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Bio-Inspired Modelling of Disease Through Delayed Strategies

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Abstract: In 2020, the reported cases were 0.12 million in the six regions to the official report of the World Health Organization (WHO). For most children infected with leprosy, 0.008629 million cases were detected under fifteen. The total infected ratio of the children population is approximately 4.4 million. Due to the COVID-19 pandemic, the awareness programs implementation has been disturbed. Leprosy disease still has a threat and puts people in danger. Nonlinear delayed modeling is critical in various allied sciences, including computational biology, computational chemistry, computational physics, and computational economics, to name a few. The time delay effect in treating leprosy delayed epidemic model is investigated. The whole population is divided into four groups: those who are susceptible, those who have been exposed, those who have been infected, and those who have been vaccinated. The local and global stability of well-known conclusions like the Routh Hurwitz criterion and the Lyapunov function has been proven. The parameters' sensitivity is also examined. The analytical analysis is supported by computer results that are presented in a variety of ways. The proposed approach in this paper preserves equilibrium points and their stabilities, the existence and uniqueness of solutions, and the computational ease of implementation.

Keywords: Leprosy disease; delayed model; stability analysis; computer results

1 Introduction

Leprosy is an infectious disease generated by a bacillus known as Mycobacterium leprae and lepromatosis. The morphology character of Mycobacterium leprae is an acid-fast, intracellular bacterium that is aerobic and rod-shaped. This long-term infection is also called Hansen's disease. Tuberculiod, Lepromatous and Borderline is the types of leprosy. There is also some simple classification of leprosy disease, known as Multibacillary and Paucibacillary leprosy. It is mostly found in poverty where there is poor nutrition. This infection is mostly prevented in thronged areas and infects those whose immune system is weak enough to fight such disease. This murderous infectious harms an infected person's nerves, respiratory, tract, skin, and eyes. As a result, nerve damage deprives the infected individual of the ability to feel pain, resulting in the loss of limbs owing to reoccurring traumas or infection from undiscovered wounds. Loss of eyesight and weakness of muscles is an aching part of this disease. Its symptoms may vary from person to person. Its first symptoms may occur within a year, and for others, it can take 20 years to exhibit any symptom. Primarily, this disease shows paler patches on the skin and sometimes nodules on the skin. It can be complex due to diagnosing it, and other times the patient remains misdiagnosed for years. The most common or usual symptoms of leprosy include a runny nose, muscle and eyesight weakness, dry scalp, skin lesions, loss of sensation in some body parts, especially fingers and toes, nasal cartilage destruction, and speech phonation and resonation of sound. These symptoms vary from person to person. The symptoms of this disease are categorized according to their stages. The incubation period is five years. The infected person may notice pale or pinkish skin patches that may lead to pain or sometimes temperature or insensitive. The next stage of leprosy can result in tissue loss, and it will affect fingers and toes to become insensitive to any pain. The immunological response of an individual varies according to the type of leprosy. Nerve damage is also a fatal symptom as it may cause loss of muscle function that may lead to paralysis and numbness. if treated early, this disease can be curable but become perpetual if delayed for months. Early discovery of the disease is significant, as it will present physical and neurological damage. The WHO recommends that those in close contact with someone who has leprosy receive preventive treatment. A single dosage of rifampicin (SDR) is suggested for adults and children who do not have leprosy or tuberculosis. This precaution treatment reduces infection up to 51% within 02 years and 30% within 06 years. The Bacillus Calmette-Guerin (BCG) is considered a great weapon against leprosy. According to WHO estimates, this vaccine (BCG) at birth reduces the cases of leprosy, and it should be recommended in countries that have repeated frequent leprosy cases. A 3–drug regimen of rifampicin, dapsone, and clofazimine is suggested to cure leprosy. On the other hand, multidrug therapy (MDT) has shown efficacy, and individuals are no longer infectious following the first monthly dose. According to the estimate of WHO in the year 2018, approximately 208. Six hundred nineteen cases were reported in different countries of the world. According to a report, in 2003, Pakistan had 20,000 leprosy cases, and its ratio decreased from 2009 to 2018. The leprosy rate is highly associated with India, the African region is considered the second most highly affected, and Brazil is third [1]. In 2004, Meima et al. investigated the effects of the current strategy to overcome leprosy, and the scenario presented that the following strategy will help reduce transmission. However, it may be slow in the process [2]. In 2004, Haanpa et al. discussed the possible dynamics, challenges in treatment, and needs in neuropathic pain, which required research work [3]. In 2006, Scollarod et al. explained the advanced research of Mycobacterium leparo [4]. In 2009, Bakker et al. discussed GIS to reduce the leprosy disease rate. GIS was a powerful tool in disease [5]. In 2010, Fischer et al. approached the comprehensive data of cluster leprosy within the household [6]. In 2012, Mushayabasa et al. presented the result of different strategies aiming to overcome leprosy disease by targeting infected and susceptible people [7]. In 2014, Barreto et al. discussed spatial analysis to explain the temporal pattern of leprosy disease in the Brazilian Amazon region.

Through spatial analysis, the leprosy rate in early childhood gives a better understating [8]. In 2014, Lastoria et al. updated leprosy aspects like dermatologists on epidemiological etiopathogenic, and clinical [9]. In 2014, Martinez et al. evaluated PCR-based techniques for diagnosing leprosy disease [10]. In 2015, white et al. suggested some areas that play an important role for future research in leprosy disease and some challenges in which leprosy patients survive and the physician is facing in the treatment of Hansen's disease [11]. In 2015, Blok et al. discussed that the eradication of leprosy is reached at the national level by 2020, but it remains in some specific regions [12]. Oli et al., 2017, investigated the dynamic of leprosy and its impact on the host population [13]. In 2018, Block et al. discussed the result of a diagnostic test to find the subclinical leprosy rate [14]. Le et al., in 2018, presented Southwest China the epidemiological characteristics of leprosy and presented the current situation of antigen-specific serum antibodies in humans who are suffering from leprosy and have different types of clinical [15]. In 2018, de Souza et al. examined the relationship link between the social degree of vulnerability and the intensity of leprosy [16]. In 2018, Mawardi et al. Leprosy morbidity reports played an important role in epidemiology because its result was authentic [17]. Using a mathematical model, Giraldo et al. described the involvement of multibacillary and paucibacillary leprosy. Some mathematical simulations are taken to analyze different stages of leprosy disease [18]. Lietman et al. estimated that the spread of leprosy in different areas of India was explained using geometric stability [19]. In 2018, Geluk et al. discussed strategies to identify leprosy reactions based on correlations that would be useful in leprosy [20]. In 2020, Gunawan et al. presented that testicular atrophy has many frequencies. The testicular function should be considered normal for leprosy patients [21]. Similar research on leprosy has been discussed in [22,23]. Numerical techniques with spatio temporal analysis of models are discussed [24,25]. In [26–36], researchers have recently focused on computer simulations of the epidemic model with various delay tactics. The following work studied the related models [37,38].

