Tech Science Press

Generating Type 2 Trapezoidal Fuzzy Membership Function Using Genetic Tuning

Siti Hajar Khairuddin, Mohd Hilmi Hasan[*], Emilia Akashah P. Akhir and Manzoor Ahmed Hashmani

Centre for Research in Data Science, Department of Computer and Information Sciences, Universiti Teknologi PETRONAS, Perak, Malaysia
[*]Corresponding Author: Mohd Hilmi Hasan. Email: mhilmi_hasan@utp.edu.my

Abstract: Fuzzy inference system (FIS) is a process of fuzzy logic reasoning to produce the output based on fuzzified inputs. The system starts with identifying input from data, applying the fuzziness to input using membership functions (MF), generating fuzzy rules for the fuzzy sets and obtaining the output. There are several types of input MFs which can be introduced in FIS, commonly chosen based on the type of real data, sensitivity of certain rule implied and computational limits. This paper focuses on the construction of interval type 2 (IT2) trapezoidal shape MF from fuzzy C Means (FCM) that is used for fuzzification process of mamdani FIS. In the process, upper MF (UMF) and lower MF (LMF) of the MF need to be identified to get the range of the footprint of uncertainty (FOU). This paper proposes Genetic tuning process, which is a part of genetic algorithm (GA), to adjust parameters in order to improve the behavior of existing system, especially to enhance the accuracy of the system model. This novel process is a hybrid approach which produces Genetic Fuzzy System (GFS) that helps to enhance fuzzy classification problems and performance. The approach provides a new method for the construction and tuning process of the IT2 MF, based on the FCM outcomes. The result is compared to Gaussian shape IT2 MF and trapezoid IT2 MF generated by the classic GA method. It is shown that the proposed approach is able to outperform the mentioned benchmarked approaches. The work implies a wider range of IT2 MF types, constructed based on FCM outcomes, and an optimum generation of the FOU so that it can be implemented in practical applications such as prediction, analytics and rule-based solutions.

Keywords: Fuzzy inference system; membership function; genetic tuning; lateral adjustment; trapezoidal MF; fuzzy C means

1 Introduction

The three main types of fuzzy membership function (MF) are Gaussian [1,2], triangular [3,4] and trapezoidal [5,6]. Each of the MFs has its own advantages and disadvantages, hence providing better performance than the other MFs in certain scenarios or applications. One way of constructing the MF is through data clustering using fuzzy C means (FCM). The limitation of

FCM is that it generates Gaussian MF, instead of linear MFs such as triangular and trapezoidal. The previous work proposed the generation of trapezoidal MF for the interval type 2 (IT2) fuzzy inference system (FIS) [7]. This paper proposes the extension of the work which is to tune the IT2 trapezoidal MF so that it could produce improved accuracy.

The purpose of Genetic Algorithm (GA) as part of producing the FIS is to generate IT2 trapezoidal MF (referred as trapMF in this paper) by using the theory of genetic tuning and lateral adjustment [8]. It will be the continuation of the previous work [7] where FT1 trapMF is constructed through the theory of Gaussian distribution. To generate IT2 trapMF, UMF and LMF of the MF need to be identified to get the range of the FOU. In this section, the LMF is constructed to get the FOU range, while the parameters of the UMF is maintained. Genetic tuning process, which is part of GA, is the process of adjusting parameters to improve the behavior of existing system, especially to enhance the accuracy of the system model [9]. This process is a hybrid approach which produces Genetic Fuzzy System (GFS) that helps to enhance fuzzy classification problems and performance. In [9], a collaborative tuning based on ignorance function and lateral adjustment is performed to improve the fuzzy rule based classification system. Ignorance functions are also famously used to generate Interval Value Fuzzy Sets (IVFS) which is the general type of fuzzy IT2. The methodology used in this research is the modified adaptation of techniques in the IVFS performance enhancement, to specifically suit trapMF.

In GFS, the concept of weak ignorance function is introduced. It is used to evaluate or measure the uncertainty of membership values [9]. A definition in [10] implies that a value of 0.5 corresponds to the complete lack of knowledge of the expert on the membership of an element to a class. In [10] the five axioms are justified. Bijections are used to construct IT2 where their amplitude is proportional to weak ignorance function having the maximum ignorance degree for initial membership degree = 0. Thus, a bijection $h : [0, 1] \rightarrow [0.5, 1]$ so that $h(0) = 0.5$ and $h(1) = 1$ is employed.

The composed MFs are usually obtained by a normalization process and they remain the same during the rule based construction process [9]. This indicates that the MFs are not properly adapted to the context of each variable which may limit the performance of the global rule set [9]. A post-processing tuning step is often used to overcome this problem. The tuning provides a variation in the shape of the MFs with the main aim is to find the best global configuration for them. The previous work in [11] had carry out the tuning step for the amplitude of the support of UMF which manages the ignorance that each IT2 represents.

Tuning process in GFS is commonly applied to improve the performance of fuzzy model. It introduces an alterations of MF shapes to improve global interaction which induces better cooperation among rules [12,13]. Thus, the real aim of tuning is to find the best global configurations of the MF [14]. Based on this goal, the research methodology takes a new direction which is using the tuning process to directly construct trap IT2, instead of using it as a post processing steps after constructing the rules for the fuzzy sets. In this way, the best range for MF amplitude is obtained, and it reduces the steps of getting the best model with better result in term of its accuracy. This has become the aim of this paper. The proposed approach will provide practical implications through IT2 FIS implementation in the areas including health prediction and classification [15], computer and network security [16], image processing and security [17], rule-based solutions and many others.

The tuning steps will involve a simultaneous step of adjusting the amplitude with varying range, by moving only two parameters (left and right) of trapMF. At the same time, lateral

adjustment based on 2-tuples fuzzy linguistic representation of the MF is also performed, where the trapMF parameters are adjusted to the left or right position based on varying range in order to achieve better adaptation of the fuzzy partitions. In order to prove the enhancement in term of performance of this methodology, the result of the fuzzy modeling through FIS will be compared with the generic tuning of trapMF whereby it follows the initial situation of genetic weak ignorance tuning [18]. It also uses triangular MF to show the initial lateral adjustment where the horizontal parameters, which exists only for trapMF, are not counted.

This paper is structured as follows; next section contains literature review, the explanation of FIS which becomes the research methodology is presented in Section 3, while results and discussion are contained in Section 4. The paper ends with conclusion in Section 5 in which the research is summarized as well as some future works are outlined.

