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Article

Optimal Beamforming for Secure Transmit in Practical Wireless Networks

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Abstract: In real communication systems, secure and low-energy transmit scheme is very important. So far, most of schemes focus on secure transmit in special scenarios. In this paper, our goal is to propose a secure protocol in wireless networks involved various factors including artificial noise (AN), the imperfect receiver and imperfect channel state information (CSI) of eavesdropper, weight of beamforming (BF) vector, cooperative jammers (CJ), multiple receivers, and multiple eavesdroppers, and the analysis shows that the protocol can reduce the transmission power, and at the same time the safe reachability rate is greater than our pre-defined value, and the analysis results are in good agreement with the simulation results. In this letter, the minimal transmit power is modeled as a non-convexity optimization that is general difficult. Our method is to transform it into a two-level non-convex problem. The outer is a univariate optimization that can be solved by the golden search algorithm. The inner is a convex optimization solved by using the CVX. The solutions are further used to improve the confidentiality rate of the system, and reduce the transmit power of the system and resource consumption in terms of the imperfect CSI. Simulations show the efficiency and robustness of the proposed protocol.

Keywords: Secure transmission; MISO system; imperfect CSI; BF vector; convex optimization

1 Introduction

Physical Layer Security (PLS) has received extensive attention in ensuring the security of data transmission. The main goal of PLS in applications is to ensure the secrecy of messages,which transmitted by legitimate receivers [1]. This is achieved by reducing the signal-to-noise ratio (SNR) of the eavesdropper. One method is to add AN to the legal signal [2–4]. Cooperative relay (CR) is useful in the secure transmit of CR-assisted multi-antenna systems. CR can act as a jammer and expand the interference range [5–7]. At the same time, the status information of the channel should also be considered. It is difficult to obtain accurate CSI because of the time-varying characteristics of the channel. Some schemes have studied robust and secure transmit with imperfect CSI [8–12]. There are two kinds of protocols with uncertain CSI. One is uncertain on one side, that is, only the status information of the eavesdropper or receiver is unknown [13–15]. The other is uncertain on both sides, which mean the CSI of both parties is completely unknown [16–19]. In addition to the secrecy performance, the power consumption of the system is also considered [20–25]. Reference [25] gives the best solution for spectral efficiency and energy consumption issues in 5G communications.

Our motivation comes from the fact that most of cooperative interferences only consider one or a few factors, while the actual factors are not involved in applications, such as imperfect receiver and imperfect CSI of eavesdropper, cooperative jammer, multiple Receivers, and multiple eavesdroppers. In this article, we propose a secure transmit protocol by considering all these factors, which provide a more reliable model for complex networks. The main consideration is coordinated interference, which ensures the confidentiality of the receiver CSI (RCSI) and the eavesdropper CSI (ECSI) under the imperfect premise. At the same time, we will reduce the power required to transmit the signal. Due to the bounded error of CSI, we propose a robust and secure transmit scheme in the MISO downlink network. To further improve the security performance, we also use multi-antenna auxiliary jammers. The main contributions made in this paper are as follows:

(1) Aiming at imperfect RCSI and ECSI, we propose a robust transmit scheme for the eavesdropping model with multi-receiver-eavesdroppers in multicast. We evaluate the minimum transmit power of the transmitter and the CR to meet the transmit reliability and confidentiality, that is, the minimum secrecy rate in the worst case is still greater than what is required to ensure the secure transmission.

(2) We introduce a slack-variable logarithm and semi-definite slack (SDR) to simplify the transmit power optimization, which is due to the non-convexity of the optimization problem involved. At the same time, the Lagrange duality is used to obtain the analytical formula of the constraint conditions. The non-convex problem further becomes a two-level optimization. The external is a univariate optimization, which is solved by using the golden search algorithm. The inner layer is a semi-definite programming (SDP) problem which will be resolved by CVX.

The rest of the paper is organized as follows. Section 2 introduces the system model. In Section 3, we establishes the problem formulationand model the robust transmit power as a non-convex minimization problem. The two-level optimization algorithms will be applied to solve the present problem. Section 4 contributes the simulation that shows the efficiency of the proposed schemes while last section concludes the paper.

2 System Model

Before introducing the system model, we introduce the parameter symbols used in this paper and their meanings, as shown in Tab. 1. The matrix is indicated in bold capital letters. Vectors are represented in bold lowercase letters.

Table 1: Notions used in this paper

Notation	Description
$\mathrm{tr}(\mathbf{A})$	The trace of matrix $\mathbf{A}$
$\mathbf{A}^{-1}$	The inverse of matrix $\mathbf{A}$
$\mathbf{A}^{H}$	Hermitian transpose of matrix $\mathbf{A}$
$\lVert\mathbf{A}\rVert$	Euclidean norm of matrix $\mathbf{A}$

(Continued)

Table 1: Continued

Notation	Description
$\mathbb{C}^N$	The spaces of N-dimensional complex vectors
$\mathbb{H}^N$	$N \times N$ Hermitian matrix
$\mathrm{E}\,(\mathrm{A})$	A's expectations
$\mathbf{Q} \succeq 0\,(\mathbf{Q} \succ 0)$	$\mathbf{Q}$ is a positive semidefinite (definite) matrix
$\mathrm{vec}\,(\mathbf{A})$	A column vector by stacking all the elements
$\mathcal{CN}\,(c,\ \mathbf{Q})$	Circularly symmetric complex distribution with mean vector c and covariance $\mathbf{Q}$
$\otimes$	Kronecker product
$rank\,(\mathbf{A})$	The rank of the matrix $\mathbf{A}$

We consider anassisting jammer (AJ)-assisted MISO system as shown in Fig. 1. Alice sends message to multiple receivers $\mathrm{Bob}_1, \ldots, \mathrm{Bob}_l$.The AJ sends AN (here, Gaussian noises will be used in what follows) for the secure transmit by confusing Keavesdroppers Eve. Due to limited feedback, assume that Alice knows the imperfect CSI of each receiver and eavesdropper. The channel estimation error Δ is bounded. The receiver is equipped with an antenna, Eves as same as the receiver. The total number of antennas for the transmitter and assisting jammer is denoted as N_t and N_j, respectively.