Delay models play a key role in real-life objects in the mathematical epidemiological field. Delay tactics are a control strategy near nature. As a result, we strive for a highly effective leprosy cure through delay tactics. Our scenario for the paper is as follows: Section 1 introduces leprosy as a model of delay. Section 2 discusses the leprosy model's equilibria. Section 3 leprosy impairs model stability on a local and global scale. Section 4, to support theoretical analysis, some numerical results are discussed in the model and future discussions.

2 Model Methodology

In this section, we have studied leprosy delayed epidemic model. At any time t, the entire population $N(t)$ is divided into four groups, with $X(t)$ being the compartment of susceptible humans, $Y(t)$ is the asymptomatic infected human, $Z(t)$ is the compartment of human infected from multibacillary, and $W(t)$ is the group of humans infected from paucibacillary as shown in Fig. 1.

The parameters of the leprosy delayed model are described as follows: ρ is the natural birth rate, β_p is the rate of interaction with a human having transmission of paucibacillary, β_m is the rate of exchange with a human having information of multibacillary, θ is the developing rate between symptomatic to the asymptomatic stage of leprosy, f represents the fraction of population who come out with multibacillary, $(1-f)$ is the fraction of population who come out with paucibacillary, μ denotes the natural death rate. The following system of delay differential equations represents the leprosy delayed epidemic model.

$$\frac{dX}{dt} = \rho - \left(\beta_m X(t-\tau)\ Z\ (t-\tau) + \beta_p X(t-\tau)\ W(t-\tau)\right) e^{-\mu\tau} - \mu X \tag{1}$$

$$\frac{dY}{dt} = \left(\beta_m X(t-\tau)\ Z\ (t-\tau) + \beta_p X(t-\tau)\ W(t-\tau)\right) e^{-\mu\tau} - \theta Y\ - \mu Y \tag{2}$$

$$\frac{dZ}{dt} = f\theta Y\ - \mu z \tag{3}$$

$$\frac{dW}{dt} = \theta(1-f)\ Y\ - \mu W \tag{4}$$

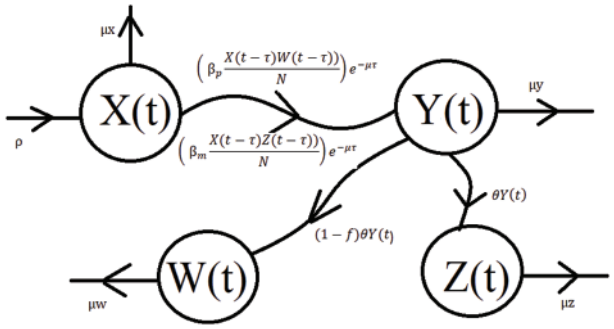


Figure 1: Flow diagram of leprosy delayed epidemic model

The initial conditions are as follows: $X = X_0 \geq 0,\ Y = Y_0 \geq 0,\ W = W_0 \geq 0,\ Z = Z_0 \geq 0.$

2.1 Positivity and Boundedness of Model

The delayed Leprosy epidemic model must maintain positivity and boundedness for practical analysis. All the state variables of the model $X(t)$, $Y(t)$, $Z(t)$, $W(t)$ must be non-negative. Therefore all the achieved outcomes of the model must be positive and bounded at any time, $t \geq 0\ \tau \leq t$ in a feasible region.

$$\mathcal{M} = \left\{(X, Y, Z, W) \epsilon R_+^4 : N(t) \leq \frac{\rho}{\mu},\ X \geq 0, Y \geq 0, Z \geq 0, W \geq 0\right\}.$$

For this purpose, we will investigate the following results.

Theorem: The solutions $(X,\ Y,\ Z,\ W) \epsilon R_+^4$ of the system (1–4) are positive at any time $t \geq 0,\ \tau \leq t$ with given non-negative initial conditions.

Proof: It is clear from the system (1–4) as follows:

$$\left.\frac{dX}{dt}\right|_{X=0} = \rho \geq 0,\ \left.\frac{dY}{dt}\right|_{Y=0} = (\beta_m XZ + \beta_p XW)e^{-\mu t} \geq 0,\ \left.\frac{dZ}{dt}\right|_{Z=0} = f\ \theta\ Y \geq 0,\ \left.\frac{dW}{dt}\right|_{W=0} = \theta(1-f)\ Y \geq 0$$

As desired.

Theorem [25]: The solutions $(S,\ E,\ I,\ V) \epsilon R_+^4$ of the system (1–4) is bounded.

Proof: Consider the function as follows:

$$N(t) = X(t) + Y(t) + Z(t) + W(t).$$

$$\frac{dN}{dt} = \rho - \mu N$$

We obtain this disparity as a result of Gronwall's inequality.

$$N(t) = N(0)\, e^{-\mu t} + \frac{\rho}{\mu},\ t \geq 0$$

$$\lim_{t\to\infty} \text{Sup}\ \ N(t) = \frac{\rho}{\mu}$$

This demonstrates that the solution to system (1–4) is constrained and falls within the feasible region $\mathcal{M}$.

2.2 Equilibria of the Model

In this section, the equilibrium points of the model are presented as follows:

Leprosy trivial equilibrium (LTE-L_0) $= \left(X^0, Y^0, W^0, Z^0\right) = (0, 0, 0, 0)$,

Leprosy free equilibrium (LFE-L_1) $= \left(X^1, Y^1, Z^1, W^1\right) = \left(\frac{\rho}{\mu}, 0, 0, 0\right)$

Leprosy existing equilibrium (LEE-L_2) $= (X^*, Y^*, Z^*, W^*)$

Where the values of X^*, Y^*, Z^* and W^* are given as follows:

$$X^* = \frac{\mu(\theta+\mu)}{\left[\beta_m f\theta + \beta_p\theta(1-f)\right]e^{-\mu\tau}}$$

$$Y^* = \frac{\left[\beta_m f\theta + \beta_p\theta(1-f)\right]\rho e^{-\mu\tau} - \mu^2(\theta+\mu)}{(\theta+\mu)\left[\beta_m f\theta + \beta_p\theta(1-f)\right]e^{-\mu\tau}}$$

$$W^* = \frac{\theta(1-f)}{\mu}\frac{\left[\beta_m f\theta + \beta_p\theta(1-f)\right]\rho e^{-\mu\tau} - \mu^2(\theta+\mu)}{(\theta+\mu)\left[\beta_m f\theta + \beta_p\theta(1-f)\right]e^{-\mu\tau}}$$

$$Z^* = \frac{f\theta}{\mu}\frac{\left[\beta_m f\theta + \beta_p\theta(1-f)\right]\rho e^{-\mu\tau} - \mu^2(\theta+\mu)}{(\theta+\mu)\left[\beta_m f\theta + \beta_p\theta(1-f)\right]e^{-\mu\tau}}$$

