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Article

Comparative Study of Valency-Based Topological Descriptor for Hexagon Star Network

Ali N. A. Koam[1], Ali Ahmad[2,*] and M. F. Nadeem[3]

[1]Department of Mathematics, College of Science, Jazan University, New Campus, Jazan, 2097, Saudi Arabia
[2]College of Computer Science & Information Technology Jazan University, Jazan, Saudi Arabia
[3]Department of Mathematics, COMSATS University Islamabad, Lahore Campus, Lahore, 54000, Pakistan
[*]Corresponding Author: Ali Ahmad. Email: ahmadsms@gmail.com
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Abstract: A class of graph invariants referred to today as topological indices are inefficient progressively acknowledged by scientific experts and others to be integral assets in the depiction of structural phenomena. The structure of an interconnection network can be represented by a graph. In the network, vertices represent the processor nodes and edges represent the links between the processor nodes. Graph invariants play a vital feature in graph theory and distinguish the structural properties of graphs and networks. A topological descriptor is a numerical total related to a structure that portray the topology of structure and is invariant under structure automorphism. There are various uses of graph theory in the field of basic science. The main notable utilization of a topological descriptor in science was by Wiener in the investigation of paraffin breaking points. In this paper we study the topological descriptor of a newly design hexagon star network. More preciously, we have computed variation of the Randic$'$ R', fourth Zagreb M_4, fifth Zagreb M_5, geometric-arithmetic GA, atom-bond connectivity ABC, harmonic H, symmetric division degree SDD, first redefined Zagreb, second redefined Zagreb, third redefined Zagreb, augmented Zagreb AZI, Albertson A, Irregularity measures, Reformulated Zagreb, and forgotten topological descriptors for hexagon star network. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structure activity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. We also gave the numerical and graphical representations comparisons of our different results.

Keywords: Topological indices; degree-based index; hexagon star network

1 Introduction

Cheminformatics is another field of modern sciences that connects chemistry, math, and other fields of science. Quantitative structure-activity relationship (QSAR) and Quantitative structure-activity relationship (QSPR) are the principle parts of cheminformatics which are useful to contemplate the physico-chemical properties of networks. A topological descriptor (TD) is a numerical total related to a structure that portray the topology of the structure and is invariant under structure automorphism. There are various uses of graph theory in the field of basic science. The main notable utilization of a TD in science was by Wiener in the investigation of paraffin breaking points [1]. From that point forward, to clarify physico-chemical properties, different TDs have been presented.

Topological descriptors (TD) are commonly partitioned into three sorts: degree, distance and spectrum based. The structures of networks can be scientifically demonstrated by a figure. The vertex represents the processor hub and an edge describes the links among processors. The topology of the figure of a network chooses the way by which any two vertices are linked by an edge. The topology of a network system can be used to obtained specific properties without a lot of stretches. The width is resolved as the most extreme separation between any two hubs in the system. The quantity of connections associated with a hub decides the level of that hub. If this number is the equivalent for all hubs in the system, the system is called regular.

TD can be effortlessly processed by utilizing the ideas of atomic topology (AT), an order dependent on the graph theory. Actually, AT has demonstrated to be a fantastic apparatus for quick and exact estimation of numerous physicochemical as well as biological properties [2,3]. So as to compute topological indices, basics of AT are utilized where chemical compound is changed over into a graph, considering the atoms and bonds are represented by vertices and edges of a graph. The basic definitions and notations are taken from the book [4]. The number of vertices adjacent to the vertex ε is the degree of ε, denoted as d_ε.

TD which are obtained through the connectivity of two-dimensional structures, deliver significant connections to various properties of these structures via QSAR/QSPR resulting from the topological networks of these structures [5]. Since last few years, numerous researchers conducted for the expansion of TD because of their significance [6–9]. For detail study of TD see [10–21].

2 Degree-Based Indices

In this section, we define some degree based topological indices $\mathbf{T}(\mathbb{H})$.

$$\mathbf{T}(\mathbb{H}) = \sum_{uv\in E(\mathbb{H})} \lambda(d_u, d_v), \tag{1}$$

- $\lambda(d_\upsilon, d_v) = (d_\upsilon d_v)^\alpha$, represents $\mathbf{T}(\mathbb{H})$ as the general, second, and second modified Randic′ indices if $\alpha \neq 0 \in \mathbb{R}$, $\alpha = 1$, and $\alpha = -1$ respectively.
- $\lambda(d_\upsilon, d_v) = (d_\upsilon + d_v)^\alpha$, represents $\mathbf{T}(\mathbb{H})$ as the general sum connectivity, sum connectivity, Zagreb and hyper Zagreb indices, if $\alpha \neq 0 \in \mathbb{R}$, $\alpha = \frac{-1}{2}$, $\alpha = 1$ and $\alpha = 2$ respectively.
- If $\lambda(d_\upsilon, d_v) = d_\upsilon^\alpha d_v^\beta + d_v^\alpha d_\upsilon^\beta$, then $\mathbf{T}(\mathbb{H})$ represents generalized Zagreb index.