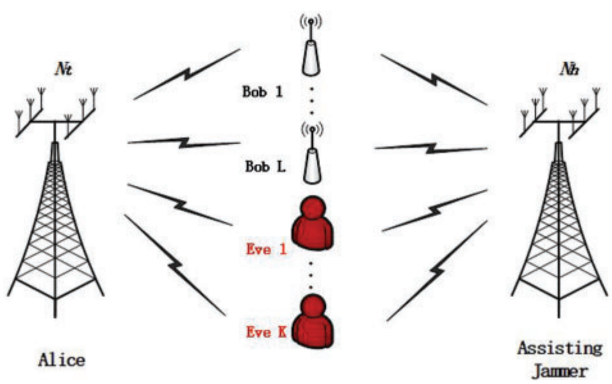


Figure 1: System model for secure transmit with assisting jammer

The signal from the transmitter is expressed by $\mathbf{x} = \mathbf{w}s$, which denotes the receiver's data stream with $E\left[|s|^2\right] = 1$. Let $\mathbf{w} \in \mathbb{C}^{N_t}$ denote the beamforming (BF) vector of the signal. Let $\mathbf{z} \in \mathbb{C}^{N_t}$ be AN generated by AJ, that is, $\mathbf{z} \sim \mathcal{CN}\,(0,\ \mathbf{Q}), \mathbf{Q} \succeq 0$.

Let $n_l \sim \mathcal{CN}\left(0,\ \sigma_{b,l}^2\right)$ and $n_k \sim \mathcal{CN}\left(0, \sigma_{e,k}^2\right)$ denote the standard complex Gaussian noises for Bob_l and Eve_k [26], respectively. Denote $\mathbf{g}_{b,l} \in \mathbb{C}^{N_j}$ and $\mathbf{g}_{e,k} \in \mathbb{C}^{N_j}$ as the channels from AJ to Bob_l and Eve_k, respectively. The signals received by Bob_l and Eve_k are respectively given by

$$y_{b,l} = \mathbf{h}_{b,l}\mathbf{x} + \mathbf{g}_{b,l}\mathbf{z} + n_{b,l}, \tag{1a}$$

$$y_{e,k} = \mathbf{h}_{e,k}\mathbf{x} + \mathbf{g}_{e,k}\mathbf{z} + n_{e,k}, \tag{1b}$$

where $\mathbf{h}_{b,l} \in \mathbb{C}^{N_t}$ and $\mathbf{h}_{e,k} \in \mathbb{C}^{N_t}$ denote the channel vectors from Alice to Bob_l and Eve_k, respectively, $l \in \mathcal{L} \triangleq \{1, \cdots, L\}$, and $k \in \mathcal{K} \triangleq \{1, \cdots, K\}$. It is assumed that all channels are independent. The SINRs of channels are respectively expressed as

$$SINR_{b,l}(\mathbf{w}, \mathbf{Q}) = \frac{\mathbf{h}_{b,l}\mathbf{w}\mathbf{w}^H\mathbf{h}_{b,l}^H}{\mathbf{g}_{b,l}\mathbf{Q}\mathbf{g}_{b,l}^H + \sigma_{b,l}^2} \tag{2a}$$

$$SINR_{e,k}(\mathbf{w}, \mathbf{Q}) = \frac{\mathbf{h}_{e,k}\mathbf{w}\mathbf{w}^H\mathbf{h}_{e,k}^H}{\mathbf{g}_{e,k}\mathbf{Q}\mathbf{g}_{e,k}^H + \sigma_{e,k}^2} \tag{2b}$$

3 Optimal Beamforming Schemes

3.1 Channel Mismatch

In this section, we consider the imperfect CSI. The wireless channels of Alice and AJ are estimated in the quadratic form. Therefore, for Alice, its channel modeling [27] is given by

$$\mathbf{H}_{b,l} = E\left[\mathbf{h}_{b,l}\mathbf{h}_{b,l}^H\right] = \tilde{\mathbf{H}}_{b,l} + \Delta_{h,b,l} = \tilde{\mathbf{h}}_{b,l}\tilde{\mathbf{h}}_{b,l}^H + \Delta_{h,b,l} \tag{3a}$$

$$\mathbf{H}_{e,k} = E\left[\mathbf{h}_{e,k}\mathbf{h}_{e,k}^H\right] = \tilde{\mathbf{H}}_{e,k} + \Delta_{h,e,k} = \tilde{\mathbf{h}}_{e,k}\tilde{\mathbf{h}}_{e,k}^H + \Delta_{h,e,k} \tag{3b}$$

where $\tilde{\mathbf{H}}_{b,l}$ and $\tilde{\mathbf{H}}_{e,k}$ are defined by

$$\tilde{\mathbf{H}}_{b,l} = \tilde{\mathbf{H}}_{b,l}\tilde{\mathbf{H}}_{b,l}^H \tag{4a}$$

$$\tilde{\mathbf{H}}_{e,k} = \tilde{\mathbf{H}}_{e,k}\tilde{\mathbf{H}}_{e,k}^H \tag{4b}$$

which are the estimated channel covariances in quadratic form, $\Delta_{h,b,l}$ and $\Delta_{h,e,k}$ are the CSI errors from Alice to Bob_l and Eve_k, respectively, and $l \in \mathcal{L}$, $k \in \mathcal{K}$.

Suppose that $\tilde{\mathbf{h}}_{b,l}$ is complex Gaussian variable [10], that is, $\tilde{\mathbf{h}}_{b,l} \sim \mathcal{CN}\left(0, \xi_{b,l}\right)$, $\tilde{\mathbf{g}}_{b,l} \sim \mathcal{CN}\left(0, \xi_{b,l}\right)$. Assume that the uncertainty is bounded in the uncertainty region of the ellipsoid [27,28] that can be modeled as

$$\mathcal{H}_{b,l} \triangleq \left\{\Delta_{h,b,l} | \Delta_{h,b,l} \in \mathbb{H}^N, \tilde{\mathbf{H}}_{b,l} + \Delta_{h,b,l} \succeq 0, a_1 \leq 1\right\} \tag{5a}$$

$$\mathcal{H}_{e,k} \triangleq \left\{\Delta_{h,e,k} | \Delta_{h,e,k} \in \mathbb{H}^N, \tilde{\mathbf{H}}_{e,k} + \Delta_{h,e,k} \succeq 0, a_2 \leq 1\right\} \tag{5b}$$

where a_1 and a_2 are defined as

$$a_1 = \mathrm{tr}\left(\Delta_{h,b,l}\mathbf{D}_{h,b,l}\Delta_{h,b,l}\right), a_2 = \mathrm{tr}\left(\Delta_{h,e,k}\mathbf{D}_{h,e,k}\Delta_{h,e,k}\right) \tag{6}$$

$\mathbf{D}_{h,b,l}$ (or $\mathbf{D}_{h,e,k}$) depends on CSI, which is from Alice to Bob_l (or Eve_k). Both of them are known. Especially, $\mathbf{D}_{h,b,l}$ and $\mathbf{D}_{h,e,k}$ can be decomposed into as follows [27]:

$$\begin{cases} \mathbf{D}_{h,b,l} = \hat{\mathbf{D}}_{h,b,l}^H\hat{\mathbf{D}}_{h,b,l} \\ \mathbf{D}_{h,b,l} \succeq 0 \end{cases}, \begin{cases} \mathbf{D}_{h,e,k} = \hat{\mathbf{D}}_{h,e,k}^H\hat{\mathbf{D}}_{h,e,k} \\ \mathbf{D}_{h,e,k} \succeq 0 \end{cases} \tag{7}$$