2 Methodology

2.1 Genetic Weak Tuning

The modification of the LMF amplitude of trapMF is based on ignorance function [9] represented by parameter W in this paper. The interval in which W can vary is [0, 0.5] where the situation without ignorance, $W = 0$ and $W = 0.5$ for maximum ignorance. The degree of ignorance will be increased from the initial value $W = 0.05$ until 0.5. Variation of degree of ignorance is depicted in Fig. 1.
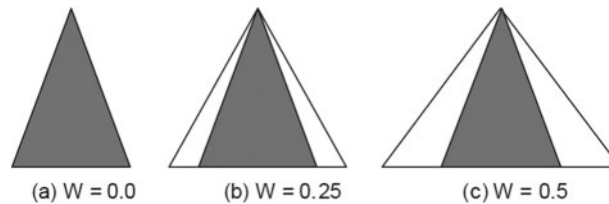

(a) W = 0.0   (b) W = 0.25   (c) W = 0.5

Figure 1: Parameter values representation in the genetic weak ignorance tuning. (a) No ignorance. (b) Initial situation. (c) Maximum ignorance [9]

The approximation of the genetic weak tuning in trapMF is initialized with the parameters for UMF. The equation of the straight line between parameter b and c is ignored due to the fact that the amplitude and lateral approximation only involve the intervals between parameter a to b, and parameter c to d. The calculation for c to d (straight vertical line of trapMF) will be similar since the MF involved in symmetrical. UMF and LMF of the interval are consistent with the following situation:

- Total certainty ($W = 0$): UMF and LMF values are the same.
- Maximum uncertainty ($W = 0.5$): Huge absence of information which causes the amplitude of UMF to be twice the amplitude of LMF.

Based on the above situation, the initial value set for W is 0.05 to test for any changes of amplitude. As mentioned, bijections are used for the constructions of IT2 so the amplitude is proportional to the weak ignorance function where degree of ignorance is considered maximum when the membership degree is 0. Specifically, we employ the following formula (Eq. (1)):

$$h(x) = \frac{1}{2}x + \frac{1}{2} \tag{1}$$

The initial FT1 trapMF is built first from the previous section, which is represented by the MF representation below:

$$\mu_A(x) = \begin{cases} 0 & if \ x < a, \\ \dfrac{x-a}{b-a} & if \ a \le x \le b, \\ 1 & if \ b < x < c, \\ \dfrac{d-x}{d-c} & if \ c \le x \le d, \\ 0 & if \ x < d, \end{cases} \tag{2}$$

In order to calculate the range of FOU for IT2 trapMF, the proposition in [9] is modified to suit with the trapezoidal shapes, whereby the parameters obtained from FT1 trapMF is combined with ignorance function, which is proportional to the MF amplitude. Fig. 2 shows the example of amplitude applied in FT1 trapMF to construct the IT2.
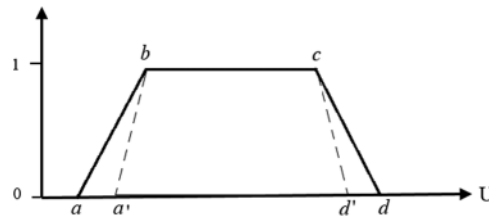

Figure 2: Amplitude tuning for IT2 trapMF

Based on Fig. 2, the amplitude is adjusted for the LMF of the FT1 trapMF at point (a', b) and (c, d'). Since the trapMF is symmetrical, calculation for point (c, d') will be the same except for the change of operator. Considering A is the amplitude of point (a, b) which is obtained from construction of trapMF, a complete algorithm to get the amplitude is:

Obtain each cluster center and sigma:

Point $a =$ aCenter $-$ aSigma $* 2$

Point $d =$ dCenter $+$ dSigma $* 2$

Point (a, b):

$$A = \frac{2}{(\text{Point } d - \text{Point } a)} \ (x - \text{Point } a)$$

$a' =$ Point $a -$ W (Point d $-$ Point a), where W > 0.

W is the parameter used for amplitude tuning to measure the certainty; $W = 0$ for total certainty, and 0.5 to be the maximum uncertainty. It can also be called weak tuning.

From A, the equation for amplitude of straight line (a', b) named B is:

$$B = \frac{2}{(\text{Point } d - \text{Point } a)(1 + 2W)} \ (x - a + W \ (\text{Point } d - \text{Point } a))$$

In order to achieve the best possible range of FOU through this weak tuning, a range of parameter W is prepared to test for its accuracy. Tab. 1 shows the range of parameter W for the trapMF amplitude accuracy testing. This range will soon be used together for MF lateral adjustments since the methodology will be combined.

Table 1: Range of weak tuning

No.	1	2	3	4	5	6	7
W (Weak tuning)	0.05	0.10	0.15	0.18	0.20	0.25	0.5 (Maximum ignorance)

2.2 Lateral Adjustments

In [19], the symbolic translation of a linguistic term is a number within the interval $[-0.5, 0.5]$ which expresses the domain of a label when it is moving between two lateral labels. The lateral displacement can be extended in the construction of MF, where the tuning of the support amplitude of MF is performed [10]. The displacement is a slight movement to the left/right of the original MF.

The adjustment is performed by considering two parameters, α and β, which represents the lateral displacement and the amplitude variation. Next, each label can be represented by a 3-tuple (s, α, β). α is a value within the interval $[-0.5, 0.5]$. β is also a number within the interval. It allows to reduce or increase the support amplitude until 50% of its initial size [19].

Fig. 3 illustrates the lateral and variation of amplitude for label S in symmetrical trap MFs. Due to similar linear parameter on the left and right side on the symmetrical triangular MF, the same adjustment can be applied to trapMF. Label S' can be represented as the 3-tuple (S, α, β). The support for this label, FL_{s2}, is computed in Eq. (3):

$$FL_S = l_s - r_s$$
$$FL_{s2} = FL_S + \beta * FL_S \tag{3}$$

where l_S and r_S are respectively the right and the left extreme of the support of S, and FL_S is the size of the support of S [19].
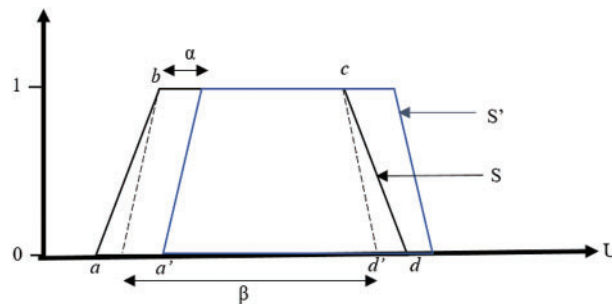

