

Study of Sugarcane Buds Classification Based on Convolutional Neural Networks

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Abstract: Accurate identification of sugarcane buds, as one of the key technologies in sugarcane plantation, becomes very necessary for its mechanized and intelligent advancements. In the traditional methods of sugarcane bud recognition, a significant amount of algorithms work goes into the location and recognition of sugarcane bud images. A Convolutional Neural Network (CNN) for classifying the bud conditions is proposed in this paper. Firstly, we convert the colorful sugarcane images into gray ones, unify the size of them, and make a TFRecord format data set, which contains 1100 positive samples and 1100 negative samples. Then, a CNN mode is developed to classify the buds into good ones or bad ones. This model contains two convolutional layers and two pooling layers, which go into two full connected layers at the end. The bud dataset is divided into three sets: training set (80%), validation set (10%) and test set (10%). After the repeated training, the structure and parameters of the model are optimized, and an optimal application model is obtained. The experiment results demonstrate that the recognition accuracy of sugarcane bud reaches to 99%, which is about 6% higher than traditional methods. It proves that the CNN model is feasible to classify sugarcane buds.

Keywords: Sugarcane bud; classification; convolutional neural networks

1 Introduction

Sugarcane is widely planted in China. According to the “China Statistical Yearbook-2019”, the area of sugarcane planted is 1,406,000 hectares. As we know that China is a big agricultural country, but the agricultural industry is backward relatively. Artificial sugarcane planting suffers from low productivity and poor economic benefits. In the process of sugarcane harvesting, transportation, seed retention and cutting, the buds are easily damaged. Therefore, the identification of the conditions of sugarcane buds becomes the key factor in the automatic sugarcane planting. And the intact seed buds of sugarcane are the basic guarantee for the budding rate of new ones. So, it is necessary to discriminate the good buds from the bad ones, such as buds dropping, dead buds, rotten buds [1,2].



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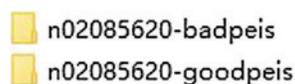
Many researches are made to detect the sugarcane nodes [3–7], but few attempt to judge the conditions of the sugarcane buds. They use the sugarcane nodes to determine indirectly the position of the sugarcane buds. Then the cutter is controlled to avoid bud damage during sugarcane cutting. Although these methods have limited the average rate of damaged buds within 2%, it is impossible to detect damaged buds before sugarcane cutting. Thus, it will greatly influence the production and the economic benefit of sugarcane. Lu [4] proposes an improved 8-points equivalent maximum inscribed circle method to separate and locate the sugarcane buds. He uses image processing technology to segment the bud based on its color and shape features. But this method generalizes not well. Huang et al. [8] propose a machine learning based method to classify the integrity of sugarcane buds. This method uses Bayesian decision model to detect whether the buds are defective or damaged. However, it needs to manually extract five classification features including the minimum gray value, the maximum gray value, the mean gray value, the standard deviation of gray value, and the gray scale median from the effective sugarcane bud area. It is undeniable fact that there are many defects in these methods. The biggest problem is that the extraction of the seed buds' characteristics has to be spotted artificially before recognition. The extraction algorithm is more complicated and the workload is heavy.

Deep learning has been widely used in image recognition and classification [9–18]. Inspired by the LeNet-5 model [19], we propose a CNN-based network model to classify the sugarcane bud conditions which reduce the work of extracting artificial seed bud features in the early stage, and improve the recognition rate. Our method mainly has two steps. The first step is to pre-process the bud images. It is to convert the color image of sugarcane into a gray one and unify the size of the image to create a data set in TFRecord format. The second step is to design and train the CNN mode. The bud data set is divided into three groups: 80% is used as the training set, 10% is used as the validation set, and 10% is used as the test set. Through the repetition training, the structure and parameters of the model are optimized. And the best application model is finally obtained which improves the recognition rate.

2 Pretreatment of Sugarcane Bud Image Data

TFRecord data file is a binary file that stores any data form by converting them into the TensorFlow form, and make the datasets of TensorFlow matching the network applications more easily. It is convenient to copy and call in training data loading as well. Converting sugarcane image data into TFRecord data sets cannot only call image data model training more quickly, but also reduce the time needed to load image data set and train section during model training. Therefore, it is necessary to encapsulate sugarcane image data set with TFRecord data set.

