

## A Stochastic Study of the Fractional Order Model of Waste Plastic in Oceans

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**Abstract:** In this paper, a fractional order model based on the management of waste plastic in the ocean (FO-MWPO) is numerically investigated. The mathematical form of the FO-MWPO model is categorized into three components, waste plastic, Marine debris, and recycling. The stochastic numerical solvers using the Levenberg-Marquardt backpropagation neural networks (LMQBP-NNs) have been applied to present the numerical solutions of the FO-MWPO system. The competency of the method is tested by taking three variants of the FO-MWPO model based on the fractional order derivatives. The data ratio is provided for training, testing and authorization is 77%, 12%, and 11% respectively. The exactness of LMQBP-NNs is observed by using the comparative performances of the obtained and the Adams-Bashforth-Moulton method. To verify the competence, validity, capability, exactness, and consistency of LMQBP-NNs, the performances have been obtained using the regression, state transitions, error histograms, correlation and mean square error.

**Keywords:** Fractional order; ocean; Adams-Bashforth-Moulton; Levenberg-Marquardt backpropagation; numerical solutions

### 1 Introduction

The plastics are a ubiquitous and versatile substantial in the global economy. The popularity of the plastic can be accredited in the field of polymers based on the durable, hydrophobic, light, bio-inert, and cheap to produce. The packaging and products of plastic have a significant role in the environmental productivity as well as economic global growth. The global manufacture of the plastics is massive, and Polyethylene is an important form of the plastic. The demand of European plastic in 2016 based on the polymer kind of high and low densities was reported 12.3%, and 17.5%, respectively. Plastic waste is affecting the environment sustainability in the developing the global economies. This can be attributed to the polymers that are low-cost in production, moldable, bioinert, hydrophobic and



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durable. Many reports [1,2] show that the global plastic production is vast. This large scale of plastic is resulting in discarded waste with the presence in the environment that can be measured through the ecological testation. The waste plastic is calculated to be approximately 150 million tons [3]. An assortment of waste plastic gets into the oceans [4], pausing a main risk to ocean systems and food chain management [5]. The wide-ranging plastic pollution is even present through the remote marine environments [6].

The plastic polymer can be organized by using the environment and plastic types [7–10]. As the massive populations placed in the atmosphere along with the absence of limited waste properties to progress the concept of “circular economy” [11]. The motivations of the natural ecosystems and reformatory models can make confidential environments using the condition of source abandoning as well as extraction of “waste” observation [12]. The major points of the circular economy are used in the technical and biological systems [13]. Numerous European strategies distress the plastic waste, which is implemented to the evolution of the circulatory economies [14,15]. In the circular budget, the recycling of plastic is important to the EU strategy [16]. The management task using the plastic waste in the economy of the EU is very important that is used to recycle in the developing countries except Europe. Hong Kong and China introduced mutually 70% of the waste plastic [17]. The collected post-consumer plastic waste is transported to EU about 46% [18]. The division of plastic is considered into three classes: plastics occupied, post-consumer and misuse of plastic waste [19]. The plastic waste management is liable by recycling, landfill, or incineration, whereas mistreated waste is disposed in the environment and merge in the oceans [20]. Mistreated waste can be used in the durable inverse association based on the economic sector.

The mathematical models are very famous due to the various disciplines of applications in the language as well as natural sciences (physics, statistics, earth science, biology, and chemistry), engineering and social science or non-physical networks. The present work aims to provide the realistic solutions for the nonlinear fractional order mathematical model using the management of waste plastic in ocean (FO-MWPO). The stochastic numerical solvers using the Levenberg-Marquardt backpropagation neural networks (LMQBP-NNs) have been applied to obtain the solutions of the FO-MWPO.

The organization of the paper is given as: Section 2 shows the design of the FO-MWPO. Section 3 presents the stochastic applications, while Section 4 describes the LMQBP-NNs structure. Section 5 provides the simulations of the FO-MWPO model, while the conclusions are drawn at the end.

## 2 The Construction of the FO-MWPO Model

In this section, the mathematical formulations of the waste plastic  $W(\phi)$ , Marine debris  $M(\phi)$  and recycling  $R(\phi)$  based on the FO-MWPO is given as [21]:

$$\begin{cases} \frac{dW(y)}{dy} = \rho + \mu R(y) - \beta M(y) W(y) - \varepsilon W(y) & W_0 = l_1, \\ \frac{dM(y)}{dy} = \beta W(y) M(y) - \pi M(y) & M_0 = l_2, \\ \frac{dR(y)}{dy} = \pi M(y) - \psi R(y) + \varepsilon W(y) - \mu R(y) & R_0 = l_3. \end{cases} \quad (1)$$

The descriptions of each parameter is provided in [Tab. 1](#).

