

Stochastic Investigations for the Fractional Vector-Host Diseased Based Saturated Function of Treatment Model

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Abstract: The goal of this research is to introduce the simulation studies of the vector-host disease nonlinear system (VHDNS) along with the numerical treatment of artificial neural networks (ANNs) techniques supported by Levenberg-Marquardt backpropagation (LMQBP), known as ANNs-LMQBP. This mechanism is physically appropriate, where the number of infected people is increasing along with the limited health services. Furthermore, the biological effects have fading memories and exhibit transition behavior. Initially, the model is developed by considering the two and three categories for the humans and the vector species. The VHDNS is constructed with five classes, susceptible humans $S_h(t)$, infected humans $I_h(t)$, recovered humans $R_h(t)$, infected vectors $I_v(t)$, and susceptible vector $S_v(t)$ based system of the fractional-order nonlinear ordinary differential equations. To solve the number of variations of the VHDNS, the numerical simulations are performed using the stochastic ANNs-LMQBP. The achieved numerical solutions for solving the VHDNS using the stochastic ANNs-LMQBP have been described for training, verifying, and testing data to decrease the mean square error (MSE). An extensive analysis is provided using the correlation studies, MSE, error histograms (EHs), state transitions (STs), and regression to observe the accuracy, efficiency, expertise, and aptitude of the computing ANNs-LMQBP.

Keywords: Nonlinear mathematical vector host disease model; fractional order; levenberg marquardt backpropagation; neural network; reference database



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1 Introduction

Human illnesses due to infectious pathogenic organisms are referred to the vector-borne diseases (VBDs), such as parasitic infections, viruses, and bacteria. Pathogens are transferred between humans or in some cases, from animals to humans via vectors. Mosquitoes, flies, insects, ticks, and snails carrying pests are the most common vectors. The VBDs are typically found in tropical and subtropical areas, particularly in such areas, where safe drinking water and sanitary are limited. The VBDs are changeable and measured as a dangerous disease, accounting for around 700,000 deaths because of vector-borne pathogenic illnesses, like leishmaniasis, dengue, schistosomiasis, Trypanosoma, cryptosporidiosis, yellow kind of fever, and trichinosis [1]. Based on the reports of WHO, 231 million declared variants of malaria were enlisted four centuries ago, culminating in 416000 causalities, whereas 228 million people were directly affected by malaria in 2018, which tends to result in 405000 causalities [2]. From 2015 to 16, the Zika epidemic was the susceptible to more than 360000 individuals in various states across the United States [3]. Ross [4] pioneered mathematical analysis of vector-borne diseases by developing the two differential systems for transferring the susceptible densities and afflicted vectors (mosquitoes) as well as hosts (people) to present the fundamental consideration of malaria spread dynamics. The basic model of Ross has been extended by Macdonald [5], who developed the concept of elementary reproduction number, which is described as a secondary case formed by an affected individual. Many researchers have expanded the Ross–Macdonald model to contain the additional features of [6–12]. The modeling system shows the alterations to label the dynamic behavior of secondary vector-borne pathogens, including Chagas disease based on the [13–16].

The computational mathematics approach based on infectious diseases has considered a valuable tool to predict the disease transmission behavior along with the prevention plans for the effective diseases. These models can help with public health preparation and response. To investigate the dynamic behavior based on the numerous vector-host communicable viruses, several fractional order systems using the standard nonlinear dynamical systems have been investigated to develop a time-dependent transmitting model for analyzing the dynamic nature of VBD [17]. Khan et al. [18] investigated the dynamic Leptospirosis disease behavior like a saturated estimated occurrence. The dictating tactics and their efficiency for the dengue co-infection system are provided [19] as well as similar kind of mathematical model for diarrhea and malaria co-infection is studied in [20]. The dynamic nature of dengue infection along with the regulator strategies in Pakistan, is presented in [21,22].

