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A Provably Secure and Efficient Remote Password Authentication Scheme Using Smart Cards

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Abstract: Communication technology has advanced dramatically amid the 21st century, increasing the security risk in safeguarding sensitive information. The remote password authentication (RPA) scheme is the simplest cryptosystem that serves as the first line of defence against unauthorised entity attacks. Although the literature contains numerous RPA schemes, to the best of the authors' knowledge, only few schemes based on the integer factorisation problem (IFP) and the discrete logarithm problem (DLP) that provided a provision for session key agreement to ensure proper mutual authentication. Furthermore, none of the previous schemes provided formal security proof using the random oracle model. Therefore, this study proposed an improved RPA scheme with session key establishment between user and server. The design of the proposed RPA scheme is based on the widely established Dolev-Yao adversary model. Moreover, as the main contribution, a novel formal security analysis based on formal definitions of IFP and DLP under the random oracle model was presented. The proposed scheme's performance was compared to that of other similar competitive schemes in terms of the transmission/computational cost and time complexity. The findings revealed that the proposed scheme required higher memory storage costs in smart cards. Nonetheless, the proposed scheme is more efficient regarding the transmission cost of login and response messages and the total time complexity compared to other scheme of similar security attributes. Overall, the proposed scheme outperformed the other RPA schemes based on IFP and DLP. Finally, the potential application of converting the RPA scheme to a user identification (UI) scheme is considered for future work. Since RPA and UI schemes are similar, the proposed approach can be expanded to develop a provably secure and efficient UI scheme based on IFP and DLP.

Keywords: Authentication scheme; discrete logarithm; factorisation; password; provable security

1 Introduction

In the 21st century, anything is possible on the internet by using applications and services, like operational networks, databases, banking services, and e-commerce, that are available to anyone, anywhere. Although users can enjoy access to the services remotely, the convenience offered is not without a cost. The communication between users and service providers often involves sensitive data dan messages being transmitted through insecure public channel. Furthermore, communication technology has progressed rapidly, thereby increasing the security risk security to protect private information. The remote password authentication (RPA) scheme is a cryptosystem that allows authorised users access to securely communicate with the service providers. Therefore, the RPA scheme serves as the first line of defence against dangerous security threats.

1.1 Related Works

In 1999, Yang et al. [1] proposed two RPA schemes with smart cards, using timestamp and nonce (random number used once). The schemes adopted the concept of an ID-based signature scheme by Shamir [2] without the need to maintain a password verification table. Furthermore, the schemes enabled users to easily select their passwords and demonstrated resistance to replay and forged login attacks. The schemes' security foundation was grounded on two cryptographic primitives: Integer Factorisation Problem (IFP) and Discrete Logarithm Problem (DLP). Nevertheless, some improved schemes [3–9] have been proposed to overcome the security concerns of Yang et al. [1] scheme while still maintaining the cryptographic primitives of IFP and DLP.

Fig. 1 presents the literature development of RPA schemes based on Yang et al. [1] scheme. The related works are defined as studies that have proposed improvements of RPA schemes and maintained the security foundation of IFP and DLP. These works are selected from the lists of citations and references of the previous studies. As an example, from Fig. 1, the enhancement scheme proposed by Shen et al. [4] was designed based on cryptanalysis of Yang et al. [1] scheme.

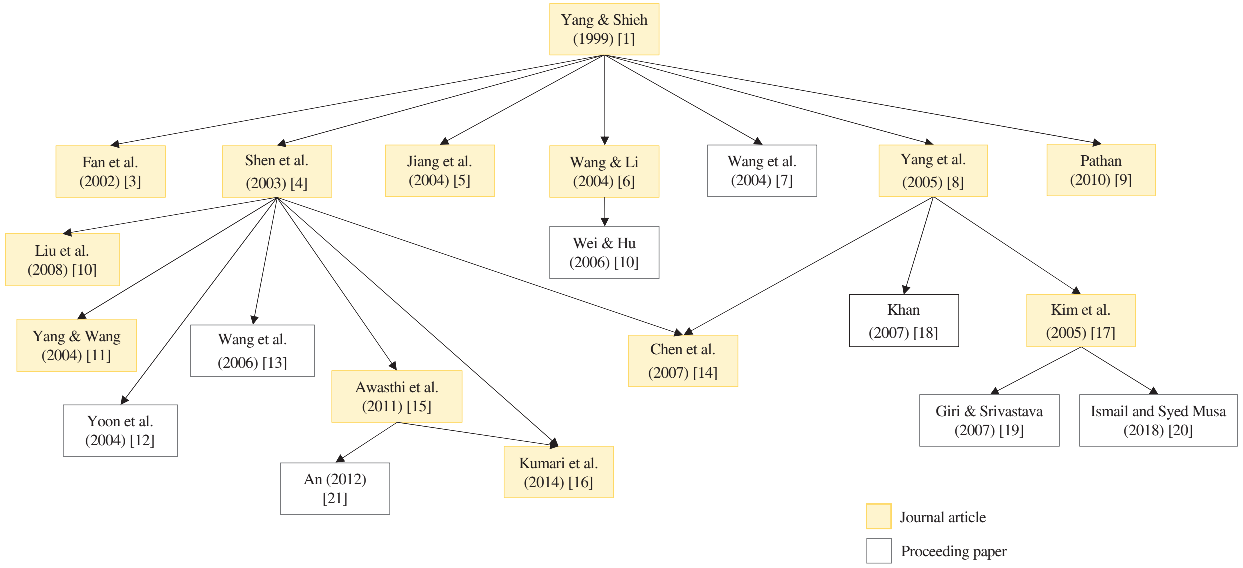


Figure 1: Development of RPA schemes based on Yang et al. [1] scheme using two cryptographic primitives (IFP and DLP)

Shen et al. [4] provided one of the most significant enhancements to scheme by Yang et al. [1], arguing that adversaries could exploit users' sensitive data through fake servers. As a result, the problem was rectified by incorporating mutual authentication between user and server. Nevertheless, the scheme was shown to be vulnerable to existing and novel security attacks, such as replay, secret-key guessing, and forgery attacks [10–13]. From there, numerous modifications [10–16] have been proposed. These studies reported their schemes to be more practical and efficient than earlier comparable schemes while maintaining a security basis of similar cryptographic primitives (i.e., IFP and DLP) during mutual authentication. Notably, Liu et al. [10] developed a novel nonce-based RPA scheme that could prevent forged login without incurring additional computational cost on the smart card.

Another notable contribution is the improved scheme by Yang et al. [8], which could withstand forgery, password-guessing, smart card loss, and replay attacks. Subsequently, Kim et al. [17] demonstrated that Yang et al. [8] scheme could not withstand previous forgery attacks. Later, Khan [18] demonstrated the vulnerabilities in [8] and presented an enhanced scheme with mutual authentication to address the problem. Nevertheless, other studies [19], [20] have shown that Kim et al. [17] is vulnerable to forgery attacks. As a result, Giri et al. [19] proposed a new scheme to resist the forgery attacks, as well as other types of threats, such as password-guessing, smart card loss, and replay attacks. The most recent related study by Ismail et al. [20] presented a new attack and proposed modifications to address the new threats.

Awasthi et al. [15] demonstrated that the scheme by Shen et al. [4] is vulnerable to forged login attacks and presented additional security concerns about the scheme by Liu et al. [10]. Hence, Awasthi et al. [15] proposed an enhanced scheme for resisting forgery attacks with reduced smart card memory storage cost. Unfortunately, the scheme was shown to be vulnerable to impersonation, insider, and password-guessing attacks by An [21], which also suggested improvements to make the scheme more secure to resist all of the mentioned attacks while supporting mutual authentication. Furthermore, Kumari et al. [16] highlighted that scheme proposed by Awasthi et al. [15] could not resist the claimed attacks. Therefore, they recommended a three-factor scheme authentication improvement with the added security of the user's fingerprint.