To make TFRecord data set, we first select 1100 bad sugarcane buds and 1100 good ones from the collected images, and then save them in the designated path respectively. The format naming is shown in Fig. 1.



n02085620-badpeis
n02085620-goodpeis

Figure 1: Naming of sugarcane bud

The good and bad sugarcane bud images are shown in Figs. 2 and 3 respectively. It is not difficult to see from the image that the biological structure of the intact seed bud is complete, and the surface or internal biological structure of the damaged seed bud is damaged and cannot germinate normally as well.

However, the original images mentioned above cannot be used directly for training. Since most of the original images have different sizes, they need to be unified in size first of all. When performing model is training, corresponding labels should be marked in the image file. The system changes the color space

into gray when converting the image data format. And the image size is modified to a same size: 250 pt × 151 pt, and a two-dimensional image label is attached to each image. The entire data set is divided into 80% training set, 10% validation set and 10% test set.



Figure 2: Bad sugarcane bud



Figure 3: Good sugarcane bud

3 Construction of Sugarcane Bud Recognition Model

3.1 Convolutional Neural Networks

Convolutional neural networks (CNN) reduce the complexity of the model by the weight sharing which is very popular in the field of image and speech recognition. The CNN-based model performs outstanding in multi-dimensional data inputting field. It has good robustness in translation, rotation and scaling image.

Image recognition based on CNN has more advantages. Compared with the traditional network structure, the biggest advantage of CNN is the two special design methods: convolutional layer and pooling layer. The convolutional neural network is connected to the upper layer by local connections and weight sharing which greatly reduces the number of model parameters. Because of this special design, the original image can be sent to CNN without the special complicated processing.

The most typical network structure is LeNet-5, which can be used in image recognition. LeNet-5 has a basic convolutional neural network modules: convolutional layer, pooling layer and fully connected layer. It is a model basis for scholars to learn or improve innovation. Fig. 4 is the detail structure of it.

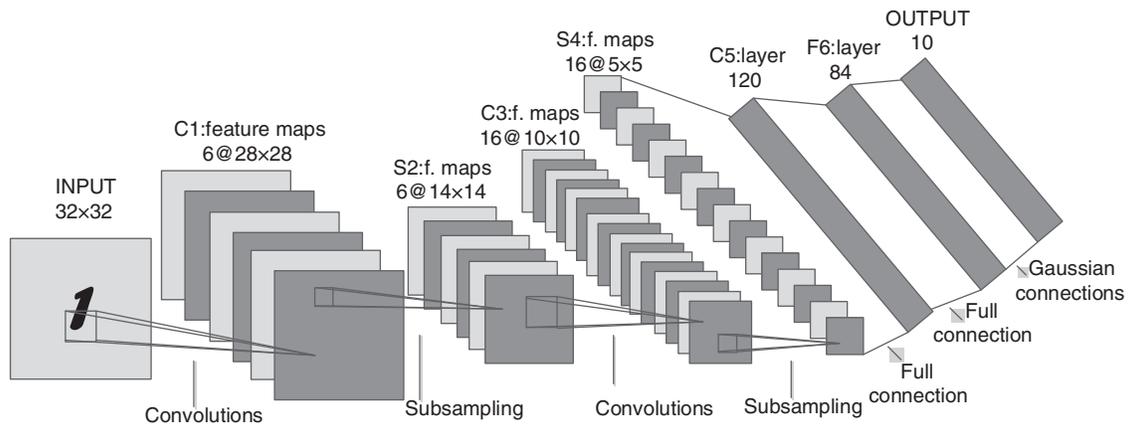


Figure 4: Architecture of LeNet-5

As can be seen from Fig. 4, the network uses 7 layers (excluding input layer). These are 2 convolutional layers, 2 pooling layers, 2 fully connected layers and 1 layer of Gaussian connection. Convolution operations are represented by Convolutions, pooling operations are represented by Subsampling, and full connection operations are represented by Full connection. The original size input is 32×32 . As a convolutional layer, C1 contains six feature maps, 28×28 each. The size of convolution kernel is 5×5 . S2 is a pooling layer, which reduces the output of the previous convolutional layer from 28×28 to 14×14 . Similarly, C3 is an other convolutional layer. S4 is an other pooling layer. And all the following layers are fully connected layers.