Similarly, from Eqs. (3a) and (3b), we get the channel model of AJ as follows:

$$\mathbf{G}_{b,l} = E\left[\mathbf{g}_{b,l}\mathbf{g}_{b,l}^H\right] = \tilde{\mathbf{G}}_{b,l} + \Delta_{g,b,l} = \tilde{\mathbf{g}}_{b,l}\tilde{\mathbf{g}}_{b,l}^H + \Delta_{g,b,l} \tag{8a}$$

$$\mathbf{G}_{e,k} = E\left[\mathbf{g}_{e,k}\mathbf{g}_{e,k}^H\right] = \tilde{\mathbf{G}}_{e,k} + \Delta_{g,e,k} = \tilde{\mathbf{g}}_{e,k}\tilde{\mathbf{g}}_{e,k}^H + \Delta_{g,e,k} \tag{8b}$$

where $\tilde{\mathbf{G}}_{b,l}$ and $\tilde{\mathbf{G}}_{e,k}$ are defined by

$$\tilde{\mathbf{G}}_{b,l} = \tilde{\mathbf{h}}_{e,k}\tilde{\mathbf{h}}_{e,k}^{h} \tag{9a}$$

$$\tilde{\mathbf{G}}_{e,k} = \tilde{\mathbf{g}}_{e,k}\tilde{\mathbf{g}}_{e,k}^{H} \tag{9b}$$

which are the estimated channel covariances in the quadratic form, $\Delta_{g,b,l}$ and $\Delta_{g,e,k}$ are CSI errors from AJ to Bob$_l$ and to Eve$_k$, respectively, and $l \in \mathcal{L}$, $k \in \mathcal{K}$. Let $\tilde{\mathbf{g}}_{b,l} \sim \mathcal{CN}\left(0, \xi_{b,l}\right)$ and $\tilde{\mathbf{g}}_{e,k} \sim \mathcal{CN}\left(0, \xi_{e,k}\right)$. Similar to Eqs. (5a) and (5b), we have

$$\mathcal{G}_{b,l} \triangleq \left\{\Delta_{g,b,l} | \Delta_{g,b,l} \in \mathbb{H}^{N}, \tilde{\mathbf{G}}_{b,l} + \Delta_{g,b,l} \succeq 0, a_3 \leq 1\right\} \tag{10a}$$

$$\mathcal{G}_{e,k} \triangleq \left\{\Delta_{g,e,k} | \Delta_{g,e,k} \in \mathbb{H}^{N}, \tilde{\mathbf{G}}_{e,k} + \Delta_{g,e,k} \succeq 0, a_4 \leq 1\right\} \tag{10b}$$

where A_3 and A_4 are defined as follows

$$a_3 = \operatorname{tr}\left(\Delta_{g,b,l}\mathbf{D}_{g,b,l}\Delta_{g,b,l}\right), a_4 = \operatorname{tr}\left(\Delta_{g,e,k}\mathbf{D}_{g,e,k}\Delta_{g,e,k}\right) \tag{11}$$

$\mathbf{D}_{g,b,l}$, and $\mathbf{D}_{g,e,k}$ can be decomposed as follows

$$\begin{cases} \mathbf{D}_{g,b,l} \succeq 0 \\ \mathbf{D}_{g,b,l} = \hat{\mathbf{D}}_{g,b,l}^{H}\hat{\mathbf{D}}_{g,b,l} \end{cases}, \begin{cases} \mathbf{D}_{g,e,k} \succeq 0 \\ \mathbf{D}_{g,e,k} = \hat{\mathbf{D}}_{g,e,k}^{H}\hat{\mathbf{D}}_{g,e,k} \end{cases} \tag{12}$$

3.2 Robust Transmit Power Minimization

The present problem is to minimize the transmit power, while the guarantee the minimum transmit secrecy rate. Here, the estimation error of CSI is considered. For the worst case of secrecy capacity, according to the principle of power minimization, the optimization function after adding AN is given by

$$Q1: \min_{\mathbf{w},\mathbf{Q}\succeq 0} \quad \operatorname{tr}\left(\mathbf{w}\mathbf{w}^{H} + \mathbf{Q}\right) \tag{13a}$$

$$s.t., \quad \min_{l\in\mathcal{L},k\in\mathcal{K}} \left\{ \min_{\substack{\Delta_{g,b,l} \in \mathcal{G}_{b,l} \\ \Delta_{hb,l} \in \mathcal{H}_{b,l}}} R_{b,l} - \max_{\substack{\Delta_{g,e,k} \in \mathcal{G}_{e,k} \\ \Delta_{h,e,k} \in \mathcal{H}_{e,k}}} R_{e,k} \right\} \geq R_s, \forall l, \forall k \tag{13b}$$

$$\operatorname{tr}\left(\mathbf{w}\mathbf{w}^{H} + \mathbf{Q}\right) \leq P_{max} \tag{13c}$$

where R_s is the pre-defined target secrecy rate. In the model, the secrecy rate is evaluated by the difference between the minimum secrecy capacity of the receiver and the maximum secrecy capacity of the eavesdropper. So, the secrecy rate obtained in the optimization is the minimum secrecy rate of the system. In our setting, the minimum secrecy rate should also be higher than the target secrecy rate. To simplify the secrecy capacity function, we introduce β is used as a slack variable. The optimization problem Q1 is equivalently written into

$$Q2: \min_{\mathbf{w},\mathbf{Q}\succeq 0} \quad \operatorname{tr}\left(\mathbf{w}\mathbf{w}^{H} + \mathbf{Q}\right) \tag{14a}$$

$$s.t. \quad \min_{\substack{\Delta_{g,b,l} \in \mathcal{G}_{b,l} \\ \Delta_{hb,l} \in \mathcal{H}_{b,l}}} \left(R_{b,l} - \log\beta\right) \geq R_s, \forall l, \tag{14b}$$

$$\max_{\substack{\Delta_{g,e,k} \in \mathcal{G}_{e,k} \\ \Delta_{h,e,k} \in \mathcal{H}_{e,k}}} R_{e,k} \leq \log \beta, \forall k \tag{14c}$$

$$\operatorname{tr}\left(\mathbf{w}\mathbf{w}^H + \mathbf{Q}\right) \leq P_{max} \tag{14d}$$

where $\log(\cdot)$ denotes the base-2 logarithmic function.