Kumari et al. [16] proposed the latest RPA scheme construction based on IFP and DLP. The study was the first to introduce a scheme that included a shared session key between the user and the server to eliminate the man-in-the-middle attack, accompanied by the most comprehensive and informal security analysis. The proposed scheme was shown to be resistant to many security attacks, including the smart card loss, replay, impersonation, forgery, offline password-guessing, denial-of-service, insider, and stolen verifier attacks. Nevertheless, the scheme's computational and communication costs were the highest among all the schemes in Fig. 1.

1.2 Motivation and Contributions

Security analysis, like that of other cryptosystems, is imperative in developing new RPA schemes. Although numerous RPA schemes based on IFP and DLP have been proposed in the literature, none of them provides security proof under the random oracle model. The security proof requirement has been fulfilled by many schemes constructed based on other cryptographic primitives in the literature, such as IFP [22], Elliptic Curve Discrete Logarithm Problem (ECDLP) [23], and chaotic maps [24]. Although the study by Kumari et al. [16] featured many security attributes, no formal security proof of its scheme was presented. Consequently, despite being the most secure among similar works, the proposed scheme had to sacrifice its performance efficiency. Therefore, the purpose of this study is two-fold. First, the aim of this study is to propose an efficient RPA scheme with session key agreement based on two cryptographic primitives (IFP and DLP). Next, the main contribution of this study is to present a formal security analysis based on the formal definitions of IFP and DLP under the random oracle model to prove the security of the proposed scheme.

1.3 Organisation of the Paper

The remainder of this paper is organised as follows. Section 2 presents the mathematical and security preliminaries. Section 3 then explains the newly proposed scheme. Section 4 presents the proposed scheme's formal and informal security analyses. Section 5 provides a comparative study of the previous schemes of [4,15,16], and the present scheme. Section 6 discusses how the RPA scheme could be used to develop a user identification (UI) scheme. Finally, Section 7 presents the conclusion and recommendation.

2 Preliminaries

This section provides a brief overview of the mathematical concepts that served as the security foundation in the development of the proposed scheme in this study, including the definitions of IFP [25], DLP [26], and the one-way hash function (e.g., MD5 [27] or SHA-256 [28]). The adversary model and security goals were also considered. Tab. 1 shows the notations and descriptions used in this paper.

Table 1: Notations and descriptions

Notation	Description
KIC	Key information centre
U_i	User i
S	Server
ID_i	Identity of user U_i
pw_i	Password of user U_i
b_i, r_i	Random integers of 160-bit length
$n = p \times q$	Public parameter of 2048-bit length, where $p = 2p_1 + 1$ and $q = 2q_1 + 1$ are 1024-bit primes, with p_1 and q_1 are large primes
e	Public-key of server S generated by KIC
d	Private key of server S generated by KIC
x	Secret parameter of server S generated by KIC
g	Primitive element of both finite prime fields $\mathbb{F}_p$ and $\mathbb{F}_q$
CID_i	Identity of user U_i's smart card
DID_i	Dynamic identity of user U_i
S_i	Secret information of user U_i
SC_i	Smart card of user U_i
$h(\cdot)$	Cryptographic one-way hash function, $h : \{0,1\}^* \rightarrow \{0,1\}^l$
T_U	Timestamp of user U_i sends login request
T_S	Timestamp of server S receives the login request
T_c	Timestamp of user U_i receives response request
ΔT	Maximum time interval for transmission delay

(Continued)

Table 1: Continued

Notation	Description
SK	Session key shared between user U_i and server S
$\Rightarrow$	Secure channel
$\rightarrow$	Public channel
$\|$	String concatenation operator
$\oplus$	Bit-wise exclusive OR (XOR) operator

2.1 IFP

Given a 2048-bit integer $n = p \times q$, find the primes p and q that are each at least 1024-bit length. If p and q are known, it will be easy to compute n. Finding p and q given n, on the other hand, is a computationally intractable problem.

2.2 DLP

Assume that g is a primitive element of a finite field $\mathbb{F}_p$ with order p. Consider the equation,

$$\beta = g^{\alpha} \bmod p, \text{ for } 1 \leq \alpha \leq p - 1. \tag{1}$$

Given g, α, and p, calculating the modular exponentiation $\beta = g^{\alpha} \bmod p$ is trivial. However, finding the exponent α given g, β, and p, it is computationally infeasible.

DLP is defined over a multiplicative group $\mathbb{Z}_n^*$ where $n = p \times q$ of order $\phi(n) = (p - 1)(q - 1)$. Consider the equation,

$$\beta = g^{\alpha} \bmod n, \text{ for } 1 \leq \alpha \leq \phi(n). \tag{2}$$

If the factorisation of order $\phi(n)$ is known and $\phi(n)$ has (small) prime factors, an instance of the DLP in $\mathbb{Z}_n^*$ can be reduced to two instances of the DLP in $\mathbb{Z}_p^*$ and $\mathbb{Z}_q^*$ using Pohlig et al. [29] algorithm. Nevertheless, it is believed that finding the exponent α is intractable for DLP in the multiplicative groups of finite fields [30].

2.3 Hash Function

A cryptographic one-way hash function h: $X = \{0, 1\}^* \rightarrow Y = \{0, 1\}^l$ has the following properties.

- The function h takes an arbitrary length input $x \in X$ and returns a fixed l-bit length message digest $y \in Y$.
- The function h is one-way; that is, given the input x, computing $h(x) = y$ is trivial. However, given y, it is computationally infeasible to find the inverse $h^{-1}(y) = x$.
- The function h is collision-resistant, which means that finding two inputs $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$ is computationally infeasible.

The SHA-256 hash function was adopted for the proposed scheme. Other secure hash algorithms, such as SHA-1, SHA-224, SHA-384, SHA-512, and SHA-512/256 [28], can also be implemented.

2.4 Adversary Model

For communications over an insecure public channel, the Dolev et al. [31] adversary model was considered. Accordingly, the following assumptions were made.

- *Assumption A1*: An adversary $\mathcal{A}$ can trap, delete, or alter the transmitted messages.
- *Assumption A2*: An adversary $\mathcal{A}$ can obtain the stored information in the smart card using power monitoring techniques [32,33].
- *Assumption A3*: An adversary $\mathcal{A}$ can guess the identity or password using the dictionary attack. However, the adversary $\mathcal{A}$ cannot guess the identity and password simultaneously using any online/offline attacks within polynomial time [34].

According to this adversary model, the following two cases as per [35] were also taken into account.

- *Case 1*: An adversary $\mathcal{A}$ can be a non-registered user who tries to perform various attacks against the authentication system.
- *Case 2*: An adversary $\mathcal{A}$ can be a registered user who tries to obtain the secret parameters of the server by which he/she can mount various attacks against the authentication system.

2.5 Security Goals

The following are the security goals of an ideal RPA scheme defined in this study that should be achieved, as listed in [36].

- *Mutual authentication*: Both the server and the user can verify the legitimacy of each other. Furthermore, no illegal users or servers can impersonate a legal user or server.
- *Session key agreement*: A session key should be created at the end of a successful mutual authentication process. Subsequently, the data transmitted between both entities should be encrypted to ensure confidentiality and secrecy.
- *User anonymity*: During data transmission over a public channel, a user's valid identity should be concealed. Even if adversary $\mathcal{A}$ can analyse login information or gain access to services, user anonymity protects user's sensitive data, such as personal details, financial information, and social circles, from unauthorised parties.