**Table 1:** The detail descriptions of each parameter of the FO-MWPO model

Parameters	Details
$R(y)$	Recycle
$W(y)$	Waste plastic
$M(y)$	Marine debris
$\pi$	Rate of Marine debris to recycle
$\beta$	Waste rate to Marine
$\rho$	New waste, which is to be recycle
$\mu$	Rate of recycle waste to reproduce
$\psi$	Rate of recycled waste to be misplaced
$\varepsilon$	Rate of waste to be recycled directly without Marine debris
$j_1, j_2, j_3$	Initial conditions
$y$	Time

The numerical investigations of the FO-MWPO have been provided through the artificial intelligence (AI) along with LMQBP-NNs. The FO-MWPO is given as:

$$\begin{cases} \frac{d^{(\alpha)} W(y)}{dy^{(\alpha)}} = \rho + \mu R(y) - \beta M(y) W(y) - \varepsilon W(y) & W_0 = l_1, \\ \frac{d^{(\alpha)} M(y)}{dy^{(\alpha)}} = \beta W(y) M(y) - \pi M(y) & M_0 = l_2, \\ \frac{d^{(\alpha)} R(y)}{dy^{(\alpha)}} = \pi M(y) - \psi R(y) + \varepsilon W(y) - \mu R(y) & R_0 = l_3, \end{cases} \quad (2)$$

where  $\alpha$  is the fractinal order derivative in the system (2), which is defined based on the definition of Rieman Liouville. By taking the fractional order  $\alpha$ , one may have better insights for superfast transients and superslow evolutions using the system dynamics to draw an appropriate inferences.

### 3 Features of the Stochastic Solvers

In this section, the stochastic numerical operators using the LMQBP-NNs are provided to solve the FO-MWPO model. The use of stochastic computing solvers through the supervised/unsupervised learning have been explored to solve the complicated, stiff, and singular models [22,23]. In recent decades, the stochastic applications have been proposed to solve the food chain model [24], singular Lane-Emden systems [25], infectious disease considering its anatomical variables [26], functional order systems [27] and HIV dynamical models [28–31]. This study is related to design and present the numerical solutions of the fractional order derivative using the numerical computing procedures based on the LMQBP-NNs. Few novel topographies of the LMQBP-NNs to present the solutions of the FO-MWPO model are described as:

- The design of the nonlinear FO-MWPO model along with its numerical solutions is presented first time in this study.

- The numerical solutions of the designed FO-MWPO are presented using the stochastic paradigms based on the proposed LMQBP-NNs.
- The proposed LMQBP-NNs is tested to solve three variants of the FO-MWPO model by taking different values of FO derivatives.
- The brilliance and correctness of the computing stochastic LMQBP-NNs scheme is checked based on the comparison of the achieved and reference performances.
- The accuracy of the LMQBP-NNs approach is achieved using the absolute error (AE), which is accomplished in good measures to solve the FO-MWPO model.
- The STs, regression, MSE, correlation and EHs values approve the trustworthiness and dependability of the designed LMQBP-NNs for solving the nonlinear FO-MWPO model.

#### 4 Proposed LMQBP-NNs Procedures

In this section, the designed LMQBP-NNs procedures are provided to solve the FO-MWPO model. The structure of the proposed LMQBP-NNs is given as:

- The essential performances based on the LMQBP-NNs are provided.
- The execution through the LMQBP-NNs is indicated to solve the FO-MWPO model.

Fig. 1 is drawn using the multi-layer optimization procedures based on the numerical stochastic paradigms of LMQBP-NNs. The stochastic procedures are given using a Matlab ‘nftool’ command with the data selection as 77% for training and 12% for testing and 11% for authorization.