Figure 3: Original lateral displacement and amplitude variation of the linguistic label S

To show the enhancement of the tuning method, the lateral adjustment is modified so that it has similar range with the applied amplitude, to ensure better stability and accuracy of the MF. Hence, the proposition of the alignment of amplitude and lateral adjustment of the IT2 trapMF

is $\alpha = (d - a) - \beta$. To ensure the validity of the amplitude and lateral adjustment cooperation, testing is done with both original adjustment with the new proposed adjustment. Fig. 4 shows the lateral displacement and amplitude variation in this research.
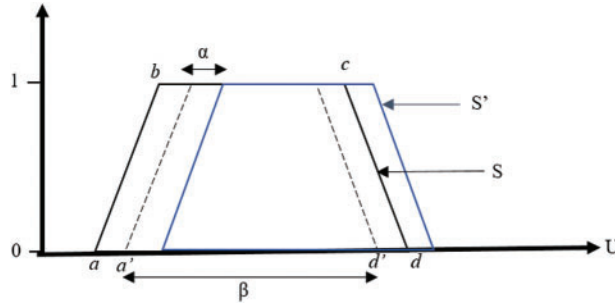

Figure 4: Proposed lateral displacement and amplitude variation

Based on Tab. 1, the initial value set for W is 0.05 to test for any changes of amplitude. Hence, the novelty of this approximation is where the adjustment of weak tuning has direct synergy with theoretical calculation of lateral tuning. While lateral tuning by theory is applied to shift MF to left or right, it is now used for parameter adjustments together with weak tuning. The motivation of this cooperation is the risk of the model to reach sub-optimal result. The cooperative approximation will fully utilize the characteristics of genetic tuning which leads to better performance of FSs in term of accuracy, specifically for trapMF IT2.

2.3 Fuzzy Inference System

Based on the Gaussian output, the parameter center, c and sigma σ, mathematical model to construct fuzzy type 1 (FT1) trapMF is performed, as explained in [7]. Next, evaluation of the proposed method is done using a function called `evalmf()` in Matlab Fuzzy Logic Tool. Algorithm 1 shows the construction of FT1 trapMF after obtaining the Gaussian output for FIS:

Algorithm 1: Proposed method to construct FT1 from FCM Gaussian output

---

1. Assign number of clusters $= N$
2. Assign cluster centers, C:

$$c_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m X_j}{\sum_{j=1}^{n} \mu_{ij}^m} \ I, j = \textit{Cluster center } i^{\text{th}} \textit{ to } j^{\text{th}}, \ m = \textit{fuzzy exponent}, \ X^n = \textit{data points}, \ \mu =$$

membership degree

3. Assign cluster sigma, σ

$$\sigma_{i = g, m, p} = \frac{\sum_{n=1}^{6147} \sqrt{(-(x_n - c_i)^2)/(2x \log(U_n)}}{6147} \ g = \textit{good}, m = \textit{moderate}, p = \textit{poor}$$

   If $\sigma = 0$, return error.
   Example: Error("`Illegal parameters`")
4. Define trapMF parameters: params a, b, c and d
5. Loop MF constructions until N cluster:
   for (int i = 0; i < N; i++)

(Continued)

6 Check for correct parameter values for trapMF
     if (a > b) or if (c > d)
     Display error
  Example: Error("`Illegal parameters`")
7. Assign x = 0.0 if x is outside of the support of a fuzzy set
     Example: if ((x < a) || (x > d))
     return `0.0`
8. Left amplitude:
  if (x < b) or x >= a && x < b
   param a = C − σ * 2
   param b = C − σ/2
   Output, y = (x − a)/(b − a)
  else (if x >= b)
   Output, y = 1
9. Right amplitude:
  if (x > c) or x > c && x <= d
    param c = C +σ * 2
   param d = C +σ/2;
   Output, y = (d − x)/(d − c)
  else (if x <= c)
   Output, y = 1
10. Evaluate trapMF using `evalMF()` function

FIS process in Algorithm 1 is repeated to generate IT2 trap FCM. Center, c and sigma, σ generated from the Gaussian output produce the UMF of the IT2 trap MF with four parameters a, b, c and d as described in [7]. Fig. 5 shows the FT1 trapMF produced by the proposed method. Afterward, LMF is generated based on the UMF and FOU. The LMF is constructed based on genetic weak tuning used to tune MFs. The parameters obtained from FT1 trapMF is combined with ignorance function, which is proportional to the MF amplitude. Fig. 6 shows the continuation of fuzzy inference process with FCM to complete the IT2 trapMF.
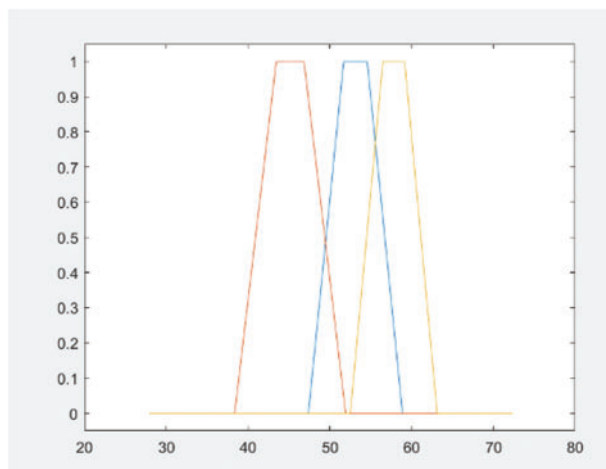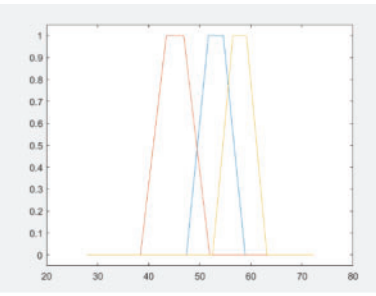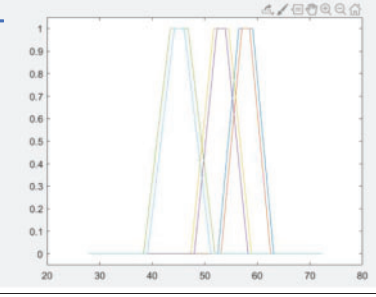