Similarly, if

$\lambda(d_\upsilon, d_v) = \sum_{uv\in E(\mathcal{G})} d_\upsilon(d_\upsilon + d_v)$, $\sum_{uv\in E(\mathcal{G})} d_v(d_\upsilon + d_v)$, $\frac{2\sqrt{d_\upsilon d_v}}{d_\upsilon + d_v}$, $\sqrt{\frac{d_\upsilon + d_v - 2}{d_\upsilon d_v}}$, $\frac{2}{d_\upsilon + d_v}$, $\frac{d_\upsilon^2 + d_v^2}{d_\upsilon d_v}$, $\frac{d_\upsilon + d_v}{d_\upsilon d_v}$, $\frac{d_\upsilon d_v}{d_\upsilon + d_v}$, $d_\upsilon d_v(d_\upsilon + d_v)$, $\left(\frac{d_\upsilon \ d_v}{d_\upsilon + d_v - 2}\right)^3$, $\frac{1}{\max\{d_\upsilon, d_v\}}$, $|d_\upsilon - d_v|$, $(d_\upsilon - d_v)^2$, $(d_\upsilon + d_v - 2)^2$, $\left(d_\upsilon^2 + d_v^2\right)$, we obtained fourth Zagreb $M_4(\mathbb{H})$, fifth Zagreb $M_5(\mathbb{H})$, geometric-arithmetic $GA(\mathbb{H})$, atom-bond connectivity $ABC(\mathbb{H})$, harmonic $H(\mathbb{H})$, symmetric division degree $SDD(\mathbb{H})$, first redefined Zagreb, second redefined Zagreb, third redefined Zagreb, augmented Zagreb $AZI(\mathbb{H})$, variation of the Randic′ $R'(\mathbb{H})$, Albertson $A(\mathbb{H})$, IRM$(\mathbb{H})$ irregularity measures, Reformulated Zagreb, and forgotten topological indices respectively.

3 Hexagon Star Network Sheet

Interconnection systems are significant in PC systems administration and used to change information between the PC and processer. In the most recent couple of years, numerous specialists structured the new interconnection systems. In an equal PC framework, interconnection organize is accustomed to expanding the exhibition. In diagram hypothesis, organize is spoken to as a chart. In this articulation, the processer spoke to by vertex and association between the units spoke to by edges. From the topology of a system, we can decide certain properties. The level of a hub is characterized as the all outnumber of connections associated with that hub. The system is supposed to be regular if each hub in the system has the same degree. In this paper, we define a new interconnection network hexagon star network. This network is a composition of triangles around a hexagon, as shown in Fig. 1.

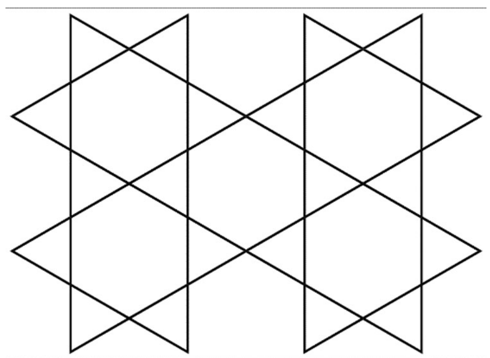

Figure 1: The hexagon star network sheet for $p = 2$, $q = 2$

4 Main Results

In this section, we give results, which are used to obtained any degree-based topological descriptors. We obtained exact results of degree-based TD for hexagon star network sheet $\mathbb{H}$. Vetrík [22] introduced a new method to calculate the topological indices and also in [23], we follow the same technique in this paper. Now, we presents a formula, which can be used to obtain any degree based TD.

Lemma 4.1 *Let* $\mathbb{H}$ *be a hexagon star network. Then* $T(\mathbb{H}) = 12pq\lambda(4,4) + 2p(4\lambda(2,4) - \lambda(4,4)) + 4q(\lambda(2,4) - \lambda(4,4))$.