According to Shannon's formula, the achievable secure rate of Bob_l and the Eve_k can be represented as

$$C_{b,l} = \log\left(1 + \frac{\operatorname{tr}\left(\left(\tilde{\mathbf{H}}_{b,l} + \Delta_{h,b,l}\right)\mathbf{w}\mathbf{w}^H\right)}{\operatorname{tr}\left(\left(\tilde{\mathbf{G}}_{b,l} + \Delta_{g,b,l}\right)\mathbf{Q}\right) + \sigma_{b,l}^2}\right) \tag{15a}$$

$$C_{e,k} = \log\left(1 + \frac{\operatorname{tr}\left(\left(\tilde{\mathbf{H}}_{e,k} + \Delta_{h,e,k}\right)\mathbf{w}\mathbf{w}^H\right)}{\operatorname{tr}\left(\left(\tilde{\mathbf{G}}_{e,k} + \Delta_{g,e,k}\right)\mathbf{Q}\right) + \sigma_{e,k}^2}\right) \tag{15b}$$

From Eqs. (14b), (14c), (15a) and (15b), we can obtain

$$Q3: \min_{\mathbf{w},\mathbf{Q}\succeq 0} \operatorname{tr}\left(\mathbf{w}\mathbf{w}^H + \mathbf{Q}\right) \tag{16a}$$

$$s.t. \min_{\substack{\Delta_{g,b,l} \in \mathcal{G}_{b,l} \\ \Delta_{hb,l} \in \mathcal{H}_{b,l}}} \left(1 + \frac{\operatorname{tr}\left(\mathbf{w}\mathbf{H}_{b,l}\mathbf{w}^H\right)}{\operatorname{tr}\left(\mathbf{G}_{b,l}\mathbf{Q}\right) + \sigma_{b,l}^2}\right) \geq \beta 2^{R_S}, \forall l \tag{16b}$$

$$\max_{\substack{\Delta_{g,e,k} \in \mathcal{G}_{e,k} \\ \Delta_{h,e,k} \in \mathcal{H}_{e,k}}} \left(1 + \frac{\operatorname{tr}\left(\mathbf{w}\mathbf{H}_{e,k}\mathbf{w}^H\right)}{\operatorname{tr}\left(\mathbf{G}_{e,k}\mathbf{Q}\right) + \sigma_{b,l}^2}\right) \leq \beta, \forall k \tag{16c}$$

$$\operatorname{tr}\left(\mathbf{w}\mathbf{w}^H + \mathbf{Q}\right) \leq P_{max}, \forall k \tag{16d}$$

The Q3 is still non-convex problem. We resort to the idea of SDR to deal with Q3. Define $\mathbf{W} = \mathbf{w}\mathbf{w}^H$. It follows that $\mathbf{W} \succeq 0, \operatorname{rank}(\mathbf{W}) = 1$. By regardless of the constraint $\operatorname{rank}(\mathbf{W}) = 1$, Q3 is written into

$$Q4: \min_{\mathbf{w},\mathbf{Q}\succeq 0} \operatorname{tr}(\mathbf{W} + \mathbf{Q}) \tag{17a}$$

$$s.t. \min_{\Delta_{g,b,l} \in \mathcal{G}_{b,l}} \left(\operatorname{tr}\left(\mathbf{G}_{b,l}\mathbf{Q}F_1\right)\right) + \min_{\Delta_{hb,l} \in \mathcal{H}_{b,l}} \left(\operatorname{tr}\left(\mathbf{W}\mathbf{H}_{b,l}\right)\right) \geq -F_1, \forall l \tag{17b}$$

$$\max_{\Delta_{h,e,k} \in \mathcal{H}_{e,k}} \left(\operatorname{tr}\left(\mathbf{W}\mathbf{H}_{e,k}\right) + \max_{\Delta_{g,e,k} \in \mathcal{G}_{e,k}} \operatorname{tr}\left(\mathbf{G}_{e,k}\mathbf{Q}F_2\right)\right) \leq -F_2, \forall k \tag{17c}$$

$$\operatorname{tr}(\mathbf{W} + \mathbf{Q}) \leq P_{max} \tag{17d}$$

where $F_1 = 1 - 2^{R_s}\beta$ and $F_2 = 1 - \beta$

Due to the semi-definite constraints in Eqs. (17b) and (17c), it is difficult to solve the problem Q5. To solve this problem, we firstly transformed into a convex form to get an accurate form. It can be stated as the following proposition.

Proposition 1: For $\Delta \in \mathbb{H}^N$ and two complex Hermitian matrices $\mathbf{B}$ and $\mathbf{D} = \mathbf{D}^H\mathbf{D}$, we have

$$\max_{\substack{\mathbf{D} \succeq 0 \\ \mathrm{tr}(\Delta \mathrm{B} \Delta) \leq 1}} \mathrm{tr}(\Delta \mathbf{B}) = 2\phi(\mathbf{B}, \mathbf{D}) \tag{18}$$

where ϕ is a function defined by

$$\phi(\mathbf{B}, \mathbf{D}) \triangleq \left\| \left(\mathbf{I} \otimes \tilde{\mathbf{D}} \right) \left(\mathbf{I} \otimes \mathbf{D} + \mathbf{D}^H \otimes \mathbf{I} \right)^{-1} \mathrm{vec}(\mathbf{B}) \right\| \tag{19}$$

Its maximum is achieved when Δ is given by

$$\Delta_{\mathrm{opt}} = \frac{\left(\mathbf{I} \otimes \tilde{\mathbf{D}} + \mathbf{D}^H \otimes \mathbf{I} \right)^{-1} \mathrm{vec}(\mathbf{B})}{\left\| \left(\mathbf{I} \otimes \tilde{\mathbf{D}} \right) \left(\mathbf{I} \otimes \mathbf{D} + \mathbf{D}^H \otimes \mathbf{I} \right)^{-1} \mathrm{vec}(\mathbf{B}) \right\|} \tag{20}$$

where $\mathbf{I}$ is the identity matrix.

Proposition 2: For $\Delta \in \mathbb{H}^N$ and two complex Hermitian matrices $\mathbf{B}$ and $\mathbf{D} = \mathbf{D}^H\mathbf{D}$, we have

$$\min_{\substack{\mathbf{D} \succeq 0 \\ \mathrm{tr}(\Delta \mathrm{B} \Delta) \leq 1}} \mathrm{tr}(\Delta \mathbf{B}) = -2\phi(\mathbf{B}, \mathbf{D}) \tag{21}$$

where its minimum is achieved when Δ is given by

$$\Delta_{\mathrm{opt}} = -\frac{\left(\mathbf{I} \otimes \mathbf{D} + \mathbf{D}^H \otimes \mathbf{I} \right)^{-1} \mathrm{vec}(\mathbf{B})}{\left\| \left(\mathbf{I} \otimes \tilde{\mathbf{D}} \right) \left(\mathbf{I} \otimes \mathbf{D} + \mathbf{D}^H \otimes \mathbf{I} \right)^{-1} \mathrm{vec}(\mathbf{B}) \right\|} \tag{22}$$

Proof: See Appendix A.