2.3 Reproduction Number

The reproduction rate of the leprosy delayed epidemic model is described in this section. It is an essential component of the model denoted by $\mathcal{R}_0$. It plays a significant role in disease dynamics. The disease will be in control when the value of $\mathcal{R}_0$ is less than 1, and if its value is greater than 1, it represents the endemic situation. From Eqs. (2) to (4), infectious and recovered populations have been taken as follows:

$$\begin{bmatrix} Y' \\ Z' \\ W' \end{bmatrix} = \begin{bmatrix} 0 & \beta_m\, Xe^{-\mu\tau} & \beta_p\, Xe^{-\mu\tau} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y \\ Z \\ W \end{bmatrix} - \begin{bmatrix} \theta+\mu & 0 & 0 \\ -f\theta & \mu & 0 \\ -\theta(1-f) & 0 & \mu \end{bmatrix} \begin{bmatrix} Y \\ Z \\ W \end{bmatrix}$$

where $\mathrm{F} = \begin{bmatrix} 0 & \beta_m\frac{\rho}{\mu}e^{-\mu\tau} & \beta_p\frac{\rho}{\mu}e^{-\mu\tau} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $V = \begin{bmatrix} \theta+\mu & 0 & 0 \\ -f\theta & \mu & 0 \\ -\theta(1-f) & 0 & \mu \end{bmatrix}$

$$FV^{-1}\big|_{L_1} = \begin{bmatrix} \dfrac{\beta_m \rho e^{-\mu\tau}\mu f\theta + \beta\rho e^{-\mu\tau}\rho\mu_m\theta(1-f)}{\mu^2(\theta+\mu)} & \dfrac{\beta_m\rho e^{-\mu\tau}}{\mu^2} & \dfrac{\beta_p\rho e^{-\mu\tau}}{\mu^2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

By the next-generation matrix method, the largest eigenvalue represents the reproduction number given by $\mathcal{R}_0 = \left(\dfrac{\beta_m f\theta + \beta_p\theta(1-f)}{\mu^2(\theta+u)}\right)\rho e^{-\mu\tau}$.

2.4 Sensitivity Analysis of Parameters

The parameters employed in the leprosy delayed epidemic model are critical to the disease's dynamics. The effects of small changes in parameters change the entire dynamic. We have addressed the sensitivity analysis of the parameters in this part. For this, we proceed as follows:

$$L_f = \frac{\dfrac{\partial\,\mathcal{R}_0}{\mathcal{R}_0}}{\dfrac{\partial f}{f}} = \frac{f}{\mathcal{R}_0}\times\left(\frac{\beta_m\theta-\beta_p\theta}{\mu^2(\theta+\mu)}\right)\rho e^{-\mu\tau} > 0, L_{\beta_m} = \frac{\beta_m}{\mathcal{R}_0}\times\left(\frac{f\theta}{\mu^2(\theta+\mu)}\right)\rho e^{-\mu\tau} > 0,$$

$$L_{\beta p} = \frac{\beta_p}{\mathcal{R}_0}\times\left(\frac{\theta(1-f)}{\mu^2(\theta+\mu)}\right)\rho e^{-\mu\tau} > 0, L_\rho = \frac{\mathrm{P}}{\mathcal{R}_0}\times\left(\frac{\beta_m f\theta+\beta_p\theta(1-f)}{\mu^2(\theta+\mu)}\right)e^{-\mu\tau} > 0,$$

$$L_\theta = \frac{\theta}{R_0}\frac{\left[\beta_m f\theta+\beta_p\theta(1-f)\right]\mu^2(\theta+\mu)\rho e^{-\mu\tau} - \mu^2\left[\beta_m f\theta+\beta_p\theta(1-f)\right]\rho e^{-\mu\tau}}{\left[\mu^2(\theta+\mu)\right]^2} > 0,$$

$$L_\mu = -\frac{\mu}{R_0}\left[\frac{\tau\mu^2(\theta+\mu)\left[\beta_m f\theta+\beta_p\theta(1-f)\right]\mathrm{P}e^{-\mu\tau} + \left(2\mu\theta+3\mu^2\right)\left[\beta_m f\theta+\beta_p\theta(1-f)\right]\rho e^{-\mu\tau}}{\left[\mu^2(\theta+\mu)\right]^2}\right] < 0,$$

From the above analysis, it can be seen that $f, \beta_m, \beta_p, \rho, \theta$ are sensitive, and the parameters μ are not sensitive.

3 Local Stability

This section discusses the local stability of leprosy delayed epidemic model at three equilibrium locations (LTE-L_0), (LFE-L_1), and (LEE-L_2).

Theorem: Leprosy trivial equilibrium points, the system (1–4) is locally asymptotically stable at (LTE-L_0), if $\mathcal{R}_0 = 1$. Otherwise, it is unstable.

Proof: The Jacobian matrix J_L of the system (1–4) is given by

$$J_L = \begin{bmatrix} -\left(\beta_m\ Z+\beta_p\ W\right)e^{-\mu\tau}-\mu & 0 & -\beta_m\ Xe^{-\mu\tau} & -\beta_p X\ e^{-\mu\tau} \\ \left(\beta_m\ Z+\beta_p\ W\right)e^{-\mu\tau} & -\theta-\mu & \beta_m\ Xe^{-\mu\tau} & \beta_p\ X\ e^{-\mu\tau} \\ 0 & f\theta & -\mu & 0 \\ 0 & \theta(1-f) & 0 & -\mu \end{bmatrix}$$

At leprosy trivial equilibrium (LTE-L_0), the Jacobean matrix is given by

$$J_L|_{L_0} = \begin{bmatrix} -\mu & 0 & 0 & 0 \\ 0 & -\theta - \mu & 0 & 0 \\ 0 & f\,\theta & -\mu & 0 \\ 0 & \theta(1-f) & 0 & -\mu \end{bmatrix}$$

Now, $\left|J_L|_{L_0} - \lambda I\right| = 0$ implies that all the Eigenvalues are negative. That is, $\lambda_1 = -\mu < 0, \lambda_2 = -(\mu + \theta) < 0, \lambda_3 = -\mu < 0$ and $\lambda_4 = -\mu < 0$. This shows that system is locally stable at (LTE-L_0).

Theorem: The system (1–4) is locally asymptotical stable at leprosy free equilibrium (LFE-L_1) $= \left(\frac{\rho}{\mu}, 0, 0, 0\right)$, if $\mathcal{R}_0 < 1$. Otherwise, the system is unstable.

Proof: The Jacobian matrix calculated at leprosy free equilibrium point (LFE-L_1) as follows:

$$J_L|_{L_1} = \begin{bmatrix} -\mu & 0 & -\beta_m \frac{\rho}{\mu} e^{-\mu\tau} & -\beta_p \frac{\rho}{\mu} e^{-\mu\tau} \\ 0 & -\theta - \mu & \beta_m \frac{\rho}{\mu} e^{-\mu\tau} & \beta_p \frac{\rho}{\mu} e^{-\mu\tau} \\ 0 & f\,\theta & -\mu_m & 0 \\ 0 & \theta(1-f) & 0 & -\mu \end{bmatrix}$$

By computing $\left|J_L|_{L_1} - \lambda I\right| = 0$, we have the Eigenvalue $\lambda = -\mu < 0$ and the determinant $\begin{vmatrix} -a_1 - \lambda & a_2 & a_3 \\ f\,\theta & -\mu - \lambda & 0 \\ a_4 & 0 & -\mu - \lambda \end{vmatrix} = 0.$

Where, $a_1 = \theta + \mu$, $a_2 = \beta_m \frac{\rho}{\mu} e^{-\mu\tau}$ $a_3 = \beta \mathrm{p}\, \frac{\rho}{\mu} e^{-\mu\tau}$ $a_4 = \theta(1-f)$.