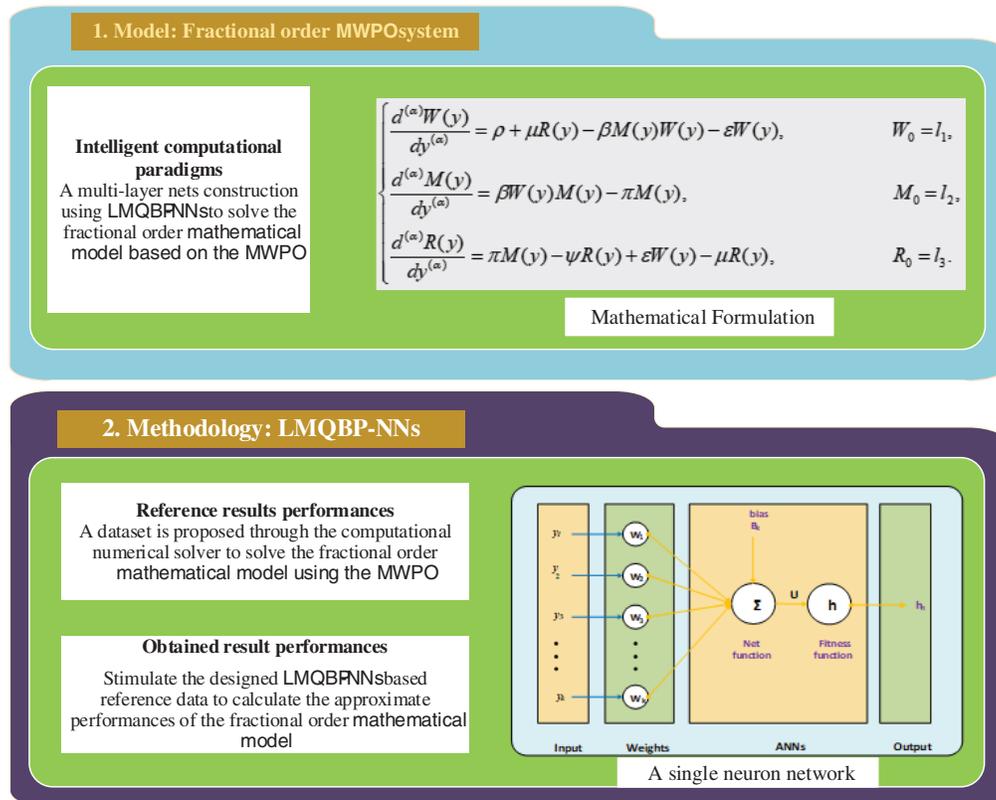
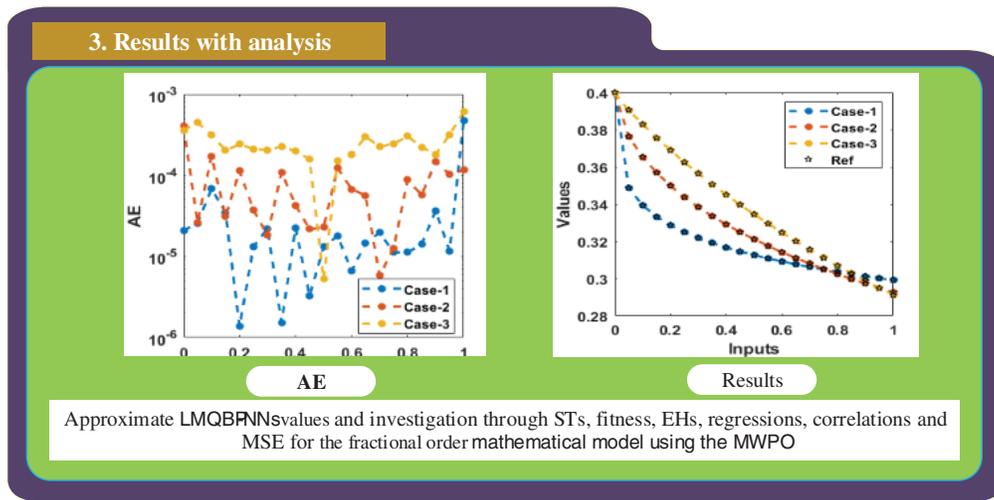


Figure 1: (Continued)



**Figure 1:** Workflow for the FO-MWPO model using the LMQBPNs

One may choose different values of these samples. If the training is selected greater than 77%, then the good results can be obtained due to the bias inputs. Additionally, the training samples are selected less than 77%, then the accuracy of the network LMQBPNs to solve the FO-MWPO degraded substantially. Therefore, the samples for bias and unbiased inputs should be set with care to avoid both premature convergence and divergence.

### 5 Results and Discussions

Three cases of the FO-MWPO model by taking different values of the FO derivatives using the designed LMQBPNs are presented in this Section

**Case 1:** Consider a FO-MWPO model is provided by taking  $\alpha = 0.3, \rho = 0.2, \mu = 0.1, \beta = 0.3, \varepsilon = 0.25, \pi = 0.4, \psi = 0.1, l_1 = 0.2, l_2 = 0.4$  and  $l_3 = 0.6$ .

$$\left\{ \begin{array}{l} \frac{d^{(0.3)}W(y)}{dy^{(0.3)}} = 0.2 - 0.3M(y)W(y) + 0.1R(y) - 0.25W(y) \quad W_0 = 0.2, \\ \frac{d^{(0.3)}M(y)}{dy^{(0.3)}} = 0.3W(y)M(y) - 0.4M(y) \quad M_0 = 0.4, \\ \frac{d^{(0.3)}R(y)}{dy^{(0.3)}} = 0.4M(y) + 0.25W(y) - 0.2R(y) \quad R_0 = 0.6. \end{array} \right. \quad (3)$$

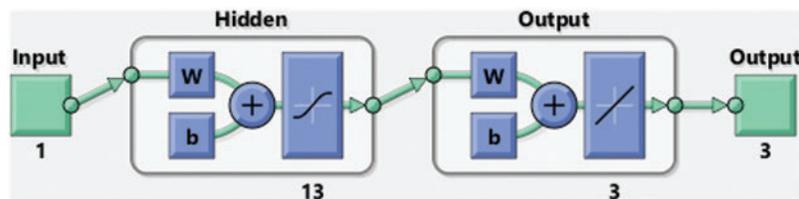
**Case 2:** Consider a FO-MWPO model is provided by taking  $\alpha = 0.6, \rho = 0.2, \mu = 0.1, \beta = 0.3, \varepsilon = 0.25, \pi = 0.4, \psi = 0.1, l_1 = 0.2, l_2 = 0.4$  and  $l_3 = 0.6$ .