3 Proposed Scheme

This section presents the proposed RPA scheme based on the security of IFP and DLP and consisted of five phases: (1) initialisation phase, (2) registration phase, (3) login phase, (4) authentication phase, and (5) password change phase. Furthermore, three entities were also considered: KIC, user U_i, and server S. In this scheme, the KIC is a trusted authority responsible for generating global parameters, computing user and server secret information, and providing new users with smart cards.

3.1 Initialisation Phase

The KIC sets up the server's public and secret parameters during the initialisation phase.

1. Generate two large primes $p = 2p_1 + 1$ and $q = 2q_1 + 1$ of 1024-bit length, where p_1 and q_1 are both primes.
2. Compute $n = p \times q$ and $\phi(n) = (p - 1) \cdot (q - 1)$.
3. Find a prime number e and integer d such that $e \cdot d \equiv 1 \bmod \phi(n)$, where e is the server S's public-key and d is the corresponding private key.

4. Find an integer g, which is a primitive element for both finite prime fields $\mathbb{F}_p$ and $\mathbb{F}_q$.
5. Decide on a secret parameter $x \in \mathbb{Z}^*_{p_1}$ or $\mathbb{Z}^*_{q_1}$ for server S and the format for identity ID of a user.
6. $KIC \Rightarrow S\ : \{d,\ x,\ ID\, format\}$.

The private key d, secret parameter x, and format of a user's ID should be safely provided to the server S. KIC is no longer needed once the system is set up, except during the registration phase when new users request to join. The integer pair p and q will not be used anymore and should be disposed of securely.

3.2 Registration Phase

In the registration phase, a new user U_i performs the following steps.

1. Choose the identity ID_i and the password pw_i.
2. Generate a random integer b_i of 160-bit length.
3. Compute $hpw_i\ =\ h(pw_i||b_i)$.
4. $U_i \Rightarrow$ KIC: $\{ID_i, hpw_i\}$.

The KIC then performs the following steps.

5. Generate $CID_i\ = h(ID_i||x)$.
6. Compute $w = h(d||x)$ and $v_i = w \oplus h(ID_i\ ||hpw_i)$.
7. Compute $h_i = g^{ID_i \cdot d \cdot x}$ mod n and $S_i = CID_i^d$ mod n.
8. Compute $j_i = h_i\ \oplus h(ID_i\ ||hpw_i)$.
9. KIC $\Rightarrow U_i : SC_i = \{n, e, g, j_i, v_i, S_i, h_i\}$.

After receiving the smart card SC_i, U_i performs the following steps.

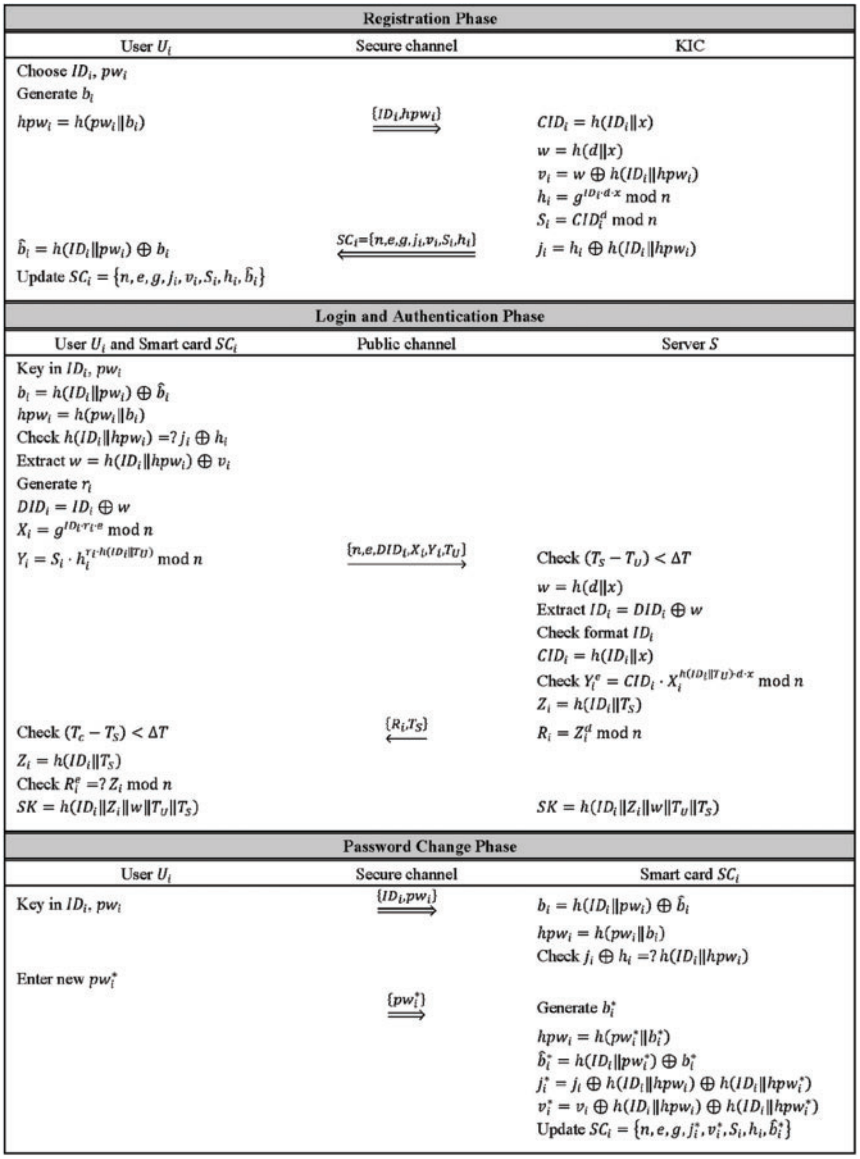
10. Compute $\hat{b}_i = h(ID_i\ ||pw_i) \oplus b_i$.
11. Update the smart card $SC_i = \{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$.

Fig. 2 depicts an overview of the proposed RPA scheme's phases.

3.3 Login Phase

When a registered user U_i wants to access the server S, the user U_i inserts the smart card SC_i into a remote terminal. The user then enters the identity ID_i and password pw_i. The following steps are taken by the smart card SC_i.

1. Extract b_i by computing $b_i = h(ID_i||pw_i) \oplus \hat{b}_i$.
2. Compute $hpw_i = h(pw_i||b_i)$.
3. Check $j_i \oplus h_i =?h(ID_ihpw_i)$. If the equation holds, server S believes that the user U_i is a valid user. Otherwise, the login request should be aborted.
4. Extract w by computing $w = h(ID_ihpw_i) \oplus v_i$.
5. Generate a random integer r_i of 160-bit length.
6. Compute dynamic identity of U_i, $DID_i = ID_i \oplus w$.
7. Compute $X_i = g^{ID_i \cdot r_i \cdot e}$ mod n and $Y_i = S_i \cdot h_i^{r_i \cdot h(ID_i||T_U)}$ mod n, where T_U is the timestamp of the user U_i when the login request is submitted.
8. $U_i \rightarrow S$: Login message $= \{n, e, DID_i, X_i, Y_i, T_U\}$.