Figure 5: FT1 trapMF

FIS (Mamdani)	FCM	Output
1. Fuzzification (transforming a crisp set to a fuzzy set through clustering) 2. Identify clustering type 3. Apply fuzzy operator (AND and OR) 4. Apply Implication Method 5. Aggregate all outputs (Max method) 6. Type Reduction 7. Defuzzify output (Centroid method)	1. Load data (Training input and output data) 2. Specify the number of cluster (CVI) 3. Specify the exponent of fuzzy partition matrix (Default value = 2) 4. Apply maximum number of iteration (Default value= 100)	FT1 Trap output: Parameters obtained: *a, b, c, d:* $$\mu_A(x)=\begin{cases} 0 & \text{if } x<a, \\ \dfrac{x-a}{b-a} & \text{if } a\le x\le b, \\ 1 & \text{if } b<x<c, \\ \dfrac{d-x}{d-c} & \text{if } c\le x\le d, \\ 0 & \text{if } x>d. \end{cases}$$
	5. Set Genetic Weak Ignorance Tuning Value (L) 6. Adapt L value to Trap MF parameters 7. Applies lateral adjustment and amplitude tuning 8. Cluster data	IT2 Trap MF output:

Figure 6: Fuzzy inference process with FCM for IT2 trapMF

Based on Fig. 6, fuzzification process determines the correlating degree of membership for UMF and LMF from the rule antecedent which generates two fuzzy values. Next, fuzzy operators are applied where the maximum value is obtained from the UMF and the minimum value is obtained from LMF [20]. In the implication phase, the rule firing will clip and scale the UMF and LMF of the output to produce the output fuzzy sets. Similar to FIS model for FT1 trapMF, max operator is used for aggregation phase which produces the final aggregated fuzzy output. The distinct phase in IT2 FIS is the type reduction. In order to generate the final crisp output, the aggregated fuzzy set is reduced to FT1 fuzzy set, which is the range with lower (C_L) and upper limit (C_R). It is frequently referred as the IT2 centroid, which is based on Eq. (4) [21]:

$$C_L \approx \frac{\sum_{i=1}^{L} x_i\mu_{umf}(xi) + \sum_{i=L+1}^{N} X_i\mu_{lmf}(xi)}{\sum_{i=1}^{L} \mu_{umf}(xi) + \sum_{i=L+1}^{N} \mu_{lmf}(xi)} \tag{4}$$

$$C_R \approx \frac{\sum_{i=1}^{R} x_i\mu_{umf}(xi) + \sum_{i=R+1}^{N} X_i\mu_{lmf}(xi)}{\sum_{i=1}^{R} \mu_{umf}(xi) + \sum_{i=R+1}^{N} \mu_{lmf}(xi)}$$

where

$N=$ output variables

$X_i=$ ith output value

$\mu_{umf}=$ UMF

$\mu_{lmf}=$ LMF

L and $R=$ switch points estimated by type reduction method.

To estimate the values for C_L and C_R, iterative type-reduction methods are used [20]. For this research, Enhanced Iterative Algorithm with Stop Condition (EIASC) is used due to its significant improvement in performance compared to other methods [20,22–24].

The approximation of the genetic weak tuning in trapMF is initialized with the parameters for UMF. The equation of the straight line between parameter b and c is ignored due to the fact that the amplitude and lateral approximation only involve the intervals between parameter a to b, and parameter c to d. The calculation for c to d (straight vertical line of trapMF) will be similar since the MF involved in symmetrical. Algorithm 2 summarizes the construction IT2 trapMF after obtaining the FT1 trapMF output for FIS:

Algorithm 2: Proposed method to construct IT2 trapMF

---

1. Obtain FT1 trap MF as UMF: a, b, c, d
2. Applies weak ignorance function from original mapping:
   Gi: [0, 1]2 → [0, 1] so that:
   $(Gi1)$ $Gi(x, y) = Gi(y, x)$ for all $x, y \in$ [0, 1];
   $(Gi2)$ $Gi(x, y) = 0$ if and only if $x=1$ or $y=1$;
   $(Gi3)$ If $x=0.5$ and $y=0.5$, then $Gi(x, y) = 1$;
   $(Gi4)$ Gi is decreasing in [0.5, 1]2;
   $(Gi5)$ Gi is increasing in [0, 0.5]2.
3. Let Let Gi : [0, 1]2 → [0, 1]. Hence, $g(x) = Gi(x, 1 - x)$
4. Assign amplitude to be proportional to the weak ignorance function:
   Maximum ignorance degree $=$ 0 MF (Initial MF)
5. Employ bijection $h(x) = \dfrac{1}{2}x + \dfrac{1}{2}$
6. Assign connection line between UMF and LMF point, $y = \dfrac{2}{c-b} (x-b)$
7. Repeat step 7 for right side of trapMF shape
8. Assign amplitude tuning, W
9. Assign $A=$ amplitude of point (a, b)
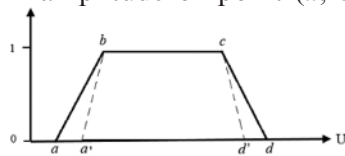
Reference:
   Point $a=$ aCenter $-$ aSigma $* 2$
   Point $d=$ dCenter $+$ dSigma $* 2$
   Point (a, b):

(Continued)

$$A = \frac{2}{(\text{Point } d - \text{Point } a)} \; (\text{x} - \text{Point } a)$$

$d' = \text{Point } a - \text{W} (\text{Point d} - \text{Point a})$, where W > 0

10. From A, the equation for amplitude of straight line (a', b) named B is:

$$B = \frac{2}{(\text{Point } d - \text{Point } a)(1 + 2\text{W})} \; (\text{x} - \text{a} + \text{W} (\text{Point } d - \text{Point } a))$$

11. Apply lateral adjustment:

α = literal displacement within the interval $[-0.5, 0.5]$

β = amplitude variation within the interval $[-0.5, 0.5]$

Assume S as one point in trapMF to be adjusted and FL_S is the size of the support of S

   $FL_S = l_s - r_s$ (l_S and r_S are respectively the right and the left extreme of the support of S)

$FL_{s2} = FL_S + \beta * FL_S$

$\alpha = (d - a) - \beta$

12. Evaluate IT2 trapMF using evalMF() function and following chromosome evaluation for error metric:

Fitness function = Accuracy rate

$$\text{MSE} = \frac{1}{2.N} \sum_{l=1}^{N} (\text{F} (xl) - yl) \, 2$$

For the purpose of testing the theory, the same dataset to generate FT1 trapMF from Gaussian output is used to ensure the continuation of research objective, which is to generate IT2 trapMF from FCM. The total of six datasets, (QoS, Iris, Abalone, HCV, BirchSet, Europe), which are popular among benchmark datasets are prepared to test for result accuracy [25–29]. Tab. 1 shows the range of W parameter used for weak tuning, with trapMF parameters involved for lateral adjustments. Two types of lateral adjustment are tested in this research, LA1 where parameter a and b of trapMF are adjusted and LA2 where all four trapMF parameters, a, b, c and d are affected in the lateral adjustments.