$$\left\{ \begin{array}{l} \frac{d^{(0.6)}W(y)}{dy^{(0.6)}} = 0.2 - 0.3M(y)W(y) + 0.1R(y) - 0.25W(y) \quad W_0 = 0.2, \\ \frac{d^{(0.6)}M(y)}{dy^{(0.6)}} = 0.3W(y)M(y) - 0.4M(y) \quad M_0 = 0.4, \\ \frac{d^{(0.6)}R(y)}{dy^{(0.6)}} = 0.4M(y) + 0.25W(y) - 0.2R(y) \quad R_0 = 0.6. \end{array} \right. \quad (4)$$

**Case 3:** Consider a FO-MWPO model is provided by taking  $\alpha = 0.9$ ,  $\rho = 0.2$ ,  $\mu = 0.1$ ,  $\beta = 0.3$ ,  $\varepsilon = 0.25$ ,  $\pi = 0.4$ ,  $\psi = 0.1$ ,  $l_1 = 0.2$ ,  $l_2 = 0.4$  and  $l_3 = 0.6$ .

$$\begin{cases} \frac{d^{(0.9)}W(y)}{dy^{(0.9)}} = 0.2 - 0.3M(y)W(y) + 0.1R(y) - 0.25W(y) & W_0 = 0.2, \\ \frac{d^{(0.9)}M(y)}{dy^{(0.9)}} = 0.3W(y)M(y) - 0.4M(y) & M_0 = 0.4, \\ \frac{d^{(0.9)}R(y)}{dy^{(0.9)}} = 0.4M(y) + 0.25W(y) - 0.2R(y) & R_0 = 0.6. \end{cases} \quad (5)$$

The numerical performances using the simulations of FO-MWPO model are described by using the computational LMQBP-NNs procedures with 13 numbers of neurons together with the data selection as 77% for training and 12% for testing and 11% for authorization. The neurons-based input, hidden and output structure are shown in Fig. 2.



**Figure 2:** Designed LMQBP-NNs for the FO-MWPO model

Figs. 3 to 5 represent the FO-MWPO model using the designed LMQBP-NNs. The performances along with the EHs values representing the best curves, verifications and authentications are provided in Fig. 3 for the FO-MWPO model. The best achieved performances of the FO-MWPO model have been provided at epochs 63, 21 and 14, which are found as  $1.5171 \times 10^{-09}$ ,  $1.0929 \times 10^{-08}$  and  $6.1108 \times 10^{-08}$ . The gradient values are calculated as  $9.8428 \times 10^{-08}$ ,  $9.996 \times 10^{-07}$  and  $3.3348 \times 10^{-06}$  for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> case. These graphical measures represent the convergence of the designed LMQBP-NNs to solve the FO-MWPO model using the LMQBP-NNs. Fig. 4 represents the results valuations and EHs for the FO-MWPO model using the LMQBP-NNs. These result valuations indicate the comparative performances of the obtained and reference solutions. The result valuations based on the training, substantiation, and testing performances for the FO-MWPO model are provided. Figs. 4d–4f indicates the EHs values, which are calculated as  $2.76 \times 10^{-05}$ ,  $5.65 \times 10^{-05}$  and  $5.15 \times 10^{-05}$  for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> case. Fig. 5 represents the correlation values for the FO-MWPO model using the designed LMQBP-NNs. The correlation values are found as 1 for the FO-MWPO model using the designed LMQBP-NNs. The training, substantiation and testing values label the exactness of the designed LMQBP-NNs procedure. The MSE convergence through the complexity, authentication, training, iterations, backpropagation, and testing is shown in Tab. 2 for the FO-MWPO model using the designed LMQBP-NNs.