Proof. The graph $\mathbb{H}$ contains $6pq + 5p + q$ vertices and $12pq + 6p$ edges. Each vertex of $\mathbb{H}$ has degree 2 or 4, vertices of $\mathbb{H}$ can be partitioned according to their degrees. Let

$$V_i = \{\upsilon \in V(\mathbb{H}) : d_\upsilon = i\}.$$

This means that the set V_i contains the vertices of degree i. The set of vertices with respect to their degrees are as follows:

$$V_2 = \{\upsilon \in V(\mathbb{H}) : d_\upsilon = 2\}$$

$$V_4 = \{\upsilon \in V(\mathbb{H}) : d_\upsilon = 4\}$$

Since, $|V_2| = 4p + 2q$ and $|V_4| = 6pq + p - q$. We partite the edges of $\mathbb{H}$ into sets based on degrees of its end vertices. Let

$$\Xi_{2,4} = \{\upsilon v \in E(\mathbb{H}) : d_\upsilon = 2, d_v = 4\}$$

$$\Xi_{4,4} = \{\upsilon v \in E(\mathbb{H}) : d_\upsilon = 4, d_v = 4\}.$$

Note that $E(\mathbb{H}) = \Xi_{2,4} \cup \Xi_{4,4}$. The number of edges incident to one vertex of degree 2 and other vertex of degree 4 is $8p + 4q$, so $|\Xi_{2,4}| = 8p + 4q$. Now, the remaining number of edges are those edges which are incident to two vertices of degree 4, i.e., $|\Xi_{4,4}| = |E(\mathbb{H})| - |\Xi_{2,4}| = 12pq - 2p - 4q$.

Hence,

$$\mathbf{I}(\mathbb{H}) = \sum_{\upsilon v \in E(\mathbb{H})} \lambda(d_\upsilon, d_v) = \sum_{\upsilon v \in \Xi_{2,4}} \lambda(2,4) + \sum_{\upsilon v \in \Xi_{4,4}} \lambda(4,4) = (8p + 4q)\lambda(2,4) + (12pq - 2p - 4q)\lambda(2,4).$$

After simplification, we get

$$\mathbf{I}(\mathbb{H}) = 12pq\lambda(4,4) + 2p(4\lambda(2,4) - \lambda(4,4)) + 4q(\lambda(2,4) - \lambda(4,4)).$$

Now we obtained the well-known degree based TD of hexagon star network in the following theorem.

Theorem 4.2 *For the hexagon star network* $\mathbb{H}$, *we have*

the general Randic′ index of $\mathbb{H}$ is,

$$R_\alpha(\mathbb{H}) = 12pq(16)^\alpha + 2p(4(8)^\alpha - (16)^\alpha) + 4q((8)^\alpha - (16)^\alpha),$$

the Randic′ index of $\mathbb{H}$ is

$$R_{\frac{-1}{2}}(\mathbb{H}) = 3pq + \frac{p}{2}\left(4\sqrt{2} - 1\right) + q\left(\sqrt{2} - 1\right),$$

the second Zagreb index of $\mathbb{H}$ is

$$R_1(\mathbb{H}) = 192pq + 32p - 32q$$

the second Zagreb index of $\mathbb{H}$ is

$$R_{-1}(\mathbb{H}) = \frac{3}{4}pq + \frac{7}{8}p + \frac{1}{4}q.$$

Proof. For $R_\alpha(\mathbb{H})$ which is the general Randic′ indices of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = (d_\upsilon d_v)^\alpha$, therefore $\lambda(2,4) = (8)^\alpha$ and $\lambda(4,4) = (16)^\alpha$. Thus by Lemma 4.1,

$$R_\alpha(\mathbb{H}) = 12pq(16)^\alpha + 2p(4(8)^\alpha - (16)^\alpha) + 4q((8)^\alpha - (16)^\alpha).$$

For $\alpha = \frac{-1}{2}$, the Randic′ index

$$R_{\frac{-1}{2}}(\mathbb{H}) = 12pq(16)^{\frac{-1}{2}} + 2p(4(8)^{\frac{-1}{2}} - (16)^{\frac{-1}{2}}) + 4q((8)^{\frac{-1}{2}} - (16)^{\frac{-1}{2}}).$$

After simplification, we get

$$R_{\frac{-1}{2}}(\mathbb{H}) = 3pq + \frac{p}{2}\left(4\sqrt{2} - 1\right) + q\left(\sqrt{2} - 1\right).$$

For $\alpha = 1$, the second Zagreb index is

$$R_1(\mathbb{H}) = 12pq(16) + 2p(4(8) - 16) + 4q(8 - 16) = 192pq + 32p - 32q.$$

For $\alpha = -1$, the second modified Zagreb index is

$$R_{-1}(\mathbb{H}) = 12pq\left(\frac{1}{16}\right) + 2p\left(\frac{4}{8} - \frac{1}{16}\right) + 4q\left(\frac{1}{8} - \frac{1}{16}\right) = \frac{3}{4}pq + \frac{7}{8}p + \frac{1}{4}q.$$

We gave graphical comparison of Theorem 4.2 in Fig. 2 and numerical values Tab. 1.