Registration Phase

User U_i	Secure channel	KIC
Choose ID_i, pw_i		
Generate b_i		
$hpw_i = h(pw_i \Vert b_i)$	$\xRightarrow{\{ID_i,hpw_i\}}$	$CID_i = h(ID_i \Vert x)$
		$w = h(d \Vert x)$
		$v_i = w \oplus h(ID_i \Vert hpw_i)$
		$h_i = g^{ID_i \cdot d \cdot x} \bmod n$
		$S_i = CID_i^d \bmod n$
$\hat{b}_i = h(ID_i \Vert pw_i) \oplus b_i$	$\xLeftarrow{SC_i=\{n,e,g,j_i,v_i,S_i,h_i\}}$	$j_i = h_i \oplus h(ID_i \Vert hpw_i)$
Update $SC_i = \{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$		

Login and Authentication Phase

User U_i and Smart card SC_i	Public channel	Server S
Key in ID_i, pw_i		
$b_i = h(ID_i \Vert pw_i) \oplus \hat{b}_i$		
$hpw_i = h(pw_i \Vert b_i)$		
Check $h(ID_i \Vert hpw_i) =? j_i \oplus h_i$		
Extract $w = h(ID_i \Vert hpw_i) \oplus v_i$		
Generate r_i		
$DID_i = ID_i \oplus w$		
$X_i = g^{ID_i \cdot r_i \cdot e} \bmod n$		
$Y_i = S_i \cdot h_i^{r_i \cdot h(ID_i \Vert T_U)} \bmod n$	$\xrightarrow{\{n,e,DID_i,X_i,Y_i,T_U\}}$	Check $(T_S - T_U) < \Delta T$
		$w = h(d \Vert x)$
		Extract $ID_i = DID_i \oplus w$
		Check format ID_i
		$CID_i = h(ID_i \Vert x)$
		Check $Y_i^e = CID_i \cdot X_i^{h(ID_i \Vert T_U) \cdot d \cdot x} \bmod n$
		$Z_i = h(ID_i \Vert T_S)$
Check $(T_c - T_S) < \Delta T$	$\xleftarrow{\{R_i,T_S\}}$	$R_i = Z_i^d \bmod n$
$Z_i = h(ID_i \Vert T_S)$		
Check $R_i^e =? Z_i \bmod n$		
$SK = h(ID_i \Vert Z_i \Vert w \Vert T_U \Vert T_S)$		$SK = h(ID_i \Vert Z_i \Vert w \Vert T_U \Vert T_S)$

Password Change Phase

User U_i	Secure channel	Smart card SC_i
Key in ID_i, pw_i	$\xRightarrow{\{ID_i,pw_i\}}$	$b_i = h(ID_i \Vert pw_i) \oplus \hat{b}_i$
		$hpw_i = h(pw_i \Vert b_i)$
		Check $j_i \oplus h_i =? h(ID_i \Vert hpw_i)$
Enter new pw_i^*		
	$\xRightarrow{\{pw_i^*\}}$	Generate b_i^*
		$hpw_i = h(pw_i^* \Vert b_i^*)$
		$\hat{b}_i^* = h(ID_i \Vert pw_i^*) \oplus b_i^*$
		$j_i^* = j_i \oplus h(ID_i \Vert hpw_i) \oplus h(ID_i \Vert hpw_i^*)$
		$v_i^* = v_i \oplus h(ID_i \Vert hpw_i) \oplus h(ID_i \Vert hpw_i^*)$
		Update $SC_i = \{n, e, g, j_i^*, v_i^*, S_i, h_i, \hat{b}_i^*\}$

Figure 2: Overview of the proposed RPA scheme

3.4 Authentication Phase

Once the server S receives the login message request at the time T_S, it proceeds with the following steps.

1. Check $(T_S - T_U) < \Delta T$, where ΔT is the allowed time transmission. If the time difference does not hold, the login request is rejected.
2. Compute $w = h(d||x)$.
3. Extract ID_i by computing $ID_i = DID_i \oplus w$.
4. Check the validity of the format for ID_i. If the format of ID_i is invalid, the login request is rejected.

5. Compute $CID_i = h(ID_i||x)$.
6. Check $Y_i^e = CID_i \cdot X_i^{h(ID_i||T_U)\cdot d\cdot x} \bmod n$. If the equation does not hold, the login request is rejected.
7. Otherwise, compute $Z_i = h(ID_i||T_S)$ and $R_i = Z_i^d \bmod n$.
8. $S \rightarrow U_i$: Response message $= \{R_i, T_S\}$.

Once the user U_i receives the response message at the time T_c, the user then performs the following steps.

9. Check $(T_c - T_S) < \Delta T$. Disconnect from the server S if the time difference does not hold.
10. Compute $Z_i = h(ID_i||T_S)$.
11. Check $R_i^e =?Z_i \bmod n$. Disconnect from the server S if the equation does not hold.
12. If mutual authentication is successful, the session key $SK = h(ID_iZ_iwT_UT_S)$ is agreed upon between the user U_i and server S.

Once the session key SK is established, the user U_i and server S can communicate with each other immediately. This step completes the mutual authentication process and eliminates the risk of the man-in-the-middle attack.

3.5 Password Change Phase

This phase enables the user U_i to change or update the password independently without interacting with the KIC or the server S. When changing the password, the user U_i inserts the smart card SC_i into the terminal and enters the identity ID_i and password pw_i. The following steps are conducted by the smart card SC_i.

1. Extract b_i by computing $b_i = h(ID_i||pw_i) \oplus \hat{b}_i$.
2. Compute $hpw_i = h(pw_i||b_i)$.
3. Check $j_i \oplus h_i =?h(ID_i||hpw_i)$. If the equation holds, the smart card SC_i believes the user U_i is a valid user and requests for a new password pw_i^*. Otherwise, the password change request is rejected.

Once the user U_i enters the new password pw_i^*, the smart card SC_i performs the following steps.

4. Generate a random integer b_i of 160-bit length.
5. Compute $hpw_i = h(pw_i^*||b_i^*)$ and $\hat{b}_i^* = h(ID_i||pw_i^*) \oplus b_i^*$.
6. Compute $j_i^* = j_i \oplus h(ID_i||hpw_i) \oplus h(ID_i||hpw_i^*)$ and $v_i^* = v_i \oplus h(ID_i||hpw_i) \oplus h(ID_i||hpw_i^*)$.
7. Replace $\hat{b}_i, j_i$, and v_i with $\hat{b}_i^*, j_i^*$, and v_i^*, respectively.
8. Update the smart card $SC_i = \{n, e, g, j_i^*, v_i^*, S_i, h_i, \hat{b}_i^*\}$.

If user U_i's smart card SC_i is lost or stolen, the user U_i must re-register with the KIC. Then, the KIC should issue a new smart card for the user U_i following the steps outlined in the registration phase.

3.6 Proof of Correctness

The propositions and proofs of correctness are presented below for the sake of completeness of the proposed scheme.

Proposition 1. If user U_i enters the correct identity ID_i and password pw_i, and Steps 1 and 2 of the login phase run well, the local user verification equation in Step 3 of the login phase will always hold. The proof is shown below.

$$
\begin{aligned}
j_i \oplus h_i &= (h_i \oplus h(ID_i||hpw_i)) \oplus h_i \\
&= h(ID_i||hpw_i) \oplus h_i \oplus h_i \\
&= h(ID_i||hpw_i)
\end{aligned}
$$

Proposition 2. If all the login phase steps and Steps 1–5 of the authentication phase run well, and the login message $\{n, e, DID_i, X_i, Y_i, T_U\}$ is properly generated, then the user authentication equation in Step 6 of the authentication phase will always hold, as shown below.

$$
\begin{aligned}
Y_i^e &= (S_i \cdot h_i^{r_i \cdot h(ID_i||T_U)})^e \bmod n \\
&= (S_i)^e \cdot (h_i^{r_i \cdot h(ID_i||T_U)})^e \bmod n \\
&= (CID_i^d)^e \cdot ((g^{ID_i \cdot d \cdot x})^{r_i \cdot h(ID_i||T_U)})^e \bmod n \\
&= CID_i \cdot (g^{ID_i \cdot r_i \cdot e})^{h(ID_i||T_U) \cdot d \cdot x} \bmod n \\
&= CID_i \cdot X_i^{h(ID_i||T_U) \cdot d \cdot x} \bmod n
\end{aligned}
$$

Proposition 3. If all the steps in the authentication phase (Steps 1–10) run well and the response message $\{R_i, T_S\}$ is properly generated, then the server authentication equation in Step 11 of the authentication phase will always be true, as shown below.

$$
\begin{aligned}
R_i^e &= (h(ID_i||T_S)^d)^e \bmod n \\
&= h(ID_i||T_S) \bmod n \\
&= Z_i \bmod n
\end{aligned}
$$

4 Security Analysis of the Proposed Scheme

This section presents the formal security proof that the proposed scheme is provably secure against an adversary $\mathcal{A}$ for deriving the private key d, secret parameter x, identity ID_i, password pw_i, and shared session key SK. The proposed scheme is also shown to provide the desired security attributes.