By evaluating the above determinant, we have the following polynomial

$\lambda^3 + b_0\lambda^2 + b_1\lambda + b_1 = 0$. Where, $b_0 = (a_1 + 2\mu)$, $b_1 = \left(2a_1\mu + \mu^2 - a_2 f\,\theta - a_3 a_4\right)$ and $b_2 = \left(a_1\mu^2 - a_2 f\,\theta\,\mu - a_3\mu a_4\right)$.

By Routh-Hurwitz Criterion for 3rd-degree polynomial, we have $b_2 b_0 > 0$ *and* $b_0 b_1 > b_2$.

Therefore, the leprosy existence equilibria (LEE-L_2) locally is stable.

Theorem: The system (1–4) is locally asymptotical stable at leprosy existence equilibrium (LEE-L_2)= (X^*, Y^*, Z^*, W^*), if $\mathcal{R}_0 > 1$ and unstable elsewhere.

Proof: The Jacobian matrix calculated at leprosy existence equilibrium point (LEE-L_2) is as follows:

$$J_{L_1} = \begin{bmatrix} -\left(\beta_m Z^* + \beta_p W^*\right) e^{-\mu\tau} - \mu & 0 & -\beta_m X^* e^{-\mu\tau} & -\beta_p X^* e^{-\mu\tau} \\ \left(\beta_m Z^* + \beta_p W^*\right) e^{-\mu\tau} & -\theta - \mu & \beta_m X^* e^{-\mu\tau} & \beta_p X^* e^{-\mu\tau} \\ 0 & f\theta & -\mu & 0 \\ 0 & \theta(1-\mathrm{f}) & 0 & -\mu \end{bmatrix}$$

$$= \begin{vmatrix} -b_1 - \mu - \lambda & 0 & -b_2 & -b_3 \\ b_1 & -b_4 - \lambda & b_2 & b_3 \\ 0 & b_5 & -b_6 - \lambda & 0 \\ 0 & b_7 & 0 & -b_6 - \lambda \end{vmatrix} = 0$$

where, $b_1 = \left(\beta_m\ Z^* + \ \beta_p\ W^*\right) e^{-\mu\tau}$, $b_2 = \beta_m\ X^* e^{-\mu\tau}$, $b_3 = \beta_p\ X^*\ e^{-\mu\tau}$, $b_4 = \theta$, $b_5 = \ \mathrm{f}\ \theta$, $b_6 = \mu$, and $b_7 = \ \theta\,(1 - \mathrm{f})$.

Now evaluating $\left|J_L|_{L_2} - \lambda I\right| = 0$, we have

$$\lambda^4 + n_1\lambda^3 + n_2\lambda^2 + n_3\lambda + n_4 = 0$$

where, $n_1 = (b_1 + b_4 + 2b_6 + \mu)$, $n_2 = (b_3b_7 - b_1b_6 - b_6\mu - {b_6}^2 - b_4b_6 + b_2b_5 - b_1b_6 - b_1b_4 - b_6\mu - b_4\mu - b_4b_6)$, $n_3 = (b_3b_6b_7 + b_3\mu b_7 + b_2b_6b_5 - b_1{b_6}^2 - b_1b_4b_6 - \mu{b_6}^2 - b_4{b_6}^2 - b_4b_6\mu + b_2b_5\mu - b_1b_4b_6 - b_4b_6\mu)$ and $n_4 = \left(b_3b_7b_6\mu + b_2b_5b_6\mu + b_1b_4{b_6}^2\mu\right)$.

By Routh-Hurwitz Criterion for 4th-degree polynomial, we have

$$n_0 > 0, n_1 > 0, n_1n_2 - n_0n_3 > 0,$$

$(n_1n_2 - n_0n_3)n_3 - {n_1}^2n_4 > 0$ and $n_4 > 0$ only if $\mathcal{R}_0 > 1$.

Hence, the leprosy existence equilibria (LEE-L_2) are locally stable.

4 Global Stability

The global stability of the leprosy delayed epidemic model is examined using the following data.

Theorem: The system (1–4) is globally asymptotical stable at leprosy trivial equilibrium (LTE-L_0), $L_0 = \left(X^0, W^0, Y^0, Z^0\right) = (0, 0, 0, 0)$, if $\mathcal{R}_0 = 1$.

Proof: Define the Volterra Lyapunov function $F : \mathcal{M} \to \mathbb{R}$ as

$$G = X + Y + W + Z \quad for\ all \quad (X, Y, W, Z)\ \epsilon\mathcal{M},$$

$$\frac{dG}{dt} = \frac{dX}{dt} + \frac{dY}{dt} + \frac{dZ}{dt} + \frac{dW}{dt}$$

$$\frac{dG}{dt} = \mathrm{P}\ -\ (\beta_m\ X\ Z + \beta_p\ X\ W)e^{-\mu\tau} -\ \mu X + (\beta_m\ X\ Z + \beta p\ X\ W)e^{-\mu\tau} - \theta\ Y\ -\ \mu\, Y + f\ \theta\ Y$$

$$-\ Z\mu + \theta\,(1 - f)\ Y\ -\ \mu\, W$$

$$= \rho - \mu\ (\mathrm{X} + \mathrm{Y} + \mathrm{Z} + \mathrm{W})$$

$$\frac{dF}{dt} \leq \rho - \mu\frac{\rho}{\mu}$$

Since, $\frac{dF}{dt} = 0$ if $R_0 = 1$ therefore (LTE-L_0) is globally asymptotically stable.

Theorem: The system (1–4) is globally asymptotically stable at leprosy free equilibrium (LFE-L_1) $= \left(\frac{\rho}{\mu}, 0, 0, 0\right)$, if $\mathcal{R}_0 < 1$.

Proof: Consider the Volterra Lyapunov function $G : \mathcal{M} \to \mathbb{R}$ defined by

$$G = \left(X - X^0\ - X^0\, log\frac{X}{X^0}\right) + Y + Z + W$$

$$\frac{dG}{dt} = (1 - \frac{X^0}{X})\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} + \frac{dw}{dt}$$

$$\frac{dG}{dt} = (X - X^0)\left[\frac{\rho}{X} - \beta_m\, Z\, e^{-\mu\tau} - \beta_p\, We^{-\mu\tau} - \mu\right] + \beta_m\, X\, ze^{-\mu\tau} + \beta_p\, X\, We^{-\mu\tau} - \theta\, y$$
$$- \mu y + f\,\theta\, Y - Z\mu + \theta(1-f)\,Y - \mu W$$

$$= \left(X - X^0\right)\left[\frac{\rho}{X} - \beta_m\, Z\, e^{-\mu\tau} - \beta_p\, We^{-\mu\tau} - \frac{\rho}{x^0} + \beta_m\, Z^0\, e^{-\mu\tau} + \beta_p\, W^0 e^{-\mu\tau}\right]$$
$$+ \beta_m X\, Z\, e^{-\mu\tau} + \beta_p\, WXe^{-\mu\tau} - (\theta + \mu)\, Y + \theta\, Y - \mu\, Z - \mu W\,.$$

$$\frac{dG}{dt} = -\frac{\left(X - X^0\right)^2 \rho}{xx^0} - \beta_m \left(Z - Z^0\right)\left(X - X^0\right) e^{-\mu\tau} - \beta_p\, \left(X - X^0\right)\left(W - W^0\right) e^{-\mu\tau}$$
$$- \mu W\left[1 - \frac{\beta_p\, X\, e^{-\mu\tau}}{\mu}\right] - Z\mu\left[1 - \frac{\beta_m\, Xe^{-\mu\tau}}{\mu}\right] - \mu Y$$

It can easily be observed that $\frac{dG}{dt} < 0$, if $\mathcal{R}_0 < 1$ and $\frac{dG}{dt} = 0$ if $X = X^1,\ Y = 0, W = 0$ *and* $Z = Z^1$. Hence (LFE-L_1) is globally asymptotically stable.