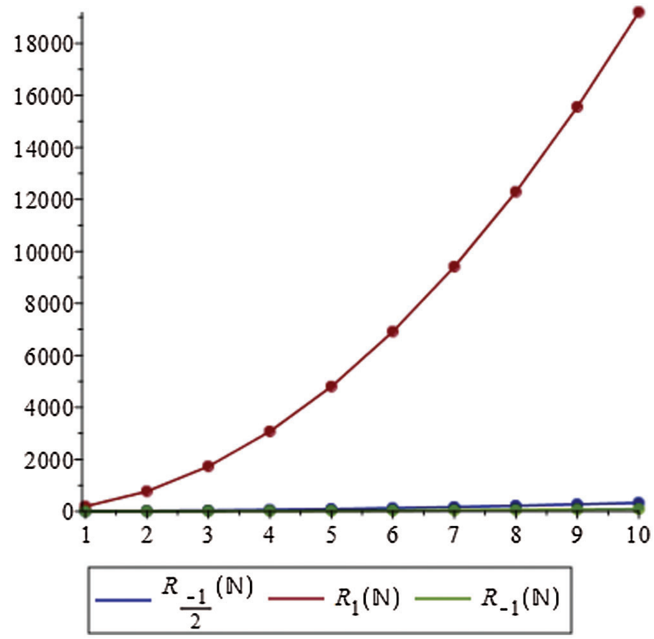

Figure 2: Graphical comparison of Theorem 4.2

Table 1: Numerical representation of Theorem 4.2

$[p, q]$	$R_{\frac{-1}{2}}$	R_1	R_{-1}
[1, 1]	5.7426	192.0	1.8750
[2, 2]	17.485	768.0	5.2500
[3, 3]	35.228	1728.0	10.125
[4, 4]	58.970	3072.0	16.500
[5, 5]	88.713	4800.0	24.375
[6, 6]	124.46	6912.0	33.750
[7, 7]	166.20	9408.0	44.625
[8, 8]	213.94	12288.0	57.0
[9, 9]	267.68	15552.0	70.875
[10, 10]	327.43	19200.0	86.250

In the next theorem, we determined general sum-connectivity index, first Zagreb index and hyper-Zagreb index of the hexagon star network $\mathbb{H}$.

Theorem 4.3 *For the hexagon star network* $\mathbb{H}$, *we have*

the general sum-connectivity index of $\mathbb{H}$ is

$$\chi_\alpha(\mathbb{H}) = 12pq(8)^\alpha + 2p(4(6)^\alpha - (8)^\alpha) + 4q((6)^\alpha - (8)^\alpha),$$

the sum-connectivity index of $\mathbb{H}$ is

$$\chi_{-\frac{1}{2}}(\mathbb{H}) = 3\sqrt{2}pq + p\left(\frac{8-\sqrt{3}}{\sqrt{6}}\right) + q\left(\frac{4-2\sqrt{3}}{\sqrt{6}}\right),$$

the first Zagreb index of $\mathbb{H}$ is

$$\chi_1(\mathbb{H}) = 96pq + 32p - 8q,$$

the hyper-Zagreb index of $\mathbb{H}$ is

$$\chi_2(\mathbb{H}) = 768pq + 160p - 112q.$$

Proof. For $\chi_\alpha(\mathbb{H})$ which is the general sum-connectivity index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = (d_\upsilon + d_v)^\alpha$, therefore $\lambda(2,4) = (6)^\alpha$ and $\lambda(4,4) = (8)^\alpha$. Thus by Lemma 4.1,

$$\chi_\alpha(\mathbb{H}) = 12pq(8)^\alpha + 2p(4(6)^\alpha - (8)^\alpha) + 4q((6)^\alpha - (8)^\alpha).$$

For $\alpha = -\frac{1}{2}$, sum-connectivity index of $\mathbb{H}$

$$\chi_{-\frac{1}{2}}(\mathbb{H}) = 12pq(8)^{-\frac{1}{2}} + 2p(4(6)^{-\frac{1}{2}} - (8)^{-\frac{1}{2}}) + 4q((6)^{-\frac{1}{2}} - (8)^{-\frac{1}{2}}).$$

After simplification, we get

$$\chi_{-\frac{1}{2}}(\mathbb{H}) = 3\sqrt{2}pq + p\left(\frac{8-\sqrt{3}}{\sqrt{6}}\right) + q\left(\frac{4-2\sqrt{3}}{\sqrt{6}}\right).$$

For $\alpha = 1$, the first Zagreb index is

$$\chi_1(\mathbb{H}) = 12pq(8)^1 + 2p(4(6)^1 - (8)^1) + 4q((6)^1 - (8)^1) = 96pq + 32p - 8q.$$

For $\alpha = 2$, the hyper-Zagreb index is

$$\chi_2(\mathbb{H}) = 12pq(8)^2 + 2p(4(6)^2 - (8)^2) + 4q((6)^2 - (8)^2) = 768pq + 160p - 112q.$$

We gave graphical comparison of Theorem 4.3 in Fig. 3 and numerical values Tab. 2.