The fractional calculus study is assumed to be a general form of traditional calculus and applied as a power tool to develop the epidemic systems. In the recent literature, the analytical solutions formulated with fractional operators have a higher level of precision and perfectly match the real statics [23–27]. Currently, the focus of the researchers based on the fractional calculus is Caputo derivatives [28], Caputo-Fabrizio [29], and Atangana-Baleanu-Caputo (ABC) [30]. Based on the fractional order, the ABC operator is to model the actual systems of communicable diseases along with its numerous characteristics [31–37]. The spreading diseases of numerous vector-host designs form a mathematical model, like dengue [38], schistosomiasis [39], zoonotic instinctive leptospirosis [40], Zika [41], and West Nile virus [42]. Furthermore, the authors considered the linear care function on the disease of vector-host communications, which is physiologically unsuitable in circumstances, where the infected grow, and the society lacks adequate health resources. As a result, in this paper, the saturated action features have been used in the vector-host system [43–45]. The purpose of this research is to present the simulation studies of the fractional-order (FO) vector-host disease nonlinear system (VHDNS)

along with the numerical treatment of the artificial neural networks (ANNs) techniques supported by Levenberg-Marquardt backpropagation (LMQBP), known as ANNs-LMQBP.

The paper is organized as follows: Section 2 represents the fractional vector host disease system. Section 3 shows the stochastic novel features. Section 4 indicates the proposed procedures based on the ANNs-LVMBP method. Section 5 provides the results, and discussion and the last Section shows the concluding remarks.

2 Fraction Order Vector-Host Disease Differential System

In this section, the dynamic nature of the vector-host disease is presented by indicating the total population of humans $N_h(t)$ and further subdivision is into three distinct classes, namely susceptible humans $S_h(t)$, afflicted humans $I_h(t)$, and healed humans $R_h(t)$ at any time t , so $N_h(t) = I_h + S_h + R_h$. The enrollment of individuals at a rate of ζ_h increases the number of individuals of susceptible humans.

It is reduced by effective contact with $\frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h}$, where χ_1 represents the disease interaction rate among susceptible human to afflicted vector and γ_1 is the concentration constant. The natural death rate is η_h and the effective contact rate is $\frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h}$, which generates in the community of infected humans.

The natural deaths η_h , the disease associated death rate ε_h , and $\frac{\kappa \omega I_h}{1 + v \omega I_h}$. It seems to be that when I or ω is very small, the diagnosis function coincides with a near-zero value, and when I is very vast, the diagnosis function reaches a finite value limit. Using such a function (treatment) will normally represent the epidemic system, so that it can be included in the evaluation of the present work. The term $\frac{\kappa}{v}$ is determined by the maximum supply of healthcare possessions per unit of time, whereas $\frac{1}{1 + v \omega I_h}$ represents the converse consequences of the infected persons who are delayed for therapies and have a significant effect on virus spread; for more information, see [18]. The people in the recoverable class are produced by the treatment function $\frac{\kappa \omega I_h}{1 + v \omega I_h}$, while η_h decreases due to natural death. This variation can be represented by the differential equation and the VHDNS model is mathematically signified as [46]:

$$\begin{cases} \frac{dS_h(t)}{dt} = -\zeta_h + \frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h} - \eta_h S_h, & S_0 = k_1, \\ \frac{dI_h(t)}{dt} = \frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h} - \eta_h I_h - \varepsilon_h I_h - \frac{\kappa \omega I_h}{1 + v \omega I_h}, & I_0 = k_2, \\ \frac{dR_h(t)}{dt} = \frac{\kappa \omega I_h}{1 + v \omega I_h} - \eta_h R_h. & R_0 = k_3. \end{cases} \quad (1)$$