By using Proposition 2 for the left side of the inequality (17b) and Proposition 1 for the left side of the inequality (17c), we have Proposition 3.

Proposition 3: The optimization problem Q4 is transformed into

$$Q5: \min_{\mathbf{W} \succeq, \mathbf{Q} \succeq 0, \beta} \mathrm{tr}(\mathbf{W} + \mathbf{Q}) \tag{23a}$$

$$s.t.\ \mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{b,l} \right) - 2\phi\left(\mathbf{W}, \mathbf{D}_{h,b,l} \right) + \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{b,l} F_1 \right) - 2\phi\left(\mathbf{Q}F_1, \mathbf{D}_{g,b,l} \right) \geq F_1, \forall l \tag{23b}$$

$$\mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{e,k} \right) - 2\phi\left(\mathbf{W}, \mathbf{D}_{h,e,k} \right) + \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{e,k} F_2 \right) - 2\phi\left(\mathbf{Q}F_2, \mathbf{D}_{g,e,k} \right) \leq -F_2, \forall k \tag{23c}$$

$$\mathrm{tr}(\mathbf{W} + \mathbf{Q}) \leq P_{max} \tag{23d}$$

Proof: See Appendix B.

Note that Q5 is still non-convex, because the existence of the slack variable. Fortunately, for a fixed β, it is a convex SDP problem. Thus, the problem Q5 is reconstructed as a two-layer optimization problem. The first layer is a successive approximation problem with slack variables β. while the inner is the convex SDP problem. Hence, Q5 will be transformed into

$$Q6: \min_{\beta} f(\beta) \tag{24a}$$

$$s.t. 1 \leq \beta \leq 1 + P_{max} \min_{l \in \mathcal{L}} \mathrm{tr}\left(\mathbf{H}_{h,b,l} \right) \tag{24b}$$

where $f(\beta)$ is a function of β, Q6 has an optimal solution when β is fixed.

Proposition 4: β is defined in $[1, 1 + P_{max}\tau]$, where $\tau = \min_{l\in\mathcal{L}} \mathrm{tr}\left(\tilde{\mathbf{H}}_{b,l}\right)$.

Proof: From the inequality (16c) and positive semi-definite matrices $\mathbf{W}, \mathbf{Q}, \mathbf{G}_{g,e,k}$, we can get

$$\frac{\mathrm{tr}\left(\mathbf{w}\mathbf{H}_{h,e,k}\mathbf{w}^{H}\right)}{\mathrm{tr}\left(\mathbf{G}_{g,e,k}\mathbf{Q}\right) + \sigma_{b,l}^{2}} \geq 0$$

It follows that $\beta \geq 1$.

The upper bound of β can be proved as follows:

$$\beta \leq 1 + \min_{\substack{l \in \mathcal{L},\, \Delta_{h,b,l} \in \mathbf{H}_{h,b,l} \\ \Delta_{g,b,l} \in \mathbf{G}_{g,b,l}}} \frac{\mathrm{tr}\left(\mathbf{H}_{h,b,l}\mathbf{W}\right)}{\sigma_{b,l}^{2} + \mathrm{tr}\left(\mathbf{G}_{g,b,l}\mathbf{Q}\right)} \tag{25a}$$

$$\leq 1 + \min_{l\in\mathcal{L}} \frac{\mathrm{tr}\left(\tilde{\mathbf{H}}_{h,b,l}\mathbf{W}\right)}{\sigma_{b,l}^{2} + \mathrm{tr}\left(\tilde{\mathbf{G}}_{g,b,l}\mathbf{Q}\right)} \tag{25b}$$

$$\leq 1 + P_{max} \min_{l\in\mathcal{L}} \mathrm{tr}\left(\tilde{\mathbf{H}}_{h,b,l}\right) \tag{25c}$$

Here, the inequality (25a) comes from the inequality (16b) and the fact that R_s may be negative. The inequality (25b) is from the inequality of $\mathrm{tr}\left(\mathbf{W}\right) \leq \mathrm{P}_{\max}$ and Eq. (4a) when $\mathbf{Q} = 0$ and $\sigma_{b,l}^{2} = 1$. This has completed the proof.

Now, in order to obtain the optimal value of β , we use the one-dimensional search variables method to calculate. Since the inner-level problem is a convex SDP, which will be solved by the CVX [29]. The outer layer problem for a fixed β is solved by using the golden search method [30]. In summary, we obtain a specific algorithm to solve Q6 as shown in Algorithm 1:

Algorithm 1: to solve Problem Q6

Input: $R_s, a_0, b_0, P_{max}, \varepsilon$.

1 Use golden search method to solve Q7 to obtain an optimal solution $\beta^{*} = \dfrac{a_i + b_i}{2}$

2 Q6 is solved by getting the optimal values of $\mathrm{W}^{*}, \mathrm{Q}_{z}^{*}, P^{*}$ using β^{*}.

3 **Output**: the optimal solution $\left(\mathbf{W}^{*},\ \mathbf{Q}^{*}\right)$

4 Simulation Results

This section provides numerical results to verify the performances of the proposed transmit scheme. The main setups are shown in Tab. 2.

Table 2: Simulation parameters

Notation	Description	Value
$N_t = N_j$	Number of antennas	4
$\sigma_{b,l}^{2} = \sigma_{e,k}^{2}$	The covariance matrix of the AN signal	1
$P_{\max}$	Total system power	$\{10\,dB, 30\,dB\}$
α_b	The parameter of Bobs 'channel uncertainty	$\{0.05, 0.08, 0.1, 0.15\}$
α_e	The parameter of Eves' channel uncertainty	$\{0.002, 0.05, 0.1, 0.15\}$
$\xi_{b,l}, \xi_{e,k}$	The covariance matrix of the channel	$\{0.02, 1\}$

The channel uncertainty is given in the following form.:

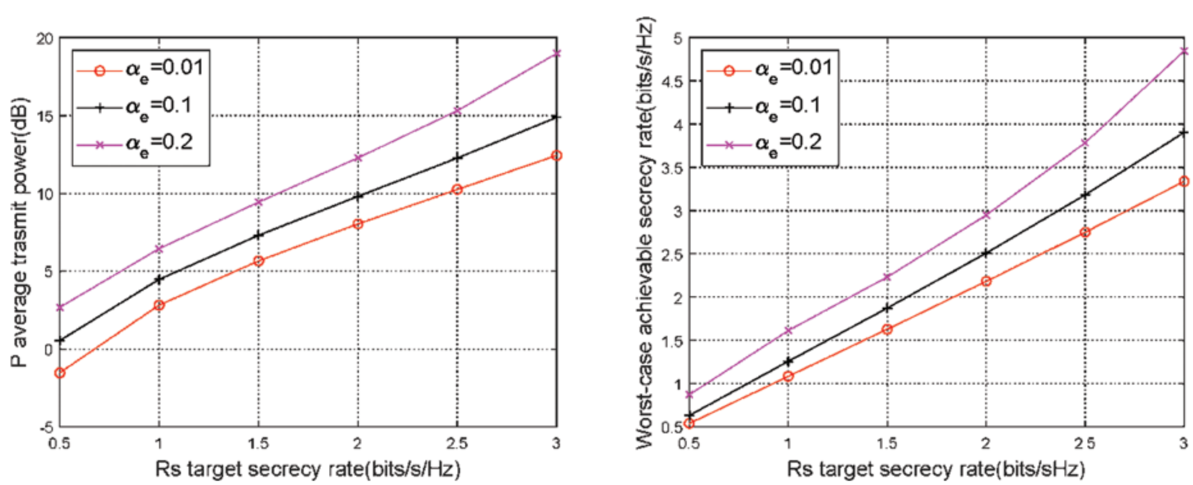
$$\mathbf{D}_{h,b,l} = \frac{I}{\gamma_{h,b,l}^2}, \quad \mathbf{D}_{g,b,l} = \frac{I}{\gamma_{g,b,l}^2} \tag{26a}$$

$$\mathbf{D}_{h,e,k} = \frac{I}{\gamma_{h,e,k}^2}, \quad \mathbf{D}_{g,e,k} = \frac{I}{\gamma_{g,e,k}^2} \tag{26b}$$

$$\gamma_{h,b,l} = \|\tilde{\mathbf{H}}_{b,l}\|\alpha_b, \gamma_{g,b,l} = \|\tilde{\mathbf{G}}_{b,l}\|\alpha_b \tag{26c}$$

$$\gamma_{h,e,k} = \|\tilde{\mathbf{H}}_{e,k}\|\alpha_e, \gamma_{g,e,k} = \|\tilde{\mathbf{G}}_{e,k}\|\alpha_e \tag{26d}$$

Figs. 2 and 3 show the relationships between R_s, P, R and α_b. Fig. 2a presents the actual total transmit power in terms of the target secrecy rate and α_e. We set $\xi_{b,l} = \xi_{e,k} = 1$. It shows that the total transmit power increases as α_e increases. Moreover, as the uncertainty of the Eve's channel increases, the proposed BF scheme requires a higher total transmit power in order to ensure R_s target secrecy rate. From Fig. 2b, the achievable secrecy rate in the worst case is higher than the target secrecy rate, which satisfies the secrecy requirement of the system in Eq. (13b). In addition, although Fig. 2b shows that the larger the α_e, the larger the actual secrecy rate when the target secrecy rate takes a certain value, which does not seem to be true. However, when we compare the actual secrecy rate, the consumed power P jointly, as shown in Fig. 3, a reasonable explanation can be obtained. As its shown in Fig. 3, the larger α_e is, the smaller the achievable secrecy rate is obtained in the system with the same power. This is because in the present scheme, the power is related to the target secrecy rate, while the achievable secrecy rate in the worst case and the power are correlated. So, the larger the channel uncertainty is, the lower the system secrecy performance is. Fortunately, this can be overcome by increasing the transmit power $P_{\max}$.



(a) (b)

Figure 2: (a) Actual average total transmit power *vs*. the pre-defined target secrecy rate. (b) Worst-case achievable secure rate *vs*. the target secrecy rate R_s for different α_e. Here, $L = 3$, $K = 4$, $\alpha_b = 0.02$, and $P_{max} = 30$ dB

Fig. 4 shows the system channel capacity and transmit power with random 100 independent experiments. There are more than 90% channels which satisfy the target secrecy rate and the power requirements. This shows that the proposed scheme should be applicable in piratical channels.

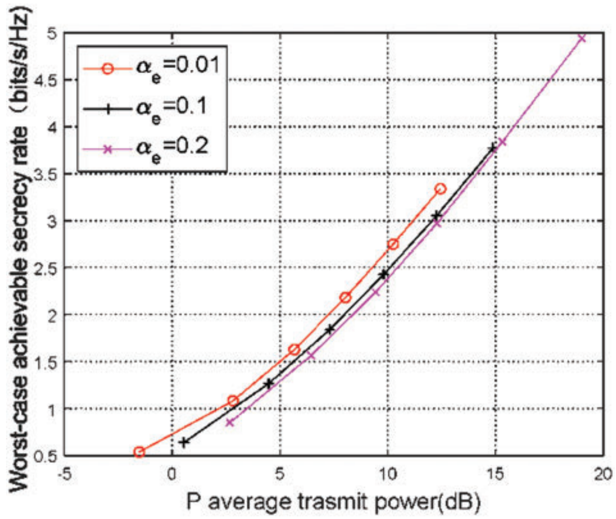


Figure 3: The worst-case achievable secrecy rate *vs.* actual average total transmit power different α_b with $L = 3, K = 4$, $\alpha_b = 0.02$ and $P_{max} = 30$ dB

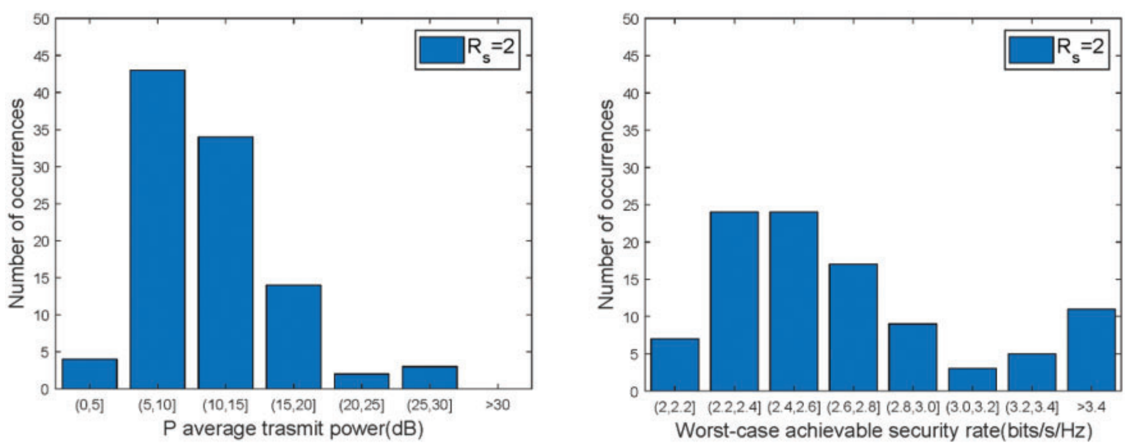
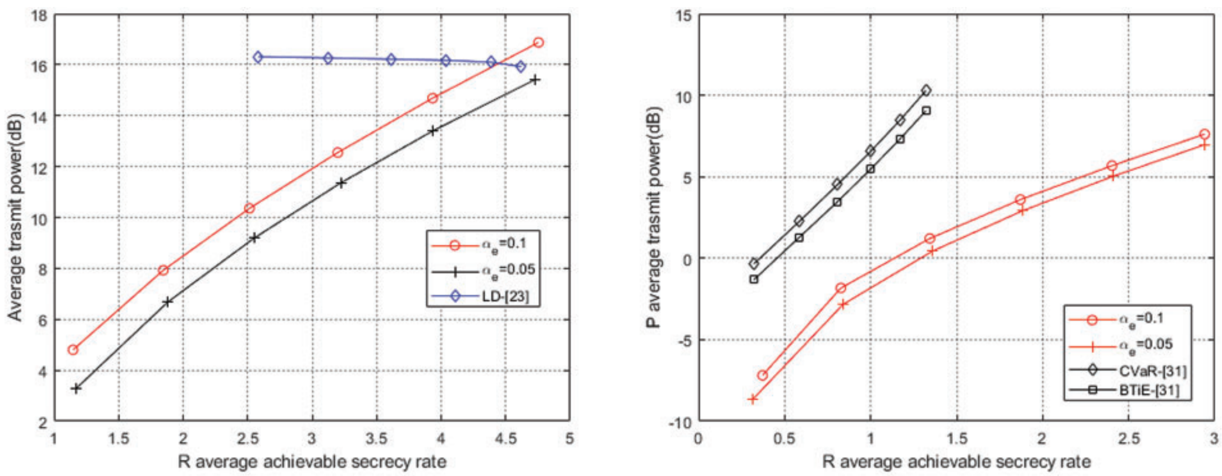