3.2 Design of Convolutional Network for Sugarcane Bud Recognition

Considering the prominent characteristics of sugarcane bud, the proposed model is designed as shown in Fig. 5. The functions of each layer are as follows.

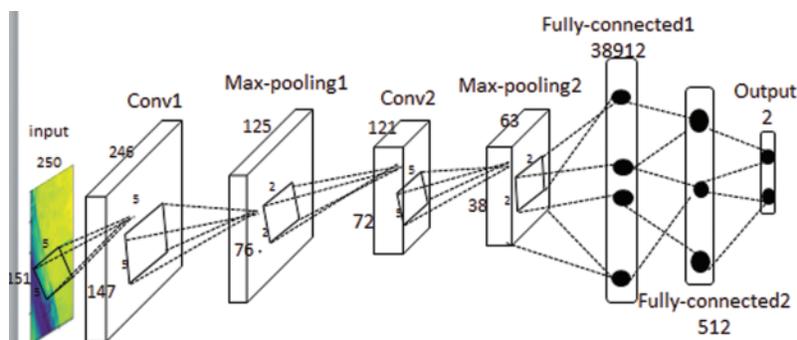


Figure 5: The structure network of sugarcane bud recognition

The first layer is the data input layer. The pixel size of the input image is 250×151 .

The second layer is Conv1 convolutional layer, which uses 32 filters. The role of convolution kernels is to extract the features of the input image which size is 5×5 . The total number of parameters to be learned for Conv1 convolutional layer is $(5 \times 5 + 1) \times 32$. If the size of the input image is 250×151 , a feature map with a size of 246×147 will be obtained after convolution by a convolution kernel with a stride of 2. Finally, the number of Conv1 convolution layer connections is $832 \times (246 \times 147)$. It can be seen that the convolution layer can greatly reduce the number of parameters to be learned by using weight sharing.

The third layer is Max-pooling1 layer (also known as the down-sampling layer). It can be seen from the Fig. 5 that Max-pooling1 layer is connected with convolutional layer Conv1. The feature map size of the Max-pooling1 layer is 1/4 of the output feature map size of the Conv1 convolutional layer.

The fourth layer is Conv2 convolution layer, which uses 64 filters. The role of convolution kernels is to extract the features of the input image. The number of parameters to be learned for Conv2 convolution is $(5 \times 5 + 1) \times 64$. And the output feature map size of Conv2 convolution is 119×70 . So, the total number of connecting lines for Conv2 convolution is $1664 \times 119 \times 70$. All the feature maps of Max-pooling1 layer are convoluted by every convolution kernel of Conv2 convolutional layer, which is the so-called fully connected.

The fifth layer is Max-pooling2 down-sampling layer. Its working principle is the same as that of the previous Max-pooling1 down-sampling layer. The feature mapping output of the Conv2 convolutional layer is processed by the same pooling method.

The sixth layer is Fully-connected1 layer, which has 38912 neurons. Every neuron is connected with all the neurons in the Max-pooling2 sampling layer, which completes the connection of the whole convolutional neural network. The number of feature maps as well as the resolution of feature maps that the network finally outputs to the sampling layer under Max-pooling2 determine the parameters that the Fully-connected1 layer needs to learn.

The seventh layer is Fully-connected2 layer. It is designed into 512 units according to the output. Fully-connected2 layer calculates the dot product of input vector and weight vector, and then adds the offset. It takes the calculated value as the input of the function, and its output is the value of each unit.

The eighth layer is output layer. In fact, this layer is a multi-class SoftMax classifier that determines the number of output nodes according to the classification task. Here, the output is set to 2, so as to achieve the goal of two classification.