First amplitude parameter W 0.05 is performed in all datasets, and according to result, IT2 trapMF is failed to be generated as the number of uncertainties becomes close 0, with values of UMF and LMF are the same except for very slight difference. Fig. 7 shows the generated trapMF with $W = 0.05$ for both lateral adjustment 1 (LA1) and 2 (LA2). No further testing is done for this parameter value.

Since this method comes from GA components, further evaluation of the output is performed using MSE to test for accuracy of FIS using the IT2 trapMF. The MSE is measured to identify the best W value for IT2 trapMF and which lateral adjustment is better suited to construct a more accurate MF. Tab. 2 summarizes the error measurement for both LA 1 and LA2.

From the error measurements, it is proven that genetic weak tuning can be used to construct IT2 trapMF, with the cooperation of lateral adjustment. Tab. 2 indicates that less error is obtained through L1, where the adjustment adapted in the weak tuning is more accurate for parameter a and d of the trapMF. Tab. 3 shows that by increasing the weak tuning until maximum ignorance $W = 5$ (not shown in Tab. 3) will decrease the accuracy of the model. Hence, the optimum value of W which successfully construct good interval for IT2 trapMF can be observed through the MF input values. $W = 0.15$ is considered to be optimum since it produces the least error, but further study on different size of dataset might result in different optimum W.
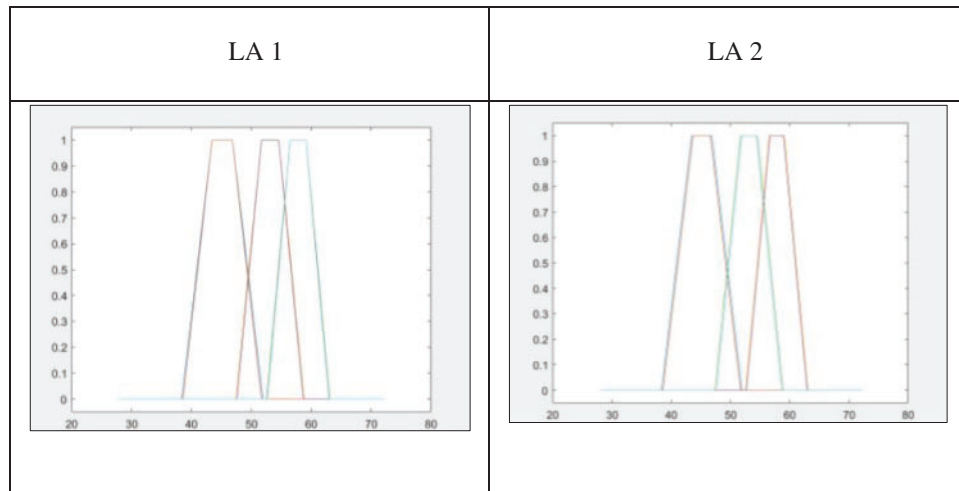
LA 1	LA 2

Figure 7: IT2 trapMF with $W = 0.05$.

Table 2: Error measurement for LA1 and LA2

	Total error	MSE	% Error
LA1	1778.53	18.935	35.46
LA2	1824.16	19.427	36.36

3 Results and Testing

In this section, the proposed method that construct IT2 Trap for FCM using both tuning methods is implemented in FIS. The output of FIS is then evaluated against the other FIS with input MFs from classic IT2FCM and trapezoidal input MFs optimized using GA (IT2 FCMTrap-GA). The proposed algorithm with the discussed two tuning methods are named IT2WTLA1-TRAP and IT2WTLA2-TRAP respectively.

Accuracy of the FIS are measured through several evaluation metrics, which are Classification accuracy, Precision, Recall, F-Measure. Confusion matrix is used to obtain the evaluation metrics. Other evaluation measurements to measure the performance are Mean Absolute Error (MAE) and Root Mean Square Error (RMSE).

Different tuning in the proposed method also being evaluated against each other to identify which tuning method has better performance. As for example, the calculation to get the percentage of difference is described in Eq. (5):

$$\%IT2\ WTLA1\text{-}TRAP/IT2\ WTLA2\text{-}TRAP = WTLA1\text{-}TRAP - WTLA2\text{-}TRAP/WTLA1\text{-}TRAP$$
$$\times 100$$

(5)