Theorem: The system (1–4) is globally asymptotical stable at leprosy existence equilibrium (LEE-L_2) = (X^*, Y^*, Z^*, W^*), if $\mathcal{R}_0 > 1$.

Proof: Consider the Volterra Lyapunov function $H : \mathcal{M} \to \mathbb{R}$ defined as

$$H = \left(X - X^* - X^* log\frac{X^*}{X}\right) + \left(Y - Y^* - Y^* log\frac{Y^*}{Y}\right) + \left(Z - Z^* - Z^* log\frac{Z^*}{Z}\right) + \left(w - w^* - w^* log\frac{w^*}{w}\right)$$

$$\frac{dH}{dt} = \left(1 - \frac{X^*}{X}\right)\frac{dX}{dt} + \left(1 - \frac{Y^*}{Y}\right)\frac{dY}{dt} + \left(1 - \frac{Z^*}{Z}\right)\frac{dZ}{dt} + \left(1 - \frac{W^*}{W}\right)\frac{dW}{dt}$$

$$= (X - X^*)\left[\frac{\rho}{X} - \beta_m\, Z\, e^{-\mu\tau} - \beta_p\ We^{-\mu\tau} - \mu\right] + (\,Y - Y^*)\left[\frac{\beta_m\, XZe^{-\mu\tau}}{Y} + \frac{\beta_p\, XW\, e^{-\mu\tau}}{Y} - (\theta + u)\right]$$
$$+ (Z - Z^*)\left[\frac{f\,\theta\, Y}{Z} - \mu\right] + (W - W^*)\left[\frac{\theta(1-f)\,Y}{W} - u\right]$$

$$= (X - X^*)\left[\frac{\rho}{X} - \beta_m\, Z\, e^{-\mu\tau} - \beta_p\, We^{-\mu\tau} - \frac{\rho}{x^*} + \beta_m Z^* e^{-\mu\tau} + \beta_p W^*\, e^{-\mu\tau}\right]$$
$$+ (Y - Y^*)\left[\frac{\beta_m\, XZe^{-\mu\tau}}{Y} + \frac{\beta_p\, XW\, e^{-\mu\tau}}{Y} - \frac{\beta_m\, XZe^{-\mu\tau}}{Y^*} - \frac{\beta_p\, XW\, e^{-\mu\tau}}{Y^*}\right]$$
$$+ (Z - Z^*)\left[\frac{f\,\theta\, Y}{Z} - \frac{f\,\theta Y}{Z^*}\right] + (W - W^*)\left[\frac{\theta(1-f)\,Y}{W} - \frac{\theta(1-f)\,Y}{Y^*}\right]$$

$$\frac{dH}{dt} = -\frac{\rho\,(X - X^*)^2}{XX^*} - \beta_m\, e^{-\mu\tau}\,(X - X^*)\,(Z - Z^*) - \beta_p\, e^{-\mu\tau}\,(X - X^*)\,(W - W^*)$$
$$- \frac{(Y - Y^*)^2}{YY^*}\,\beta_m\, XZ\, e^{-\mu\tau} - \frac{(Y - Y^*)^2}{YY^*}\,\beta_p XW\, e^{-\mu\tau} - \frac{(Z - Z^*)^2}{ZZ^*} f\,\theta\, Y - \frac{(W - W^*)^2\,\theta\,(1 - f)\,Y}{WW^*}$$

As $\frac{dH}{dt} \leq 0$ for $\mathcal{R}_0 < 1$, also $\frac{dH}{dt} = 0$ only if $X = X^*$, $Y = Y^*$, $Z = Z^*$ *and* $W = W^*$.

So, (LEE-L_2) is globally asymptotically stable.

5 Computer Simulations

The numerical treatment of the leprosy delay delayed epidemic model from the parameter value is presented. The parameter values and their data analysis is presented in Fig. 2.

$\rho = 0.5,\ \beta_p = 2.3, \beta_m = 2.5,\ \theta = 0.19, \mu = 0.5, f = 0.7, h = 0.1, t = 0, \tau = 0, X = 0.4,\quad Y = 0.1,$

$Z = 0.2$ *and* $W = 0.3$.

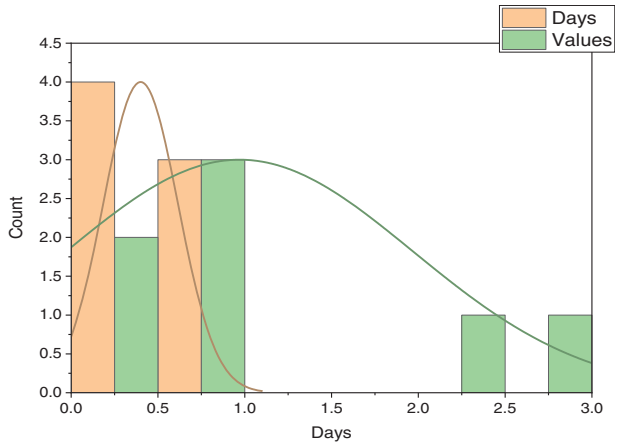


Figure 2: Data analysis behavior of the disease

Example 1: (Without delay, simulation at leprosy-free equilibrium (PFE-P 1).

Fig. 3a through Fig. 3d illustrate the solution of the system (1–4) at the leprosy-free equilibrium (PFE $-P_1$), $P_1 = \left(X^1, Y^1, Z^1, W^1\right) = (1, 0, 0, 0)$ with the model's starting data $X\,(0) = 0.5, Y\,(0) = 0.2$, $Z\,(0) = 0.1$, $W\,(0) = 0.1$. As a result, when the system (1–4) enters a disease-free state without delay, the reproduction number is $R_0 = 0.7930 < 1$. Additionally, as desired, Fig. 3e displays the system's combined behavior (1–4) without the delay effect.

