCM-MT turned into created to decide the IBFS of TPs. The TOCM-MT is coded using the C++ programming language and may accomplish an all out expense, which incorporates values similar or nearer to the right association. In Ref. [4], a method was developed for solving the feasibility of a new fuzzy method and finding the IBFS for a fuzzy TP. In Ref. [5], an algorithm was developed for solving the intuitionistic fuzzy TP. In Ref. [6], an innovative approach was presented to optimize TPs using generalized trapezoidal numbers in a fuzzy environment. After this pioneering work, many authors [7–10] proposed many different methods for obtaining the solution of fuzzy TPs. In Ref. [11], the idea of the optimum answer for TPs with a fuzzy constant expressed as fuzzy numbers was advanced. A solution for a fully fuzzy TP was presented in Ref. [12], while a solution for a fuzzy TP using the zero-points maximum allocation method was given in Muruganandam and Srinivasan [13]. A fuzzy linear fractional programming method was given in [14–16], in which utilization of fuzzy TPs is additionally applicable to being modeled and to solve important global TPs. A fuzzy TP may be a TP in which the transportation prices and provide-and-demand area units are fuzzy quantities. In Ref. [17], a solution for a fully fuzzy TP was presented. In Ref. [18], a solution of a fuzzy TP problem using the zero-point maximum allocation method was given. Zadeh [19] advanced the idea of a higher cognitive process in fuzzy surroundings and Zimmermann [20] developed a fuzzy set theory.

2 Preliminaries

In this section, we are using various basic definitions.

2.1 Definition – Fuzzy Set

The fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A \text{ and } \mu_{\tilde{A}}(x) \in [0, 1]\}$. Where $\mu_{\tilde{A}}(x)$ is called the membership function [16].

2.2 Definition – Octagonal Fuzzy Number

A fuzzy number $\tilde{A}$ is a octagonal fuzzy number denoted by $\tilde{A}_w$,

$\tilde{A}_w = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ are where $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ real numbers and its membership function [19] are $\mu_{\tilde{A}_w}$ given by

$$\mu_{\tilde{A}_w} = \begin{cases} 0 & x < a_1 \\ k \left(\frac{x - a_1}{a_2 - a_1} \right) & a_1 \leq x \leq a_2 \\ k & a_2 \leq x \leq a_3 \\ k + (1 - k) \left(\frac{x - a_3}{a_4 - a_3} \right) & a_3 \leq x \leq a_4 \\ 1 & a_4 \leq x \leq a_5 \\ k + (1 - k) \left(\frac{a_6 - x}{a_6 - a_5} \right) & a_5 \leq x \leq a_6 \\ k & a_6 \leq x \leq a_7 \\ k \left(\frac{a_8 - x}{a_8 - a_7} \right) & a_7 \leq x \leq a_8 \\ 0 & x \geq a_8 \end{cases}$$

where $0 < k < 1$

2.3 Definition – Ranking Technique

A positioning procedure that fulfills praise, linearity and additive homes offers consequences that depend upon human instinct. If $\tilde{a}$ is a fuzzy number [16,17], after that the place is distinct by

$$R(\tilde{a}) = \frac{1}{i} \sum_{i=1}^n a_i, \quad i = 1 \text{ to } 8.$$

3 Formulation of Fuzzy Linear Programming

The fuzzy linear programming (LP) model [18–20], can be expressed as follows:

$$\text{“Minimize } Z = \sum_{i=0}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} \leq \tilde{a}_i, \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \geq \tilde{b}_j, \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0,$$

$$i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

Here, for all “ $\tilde{a}_i$ ” and “ $\tilde{b}_j$ ” are assumed to be positive, “ $\tilde{a}_i$ ” are commonly known as resources, and “ $\tilde{b}_j$ ” are commonly known as demands. The fuzzy costs, “ $\tilde{c}_{ij}$ ”, are all non-negative. If “ $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ ”, then the given problem is balanced TP. If this above mentioned condition is not satisfied, introduced a dummy origin or destination is to balance the problem. The proposed new method does not require the TP to be balanced later; it can be applied to both balanced and unbalanced TPs. The constraint set (1) is represented as “ $\leq$ ” and the constraint set (2) as “ $\geq$ ”, instead of “ $=$ ”, for both cases in a balanced problem. This is one of the advantages of the proposed method, which is called the “stepping stone” method. The unbalanced problem is not required to be converted into a balanced problem, and no dummy origin or destination is introduced, so time and space are saved.

Let the TP consist of m origins and n destinations, where

$\tilde{x}_{ij}$ = is the number of goods transported from the i th origin to the j th destination

$\tilde{c}_{ij}$ is cost involved in transporting per unit product from the i th origin to the j th destination,

$\tilde{a}_i$ is the number of units available at the i th origin and

$\tilde{b}_j$ is the number of units required at the j th destination

4 Mathematical Formulation of Fuzzy TP

Mathematically, a TP can be stated as follows.