3.3 Selection of Activation Function

Linear transformation cannot meet the processing of complex data. And the use of nonlinear transformation can better deal with complex data. The activation function is used in neural network to realize non-linear transformation. Among the three commonly used activation functions, the Sigmoid function [20] and the hyperbolic tangent function Mao et al. [21] are saturated nonlinear function, and the ReLU function [22–24] is unsaturated nonlinear function. Compared with saturated nonlinear function, unsaturated nonlinear function can solve the problem of gradient disappearance and explosion, and can accelerate the convergence speed of model. The activation function used in this paper is the ReLU function. The main reason is that ReLU function is faster and more effective than Sigmoid function and TanH function in solving gradient descent in the designed convolutional neural network. In the case of a large number of training sets, it can greatly reduce the training time of the model. The image of the ReLU function is shown in Fig. 6.

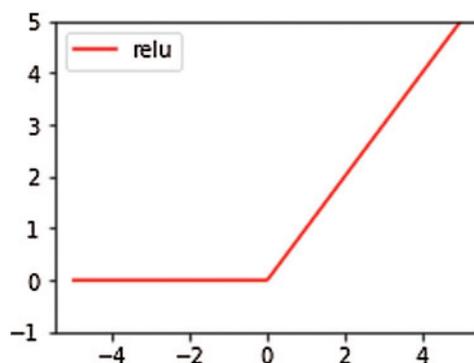


Figure 6: Image of the ReLU function

3.4 Selection of the Loss Function

The cross-entropy function is a very common probability distribution function in the loss function. The model is optimized by calculating the error between the predicted output and the truth. Introducing cross-entropy to measure the training effect of the neural network can solve many problems in the process of neural network classification. In the neural network multi classification problem, the usual method is to set n output nodes and get an n -dimensional array. Each output node in the array corresponds to a label category. The best expectation is that if a sample belongs to category L , the output dimension value of the node corresponding to this category should be 1, and the output of other nodes should be 0. The model in this paper carries out two classifications of sugarcane buds, so its value is a two-dimensional array. If the input image is an image with bad bud, it belongs to category 0. Then the two-dimension value of the network output node is $[1,0]$. Among them, 1 is the probability of damaged seed bud, which is the result we expect to output.

Cross-entropy is of great significance in defining neural networks as it is used to estimate the length of the average code. The cross entropy is defined by giving two probability distributions p and q as follows:

$$H(p, q) = - \sum_x p(x) \log q(x) \quad (1)$$

In the Eq. (1), $p(x)$ and $q(x)$ represent the probability distribution of training samples and predictions respectively. Using cross-entropy as an evaluation metric can make the model output distribution and the training sample distribution as consistent as possible. Many model training results show that the smaller the cross-entropy, the closer the probability distribution of p and q , and the more accurate the expected value of the model. Therefore, the paper uses cross-entropy to measure the error between the output and the truth, and optimizes the model based on this.

4 Results Analysis

4.1 Model Test

After getting the training model, we use the validation set to test its performance, as shown in Fig. 7.

In Fig. 7, the model accuracy calculated on the validation set each time is obtained by reading the latest saved model. The model is tested every 5 epochs. After the training epoch reaches 90 times, it can be seen that the loss value of the iteration is reduced from 508.11234 to 0.6931472. Although the loss value has fluctuated, it still tends to be smaller overall. It can also be seen from Fig. 7 that the accuracy of the validation set test increased from 0.54 to 0.94 after 35 epochs. The accuracy of the final model is always around 0.99. Since the background of the bud is relatively simple, the training model can quickly learn