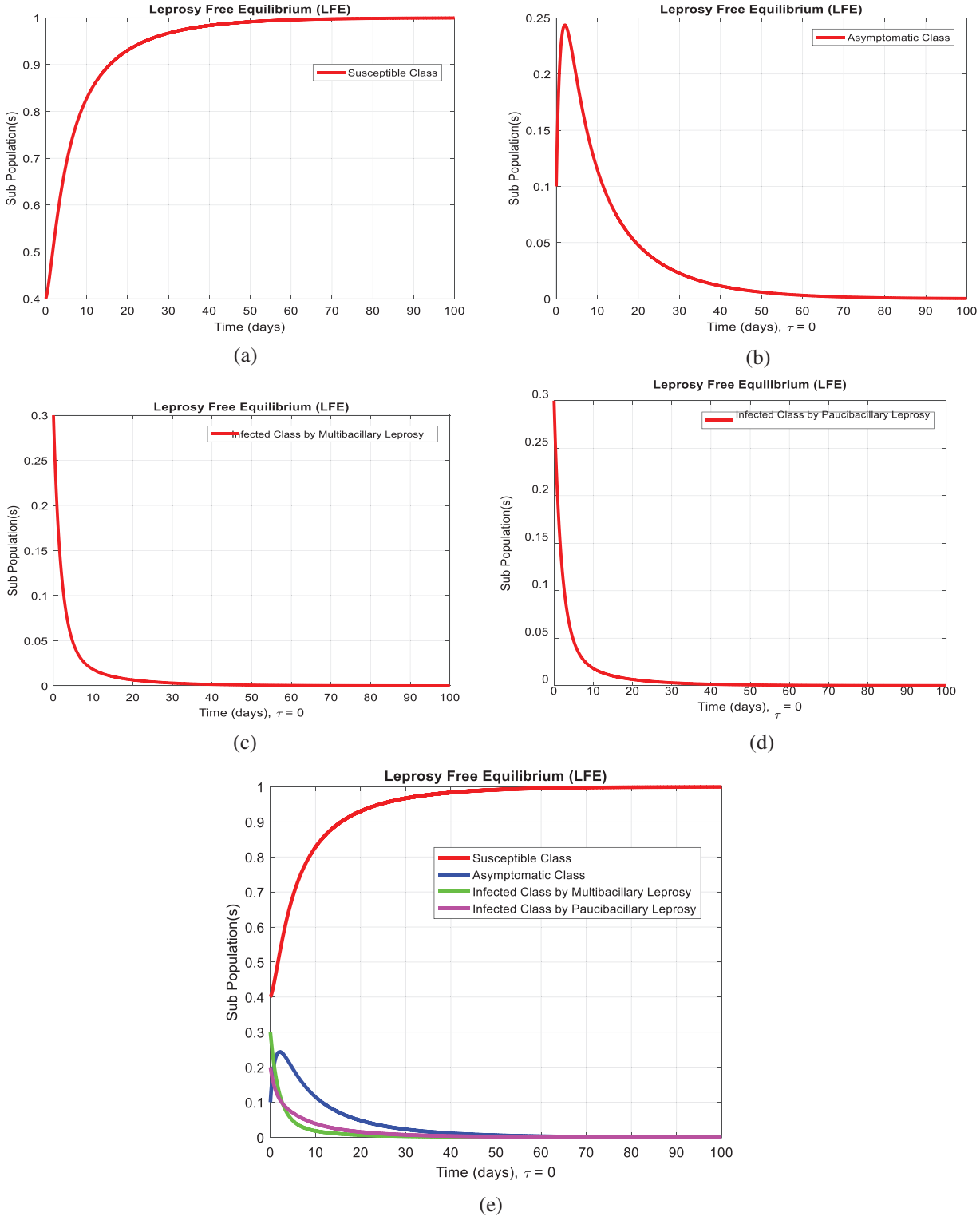


Figure 3: Plots of time for the system (1–4) at the model's leprosy-free equilibrium

Example 2: (Simulation at leprosy existing equilibrium (LEE $-L_2$) without delay).

Figs. 4a to 4d illustrate the solution of the system (1–4), at leprosy existing equilibrium (LEE $-L^*$), $L^* = (X^*, Y^*, Z^*, W^*) = (0.7442, 0.1854, 0.04931, 0.02114)$, Therefore, the system (1–4) converges to L^*, without delay, the reproduction number is $R_0 = 1.3438 > 1$. Additionally, as desired, Fig. 4e depicts the combined behavior of the system (1–4) without regard for the delay effect.

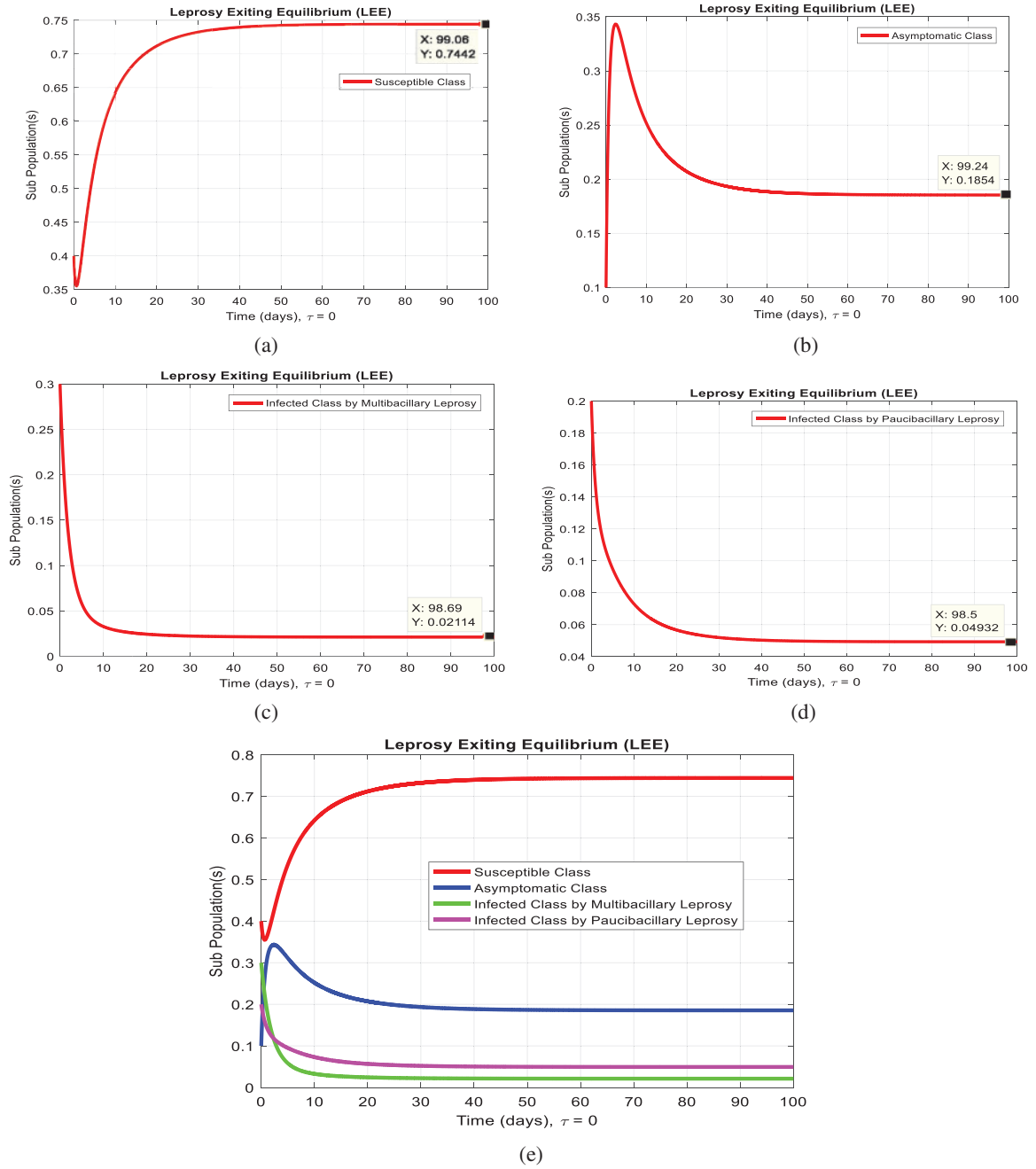


Figure 4: Plots of time for the system (1–4) at the model's Leprosy existing equilibrium

Example 3: (Simulations at the state of leprosy with a temporal delay effect).