4.1 Formal Security Proof

The formal security analysis of the proposed scheme, which is based on the random oracle model, is explained below. Specifically, the proposed scheme's formal security proof adopted the approach taken by [22,37–39]. To begin, the formal definitions of the collision-resistant cryptographic one-way hash function [39], IFP [22] and DLP [40,41] are given.

Definition 1. A secure collision-resistant one-way hash function

$$h : \{0, 1\}^* \rightarrow \{0, 1\}^l \tag{3}$$

is a deterministic algorithm that takes an arbitrary length input $x \in \{0, 1\}^*$ binary string and yields a fixed-length l-bit binary string output $h(x) \in \{0, 1\}^l$.

An adversary $\mathcal{A}$'s advantage in finding a collision is given as

$$Adv_{\mathcal{A}}^{Hash}(t_1) = \Pr[(x_1, x_2) \Leftarrow_R \mathcal{A} : x_1 \neq x_2 \wedge \ h(x_1) = h(x_2)], \tag{4}$$

where $\Pr[E]$ is the probability of an event E in a random experiment and $(x_1, x_2) \Leftarrow_R \mathcal{A}$ denotes a randomly selected pair (x_1, x_2) by the adversary $\mathcal{A}$. As a result, the adversary $\mathcal{A}$ can be probabilistic. The adversary $\mathcal{A}$ computes the probability in the advantage over the random choices with the execution time t_1. If $Adv_{\mathcal{A}}^{Hash}(t_1) \leq \varepsilon_1$, for any sufficiently small $\varepsilon_1 > 0$, the one-way function $h(\cdot)$ is collision-resistant.

Definition 2. Assume that Gen_F is a polynomial time algorithm with input security parameter 1^ρ and outputs (n, p, q), where $n = p \times q$, and p and q are ρ-bit distinct primes. Given n, the integer factorisation assumption relative to Gen_F states that it is computationally infeasible to derive the prime factors p and q, except with a negligible probability in ρ.

For any adversary $\mathcal{A}$ of probabilistic-polynomial time (PPT), its factorisation advantage is given by

$$Adv_{Gen_F,\mathcal{A}}^{IFP}(\rho) = \Pr[(n, p, q) \Leftarrow Gen_F(1^\rho) : \mathcal{A}(n) = \{p, q\}]. \tag{5}$$

The integer factorisation assumption states that $Adv_{Gen_F,\mathcal{A}}^{IFP}(\rho)$ is negligible in ρ for every PPT adversary $\mathcal{A}$. That is, the $(t_{IFP}, \varepsilon_{IFP})$-IFP assumption holds if $Adv_{Gen_F,\mathcal{A}}^{IFP}(\rho) \leq \varepsilon_{IFP}(\rho)$, for any sufficiently small $\varepsilon_{IFP} > 0$ with its running time is at most t_{IFP}.

Definition 3. Let G be an order n cyclic group, g be a generator of G, and A be an algorithm that returns an integer in $\mathbb{Z}_n$. The following experiment $EXP_{G,g}^{DLP}(A)$ in Algorithm 1 is considered.

Algorithm 1: $EXP_{G,g}^{DLP}(A)$

```
Select $u \in_R \mathbb{Z}_n$
Compute $U \leftarrow g^u \bmod n$
Compute $u^* \leftarrow A(U)$
if $g^{u^*} = U \bmod n$ then
    return 1 (Success)
else
    return 0 (Failure)
end if
```

The DLP advantage of algorithm A with execution time t is defined as

$$Adv_{G,g}^{DLP}(t) = \Pr[EXP_{G,g}^{DLP}(A) = 1]. \tag{6}$$

If the DLP advantage of any adversary $\mathcal{A}$ in terms of time complexity is small, the DLP is hard in G. Hence, DLP is computationally infeasible if $Adv_{G,g}^{DLP}(t) \leq \varepsilon_{DLP}$, for any sufficiently small $\varepsilon_{DLP} > 0$.

For this security proof, the adversary $\mathcal{A}$ is assumed to have access to the following three random oracles listed below.

- $\mathcal{O}racle\mathcal{H}$: This oracle outputs the string x from a hash value $y = h(x)$.
- $\mathcal{O}racle\mathcal{F}$: This oracle outputs the private key d of the server S from the values n and e.
- $\mathcal{O}racle\mathcal{D}$: This oracle outputs the value $x \in \mathbb{Z}_n$ from the value $h = g^x \bmod n$, where g is the generator in G of order n.

The three theorems and proof of formal security analysis are then presented as follows.

Theorem 1. If the cryptographic one-way hash function $h(\cdot)$ *behaves like a true random oracle, and integer factorisation and discrete logarithm are computationally hard problems, then the proposed RPA scheme is provably secure against an adversary $\mathcal{A}$ for deriving the private key d and secret parameter x of server S.*

Proof. Initially, an adversary $\mathcal{A}$ is constructed with the ability to derive private key d and secret parameter x of the server S by running the algorithm $ALG^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}$, as shown in Algorithm 2 for the proposed RPA scheme. By *Assumption A2,* suppose that the adversary $\mathcal{A}$ can extract $\{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$ from the smart card using power monitoring techniques. By *Assumption A1*, it is further assumed that the adversary $\mathcal{A}$ intercepts login message $\{n, e, DID_i, X_i, Y_i, T_U\}$ and response message $\{R_i, T_S\}$ at the time T_U and T_S, respectively.

Algorithm 2: $ALG^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}$ for deriving private key d and secret parameter x of server S

```
Input: $n$, $e$, $g$, $j_i$, $v_i$, $S_i$, $h_i$, $DID_i$, $X_i$, $Y_i$, $T_U$
Output: 0 or 1
Compute $CID_i = S_i^e \bmod n$
Call $\mathcal{O}racle\mathcal{H}$ on input $CID_i$ to retrieve identity $ID_i^*$ and secret parameter $x^*$ as $(ID_i^*||x^*) \leftarrow \mathcal{O}racle\mathcal{H}(CID_i)$
Compute $h(ID_i||hpw_i) = j_i \oplus h_i$
Call $\mathcal{O}racle\mathcal{H}$ on input $h(ID_i||hpw_i)$ to retrieve identity $ID_i^{**}$ and $hpw_i^*$ as $(ID_i^{**}||hpw_i^*) \leftarrow \mathcal{O}racle\mathcal{H}(h(ID_i||hpw_i))$
if $ID_i^* \neq ID_i^{**}$ then
    return 0 (Failure)
else
    Compute $w = h(ID_i||hpw_i) \oplus v_i$ and $w^* = DID_i \oplus ID_i^*$
    Call $\mathcal{O}racle\mathcal{H}$ on input $w$ to retrieve the private key $d^*$ and secret parameter $x^{**}$ as $(d^*||x^{**}) \leftarrow \mathcal{O}racle\mathcal{H}(w)$
    Call $\mathcal{O}racle\mathcal{F}$ on input $e$ and $n$ to retrieve the private key $d^{**}$ as $d^{**} \leftarrow \mathcal{O}racle\mathcal{F}(e, n)$
    if $w = w^*$ and $d^* = d^{**}$ then
        Call $\mathcal{O}racle\mathcal{D}$ on input $h_i$, $g$, $ID_i^*$, and $d^*$ to retrieve the secret parameter $x^{***}$ as $x^{***} \leftarrow \mathcal{O}racle\mathcal{D}(h_i, g, ID_i^*, d^*)$
        if $x^* = x^{**} = x^{***}$ then
            if $Y_i^e = CID_i \cdot X_i^{h(ID_i||T_U)\cdot d\cdot x} \bmod n$ then
                Accept $d^*$ and $x^*$ as the correct private key and secret parameter of server $S$, respectively
                return 1 (Success)
            else
                return 0 (Failure)
            end if
        else
            return 0 (Failure)
        end if
    else
        return 0 (Failure)
    end if
end if
```