The vector society is denoted by N_v , and it is separated into two subclasses: S_v susceptible vector and I_v afflicted vector. As a result, $N_v = I_v + S_v$. The susceptible vector sequence is created by the birth rate v , which is reduced by the contact rate $\frac{\lambda S_v I_v}{1 + \rho I_v}$ and normal death rate v . The contact rate $\frac{\lambda S_v I_v}{1 + \rho I_v}$ generates the afflicted populations, while the natural fatality rate η_v decreases it. This discussion leads to the following differential equation:

$$\begin{cases} \frac{dS_v(t)}{dt} = -\zeta_v + \frac{\lambda S_v I_v}{1 + \rho I_v} - \eta_v S_v, & S_0 = k_4, \\ \frac{dI_v(t)}{dt} = \frac{\lambda S_v I_v}{1 + \rho I_v} - \eta_v S_v. & I_0 = k_5, \end{cases} \quad (2)$$

where k_1, k_2, k_3, k_4 , and k_5 be the initial conditions of Eqs. (1) and (2). After combining these two equations, the SIRSI system is obtained as:

$$\begin{cases} \frac{d^\tau S_h(t)}{dt^\tau} = \zeta_h - \frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h} - \eta_h S_h, & S_0 = k_1, \\ \frac{d^\tau I_h(t)}{dt^\tau} = \frac{S_h \gamma_1 I_h}{1 + \chi_1 I_h} - \eta_h I_h - \varepsilon_h I_h - \frac{\kappa \omega I_h}{1 + \nu \omega I_h}, & I_0 = k_2, \\ \frac{d^\tau R_h(t)}{dt^\tau} = \frac{\kappa \omega I_h}{1 + \nu \omega I_h} - \eta_h R_h, & R_0 = k_3, \\ \frac{d^\tau S_v(t)}{dt^\tau} = \zeta_v - \frac{\lambda S_v I_v}{1 + \rho I_v} - \eta_v S_v, & S_0 = k_4, \\ \frac{d^\tau I_v(t)}{dt^\tau} = \frac{\lambda S_v I_v}{1 + \rho I_v} - \eta_v S_v. & I_0 = k_5. \end{cases} \quad (3)$$

This system, τ shows the FO derivative of the SIRSI model. In Eq. (3), FO-SIRSI system parameters are denoted by small Greek and English characters. Each model parameter has a particular value, calculated using the actual information given in [46]. Fig. 1 illustrates the visual effects of every phase of the epidemic.

3 Novel Stochastic Solvers Features

The numerical measures through the ANNs-LMQBP are proposed for solving the FO-SIRSI system. The stochastic solvers have been exploited using local and global search efficiencies based on the complicated, singular, and stiff models [47–49]. Few more schemes of the stochastic schemes are third-order nonlinear singular models [50], and fractional-order singular models [51–54]. In this study, the solutions of the fraction order VHDNS based on the SIRSI have been proposed using the ANNs-LMQBP. Recently, stochastic solvers have been presented to solve the fractional dynamical models. Few of them are dynamical nonlinear susceptible infected and quarantine differential model [55], immune-chemotherapeutic treatment for breast cancer [56], nonlinear prey-predator system [57], SIDARTHE COVID-19 pandemic differential model [58], Bagley–Torvik mathematical model [59] and seventh order singular system [60]. It is found that the time FO derivatives have been applied to different conditions in various applications. The FO derivative represents the framework based on remembrance [61]. Few novel features of the ANNs-LMQBP are presented as:

- A preliminary design of its FO-VHDNS is presented based on the nonlinear mathematical form of the SIRSI effects.
- The solutions of the FO-VHDNS system using the nonlinear mathematical form of the SIRSI model have never been presented through the stochastic solvers.
- The numerical stochastic measures based on the ANNs-LMQBP have been presented for the FO-VHDNS using the nonlinear mathematical form of the SIRSI.
- The comparisons of the obtained results through ANNs-LMQBP have been presented with the reference (Adams–Bashforth–Moulton) solutions to authenticate the excellence of stochastic computing solvers.