Figure 4: (a) Number of occurrences in terms of actual average total transmit power P with random 100 independent experiments. (b) Number of occurrences in terms of worst-case achievable secrecy ratewith random 100 independent experiments. Here, $L = 3$, $K = 4$, $\alpha_b = \alpha_e = 0.05$, $P_{max} = 30\,dB$ and $R_s = 2$ bit/s/Hz

Now, we compare the present schemes with previous schemes to show its performances. We mainly consider the LD system model in ref. [23] and CVaR and BTiE in ref. [31], while other schemes may not be consistent with the communication model in this paper or have inconsistent optimization objectives, so no more schemes are selected for comparison.The transmit power and achievable secrecy rate in worst case will be simulated.

Fig. 5a shows a comparison between the present scheme and its in ref. [23] for one receiver and one eavesdropper. In Fig. 5a, the present scheme costs less power than the LD scheme when the system secrecy rate is less than 4.4 provided that the secrecy rate in the worst case reaches the target. However, when the achievable secrecy rate is larger than 4.4, we get converse result. So, when the system secrecy rate is less than 3.8, the present scheme is better than its in ref. [23] to guarantee both the secrecy and power consumption Fig. 5b shows the comparison between the present scheme and the two approaches in ref. [31] for one receiver and multiple eavesdroppers. In both models, the simulations are completed with the same total transmit power of $P_{max} = 30\,dB$. Here, $\xi_{b,l} = \xi_{e,k} = 0.002$. In Fig. 5b, the present scheme achieves the bettersecrecyperformance when three schemes have the same transmit power. Hence, these results show when both power loss and achievable secrecy rate are involved, our transmit scheme is better than both schemes in refs. [23,31]. Other parameter effects and performance indicators were not considered in this simulation, but it is worth considering.



(a) (b)

Figure 5: (a) Achievable secrecy rate *vs*. average transmit power with $L = 1, K = 1$ (b) Average transmit power *vs*. achievable secrecy rate with $L = 1$ and $K = 3$

5 Conclusion

In this paper, we investigate the robust transmit BF design for MISO wiretapping channels with the imperfect CSI and the minimum transmit power. The covariance-based CSIs of both legitimate receivers and eavesdroppers are imperfect, where the CSI error is restricted to the ellipsoidal model. The communication models are assisted by AJs. We jointly optimized the covariance of the interference signal generated by the auxiliary node and the beamforming vector of the source node in the AJ-assisted system. The SDR method is firstly used to approximate the present non-convex optimization. We then obtained the equivalent tractable semi-infinite constraints by using the Lagrange duality. This transforms the original non-convex optimization into a two-level optimization with univariate optimization in the outer layer and convex SDP in the inner layer. Simulation results show the present scheme is better than previous schemes. This provides an efficient transmit scheme in practical systems.

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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Appendix A.

It can be verified that the convex problem (18) satisfies Slater's conditions [27]. Hence the strong duality holds between (18) and its dual problem, i.e., they have the same objective value. The Lagrangian of (18) can be written as

$$\mathcal{L}(\Delta,\lambda) = \mathrm{tr}(\Delta\mathbf{A}) - \lambda(\Delta\mathbf{D}\Delta - 1) > 0 \tag{27}$$

λ is the dual variable corresponding to the inequality constraint. Differentiating (27) with respect to Δ and setting the derivative to zero, we have

$$\frac{\partial\mathcal{L}(\Delta,\lambda)}{\partial\Delta} = \mathbf{A} - \lambda\Delta - \lambda\Delta\mathbf{D} = 0 \tag{28}$$

After applying the identity $\mathrm{vec}(\mathbf{BXC}) = \left(\mathbf{C}^T \otimes \mathbf{B}\right)\mathrm{vec}(\mathbf{X})$, we have the following equality

$$\lambda\mathrm{vec}(\Delta) = \mathbf{M}\mathrm{vec}(\mathbf{A}), \mathbf{M} = \left(\mathbf{I}\otimes\mathbf{C} + \mathbf{C}^T \otimes \mathbf{I}\right)^{-1} \tag{29}$$

The Lagrangian dual function for the problem (18) can be written as

$$\mathcal{L} = 2\lambda\mathrm{tr}\left(\Delta\mathbf{D}\Delta^H\right) - \lambda\left(\mathrm{tr}\left(\Delta\mathbf{D}\Delta^H\right) - 1\right) = \lambda\mathrm{tr}\left(\Delta\mathbf{D}\Delta^H\right) + \lambda \overset{(1)}{=} 2\lambda \tag{30}$$

where tr $\left(\Delta \mathbf{D} \Delta^H\right) = 1$ if it is in the optimum point, therefore, step 1 in the above formula holds. Due to $\Delta = \Delta^H, \mathbf{D} = \mathbf{D}^{\sim H}\tilde{\mathbf{D}}$, we can get $\left\|\tilde{\mathbf{D}}\Delta\right\|^2 = 1$, through vec $(\mathbf{BXC}) = \left(\mathbf{C}^T \otimes \mathbf{B}\right)$ vec $(\mathbf{X})$, we can get follow equality:

$$\left\|\tilde{\mathbf{D}}\Delta\right\|^2 = \left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}\right) \mathrm{vec}\,(\Delta)\right\| = 1 \tag{31}$$

By combining Eqs. (29) and (31) we can get the value of λ:

$$\lambda = \lambda \left\|\tilde{\mathbf{D}}\Delta\right\|^2 = \left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}\right) \mathbf{M}\mathrm{vec}\,(\mathbf{A})\right\| \tag{32}$$

By combining Eqs. (30) and (32), we can get

$$\max \mathrm{tr}\,(\Delta B) = 2 \left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}\right) \left(\mathbf{I} \otimes \mathbf{C} + \mathbf{C}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{A})\right\| \tag{33}$$

As a result, we proved Proposition 1 and the optimal Δ, denoted by Δ_{opt} which completes the proof.