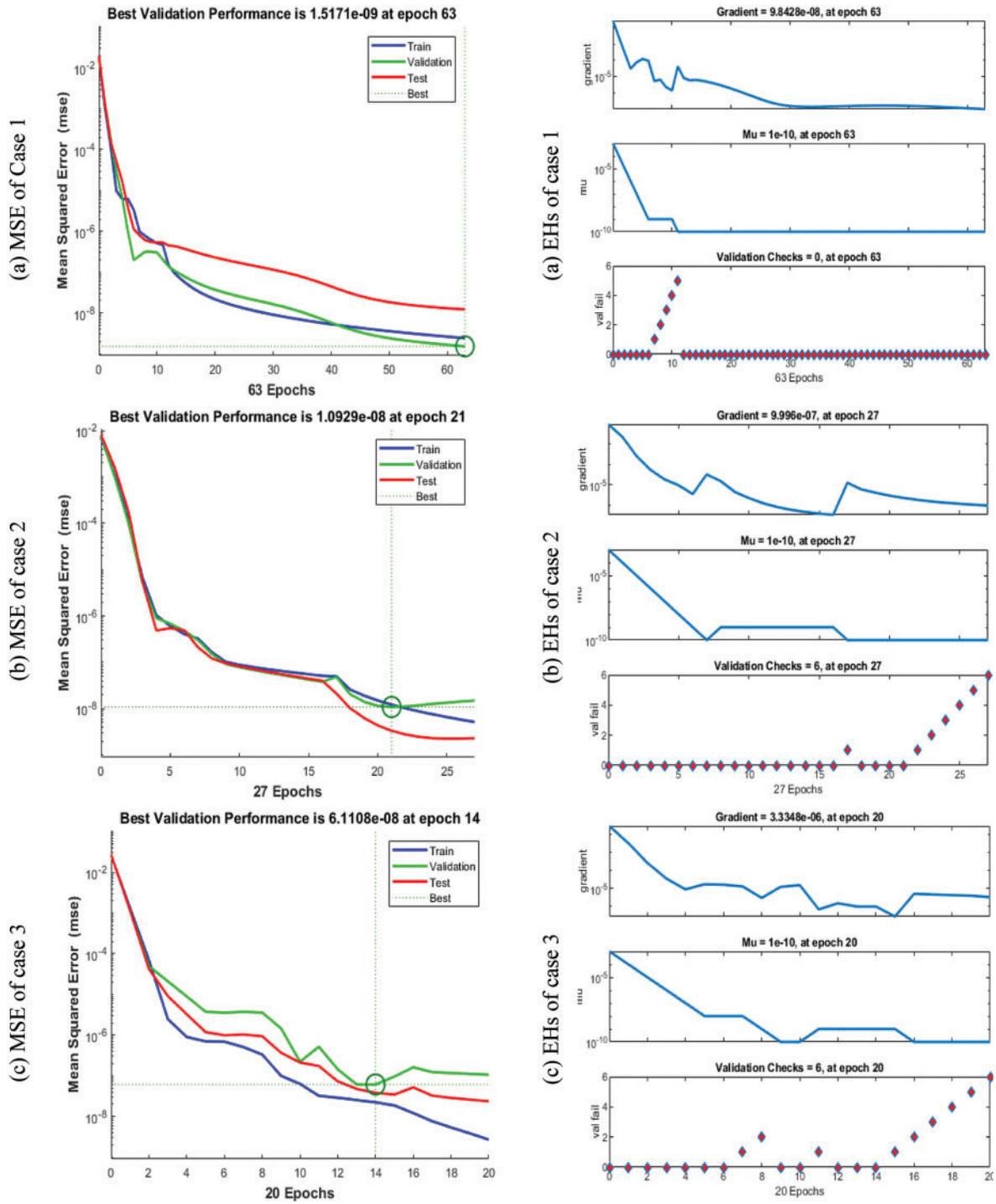


Figure 3: STs and MSE values for the FO-MWPO model