In this section, we have taken the effect of the system (1–4) at leprosy existing equilibrium of the model with effective use of artificial delay tactics. Figs. 5a to 5e, we can observe the susceptibility of humans increases with the help of delay tactics. On the other hand, we can keep the infectivity of leprosy patients decreasing and even converging to zero.

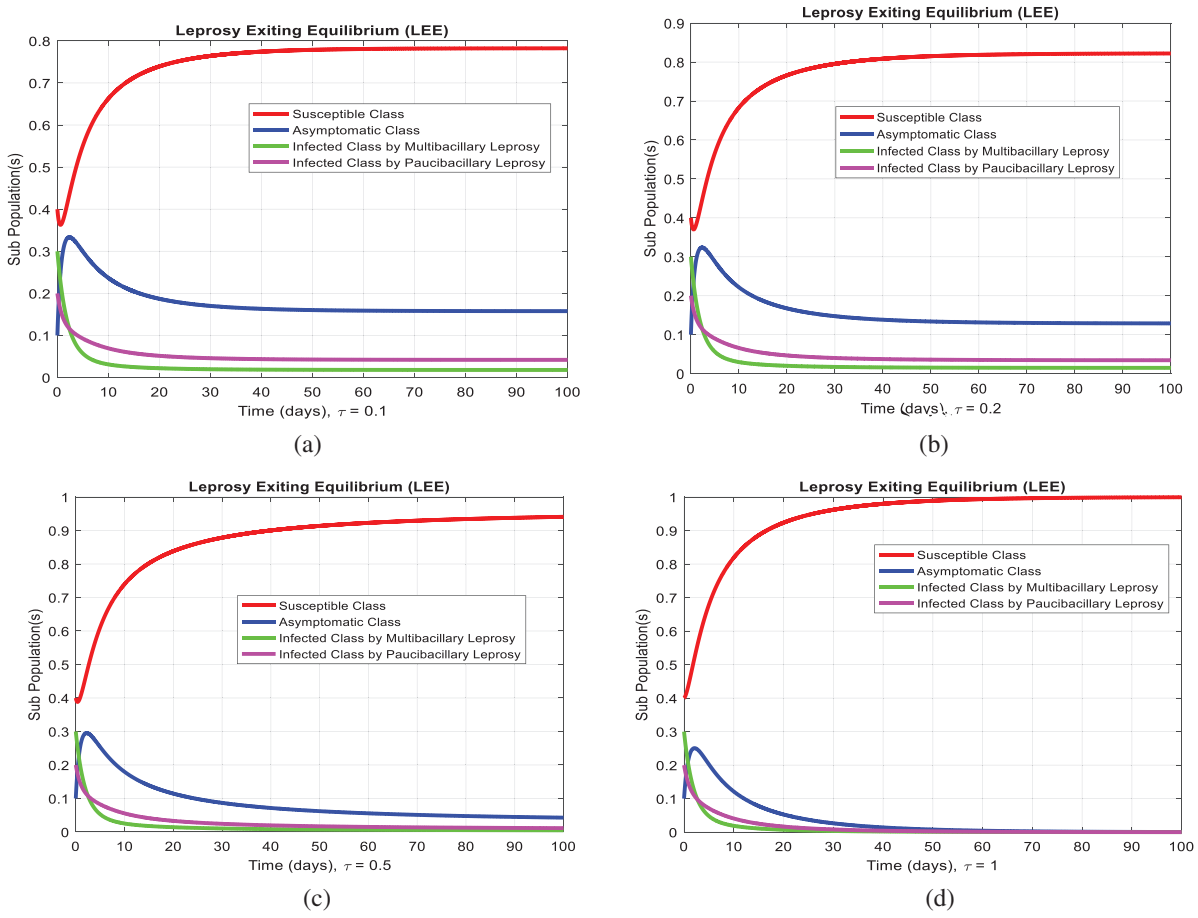


Figure 5: Time graphs of the system (1–4) in its leprosy-free state with successful employment of delay tactics

Example 6: (Effectiveness of delaying techniques on the model's reproduction rate).

Let $\tau = 0.9955$. The reproduction value of the leprosy virus drops, which moves the system from its current state to one where there is no leprosy. Nonetheless, Fig. 6 demonstrates that, when necessary, the delay technique can overcome the infectivity of leprosy viruses.

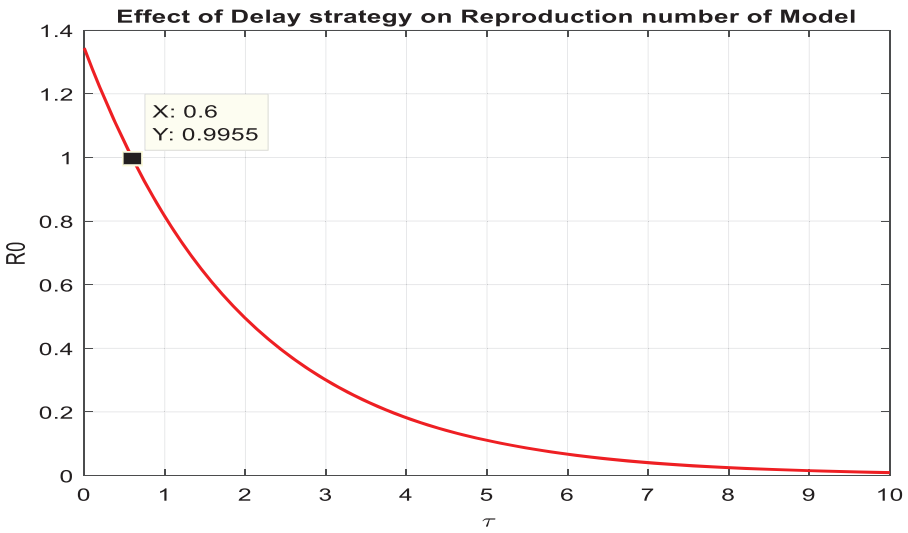
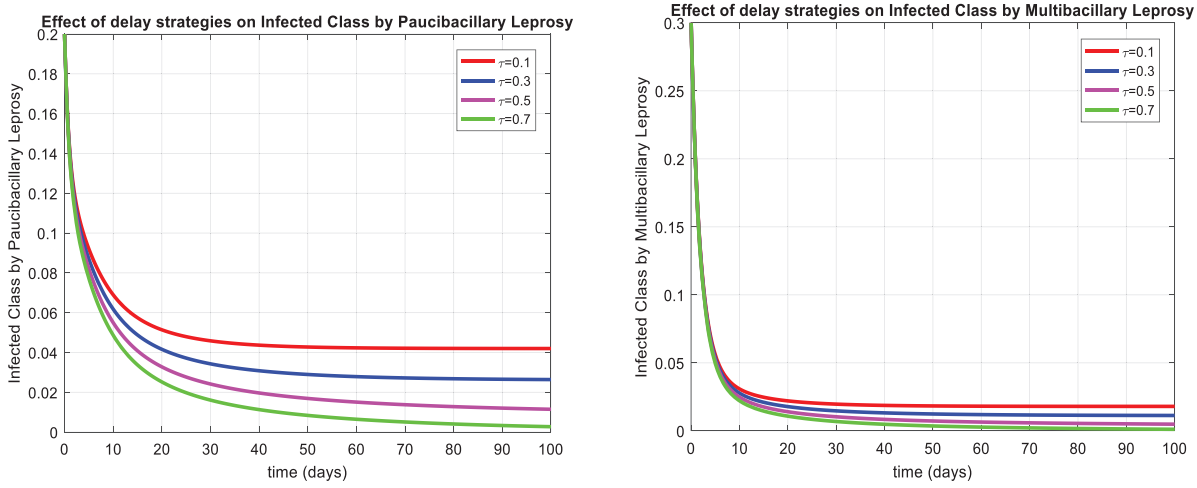