Theorem 4.4 *For the hexagon star network* $\mathbb{H}$, *we have*

the geometric-arithmetic index of $\mathbb{H}$,

$$GA(\mathbb{H}) = 12pq + 2p\left(\frac{8\sqrt{2}}{3} - 1\right) + 4q\left(\frac{2\sqrt{2}}{3} - 1\right)$$

the atom-bond connectivity index of $\mathbb{H}$,

$$ABC(\mathbb{H}) = 3\sqrt{6}pq + p\left(\frac{8\sqrt{2}-\sqrt{6}}{2}\right) + q\left(2\sqrt{2} - \sqrt{6}\right)$$

the augmented Zagreb index of $\mathbb{H}$,

$$AZI(\mathbb{H}) = \frac{2048}{9}pq + \frac{704}{27}p - \frac{1184}{27}q.$$

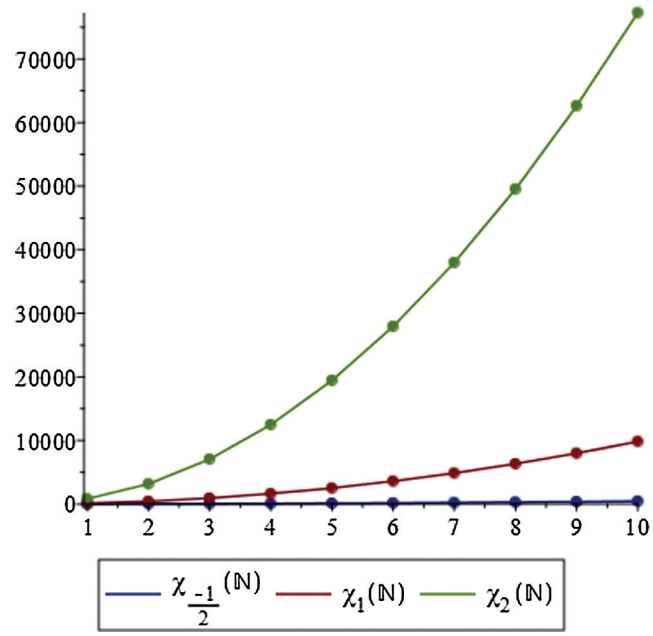

Figure 3: Graphical comparison of Theorem 4.3

Table 2: Numerical representation of Theorem 4.3

$[p, q]$	$\chi_{\frac{-1}{2}}$	χ_1	χ_2
[1, 1]	7.0202	120	816
[2, 2]	22.525	432	3168
[3, 3]	46.516	936	7056
[4, 4]	78.992	1632	12480
[5, 5]	119.94	2520	19440
[6, 6]	169.39	3600	27936
[7, 7]	227.33	4872	37968
[8, 8]	293.75	6336	49536
[9, 9]	368.65	7992	62640
[10, 10]	452.04	9840	77280

Proof. For $GA(\mathbb{H})$ which is the geometric-arithmetic index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_\nu) = \frac{2\sqrt{d_\upsilon d_\nu}}{d_\upsilon + d_\nu}$, therefore $\lambda(2,4) = \frac{2\sqrt{2}}{3}$ and $\lambda(4,4) = 1$. Thus by Lemma 4.1,

$$GA(\mathbb{H}) = 12pq + 2p\left(\frac{8\sqrt{2}}{3} - 1\right) + 4q\left(\frac{2\sqrt{2}}{3} - 1\right)$$

For $ABC(\mathbb{H})$ which is the atom-bond connectivity index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_\nu) = \sqrt{\frac{d_\upsilon + d_\nu - 2}{d_\upsilon d_\nu}}$, therefore $\lambda(2,4) = \frac{1}{\sqrt{2}}$ and $\lambda(4,4) = \frac{\sqrt{6}}{4}$. Thus by Lemma 4.1,

$$ABC(\mathbb{H}) = 12pq\left(\frac{\sqrt{6}}{4}\right) + 2p\left(4\left(\frac{1}{\sqrt{2}}\right) - \frac{\sqrt{6}}{4}\right) + 4q\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{6}}{4}\right)$$

After simplification, we get

$$ABC(\mathbb{H}) = 3\sqrt{6}pq + p\left(\frac{8\sqrt{2} - \sqrt{6}}{2}\right) + q\left(2\sqrt{2} - \sqrt{6}\right).$$

For $AZI(\mathbb{H})$ which is the augmented Zagreb index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \left(\frac{d_\upsilon\ d_v}{d_\upsilon + d_v - 2}\right)^3$, therefore $\lambda(2,4) = 8$ and $\lambda(4,4) = \frac{512}{27}$. Thus by Lemma 4.1,

$$AZI(\mathbb{H}) = 12pq\left(\frac{512}{27}\right) + 2p\left(4(8) - \frac{512}{27}\right) + 4q\left(8 - \frac{512}{27}\right) = \frac{2048}{9}pq + \frac{704}{27}p - \frac{1184}{27}q.$$

We gave graphical comparison of Theorem 4.4 in Fig. 4 and numerical values Tab. 3.