the features of the training data, and can accurately recognize the conditions of the test image. The experimental results show that the trained model has a good performance. If the choice of learning rate is not good, it is also likely to have a tendency of fitting. There are two ways to prevent the model from overfitting during training. One is to increase the training set, the other is to use the dropout operation. Fig. 8 shows the accuracy of the test set using the dropout operation.

```
epoch= 0 loss = 508.11234 accuracy= 0.54
epoch= 5 loss = 278.40906 accuracy= 0.34
epoch= 10 loss = 319.55832 accuracy= 0.52
epoch= 15 loss = 300.7409 accuracy= 0.6
epoch= 20 loss = 210.8685 accuracy= 0.6
epoch= 25 loss = 158.58916 accuracy= 0.74
epoch= 30 loss = 64.57806 accuracy= 0.84
epoch= 35 loss = 52.988693 accuracy= 0.94
epoch= 40 loss = 3.1207726 accuracy= 0.98
epoch= 45 loss = 1.1463218 accuracy= 1.0
epoch= 50 loss = 2.6897454 accuracy= 0.98
epoch= 55 loss = 15.204468 accuracy= 1.0
epoch= 60 loss = 0.7367603 accuracy= 0.98
epoch= 65 loss = 0.6931472 accuracy= 0.98
epoch= 70 loss = 0.6792842 accuracy= 0.98
epoch= 75 loss = 0.6931472 accuracy= 1.0
epoch= 80 loss = 11.589621 accuracy= 1.0
epoch= 85 loss = 4.566599 accuracy= 0.98
epoch= 90 loss = 0.6931472 accuracy= 1.0
epoch= 95 loss = 0.6931472 accuracy= 1.0
training finished
```

Figure 7: Training loss value and verification accuracy

If the system model is overfitting, and the number of data sets does not increase, the loss value will no longer decrease regardless of the number of training iterations, and the test accuracy will always be 1.0. It can be seen from Figs. 7 and 8 that the accuracy of the model on the validation set and the test set is not much different, which indicates that the model has a slight overfitting.

```
In round 0 tests, the accuracy rate was: 1.0
In round 10 tests, the accuracy rate was: 0.9945454543287103
In round 20 tests, the accuracy rate was: 0.9961904769852048
In round 30 tests, the accuracy rate was: 0.9967741947020253
In round 40 tests, the accuracy rate was: 0.9965853662025638
In round 50 tests, the accuracy rate was: 0.9964705892637664
In round 60 tests, the accuracy rate was: 0.9963934441081813
In round 70 tests, the accuracy rate was: 0.9960563401101341
In round 80 tests, the accuracy rate was: 0.9965432117014755
In round 90 tests, the accuracy rate was: 0.9969230785474672
Total accuracy was: 0.996600014543533
testing finished
```

Figure 8: Accuracy list of 100 rounds of tests

Finally, the model can achieve 95% accuracy through a small number of iterations. The use of the dropout function can prevent the model from overfitting and further improve the accuracy of the model. Fig. 9 shows the training loss curve, and Fig. 10 shows the test accuracy rate curve. The accuracy rate of the test set is also very high, and the sample test rate can generally reach to 0.99, which shows that the performance of the training model is good. We compare the accuracy of sugarcane buds obtained by the CNN model and the Bayesian decision model [8] as shown in Tab. 1. It can be seen that the CNN model has better performance when the test samples are approximately equal.