“Minimize

$$z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \tag{1}$$

subject to

$$\left. \begin{array}{l} \sum_{j=1}^n x_{ij} = a_i \quad j = 1, 2, \dots, n \\ \sum_{i=1}^m x_{ij} = b_j \quad i = 1, 2, \dots, m \\ x_{ij} \geq 0 \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \end{array} \right\} \quad (2)$$

where c_{ij} is the cost of transportation of a unit from the i th resource to the j th destination, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the i th origin to the j th destination. A visible necessary and sufficient condition that the linear programming problem given in (1) has a solution

$$\sum_{i=1}^n a_i = \sum_{j=1}^m b_j \quad (3)$$

Mathematically, a fuzzy TP can be explicitly expressed as follows.

Minimize

$$z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (4)$$

subject to

$$\left. \begin{array}{l} \sum_{j=1}^n x_{ij} = \tilde{a}_i \quad j = 1, 2, \dots, n \\ \sum_{i=1}^m x_{ij} = \tilde{b}_j \quad i = 1, 2, \dots, m \\ x_{ij} \geq 0 \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \end{array} \right\} \quad (5)$$

in which the transportation costs $\tilde{c}_{ij}$, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities are fuzzy quantities. An obvious necessary and sufficient condition for the fuzzy linear programming problem given in (4) and (5) to each a solution is that

$$\sum_{i=1}^n \tilde{a}_i \simeq \sum_{j=1}^m \tilde{b}_j \quad (6)$$

This problem can also be represented as shown in Tab. 1.

Table 1: Problem Statement

	1		n	Supply
1	$\tilde{c}_{11}$		$\tilde{c}_{1n}$	$\tilde{a}_1$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
M	$\tilde{c}_{m1}$		$\tilde{c}_{mn}$	$\tilde{a}_m$
Demand	$\tilde{b}_1$		$\tilde{b}_n$	

5 Method for Solving TP

The following steps are used to solve the TP

Step – 1: From the given TP, switch the values in fuzzy to crisp using the ranking function.

Step – 2: Find the minimum cell values from each of the cell value of every row/column of the TP and place them on the right-top/right-bottom of the corresponding cost.

Step – 3: Add the right-top and right-bottom values and place the summation value in the corresponding cell value.

Step – 4: Identify the smallest element in each row and column of the transportation table and subtract from their corresponding row and column.

Step – 5: Calculate the sum the values in the rows and columns. Choose the maximum value and assign the minimum value of supply/demand in the smallest element of the rows and columns. Remove the columns or rows corresponding to where the supply or demand is satisfied.

Step – 6: Repeat steps 4 and 5 until the supply-and-demand is totally met.

Step – 7: Replace the original transportation value with the satisfied cell value.

Step – 8: Workout the minimum cost.

(i.e.), Total Cost = “ $\sum \sum C_{ij}X_{ij}$ ”

5.1 Numerical Example

Tab. 2 shows the fuzzy transportation table

Table 2: Fuzzy Transportation Table

	O ₁	O ₂	O ₃	O ₄	
S ₁	[-4,0,1,2,3,4,5,6]	[-3,1,2,3,4,5,6,7]	[5,6,8,10,11,14,16,18]	[2,4,5,6,7,9,10,13]	[1,2,3,5,7,9,10,11]
S ₂	[2,3,6,8,9,11,12,13]	[-4,-3,-2,-1,0,1,2,6]	[1,2,3,5,7,9,10,11]	[2,3,6,8,9,11,12,13]	[2,3,6,8,9,11,12,13]
S ₃	[0,1,2,4,5,8,9,11]	[2,3,6,8,9,11,12,13]	[9,10,12,14,16,18,20,21]	[4,5,6,7,8,13,14,16]	[5,7,8,9,11,12,13,15]
	[2,4,5,6,7,9,10,13]	[0,1,2,4,5,8,9,11]	[-3,1,2,3,4,5,6,7]	[-4,0,1,2,3,4,5,6]	

5.2 Solution

The octagonal fuzzy numbers are converted into crisp numbers presented in Tab. 3 with using the proposed ranking method.

Table 3: Iteration – 1

	O ₁	O ₂	O ₃	O ₄	
S ₁	2	3	11	7	6
S ₂	8	0	6	8	8
S ₃	5	8	15	9	10
	7	5	3	2	

With the help of the proposed algorithm, the optimal solution is obtained and shown in [Tab. 4](#)

Table 4: Optimal solution

	O ₁	O ₂	O ₃	O ₄
S ₁		<u>3</u>	<u>11</u>	6
		4	2	
S ₂	<u>8</u>		<u>6</u>	<u>8</u>
	7		1	2
S ₃		<u>8</u>		10
		1		
	7	5	3	2

The total cost of the IBFS is calculated as a below total cost of 120, which is a basic feasible solution. The solution was obtained using the NCM, LCM, VAM and MODI/stepping stone methods separately. Hence, the basic feasible solution obtained using the proposed method is a free solution.

6 Conclusions

In this research paper, a method for solving octagonal fuzzy numbers is projected to solve the fuzzy optimal solutions of fuzzy TPs. Since the projected algorithm is a new and easy way to switch the uncertainty in the crisp environment numerical examples are presented that show that the proposed this technique is very easy to understand and to apply to classical TPs to obtain an optimum solution. In the future, the proposed method can be applied to real-world problems in the fields that include job assignments and job scheduling.

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