Figure 6: Comparison graph showing the model's reproduction rate and artificial delay duration

Example 5: (Replication of the delay term's effect on the infectious class).

The varying values of τ demonstrate that infective classes of people are reduced or even wiped out of the population. Fig. 7 illustrates the appropriate delay strategies, such as immunizations, for various age groups.



(a) (b)

Figure 7: The influence of the delay term on the model's infectious class is visualized

Example 6: 2 phase plots with delay effects.

Using artificial delay techniques, we plotted the model's two-dimensional behavior. However, these simulations are based on the interaction between the model's susceptible class and the rest of the model's compartments. Figs. 8a–8c shows that the human immune system is dominant over the immune system of infected individuals, as intended.

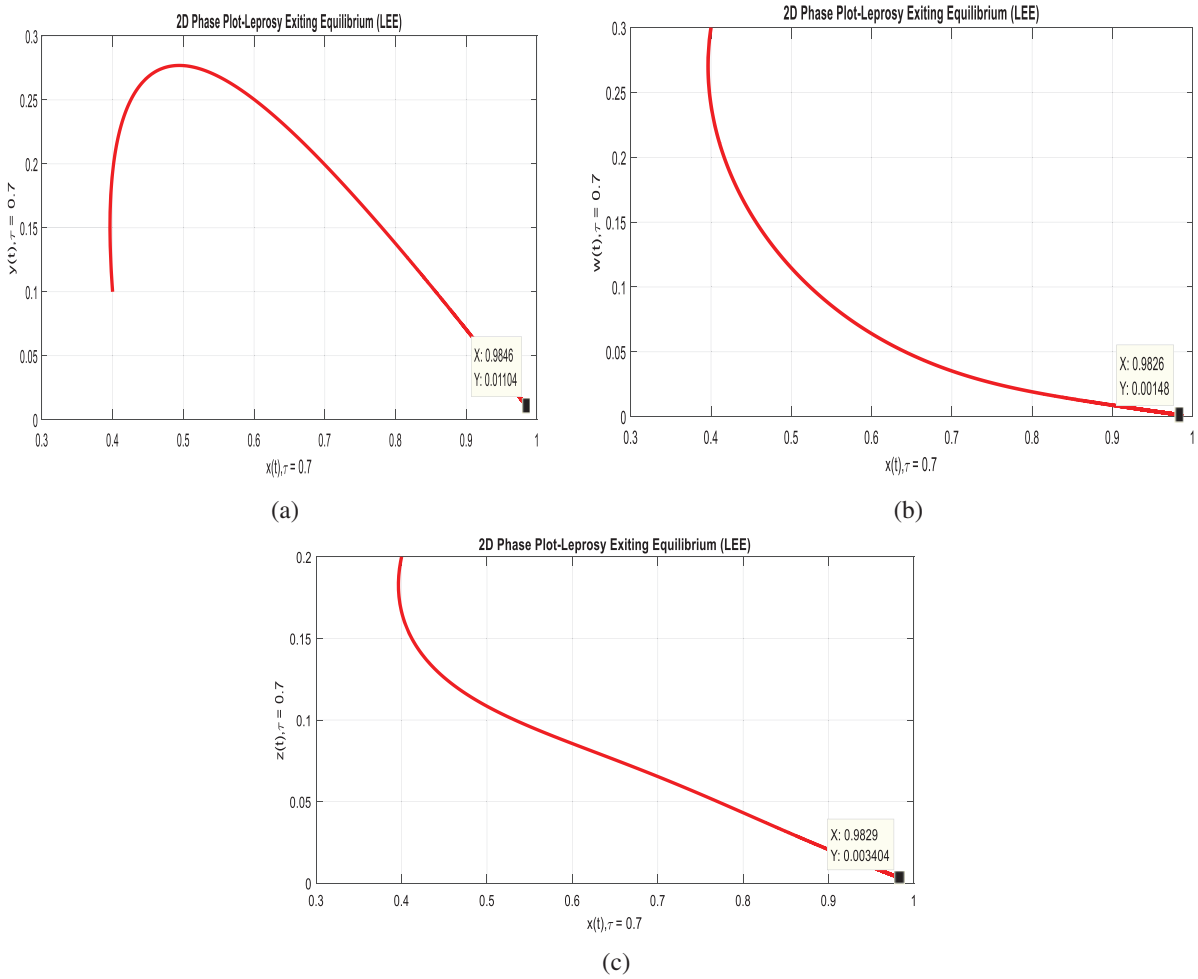


Figure 8: The system's two-dimensional phases (1–4) with a delay effect

6 Conclusion

However, in delayed differential equations (DDEs), the past determines the system's evolution at a specific time instant. The introduction of such time delays significantly increases the complexity of a differential model. Therefore, stability and bifurcation analysis of these models are essential for studying their qualitative behavior. These models haven't been appropriately investigated for parameter identifiability or sensitivity analysis. The application of DDEs with state-dependent delays is a very recent topic in mathematics that might result in significant advances. This article examined the dynamics of leprosy disease through the successful use of delay techniques. As a group, we split the whole population into three groups:

- People who were susceptible to the virus.
- People are exposed to the virus.
- People who had been infected or had been vaccinated.

The reproduction number for the leprosy delayed epidemic model and sensitivity analysis on the reproduction number-related parameters are examined. Additionally, using well-known mathematical

results, the demonstration of the model's local and global stabilities at its equilibrium states, namely leprosy-free and leprosy-existing equilibrium. It is concluded that the desired research can control the dynamic of leprosy by positively using different effective delay techniques like treating infected people, keeping distance from the infected people, by immediate and annual examinations for a minimum of five years after the last meeting with an infected person.

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