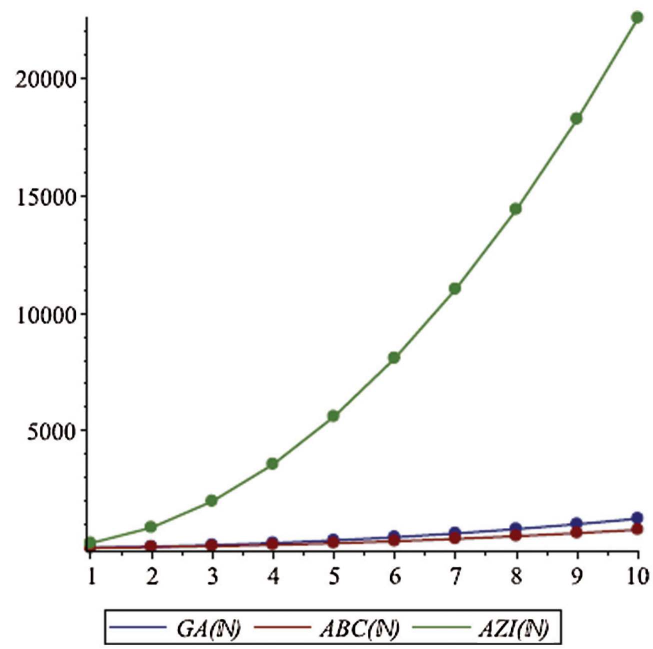

Figure 4: Graphical comparison of Theorem 4.4

Table 3: Numerical representation of Theorem 4.4.

$[p, q]$	$GA(\mathbb{H})$	$ABC(\mathbb{H})$	$AZI(\mathbb{H})$
[1, 1]	17.314	12.159	209.78
[2, 2]	58.627	39.016	874.67
[3, 3]	123.94	80.570	1994.7
[4, 4]	213.25	136.82	3569.8
[5, 5]	326.57	207.77	5600.0

Table 3 (continued).

$[p, q]$	$GA(\mathbb{H})$	$ABC(\mathbb{H})$	$AZI(\mathbb{H})$
$[6, 6]$	463.88	293.41	8085.3
$[7, 7]$	625.20	393.76	11026.0
$[8, 8]$	810.51	508.79	14421.0
$[9, 9]$	1019.8	638.53	18272.0
$[10, 10]$	1253.1	782.96	22578.0

Theorem 4.5 *For the hexagon star network* $\mathbb{H}$, *we have*

the symmetric division degree index of $\mathbb{H}$,

$SDD(\mathbb{H}) = 24pq + 16p + 2q$.

the Albertson index of $\mathbb{H}$,

$A(\mathbb{H}) = 16p + 8q$

the harmonic index of $\mathbb{H}$

$H(\mathbb{H}) = 3pq + \frac{13}{6}p + \frac{1}{3}q$.

Proof. For $SDD(\mathbb{H})$ which is the symmetric division degree index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \frac{d_\upsilon^2 + d_v^2}{d_\upsilon d_v}$, therefore $\lambda(2, 4) = \frac{5}{2}$ and $\lambda(4, 4) = 2$. Thus by Lemma 4.1, $SDD(\mathbb{H}) = 12pq(2) + 2p\left(4.\frac{5}{2} - 2\right) + 4q\left(\frac{5}{2} - 2\right) = 24pq + 16p + 2q$.

For $A(\mathbb{H})$ which is the Albertson index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = |d_\upsilon - d_v|$, therefore $\lambda(2, 4) = 2$ and $\lambda(4, 4) = 0$. Thus by Lemma 4.1,

$A(\mathbb{H}) = 16p + 8q$.

For $H(\mathbb{H})$ which is the harmonic index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \frac{2}{d_\upsilon + d_v}$, therefore $\lambda(2, 4) = \frac{1}{3}$ and $\lambda(4, 4) = \frac{1}{4}$. Thus by Lemma 4.1,

$$H(\mathbb{H}) = 12pq\left(\frac{1}{4}\right) + 2p\left(4.\frac{1}{3} - \frac{1}{4}\right) + 4q\left(\frac{1}{3} - \frac{1}{4}\right) = 3pq + \frac{13}{6}p + \frac{1}{3}q.$$

We gave graphical comparison of Theorem 4.5 in Fig. 5 and numerical values Tab. 4.