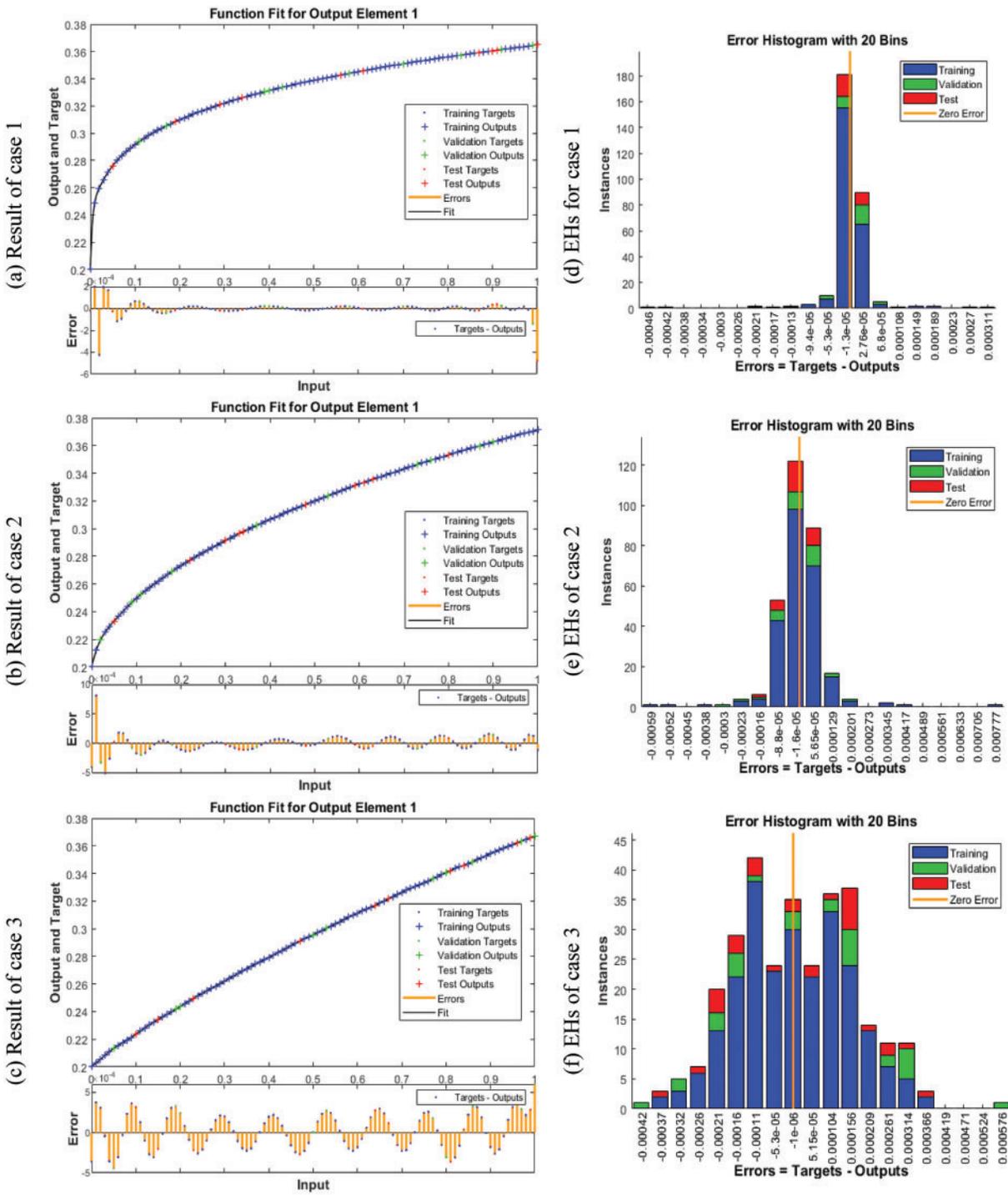
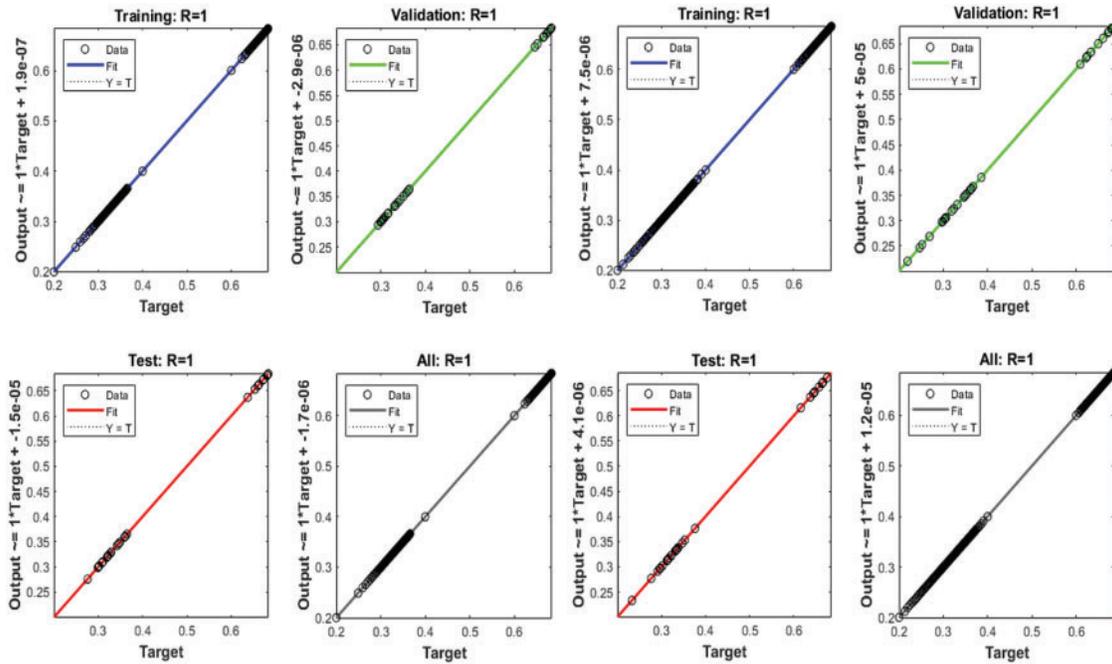
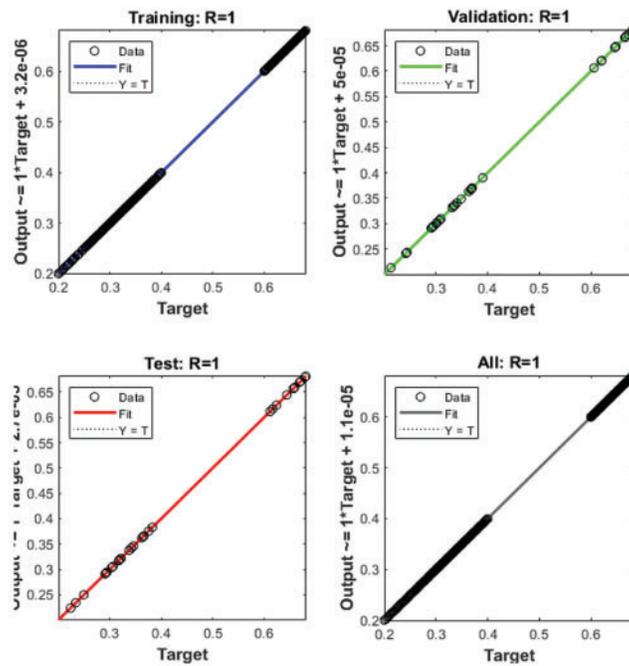