The success probability of $ALG^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}$ is given by $Succ^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}} = 2\Pr[ALG^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}} = 1] - 1$. Additionally, the advantage for $ALG^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}$ is given by $Adv^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}(t, q_H, q_F, q_D) = \max_{\mathcal{A}}\{Succ^{\mathcal{O}racle}_{\mathcal{A},\mathcal{PAS}}\}$, where the maximum of all adversary $\mathcal{A}$ is taken with execution time t and the number of queries q_H, q_F, and q_D made to $\mathcal{O}racle\mathcal{H}$, $\mathcal{O}racle\mathcal{F}$, and $\mathcal{O}racle\mathcal{D}$, respectively.

According to $ALG_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$, if an adversary $\mathcal{A}$ could obtain the inverse of the cryptographic one-way hash function $h(\cdot)$ and solve the IFP and DLP, then the adversary $\mathcal{A}$ can successfully find the private key d and secret parameter x of server S using the oracles $\mathcal{O}racle\mathcal{H}$, $\mathcal{O}racle\mathcal{F}$, and $\mathcal{O}racle\mathcal{D}$, and wins the game. However, it is computationally infeasible for the adversary $\mathcal{A}$ as the advantage is negligible in polynomial time. By Definitions 1, 2, and 3, $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{H}}(t_1) \leq \varepsilon_1$, $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{F}}(t_2) \leq \varepsilon_2$, and $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{D}}(t_3) \leq \varepsilon_3$, for any sufficiently small $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3 > 0$. Since $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H, q_F, q_D)$ depends on all $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{H}}(t_1)$, $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{F}}(t_2)$, and $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{D}}(t_3)$, it must be that $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H, q_F, q_D) \leq \varepsilon$, for any sufficiently small $\varepsilon > 0$. As a result, the theorem is proven.

Theorem 2. If the cryptographic one-way hash function $h(\cdot)$ *behaves like a true random oracle, then the proposed RPA scheme is provably secure against an adversary* $\mathcal{A}$ *for deriving the identity* ID_i *and password* pw_i *of user* U_i.

Proof. An adversary $\mathcal{A}$ is constructed who can derive the identity ID_i and password pw_i of user U_i by running the algorithm $ALG2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ of the proposed RPA scheme, as presented in Algorithm 3. Suppose that the adversary $\mathcal{A}$ can obtain the secret values $\{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$ stored in a lost or stolen smart card, as shown in Theorem 1.

Algorithm 3: $ALG2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ for deriving identity ID_i and password pw_i of user U_i

```
Input: $n, e, j_i, S_i, h_i, \hat{b}_i$
Output: 0 or 1
    Compute $CID_i = S_i^e \bmod n$
    Call $\mathcal{O}racle\mathcal{H}$ on input $CID_i$ to retrieve identity $ID_i^*$ and secret parameter $x^*$ as $(ID_i^*||x^*) \leftarrow \mathcal{O}racle\mathcal{H}(CID_i)$
    Compute $h(ID_i||hpw_i) = j_i \oplus h_i$
    Call $\mathcal{O}racle\mathcal{H}$ on input $h(ID_i||hpw_i)$ to retrieve identity $ID_i^{**}$ and $hpw_i^*$ as $(ID_i^{**}||hpw_i^*) \leftarrow \mathcal{O}racle\mathcal{H}(h(ID_i||hpw_i))$
    if $ID_i^* \neq ID_i^{**}$ then
        return 0 (Failure)
    else
        Call $\mathcal{O}racle\mathcal{H}$ on input $hpw_i^*$ to retrieve the identity $pw_i^*$ and $b_i^*$ as $(pw_i^*||b_i^*) \leftarrow \mathcal{O}racle\mathcal{H}(hpw_i^*)$
        Compute $\hat{b}_i^* = h(ID_i^*||pw_i^*) \oplus b_i^*$
        if $\hat{b}_i^* = \hat{b}_i$ then
            Accept $ID_i^*$ and $pw_i^*$ as the correct identity and password of user $U_i$, respectively
            return 1 (Success)
        else
            return 0 (Failure)
        end if
    end if
```

The success probability of $ALG2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ is defined as $Succ2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle} = 2\Pr[ALG2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle} = 1] - 1$ and its advantage is $Adv2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H) = \max_{\mathcal{A}}\{Succ2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}\}$, where the maximum of all adversary $\mathcal{A}$ is taken with execution time t and the number of queries q_H made to $\mathcal{O}racle\mathcal{H}$.

Consider $ALG2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ given in Algorithm 3, the adversary $\mathcal{A}$ will successfully obtain identity ID_i and password pw_i, and will thus win the game only if the adversary $\mathcal{A}$ can calculate the inverse of the cryptographic one-way hash function $h(\cdot)$. Nevertheless, it is a computationally infeasible problem for any adversary due to the collision-resistant property of $h(\cdot)$. Since $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{H}}(t_1) \leq \varepsilon_1$ for any sufficiently small $\varepsilon_1 > 0$ by Definition 1, then it must be that $Adv2_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H) \leq \varepsilon$, for any sufficiently small $\varepsilon > 0$. Therefore, the proposed RPA scheme is provably secure against an adversary $\mathcal{A}$ for deriving identity ID_i and pw_i of user U_i.

Theorem 3. If the cryptographic one-way hash function $h(\cdot)$ *behaves like a true random oracle, then the proposed RPA scheme is provably secure against an adversary* $\mathcal{A}$ *for deriving the shared session key* SK *between user* U_i *and server* S.

Proof. Suppose that an adversary can derive the shared session key SK by running the algorithm $ALG3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ against the proposed RPA scheme, as described in Algorithm 4. As in Theorem 1 and Theorem 2, suppose that the adversary $\mathcal{A}$ can extract the information $\{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$ from a lost or stolen smart card, and intercept login message $\{n, e, DID_i, X_i, Y_i, T_U\}$ and response message $\{R_i, T_S\}$.

Given the success probability of $ALG3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ is $Succ3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle} = 2\Pr[ALG3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle} = 1] - 1$, and the advantage as $Adv3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H) = \max_{\mathcal{A}}\{Succ3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}\}$, where the maximum of all adversary $\mathcal{A}$ is taken with execution time t and the number of queries q_H made to $\mathcal{O}racle\mathcal{H}$.

Based on $ALG3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$, if the adversary $\mathcal{A}$ can evaluate the inverse of a collision-resistant one-way hash function $h(\cdot)$, the adversary $\mathcal{A}$ can successfully derive the shared session key SK by using $\mathcal{O}racle\mathcal{H}$ and wins the game. Nevertheless, Definition 1 states that $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{H}}(t_1) \leq \varepsilon_1$ for any sufficiently small $\varepsilon_1 > 0$. Then, it must be that $Adv3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}(t, q_H) \leq \varepsilon$, for any sufficiently small $\varepsilon > 0$, since it depends on $Adv_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle\mathcal{H}}(t_1)$. Hence, the proposed RPA scheme is shown to be provably secure against an adversary $\mathcal{A}$ for deriving shared session key SK between the user U_i and server S.