- The absolute error (AE) in good measures has been achieved for the FO-VHDNS using the nonlinear mathematical form of the SIRSI.
- The reliability and consistency of the developed ANNs-LMQBP for solving the FO-VHDNS system are validated by using the regression, STs, MSE, EHs, and similarity performances

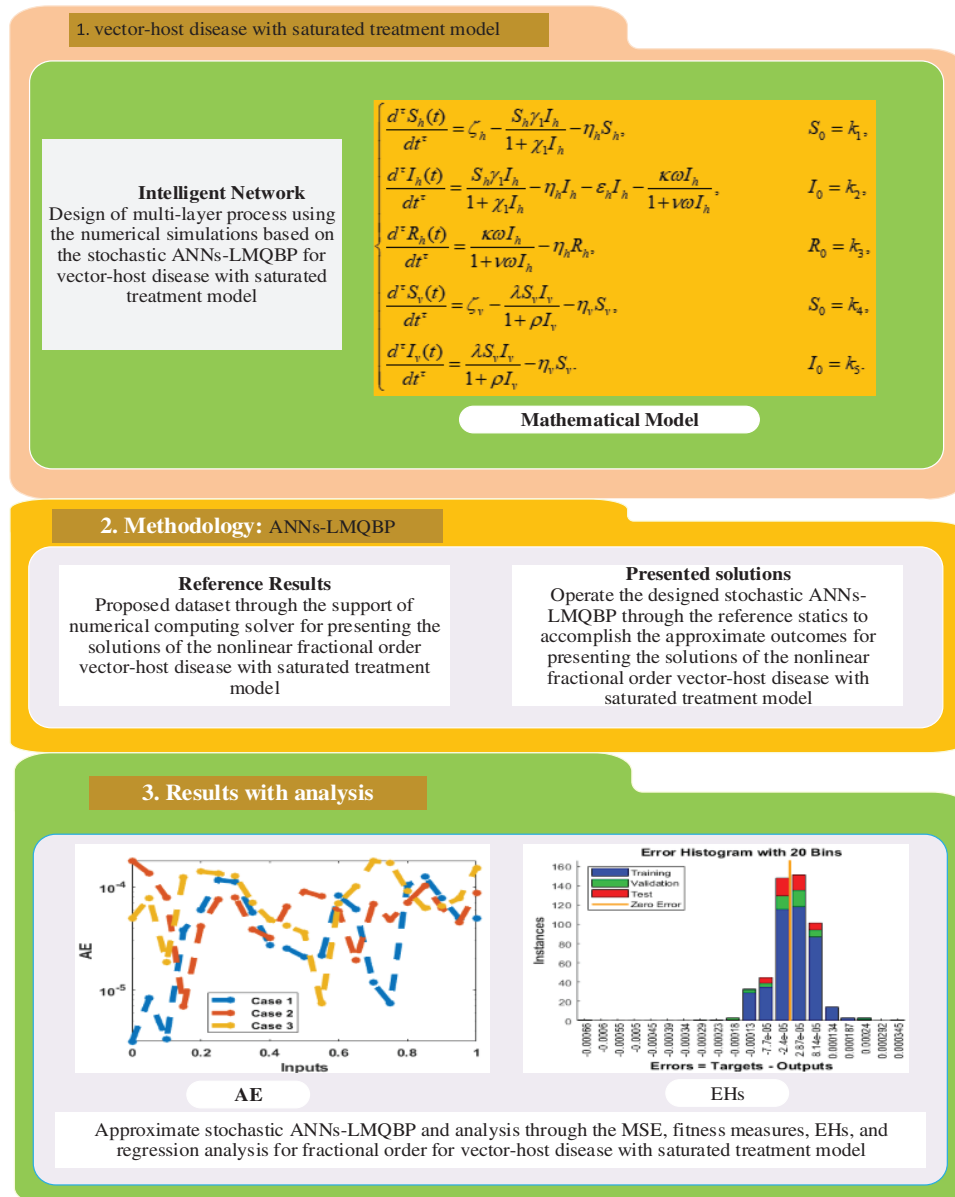


Figure 1: Workflow-based fractional-order derivative of the mathematical VHDNS model using the ANNs-LVMBP method

4 Proposed Procedures: ANNs-LVMBP Method

The ANNs-LVMBP scheme is provided in two steps to solve the FO-VHDNS system using the nonlinear mathematical form of the SIRSI model. First the basic procedures of the ANNs-LVMBP operator performances are introduced along with the designed structure of the FO-VHDNS system using the nonlinear mathematical form of the SIRSI model.