Figure 4: Results valuations and EHs for the FO-MWPO model



(a) Regression of Case 1

(b) Regression of Case 2



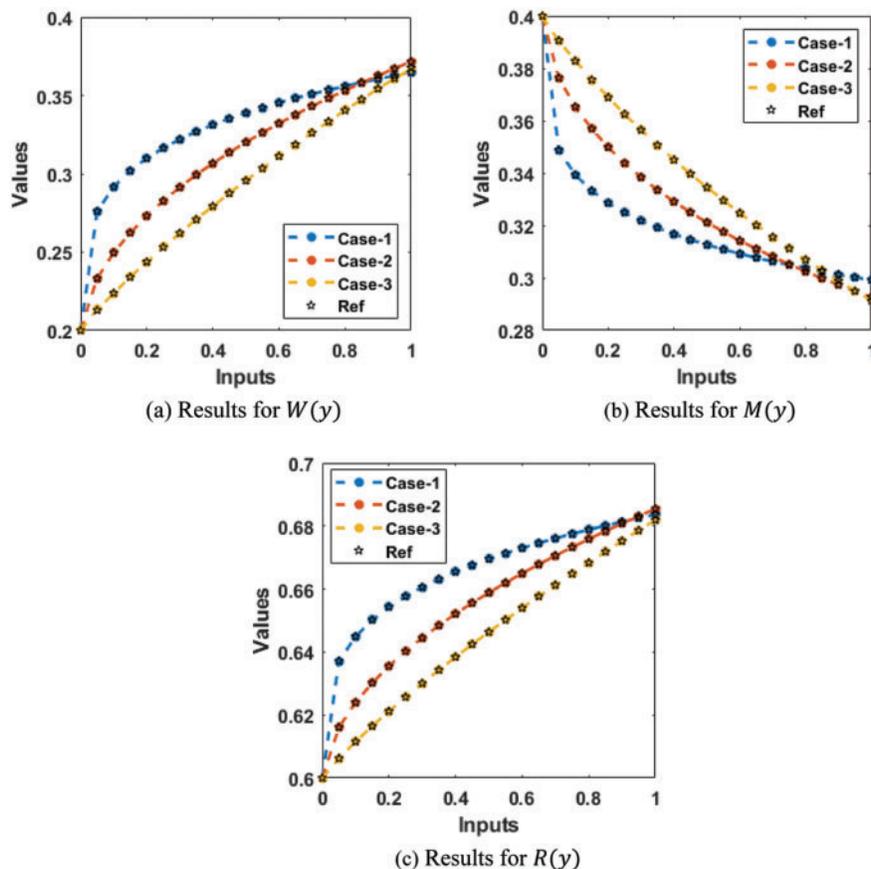
(c) Regression of Case 3

Figure 5: Regression values for the FO-MWPO model

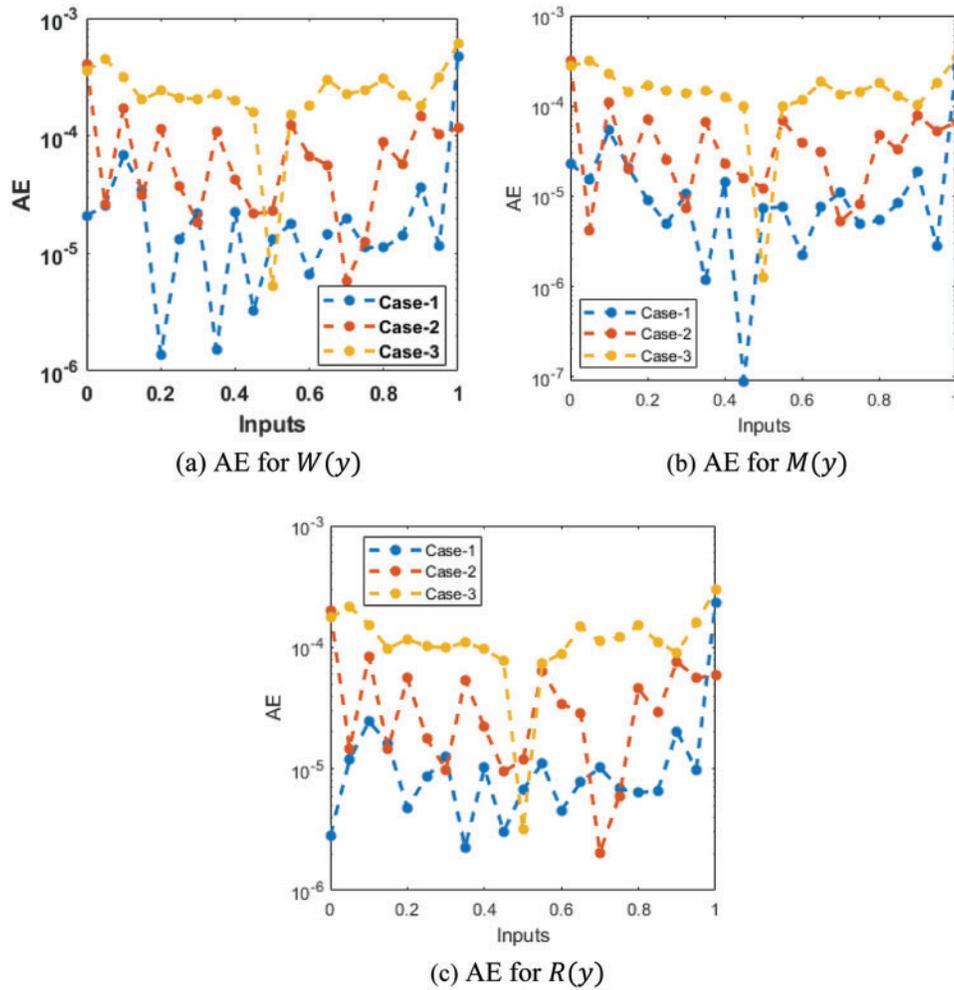
**Table 2:** LMQBP-NNs procedure for the FO-MWPO model