Theorem 4.6 *For the hexagon star network* $\mathbb{H}$, *we have*

the first redefined Zagreb index of $\mathbb{H}$,

$ReZG_1(\mathbb{H}) = 6pq + 5p + q$.

the second redefined Zagreb index of $\mathbb{H}$,

$ReZG_2(\mathbb{H}) = 24pq + \frac{20}{3}p - \frac{8}{3}q$

the third redefined Zagreb index of $\mathbb{H}$

$ReZG_3(\mathbb{H}) = 1536pq + 128p - 320q.$

Proof. For $ReZG_1(\mathbb{H})$ which is the first redefined Zagreb index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_\nu) = \frac{d_\upsilon + d_\nu}{d_\upsilon d_\nu}$, therefore $\lambda(2,4) = \frac{3}{4}$ and $\lambda(4,4) = \frac{1}{2}$. Thus by Lemma 4.1,

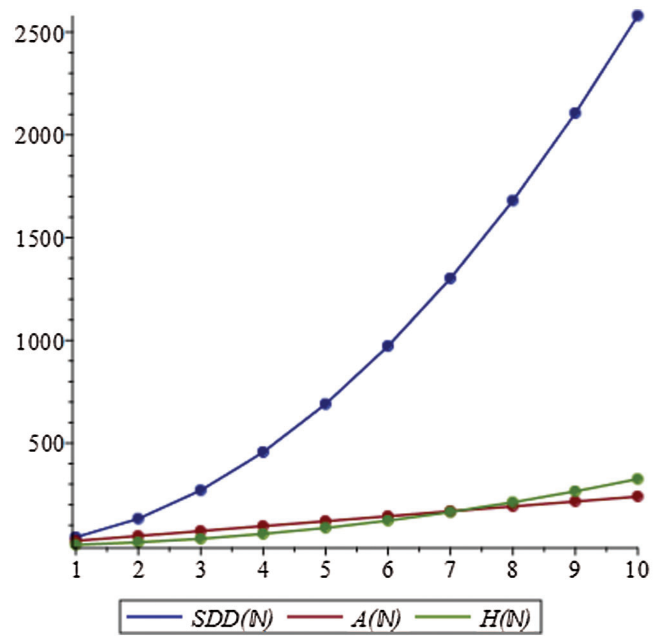

Figure 5: Graphical comparison of Theorem 4.5

Table 4: Numerical representation of Theorem 4.5

$[p,q]$	$SDD(\mathbb{H})$	$A(\mathbb{H})$	$H(\mathbb{H})$
$[1,1]$	42	24	5.5000
$[2,2]$	132	48	17.0
$[3,3]$	270	72	34.500
$[4,4]$	456	96	58.0
$[5,5]$	690	120	87.500
$[6,6]$	972	144	123.0
$[7,7]$	1302	168	164.50
$[8,8]$	1680	192	212.0
$[9,9]$	2106	216	265.50
$[10,10]$	2580	240	325

$$ReZG_1(\mathbb{H}) = 12pq\left(\frac{1}{2}\right) + 2p\left(4.\frac{3}{4} - \frac{1}{2}\right) + 4q\left(\frac{3}{4} - \frac{1}{2}\right) = 6pq + 5p + q.$$

For $ReZG_2(\mathbb{H})$ which is the second redefined Zagreb index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \frac{d_\upsilon d_v}{d_\upsilon + d_v}$, therefore $\lambda(2,4) = \frac{4}{3}$ and $\lambda(4,4) = 2$. Thus by Lemma 4.1,

$$ReZG_2(\mathbb{H}) = 12pq(2) + 2p\left(4.\frac{4}{3} - 2\right) + 4q\left(\frac{4}{3} - 2\right) = 24pq + \frac{20}{3}p - \frac{8}{3}q.$$

For $ReZG_3(\mathbb{H})$ which is the third redefined Zagreb index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = d_\upsilon d_v(d_\upsilon + d_v)$, therefore $\lambda(2,4) = 48$ and $\lambda(4,4) = 128$. Thus by Lemma 4.1,

$$ReZG_3(\mathbb{H}) = 12pq(128) + 2p(4(48) - 128) + 4q(48 - 128) = 1536pq + 128p - 320q.$$

We gave graphical comparison of Theorem 4.6 in Fig. 6 and numerical values Tab. 5.