Fig. 1 shows the multi-layer performances of the optimization using the stochastic ANNs-LVMBP. The ANNs-LVMBP procedures are assembled in MATLAB through the 'nftool' process, with data chosen as 74% for training, 12% for testing, and 14% for authorization.

5 Results and Discussions

The numerical results with three FO-SIRSI cases using the ANNs-LVMBP method are drawn in this section. These cases have been presented by using the variations of the FO to solve the model using the stochastic schemes.

Case 1: Consider the FO-SIRSI model by taking the $\tau = 0.5$, $\zeta_h = 0.0002$, $\gamma_1 = 0.000044$, $\chi_1 = 0.003$, $\eta_h = 0.00020$, $v = 0.4$, $\varepsilon_h = 0.002$, $\kappa = 0.1$, $\zeta_v = 0.008$, $\lambda = 0.007$, $\rho = 0.002$, $\eta_v = 0.2$, $S_0 = 0.01$, $I_0 = 0.01$, $R_0 = 0.01$, $\omega = 0.1$, $S_0 = 0.01$ and $I_0 = 0.01$ is given as:

$$\left\{ \begin{array}{l} \frac{d^{0.5} S_h(t)}{dt^{0.5}} = 0.0002 - \frac{0.000044 S_h I_h}{1 + 0.003 I_h} - 0.00020 S_h \quad S_0 = 0.01, \\ \frac{d^{0.5} I_h(t)}{dt^{0.5}} = \frac{0.000044 S_h I_h}{1 + 0.003 I_h} - 0.0022 I_h - \frac{0.01 I_h}{1 + 0.04 I_h} \quad I_0 = 0.01, \\ \frac{d^{0.5} R_h(t)}{dt^{0.5}} = \frac{0.01 I_h}{1 + 0.04 I_h} - 0.00020 R_h \quad R_0 = 0.01, \\ \frac{d^{0.5} S_v(t)}{dt^{0.5}} = 0.008 - \frac{0.007 S_v I_v}{1 + 0.002 I_v} - 0.2 S_v \quad S_0 = 0.01, \\ \frac{d^{0.5} I_v(t)}{dt^{0.5}} = \frac{0.007 S_v I_v}{1 + 0.002 I_v} - 0.2 S_v \quad I_0 = 0.01. \end{array} \right. \quad (4)$$

Case 2: Consider the FO-SIRSI model by taking the $\tau = 0.6$, $\zeta_h = 0.0002$, $\gamma_1 = 0.000044$, $\chi_1 = 0.003$, $\eta_h = 0.00020$, $v = 0.4$, $\varepsilon_h = 0.002$, $\kappa = 0.1$, $\zeta_v = 0.008$, $\lambda = 0.007$, $\rho = 0.002$, $\eta_v = 0.2$, $S_0 = 0.01$, $I_0 = 0.01$, $R_0 = 0.01$, $\omega = 0.1$, $S_0 = 0.01$ and $I_0 = 0.01$ is presented as:

$$\left\{ \begin{array}{l} \frac{d^{0.6} S_h(t)}{dt^{0.6}} = 0.0002 - \frac{0.000044 S_h I_h}{1 + 0.003 I_h} - 0.00020 S_h \quad S_0 = 0.01, \\ \frac{d^{0.6} I_h(t)}{dt^{0.6}} = \frac{0.000044 S_h I_h}{1 + 0.003 I_h} - 0.00020 I_h - 0.002 I_h - \frac{0.01 I_h}{1 + 0.04 I_h} \quad I_0 = 0.01, \\ \frac{d^{0.6} R_h(t)}{dt^{0.6}} = \frac{0.01 I_h}{1 + 0.04 I_h} - 0.00020 R_h \quad R_0 = 0.01, \\ \frac{d^{0.6} S_v(t)}{dt^{0.6}} = 0.008 - \frac{0.007 S_v I_v}{1 + 0.002 I_v} - 0.2 S_v \quad S_0 = 0.01, \\ \frac{d^{0.6} I_v(t)}{dt^{0.6}} = \frac{0.007 S_v I_v}{1 + 0.002 I_v} - 0.2 S_v \quad I_0 = 0.01. \end{array} \right. \quad (5)$$