Algorithm 4: $ALG3_{\mathcal{A},\mathcal{PAS}}^{\mathcal{O}racle}$ for deriving session key SK shared between user U_i and server S

```
Input: $n, e, j_i, v_i, S_i, h_i, DID_i, X_i, Y_i, T_U, R_i, T_S$
Output: 0 or 1
Compute $CID_i = S_i^e \bmod n$
Call $\mathcal{O}racle\mathcal{H}$ on input $CID_i$ to retrieve identity $ID_i^*$ and secret parameter $x^*$ as $(ID_i^*||x^*) \leftarrow \mathcal{O}racle\mathcal{H}(CID_i)$
Compute $h(ID_i||hpw_i) = j_i \oplus h_i$ and $w = h(ID_i||hpw_i) \oplus v_i$
Call $\mathcal{O}racle\mathcal{H}$ on input $h(ID_i||hpw_i)$ to retrieve identity $ID_i^{**}$ and $hpw_i^*$ as $(ID_i^{**}||hpw_i^*) \leftarrow \mathcal{O}racle\mathcal{H}(h(ID_i||hpw_i))$
if $ID_i^* \neq ID_i^{**}$ then
    return 0 (Failure)
else
    Compute $w^* = DID_i \oplus ID_i^*$ and $Z_i^* = h(ID_i^*||T_S)$
    if $w = w^*$ and $Z_i^* = R_i^e \bmod n$ then
        Successfully compute the session key $SK = h(ID_i^* Z_i^* w^* T_U T_S)$ shared between user $U_i$ and server $S$
        return 1 (Success)
    else
        return 0 (Failure)
    end if
end if
```

4.2 Security Attributes

This section further analyses the security attributes offered by the proposed RPA scheme.

4.2.1 No Data Storage in Server S

The proposed scheme preserves the "no data storage" feature of Kumari et al. [16] scheme. By using the information provided by the login message request, private key d, and secret parameter x, the server S can perform all the calculations to authenticate the validity of the user U_i.

4.2.2 Mutual Authentication

The proposed scheme includes mutual authentication steps for verifying the legitimacy of the user U_i and server S. The server S authenticates the user U_i by checking the user authentication equation $Y_i^e = CID_i \cdot X_i^{h(ID_i||T_U)\cdot d\cdot x} \bmod n$. A valid user U_i will pass the authentication since the identity ID_i must follow the specified ID format.

Next, the user U_i checks the legitimacy of server S by verifying $R_i^e = Z_i \bmod n$. Since the user's identity is not transmitted explicitly in the public channel, the adversary $\mathcal{A}$ does not know the value of ID_i. Therefore, any malicious user cannot compute the value of $Z_i = h(ID_i||T_S)$. As a result, the proposed scheme can attain mutual authentication.

4.2.3 Session Key Agreement

After completing the mutual authentication process, both the user U_i and server S will establish a shared session key $SK = h(ID_iZ_iwT_UT_S)$. Since the adversary $\mathcal{A}$ does not know ID_i, Z_i, and w, the session key SK cannot be directly computed due to the cryptographic collision-resistant one-way hash function. As a result, the proposed scheme can protect the secrecy of shared session keys.

4.2.4 User Anonymity

According to *Assumption A2*, the adversary $\mathcal{A}$ may extract information $\{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$ from the smart card SC_i. The identity ID_i is contained in the parameters j_i, v_i, S_i, and h_i. Nevertheless, the adversary $\mathcal{A}$ is unable to derive identity ID_i since the adversary $\mathcal{A}$ needs to invert the output of a collision-resistant one-way hash function. This is only possible for an adversary with a negligible probability in polynomial time, as proven in Theorem 2. As a result, the proposed scheme can preserve user anonymity.

4.2.5 Local Password Verification

The proposed scheme offers an incorrect input detection feature. Before logging into the server S, the smart card SC_i verifies the legality of identity ID_i and password pw_i. The verification equation $j_i \oplus h_i = h(ID_i||hpw_i)$ will detect if a user U_i inputs the identity ID_i or password pw_i, or both incorrectly by mistake. Without knowing ID_i, pw_i, and b_i, the adversary $\mathcal{A}$ is unable to correctly calculate $h(ID_i||hpw_i)$ and subsequently, the verification $j_i \oplus h_i = h(ID_i||hpw_i)$ will fail. Therefore, the proposed scheme can block illegal access using local password verification.

4.2.6 Password Changeability

The extra "password change" phase in the proposed scheme grants users the convenience to change or update their passwords locally. This phase can be done without interacting with the KIC or the server S.

4.2.7 User-Friendliness

The proposed scheme permits the user U_i to freely choose the identity ID_i and password pw_i. The user U_i can easily change or update the password pw_i without communicating with server S within minimal time without having to go through the registration phase. As a result, the proposed scheme is hassle-free and user-friendly.

5 Performance Comparison and Analysis

The endorsement of a new RPA scheme should be supported by careful analysis of its performance. For this purpose, the proposed scheme was compared with similar RPA schemes [4,15,16]. These schemes are chosen according to the security attributes offered, which are mutual authentication and no data storage in the server. Furthermore, since the aim of this study is to propose an efficient RPA scheme, it is considerable to compare its performance to the most recent scheme by Kumari et al. [16] that is found in the literature. The security attributes and efficiency of all schemes considered are investigated in this section.

Tab. 2 compares all schemes based on the security attributes discussed in Section 4. According to Tab. 2, the proposed scheme and the scheme by Kumari et al. [16] outperformed the schemes by Shen et al. [4] and Awasthi et al. [15]. All of the security attributes of [16] were retained in the proposed scheme, including no storage of data in server S, mutual authentication, session key agreement, user anonymity, local password verification, password changeability, and user-friendliness. Furthermore, unlike the other schemes, the proposed scheme includes a formal security analysis. As a result, the proposed RPA scheme outperformed other considered schemes in terms of security attributes.

Table 2: Comparison of schemes based on security attributes

Security attribute	Schemes			
	[4]	[15]	[16]	Proposed
Formal security proof (provable security)	✗	✗	✗	✓
No data storage in server S	✓	✓	✓	✓
Mutual authentication	✓	✓	✓	✓
Session key agreement	✗	✗	✓	✓
User anonymity (user protection)	✗	✗	✓	✓
Local password verification	✗	✗	✓	✓

(Continued)

Table 2: Continued

Security attribute	Schemes			
	[4]	[15]	[16]	Proposed
Password changeability	✗	✗	✓	✓
User-friendliness	✗	✗	✓	✓

Note: ✓: Provide the security attribute, ✗: Does not provide the security attribute.

The assessment assumptions for the efficiency analysis were based on [17,29]. Assuming that each value of $\{ID_i, pw_i, b_i, r_i\}$ is 160-bit long, the output message digests of secure one-way hash function (SHA-256 [28]) $\{CID_i, DID_i, SK, w, hpw_i, v_i, j_i, \hat{b}_i\ \}$ are 256-bit long, and the timestamps $\{T_U, T_S, T_c\}$ are 32-bit long. The modular operation of mod n is 2048-bit long, and the modular exponentiation is regarded as the most expensive operation. Hence, the values $\{n, e, d, S_i, h_i, X_i, Y_i, R_i\}$ are 2048-bit and $\{x, g\}$ are 1024-bit. The exclusive OR ($\oplus$) operation involves very few computations and hence is negligible. The time complexity with the exponential operation (T_e), modular multiplication operation (T_m), hashing operation (T_h), and exclusive OR operation ($\oplus$) can be roughly expressed as $T_e \gg T_m \approx T_h > \oplus$. For ease of time complexity comparison between schemes, the approximation of execution time complexity of T_e and T_h in terms of T_m is assumed as $T_e \approx 240\ T_m$ and $T_h \approx T_m$ [42]. Tab. 3 shows the transmission/computational cost and time complexity for all considered schemes.