Case	MSE			Gradient	Performance	Iterations	Mu	Time
	Training	Testing	Authentication					
1	$2.41 \times 10^{-09}$	$1.21 \times 10^{-08}$	$1.51 \times 10^{-09}$	$9.84 \times 10^{-08}$	$2.42 \times 10^{-09}$	63	$1 \times 10^{-10}$	06
2	$1.22 \times 10^{-08}$	$3.35 \times 10^{-09}$	$1.09 \times 10^{-08}$	$1 \times 10^{-06}$	$5.19 \times 10^{-09}$	27	$1 \times 10^{-10}$	06
3	$2.21 \times 10^{-08}$	$3.39 \times 10^{-08}$	$6.11 \times 10^{-08}$	$3.33 \times 10^{-06}$	$2.69 \times 10^{-09}$	20	$1 \times 10^{-10}$	03

The comparison plots and AE performances are shown in Figs. 6 and 7. The obtained numerical results have been provided to solve the nonlinear FO-MWPO model using the designed LMQBP-NNs. The overlapping of the obtained and reference solutions has been provided in Fig. 6. These overlapping of the results authenticates the correctness of the designed LMQBP-NNs to solve the nonlinear FO-MWPO model. The values of the AE have been performed to solve the nonlinear FO-MWPO model in Fig. 7. The AE for the waste plastic material  $W(y)$  calculated as  $10^{-05}$  to  $10^{-06}$ ,  $10^{-04}$  to  $10^{-06}$  and  $10^{-04}$  to  $10^{-05}$  for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> case. The AE for Marine debris  $M(y)$  calculated as  $10^{-05}$  to  $10^{-07}$ ,  $10^{-04}$  to  $10^{-05}$  and  $10^{-03}$  to  $10^{-06}$  for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> case. Similarly, the AE for the recycle  $R(y)$  calculated as  $10^{-04}$  to  $10^{-07}$ ,  $10^{-04}$  to  $10^{-06}$  and  $10^{-03}$  to  $10^{-05}$  for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> case. These best calculated AE performances indicate the exactness of the designed LMQBP-NNs to solve the FO-MWPO.



**Figure 6:** Comparison of the result for the FO-MWPO model



**Figure 7:** AE for the FO-MWPO model

### 6 Concluding Remarks

The study shows the design of the fractional order mathematical model based on the MWPO. The investigations related to fractional order have been presented to get more realistic results of the ocean model. The designed system is divided into, waste plastic  $W(y)$ , Marine debris  $M(y)$  and recycle  $R(y)$ . The solution of the FO-MWPO model have never been applied before to solve the stochastic procedures of the artificial neural networks based Levenberg-Marquardt backpropagation. Three different variations of the fractional order derivatives have been described to solve the FO-MWPO model. The data is applied to present the performances of the FO-MWPO model as selected 77% for training and 12% for testing and 11% for authorization. Thirteen neurons have been discussed to present the numerical solutions of the nonlinear FO-MWPO model. The comparison of the numerical observations of the FO-MWPO model with the Adams-Bashforth-Moulton shows the correctness of the designed scheme. The AE is performed in good measures that is calculate  $10^{-4}$  to  $10^{-6}$  for each parameter of the model. The reduction of MSE is obtained using the obtained numerical performances through the LMQBP-NNs. The dependability and ability of LMQBP-NNs is obtained through the numerical performances that is illustrated in the plots of STs, regression, EHs, correlation and MSE.

The precision of LMQBP-NNs is obtained using the designed FO-MWPO model with the matching performances of reference and obtained solutions.

In future study, the LMQBP-NNs will be applied to present the numerical outcomes of the fluid, fractional and longren-wave systems [32–49].

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