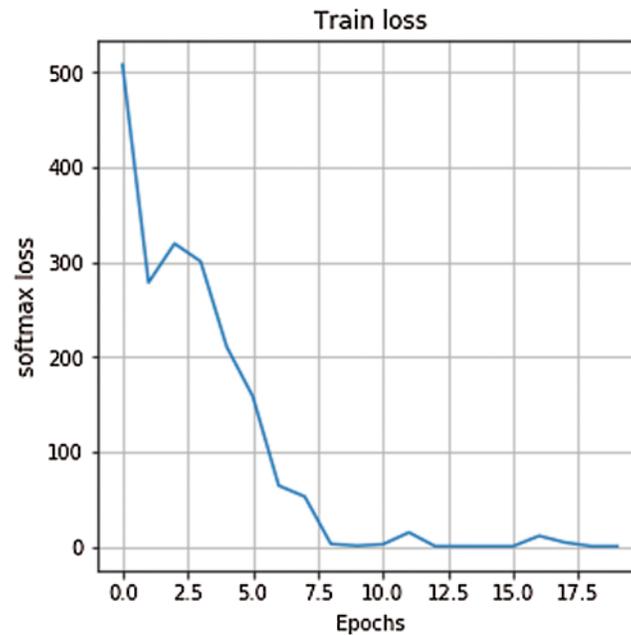


Figure 9: Training loss curve

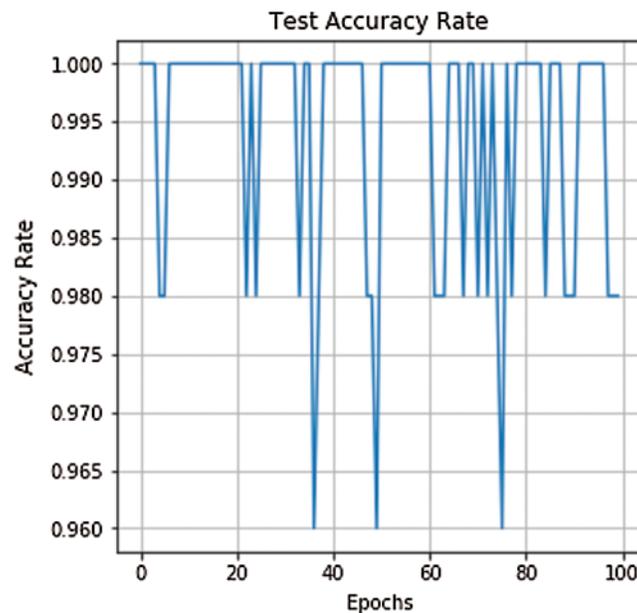


Figure 10: Test accuracy curve

4.2 Visualization of Convolution Feature

To optimize the model, the convolutional neural networks first propagate the extracted image features forward. Then the difference between the model output and the data label is backpropagated to adjust the network parameters [25]. Since CNN is a black box network, it will increase the difficulty of optimizing network weights. However, if we can display the image features [26] learned by CNN in various layers

of networks in the form of images, it will help us to optimize the network parameters more easily. The original input image in the network is shown in [Fig. 11](#).

Table 1: Accuracy of bud classification

Model	Test number	Average accuracy/%
Bayes	215	92.87
CNN	220	99.66

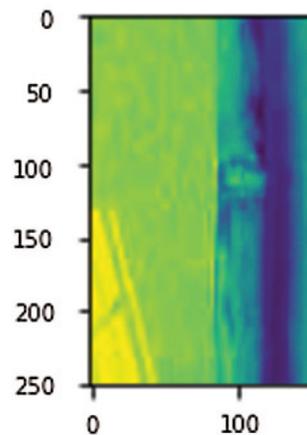


Figure 11: Sugarcane image on input

In the model, 32 filters are used for the first convolution layer Conv1 and the first pooling layer Pool1, 64 filters are used for the second convolution layer Conv2 and the second pooling layer Pool2. Convolution kernels of 5×5 are used for Conv1 and Conv2, while convolution kernels of 2×2 are used for Pool1 and Pool2. We select 9 feature maps from each layer, and the visualization results are shown in [Fig. 12](#).

It can be seen from [Fig. 12](#) that the filter of the convolutional layer Conv1 mainly extracts the underlying feature information of the color, texture and direction of the image. The filter of the pooling layer Pool1 further extracts the image texture and direction on the basis of Conv1. Pool1 can reduce the parameter value of the model by reducing the size of the feature map. Similarly, Conv2 continues to extract deeper texture and direction information from the feature map output by Pool1. This is more conducive to the model to learn the main features of the image. The visual patterns of lines, directions, and textures observed in [Fig. 12](#) are very regular, which indicates that the features extracted by the model are very obvious.