Table 3: IT2 trapMF after weak tuning and lateral adjustment

W	LA 1	LA2
0.10		
0.15		
0.18		
0.2		
0.25		
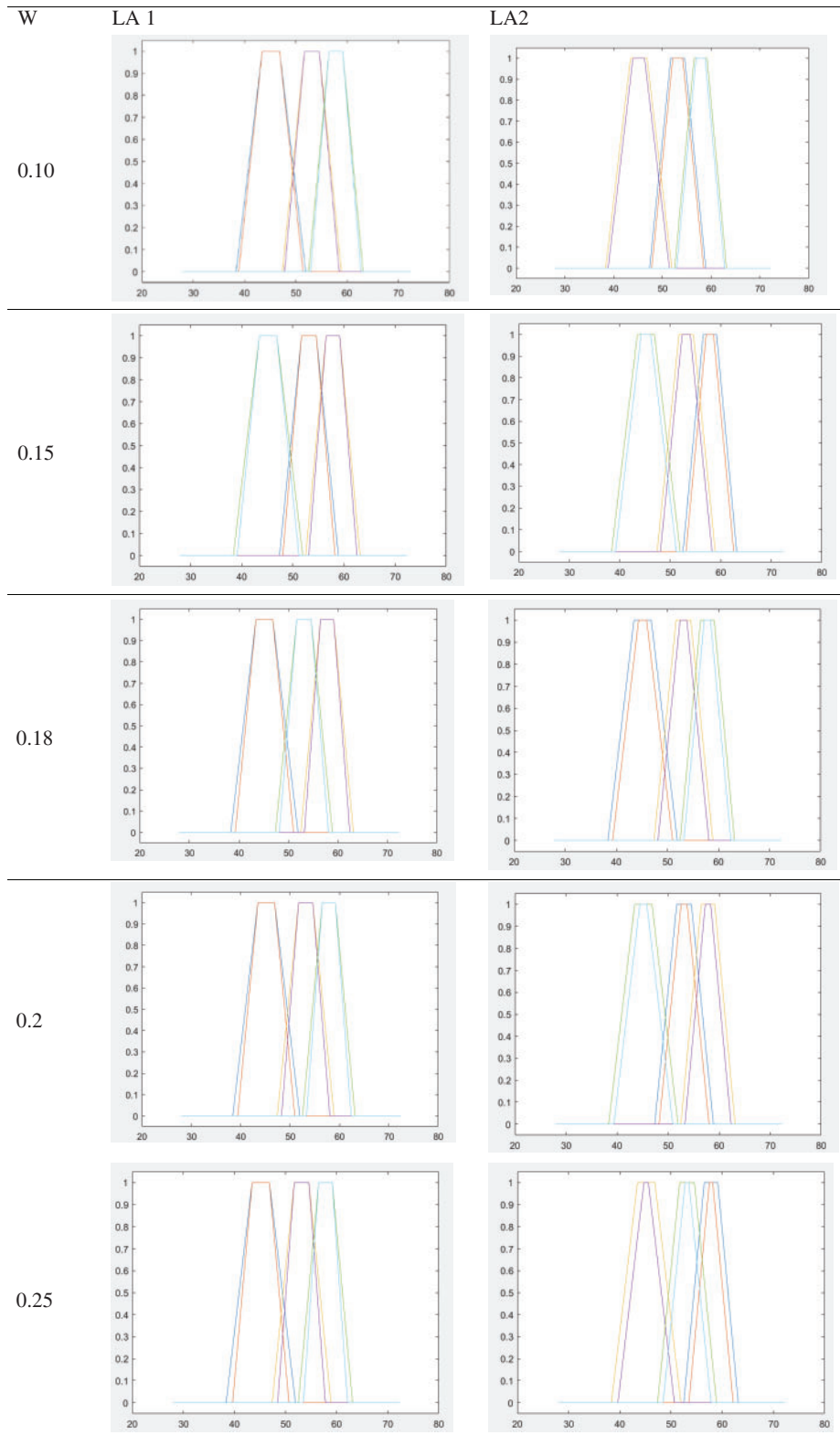
(Continued)

Table 3: Continued

| 0.5 | | |
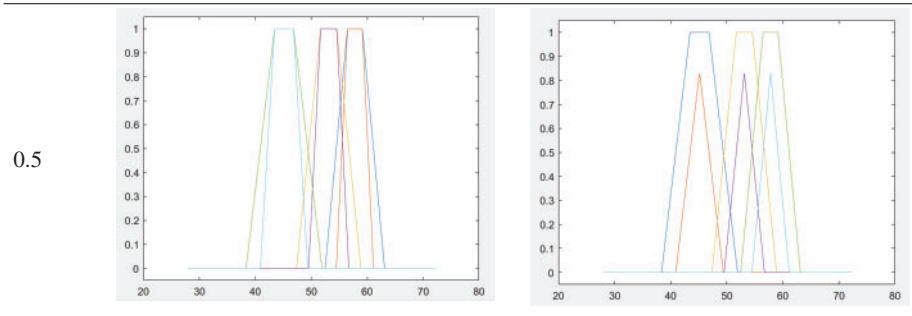
Table 4: FIS analysis result

	Accuracy	Precision	Recall	F-Measure
IRIS				
IT2 WTLA1-TRAP	70.331	69.342	79.633	74.132
IT2 WTLA2-TRAP	69.324	72.345	63.823	67.817
IT2 FCMTrap-GA	70.121	71.923	62.934	67.129
IT2FCM	64.233	69.012	60.123	64.262
% IT2 WTLA1-TRAP/IT2 WTLA2-TRAP	1.43	−4.33	19.85	8.52
% IT2 WTLA1-TRAP/IT2 FCMTrap-GA	0.30	−3.72	20.97	9.45
% IT2 WTLA1-TRAP/IT2FCM	8.67	0.48	24.50	13.31
% IT2 WTLA2-TRAP/IT2 FCMTrap-GA	−1.15	0.58	1.39	1.02
% IT2 WTLA2-TRAP/IT2FCM	7.34	4.61	5.80	5.24
ABALONE				
IT2 WTLA1-TRAP	87.332	78.233	75.231	76.703
IT2 WTLA2-TRAP	82.103	79.023	71.213	74.915
IT2 FCMTrap-GA	77.234	64.234	62.923	63.572
IT2FCM	66.123	60.623	59.444	60.028
% IT2 WTLA1-TRAP/IT2 WTLA2-TRAP	5.99	−1.01	5.34	2.33
% IT2 WTLA1-TRAP/IT2 FCMTrap-GA	1.29	1.56	1.59	1.57
% IT2 WTLA1-TRAP/IT2FCM	24.29	22.51	20.98	21.74
% IT2 WTLA2-TRAP/IT2 FCMTrap-GA	5.93	18.71	11.64	15.14
% IT2 WTLA2-TRAP/IT2FCM	19.46	23.28	16.53	19.87
HCV				
IT2 WTLA1-TRAP	67.623	71.274	78.233	74.592
IT2 WTLA2-TRAP	60.823	63.673	60.213	61.895
IT2 FCMTrap-GA	60.342	59.234	58.232	58.729
IT2FCM	60.344	60.456	59.001	59.720
% IT2 WTLA1-TRAP/IT2 WTLA2-TRAP	10.06	10.66	23.03	17.02
% IT2 WTLA1-TRAP/IT2 FCMTrap-GA	1.66	1.69	1.72	1.70
% IT2 WTLA1-TRAP/IT2FCM	10.76	15.18	24.58	19.94
% IT2 WTLA2-TRAP/IT2 FCMTrap-GA	0.79	6.97	3.29	5.12
% IT2 WTLA2-TRAP/IT2FCM	0.79	5.05	2.01	3.51