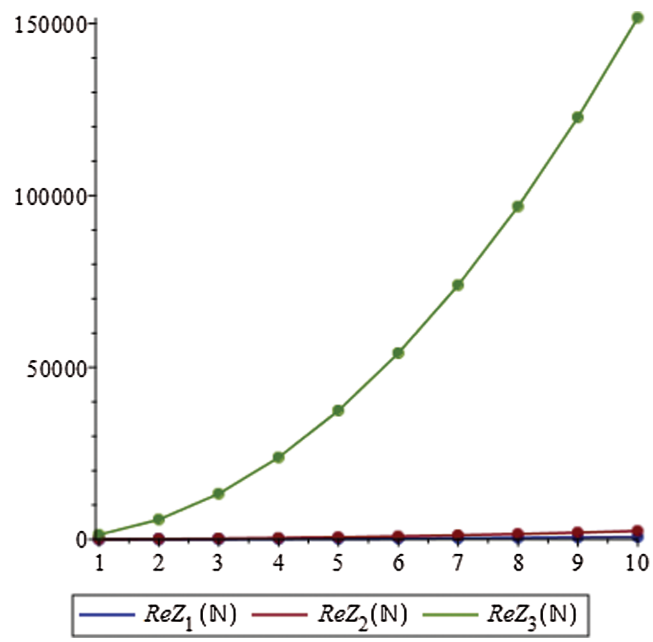

Figure 6: Graphical comparison of Theorem 4.6

Table 5: Numerical representation of Theorem 4.6

$[p, q]$	$ReZG_1$	$ReZG_2$	$ReZG_3$
$[1, 1]$	12	28	1344
$[2, 2]$	36	104	5760
$[3, 3]$	72	228	1324
$[4, 4]$	120	400	23808
$[5, 5]$	180	620	37440
$[6, 6]$	252	888	54144

(Continued)

Table 5 (continued).

$[p, q]$	$ReZG_1$	$ReZG_2$	$ReZG_3$
$[7, 7]$	336	1204	73920
$[8, 8]$	432	1568	96768
$[9, 9]$	540	1980	122688
$[10, 10]$	660	2440	151680

Theorem 4.7 *For the hexagon star network* $\mathbb{H}$, *we have*
the Randic′ index of $\mathbb{H}$,

$$R'(\mathbb{H}) = 3pq + \frac{3}{2}p.$$

the Reformulated Zagreb index of $\mathbb{H}$,

$$RZ(\mathbb{H}) = 432pq + 56p - 80q$$

the forgotten index of $\mathbb{H}$

$$F(\mathbb{H}) = 384pq + 96p - 48q.$$

the irregularity measures of $\mathbb{H}$

$$IRM(\mathbb{H}) = 32p + 16q.$$

Proof. For $R'(\mathbb{H})$ which is the Randic′ index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \frac{1}{\max\{d_\upsilon, d_v\}}$, therefore $\lambda(2,4) = \frac{1}{4}$ and $\lambda(4,4) = \frac{1}{4}$. Thus by Lemma 4.1,

$$R'(\mathbb{H}) = 12pq\left(\frac{1}{4}\right) + 2p\left(4\left(\frac{1}{4}\right) - \frac{1}{4}\right) + 4q\left(\frac{1}{4} - \frac{1}{4}\right) = 3pq + \frac{3}{2}p.$$

For $RZ(\mathbb{H})$ which is the Reformulated Zagreb index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = (d_\upsilon + d_v - 2)^2$, therefore $\lambda(2,4) = 16$ and $\lambda(4,4) = 36$. Thus by Lemma 4.1,

$$RZ(\mathbb{H}) = 432pq + 56p - 80q.$$

For $F(\mathbb{H})$ which is the forgotten index of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = \left(d_\upsilon^2 + d_v^2\right)$, therefore $\lambda(2,4) = 20$ and $\lambda(4,4) = 32$. Thus by Lemma 4.1,

$$F(\mathbb{H}) = 384pq + 96p - 48q.$$

For $IRM(\mathbb{H})$ which is the irregularity measures of $\mathbb{H}$, we have $\lambda(d_\upsilon, d_v) = (d_\upsilon - d_v)^2$, therefore $\lambda(2,4) = 4$ and $\lambda(4,4) = 0$. Thus by Lemma 4.1,

$$IRM(\mathbb{H}) = 32p + 16q.$$

4 Conclusion

The study of graphs and networks through topological descriptors is important to understand their underlying topologies. Such investigations have a wide range of applications in cheminformatics, bioinformatics and biomedicine fields, where various graph invariants based assessments are used to deal with several challenging schemes. In the analysis of the quantitative structure property relationships (QSPRs) and the quantitative structureactivity relationships (QSARs), graph invariants are important tools to approximate and predicate the properties of the biological and chemical compounds. In this paper, we study the valency-based topological descriptor for hexagon star network.

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