Table 3: Comparison of schemes based on transmission/computation cost and time complexity

Memory/Cost	Scheme			
	[4]	[15]	[16]	Proposed
C_1 (in bits)	9632	9376	6144	9984
C_2 (in bits)	11744	11488	11584	10560
C_3	$2\ T_e + 1\ T_m + 1\ T_h$	$2\ T_e + 1\ T_m + 1\ T_h$	$2\ T_e + 1\ T_m + 5\ T_h$	$2\ T_e + 2\ T_m + 5\ T_h$
C_4	$3\ T_e + 3\ T_m + 2\ T_h$	$3\ T_e + 3\ T_m + 2\ T_h$	$4\ T_e + 2\ T_m + 8\ T_h$	$3\ T_e + 4\ T_m + 6\ T_h$
C_5	$3\ T_e + 1\ T_m + 3\ T_h$	$3\ T_e + 1\ T_m + 3\ T_h$	$4\ T_e + 2\ T_m + 5\ T_h$	$3\ T_e + 3\ T_m + 5\ T_h$
$C_3 + C_4 + C_5$	$8\ T_e + 5\ T_m + 6\ T_h$	$8\ T_e + 5\ T_m + 6\ T_h$	$10\ T_e + 5\ T_m + 18\ T_h$	$8\ T_e + 9\ T_m + 16\ T_h$
Total cost	$\approx 1931\ T_m$	$\approx 1931\ T_m$	$\approx 2433\ T_m$	$\approx 1945\ T_m$

Note: T_e: Exponential operation time complexity, T_m: Modular multiplication operation time complexity, T_h: Hashing operation time complexity.

In the proposed scheme, the parameters $\{n, e, g, j_i, v_i, S_i, h_i, \hat{b}_i\}$ are stored within the smart card SC_i. The memory storage required for the smart card is $C_1 = (4\ \times 2048) + (1024) + (3\ \times 256) = 9984$-bit, which is the highest among other schemes, particularly 352-bit more than Shen et al. [4]. The transmission cost C_2 is the memory space of the login message, $\{n, e, DID_i, X_i, Y_i, T_U\}$ and response message, $\{R_i, T_S\}$ that are exchanged during the login and authentication phases. For the proposed scheme, its $C_2 = (5 \times 2048) + (256) + (2 \times 32) = 10560$-bit, which is the lowest among other schemes, particularly 928-bit less than Awasthi et al. [15]. The computational cost C_3 is the total time complexity of operations executed during the registration phase, $C_3 = 2\ T_e + 2\ T_m + 5\ T_h$. The computational cost of smart card SC_i and server S are $C_4 = 3\ T_e + 4\ T_m + 6\ T_h$ and $C_5 = 3\ T_e + 3\ T_m + 5\ T_h$, respectively (exhibit the time spent during the authentication phase and session key agreement).

Based on Tab. 3, the total computational costs ($C_3 + C_4 + C_5$) of the schemes of Shen et al. [4] and Awasthi et al. [15] are both $8\ T_e + 5\ T_m + 6\ T_h \approx 1931\ T_m$. While, the total computational costs for schemes of Kumari et al. [16] and the proposed scheme are $10\ T_e + 5\ T_m + 18\ T_h \approx 2433\ T_m$ and $8\ T_e + 9\ T_m + 16\ T_h \approx 1945\ T_m$, respectively. Compared with the schemes by Shen et al. [4] and Awasthi et al. [15], the proposed scheme is less efficient with 14 T_m higher computational cost. In Fig. 3, the bar chart presents the efficiency of the proposed scheme over other considered schemes. It is clear that the proposed scheme is more efficient than Kumari et al. [16]. The total computational cost of Kumari et al. [16] has been significantly reduced by 20% in the proposed scheme.

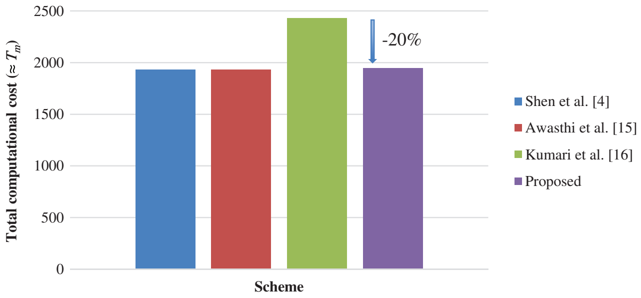


Figure 3: Comparison of schemes based on total computational cost ($C_3 + C_4 + C_5$)

As provided in Tab. 2, both the proposed scheme and Kumari et al. [16] require extra steps for session key agreement, which explains the higher computational cost when compared to the schemes by Shen et al. [4] and Awasthi et al. [15] in Tab. 3. It is worth noting that, as shown in Tab. 3, the proposed scheme requires larger smart card memory storage, particularly 3480-bit more than Kumari et al. [16]. However, this is justified because the proposed RPA scheme significantly reduced the transmission cost by 1024-bit as compared to Kumari et al. [16]. Additionally, the total computational cost improved to 1945 T_m, which is 488 T_m less than Kumari et al. [16]. Based on the security attributes, communication cost, and time complexity, it can be concluded that the proposed scheme outperformed all other schemes considered.

6 Application

This section discusses the proposed approach's potential applicability in developing a UI scheme. The UI scheme can be considered a simpler algorithm used to distinguish unique users prior to the authentication process. Most RPA schemes require two or more factors (e.g., password, smart card, and fingerprint), whereas UI schemes just need the user's identity. Figs. 4a and 4b show the flowcharts for the RPA and UI schemes, respectively. At a glance, the phases in the RPA and UI schemes appear similar, except that the UI scheme does not require a login phase. Some parameters can be removed while retaining the cryptographic primitives of IFP and DLP, depending on the security goals and purposes. Therefore, it would be interesting to investigate the prospect of converting the proposed RPA scheme into an improved UI scheme with provable security.

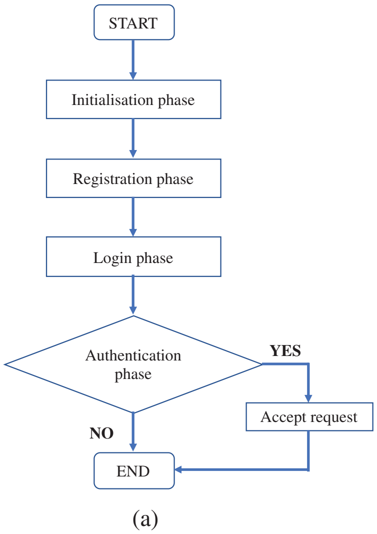


Figure 4: Process flowchart for RPA and UI schemes (a) RPA scheme (b) UI scheme

7 Conclusion

The aim of this study is to primarily propose an efficient RPA scheme that offers session key establishment between user and server. The widely established Dolev-Yao adversary model was considered in the development of the proposed scheme, which attained the desired security attributes of Kumari et al., such as no data storage in server S, user anonymity, local password verification, password changeability, and user-friendliness. Furthermore, as the main contribution, a formal security proof of the proposed scheme was presented based on the random oracle model using formal definitions of IFP and DLP. Although the proposed scheme required higher smart card memory than other similar schemes by Shen et al., Awasthi et al. and Kumari et al., this was acceptable owing to its much-reduced transmission/computation cost and time complexity than Kumari et al.'s scheme. The performance analysis proved that the proposed RPA scheme is noticeably better than Kumari et al., given that it can provide the same security attributes. Future work will investigate the use of two cryptographic primitives (IFP and DLP) in the development of UI schemes. Since the phases in RPA and UI schemes are similar, it would be interesting to examine the potential application, particularly in terms of security and performance. Expectantly, this should aid in the design of an efficient and provably secure UI scheme.

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