Referring to Tab. 4, the improvement of performance for the proposed algorithm is evident. In the tables, negative (−) values indicate lower values (lower performance) for the evaluation metrics. For example, in Iris dataset, IT2FCMtrap-GA has 1.15% higher accuracy than IT2 WTLA2-TRAP. The small difference in percentage shows less impact of performance evaluation since IT2 WTLA1-TRAP shows improvement in accuracy by 0.3% and 8.67% compared to IT2FCMtrap-GA and IT2FCM respectively. Looking at average performance for dataset Iris, IT2 WTLA1-TRAP has good improvement in term of accuracy compared to IT2 WTLA2-TRAP and the other two algorithms. Abalone and HCV datasets show promising performance for both IT2 WTLA1-TRAP and IT2 WTLA2-TRAP. IT2 WTLA1-TRAP has slightly lower values by 4.33% and 1.01% compared to IT2 WTLA2-TRAP for Iris and Abalone dataset, respectively. The precision performs better in dataset HCV with 10.66% higher precision. Tab. 5 shows the results of MAE and RMSE for all datasets. Shaded cells in the table indicate the lowest value. Cells with lighter shades indicate second lowest value. Two sets of datasets are evaluated: datasets without noise and datasets with 5% noise. Based on the results, IT2 WTLA1 TRAP obtains the lowest value of both RMSE and MAE by about 83.3% from all the values compared in the evaluation. IT2 WTLA1 TRAP achieves stronger performance by having all lower values compared to IT2 WTLA2 TRAP. This indicates the best tuning method for the proposed algorithm is IT2 WTLA1 TRAP. In overall observation, IT2 WTLA2 TRAP has promising performance whereby it achieves 75% lower MAE and RMSE values compared to IT2 FCM Trap-GA and IT2 FCM Gaussian. It is also proven that IT2 WTLA1 TRAP and IT2 WTLA2 TRAP are executing well even with noise. Both methods attain good performance with 5% noise with minimal decline of performance. As an example, MAE value for WTLA1 TRAP has increased value (less performance) between 1.2% and 14%. For RMSE, declined performance are between 0.2% and 17%.

Table 5: MAE and RMSE results

	MAE		RMSE	
IRIS	Without noise	5% Noise	Without noise	5% Noise
IT2 WTLA1 TRAP	0.4002	0.4122	0.4212	0.5092
IT2 WTLA2 TRAP	0.4023	0.4132	0.5024	0.5087
IT2 FCM Trap-GA	0.3923	0.4192	0.4942	0.5102
IT2 FCM Gaussian	0.4566	0.4551	0.5261	0.5257
ABALONE				
IT2 WTLA1 TRAP	0.0234	0.0124	0.1832	0.1867
IT2 WTLA2 TRAP	0.7234	0.7192	0.8232	0.8512
IT2 FCM Trap-GA	1.9168	1.9221	2.7440	2.7102
IT2 FCM Gaussian	2.2697	2.3023	2.8880	2.9023
HCV				
IT2 WTLA1 TRAP	22.8342	20.0421	23.6723	23.2823
IT2 WTLA2 TRAP	23.2368	23.1928	28.9316	28.9011
IT2 FCM Trap GA	22.7154	29.7639	55.8020	55.9350
IT2 FCM Gaussian	40.2624	40.2104	73.5748	73.5348

Statistical test is carried out after obtaining all results to further analyze the significance of the experiment results. One-way analysis of variance or ANOVA (Tab. 6) is utilized to identify

whether there is any statistically significant dissimilarity between the means of independent labels or groups. From Tab. 6, it is proven that the null hypothesis is rejected based on higher F Values as compared to F Critical values for all datasets. In addition, p value less than 0.05 is achieved which indicates that the test is statistically significant different.

Table 6: One-way ANOVA

Source of variation	SS	df	MS	F	P-value	F crit
IRIS						
Between groups	207.1816	3	69.06053	191.7236	5.14E−87	2.61985358
Within groups	214.6844	596	0.360209			
Total	421.866	599				
HCV						
Between groups	24014458	3	8004819	446.1219	2.02E−229	2.60868638
Within groups	42202218	2352	17943.12			
Total	66216676	2355				
ABALONE						
Between groups	79632.73	3	26544.24	2583.307	0.002	2.60544073
Within groups	171638.5	16704	10.2753			
Total	251271.3	16707				

4 Conclusions

In this paper, we presented a method to generate IT2 trapMF by using approximation of genetic weak tuning and lateral adjustment. The UMF is initially prepared, and the LMF is approximated by the combination of the two elements of genetic algorithm. The result shows that it successfully generates symmetrical trapMF with minimum W value of 0.10. From the results, it shows that weak tuning and lateral adjustment with two lower limit a and upper limit d of trapMF is more accurate than involving all four parameters of trapMF. The contribution of this work includes providing alternative IT2 MFs, constructed based on FCM outcomes, that can be used especially in the event that Gaussian MF is not performing well. Furthermore, the proposed approach provides an optimum way to construct the trapMF. By having more options of MFs, IT2 FIS can be applied to a wider range of practical applications such as prediction, analytics and rule-based solutions. However, further study on optimum W value can be done to generate different shapes of MF, either symmetrical or asymmetrical. Based on the forecasting results, it shows that it gives better performance in term of accuracy as compared to other methods. Another limitation of this work is that the proposed method is only tested on mamdani FIS. Furthermore, amplitude tuning, and lateral adjustments performed to construct IT2 Trap MF are applied simultaneously; separate execution of the two algorithms was not investigated.

There are several areas of improvement that can be incorporated in the present work and can be further extended. The possible future directions are discussed as follows:

(1) Different genetic tuning methods other than amplitude and lateral adjustments can be explored to assist on IT2 MF constructions for different shapes of MFs.

(2) Investigations on single amplitude tuning or lateral adjustments to construct IT2 trapMF with larger range without limited to weak ignorance function.

(3) Introducing a method that can heuristically produce the correct trapezoidal shapes given any types of data. The improvement in term of accuracy or precision, for example, can be further explored.

(4) Optimizing IT2 FIS results through the application of other than GA in MF generation processes.

(5) Investigating of the IT2 MFs generated by FCM being applied in the general fuzzy type 2 FIS. The performance effects in various general fuzzy type 2 applications can be further explored.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

References

[1] E. Rubio and O. Castillo, "Interval type-2 fuzzy clustering for membership function generation," in *2013 IEEE Workshop on Hybrid Intelligent Models and Applications (HIMA)*, Singapore, pp. 13–18, 2013.

[2] J. E. Rodríguez-Sánchez, M. G. Orozco-del-Castillo, A. Rodríguez-Castellanos, R. Ávila-Carrera and C. Valle-Molina, "A fuzzy inference system applied to estimate the error in the recovery of the Green's function by means of seismic noise correlations," *Journal of Geophysics and Engineering*, vol. 15, pp. 2110–2123, 2018.