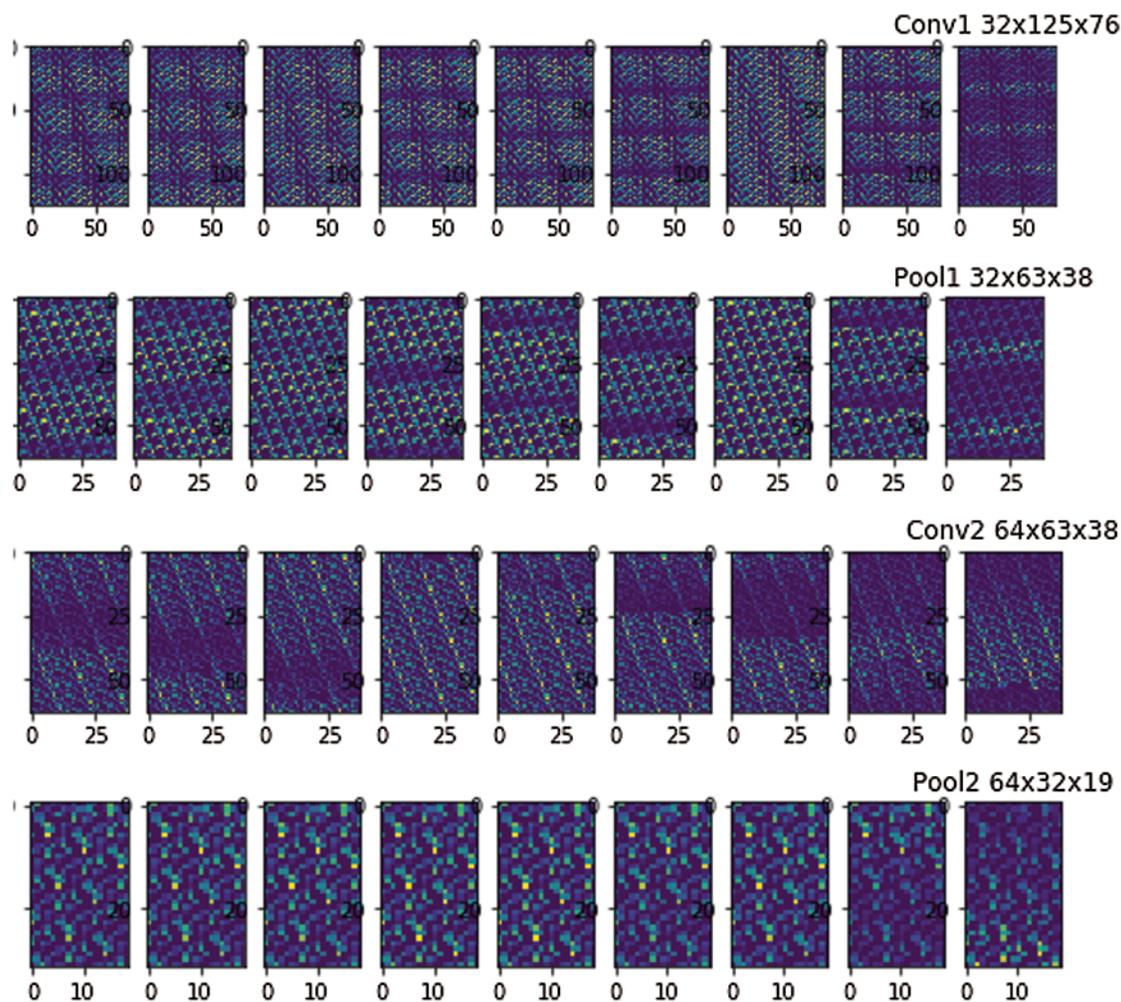


Figure 12: Visualization results of convolution and pooling

5 Conclusion

In this paper, the CNN model is constructed to recognize the conditions of sugarcane buds. Compared with the traditional sugarcane bud recognition technology, this method greatly reduces the work of artificially extracting seed bud characteristics during image recognition. The model designed in this paper has better recognition accuracy than the traditional image processing method. There are two reasons for the high recognition rate. One is that the sugarcane bud images are collected under a single background, so the characteristics of the bud are not disturbed by complex background features. It can be seen from Fig. 12 that the extracted image features are very regular. The second is that deep learning has excellent performance in image recognition. The experiments results show that the recognition accuracy of the model reaches to 99%. A higher recognition rate will effectively reduce the production cost and improve the economic benefits of sugarcane.

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