The proof of Proposition 2 is similar to that of Proposition 1.

Appendix B.

Use Proposition 2, Eqs. (3a) and (5a) for the left side of the inequality (17b)

$$\begin{aligned}\min_{\Delta_{g,b,l} \in \mathcal{G}_{b,l}} \left(\mathrm{tr}\left(\mathbf{G}_{b,l}\mathbf{Q}F_1\right)\right) &= \min_{\Delta_{g,b,l} \in \mathcal{G}_{b,l}} \left(\mathrm{tr}\left(\tilde{\mathbf{G}}_{b,l}\mathbf{Q}F_1 + \Delta_{g,b,l}\mathbf{Q}F_1\right)\right) \\ &= \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{b,l}F_1\right) - 2\phi\left(\mathbf{Q}F_1, \mathbf{D}_{g,b,l}\right)\end{aligned} \tag{34a}$$

$$\begin{aligned}\min_{\Delta_{h,b,l} \in \mathcal{H}_{b,l}} (\mathrm{tr}(\mathbf{W}\mathbf{H}_{b,l})) &= \min_{\Delta_{h,b,l} \in \mathcal{H}_{b,l}} (\mathrm{tr}(\tilde{\mathbf{W}}\mathbf{H}_{b,l} + \Delta_{h,b,l}\mathbf{W})) \\ &= \mathrm{tr}(\tilde{\mathbf{W}}\mathbf{H}_{b,l}) - 2\phi(\mathbf{W}, \mathbf{D}_{h,b,l})\end{aligned} \tag{34b}$$

The above equation holds when $\Delta_{g,b,l}$ and $\Delta_{h,b,l}$ takes the following optimal values:

$$\Delta_{g,b,l|\mathrm{opt}} = \frac{\left(\mathbf{I} \otimes \mathbf{D}_{g,b,l} + \mathbf{D}_{g,b,l}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{QF}_1)}{\left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}_{g,b,l}\right) \left(\mathbf{I} \otimes \mathbf{D}_{g,b,l} + \mathbf{D}_{g,b,l}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{QF}_1)\right\|} \tag{35a}$$

$$\Delta_{h,b,l|\mathrm{opt}} = \frac{\left(\mathbf{I} \otimes \mathbf{D}_{h,b,l} + \mathbf{D}_{h,b,l}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{W})}{\left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}_{h,b,l}\right) \left(\mathbf{I} \otimes \mathbf{D}_{h,b,l} + \mathbf{D}_{h,b,l}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{W})\right\|} \tag{35b}$$

Similarly, using Proposition 1 and Eqs. (3b) and (5b), from the inequality (17c) we get

$$\max_{\Delta_{h,e,k} \in \mathcal{H}_{e,k}} \left(\mathrm{tr}\left(\mathbf{W}\mathbf{H}_{e,k}\right)\right) = \max_{\Delta_{h,e,k} \in \mathcal{H}_{e,k}} \left(\mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{e,k} + \mathbf{W}\Delta_{h,e,k}\right)\right) = \mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{e,k}\right) - 2\phi\left(\mathbf{W}, \mathbf{D}_{h,e,k}\right) \tag{36a}$$

$$\max_{\Delta_{g,e,k} \in \mathcal{G}_{e,k}} \left(\mathrm{tr}\left(\mathbf{G}_{e,k}\mathbf{Q}F_2\right)\right) = \max_{\Delta_{g,e,k} \in \mathcal{G}_{e,k}} \left(\mathrm{tr}\left(\tilde{\mathbf{G}}_{e,k}\mathbf{Q}F_2\right) + \Delta_{g,e,k}\mathbf{Q}F_2\right)\right) = \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{e,k}F_2\right) - 2\phi\left(\mathbf{Q}F_2, \mathbf{D}_{g,e,k}\right) \tag{36b}$$

Eqs. (36a) and (36b) holds when $\Delta_{h,e,k}$ and $\Delta_{g,e,k}$ take the following optimal values:

$$\Delta_{h,e,k|\mathrm{opt}} = -\frac{\left(\mathbf{I} \otimes \mathbf{D}_{h,e,k} + \mathbf{D}_{h,e,k}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{W})}{\left\|\left(\mathbf{I} \otimes \tilde{\mathbf{D}}_{h,e,k}\right) \left(\mathbf{I} \otimes \mathbf{D}_{h,e,k} + \mathbf{D}_{h,e,k}^H \otimes \mathbf{I}\right)^{-1} \mathrm{vec}\,(\mathbf{W})\right\|} \tag{37a}$$

$$\Delta_{g,e,k|\mathrm{opt}} = -\frac{\left(\mathbf{I}\otimes\mathbf{D}_{g,e,k}+\mathbf{D}_{g,e,k}^{H}\otimes\mathbf{I}\right)^{-1}\mathrm{vec}\left(\mathbf{Q}\mathbf{F}_2\right)}{\left\|\left(\mathbf{I}\otimes\tilde{\mathbf{D}}_{g,e,k}\right)\left(\mathbf{I}\otimes\mathbf{D}_{g,e,k}+\mathbf{D}_{g,e,k}^{H}\otimes\mathbf{I}\right)^{-1}\mathrm{vec}\left(\mathbf{Q}\mathbf{F}_2\right)\right\|} \tag{37b}$$

Thus we can transform inequality (17b) and inequality (17c) into the following form:

$$\mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{b,l}\right) - 2\phi\left(\mathbf{W},\mathbf{D}_{h,b,l}\right) + \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{b,l}F_1\right) - 2\phi\left(\mathbf{Q}F_1,\mathbf{D}_{g,b,l}\right) \geq F_1, \forall l \tag{38a}$$

$$\mathrm{tr}\left(\mathbf{W}\tilde{\mathbf{H}}_{e,k}\right) - 2\phi\left(\mathbf{W},\mathbf{D}_{h,e,k}\right) + \mathrm{tr}\left(\mathbf{Q}\tilde{\mathbf{G}}_{e,k}F_2\right) - 2\phi\left(\mathbf{Q}F_2,\mathbf{D}_{g,e,k}\right) \geq -F_2, \forall k \tag{38b}$$

Combining optimization problem Q4 with inequalities (38a) and (38b) , we can obtain optimization problem Q5.