[3] K. A. Ahmad, S. L. S. Abdullah, O. Mahmod and M. A. Bakar, "Induction of membership function and fuzzy rules for Harumanis classification," *Journal of Fundamental and Applied Sciences*, vol. 10, no. 1S, pp. 1202–1215, 2018.

[4] U. Ghani, I. Bajw and A. Ashfaq, "A fuzzy logic based intelligent system for measuring customer loyalty and decision making," *Symmetry*, vol. 10, no. 12, pp. 1–19, 2018.

[5] P. Pancardo, J. A. Hernández-Nolasco and F. Acosta-Escalante, "A fuzzy logic-based personalized method to classify perceived exertion in workplaces using a wearable heart rate sensor," *Mobile Information Systems*, vol. 2018, pp. 1–7, 2018.

[6] I. Subbotin, "Trapezoidal fuzzy logic model for learning assessment," 2014. [Online]. Available: https://www.researchgate.net/publication/263582488_Trapezoidal_Fuzzy_Logic_Model_for_Learning_Assessment.

[7] J. Tyagi and N. Verma, "Optimization of fuzzy C means clustering using genetic algorithm for an image," *International Journal of Computer Applications*, vol. 121, no. 17, pp. 29–32, 2015.

[8] S. H. Khairuddin, M. H. Hasan and M. A. Hashmani, "Integration of cluster centers and gaussian distributions in fuzzy c-means for the construction of trapezoidal membership function," *Mathematics and Statistics*, vol. 8, no. 5, pp. 559–565, 2020.

[9]   J. Sanz, A. Fernández, H. Bustince and F. Herrera, "A genetic tuning to improve the performance of fuzzy rule-based classification systems with interval-valued fuzzy sets: Degree of ignorance and lateral position," *International Journal of Approximate Reasoning*, vol. 52, no. 11, pp. 751–766, 2011.

[10] R. Alcalá, J. Alcalá-Fdez, M. J. Gacto and F. Herrera, "Rule base reduction and genetic tuning of fuzzy systems based on the linguistic 3-tuples representation," *Soft Computing*, vol. 11, no. 5, pp. 401–419, 2007.

[11] J. A. Sanz, A. Fernández, H. Bustince and F. Herrera, "Improving the performance of fuzzy rule-based classification systems with interval-valued fuzzy sets and genetic amplitude tuning," *Information Sciences*, vol. 180, no. 19, pp. 3674–3685, 2010.

[12] J. Qin and X. Lew, "Type-2 fuzzy sets and its extensions," *Type-2 Fuzzy Decision-Making Theories, Methodologies and Applications*, pp. 1–11, 2019.

[13] S. Guillaume, "Designing fuzzy inference systems from data: An interpretability-oriented review," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 3, pp. 426–443, 2001.

[14] A. Kowalczyk and A. Pelikant, "Implementation of automatically generated membership functions based on grouping algorithms," in *EUROCON, 2007—The Int. Conf. on Computer as a Tool*, Warsaw, Poland, 2007.

[15] P. N. Srinivasu, J. G. SivaSai, M. F. Ijaz, A. K. Bhoi, W. Kim *et al.,* "Classification of skin disease using deep neural networks with MobileNet V2 and LSTM," *Sensors*, vol. 21, no. 8, pp. 2852, 2021.

[16] R. Panigrahi, S. Borah, A. K. Bhoi, M. F. Ijaz, M. Pramanik *et al.,* "A consolidated decision tree-based instrution detection system for binary and multiclass imbalanced datasets," *Mathematics*, vol. 9, no. 7, pp. 751, 2021.

[17] J. Tamang, J. D. D. Nkapkop, M. F. Ijaz, P. K. Prasad, N. Tsafack *et al.,* "Dynamical properties of ion-acoustic waves in space plasma and its application to image encryption," *IEEE Access*, vol. 9, pp. 18762–18782, 2021.

[18] F. Gomide, *Uncertain rule-based fuzzy logic systems: Introduction and new directions*. Cham, Switzerland: Springer International Publishing, 2003.

[19] R. Alcalá, J. Alcalá-Fdez and F. Herrera, "A proposal for the genetic lateral tuning of linguistic fuzzy systems and its interaction with rule selection," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 4, pp. 616–635, 2007.

[20] D. Wu and M. Nie, "Comparison and practical implementation of type-reduction algorithms for type-2 fuzzy sets and systems," in *2011 IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE 2011)*, Taipei, Taiwan, 2011.

[21] J. M. Mendel, H. Hagras, W. W. Tan, W. W. Melek and H. Ying, *Introduction to type-2 fuzzy logic control: Theory and applications*. Hoboken, New Jersey, USA: Wiley-IEEE Press, 2014.

[22] C. E. Celemin and M. A. Melgarejo, "A proposal to speed up the computation of the centroid of an interval type-2 fuzzy set," *Advances in Fuzzy Systems*, vol. 2013, pp. 1–18, 2013.

[23] W. E. Sari, O. Wahyunggoro, S. Fauziati and A. I. Cahyadi, "State of charge estimation of lithium polymer battery using ANFIS and IT2FLS," in *Proc.—2016 2nd Int. Conf. on Science and Technology-Computer, ICST 2016*, Yogyakarta, Indonesia, 2017.

[24] C. Chen, R. John, J. Twycross and J. M. Garibaldi, "A direct approach for determining the switch points in the Karnik–Mendel algorithm," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 1079–1082, 2018.

[25] R. A. Fisher, "The use of multiple measurements in taxonomics problems," *Annals of Human Genetics*, vol. 7, no. 2, pp. 87–193, 1936.

[26] W. Nash, T. L. Sellers, S. R. Talbot, A. J. Cawthorn and W. B. Ford, "The Population Biology of Abalone (_Haliotis_ species) in Tasmania. I. Blacklip Abalone (_H. rubra_) from the North Coast and Islands of Bass Strait," Sea Fisheries Division, Technical Report No. 48, 1994.

[27] G. Hoffmann, A. Bietenbeck, R. Lichtinghagen and F. Klawonn, "Using machine learning techniques to generate laboratory diagnostic pathways—A case study," *Journal of Laboratory and Precision Medicine*, vol. 3, no. 6, pp. 1–10, 2018.

[28] T. Zhang, R. Ramakrishnan and M. Livny, "BIRCH: A new data clustering algorithm and its applications," *Data Mining and Knowledge Discovery*, vol. 1, pp. 141–182, 1997.

[29] P. Fränti, M. Rezaei and Q. Zhao, "Centroid index: Cluster level similarity measure," *Pattern Recognition*, vol. 47, no. 9, pp. 3034–3045, 2014.