Case 3: Consider the FO-SIRSI model by taking the $\tau = 0.7$, $\zeta_h = 0.0002$, $\gamma_1 = 0.000044$, $\chi_1 = 0.003$, $\eta_h = 0.00020$, $v = 0.4$, $\varepsilon_h = 0.002$, $\kappa = 0.1$, $\zeta_v = 0.008$, $\lambda = 0.007$, $\rho = 0.002$, $\eta_v = 0.2$, $S_0 = 0.01$, $I_0 = 0.01$, $R_0 = 0.01$, $\omega = 0.1$, $S_0 = 0.01$ and $I_0 = 0.01$ is described as:

$$\begin{cases}
\frac{d^{0.7}S_h(t)}{dt^{0.7}} = 0.0002 - \frac{0.000044S_hI_h}{1 + 0.003I_h} - 0.00020S_h & S_0 = 0.01, \\
\frac{d^{0.7}I_h(t)}{dt^{0.7}} = \frac{0.000044S_hI_h}{1 + 0.003I_h} - 0.00020I_h - 0.002I_h - \frac{0.01I_h}{1 + 0.04I_h} & I_0 = 0.01, \\
\frac{d^{0.7}R_h(t)}{dt^{0.7}} = \frac{0.01I_h}{1 + 0.04I_h} - 0.00020R_h & R_0 = 0.01, \\
\frac{d^{0.7}S_v(t)}{dt^{0.7}} = 0.008 - \frac{0.007S_vI_v}{1 + 0.002I_v} - 0.2S_v & S_0 = 0.01, \\
\frac{d^{0.7}I_v(t)}{dt^{0.7}} = \frac{0.007S_vI_v}{1 + 0.002I_v} - 0.2S_v & I_0 = 0.01.
\end{cases} \quad (6)$$

The numerical representations of the FO mathematical bone disease model are discussed using the ANNs-LVMBP method with 15 neurons along with the data selection is chosen as 74%, 12% and 14%, for training, certification, and testing. The structure of the input, hidden, and output neurons are depicted in Fig. 2.

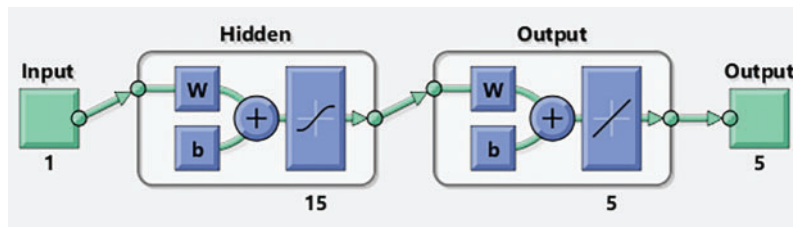


Figure 2: Designed ANNs-LVMBP method for VHDNS

The plots are using the ANNs-LVMBP method for the VHDNS model based on the FO-SIRSI are shown in Figs. 3–5. The graphical visualizations are illustrated in Figs. 3 and 4 to investigate the STs best measures. The MSE and STs for training, ideal curves, and confirmation are produced in Fig. 3 to solve the FO-SIRSI system. On behalf of these accomplishments based on the FO-SIRSI are provided at epochs 6, 6, and 6, the derived values are $9.9334e^{-09}$, $6.5823e^{-09}$, and $8.6883e^{-09}$, respectively. The curve values are also provided in Fig. 3 for the VHDNS model based on the FO-SIRSI. These cure performances have been provided as $2.6797e^{-08}$ for Case 1, $1.6746e^{-08}$ for Case 2, and $3.6855e^{-08}$ for Case 3. These visualization tools show the convergence of recommended ANNs-LVMBP for the VHDNS model based on the FO-SIRSI. Fig. 4 shows the results and EHs performances of the FO system. The EHs for cases 1, 2, and 3 are predicted as $-2.4e^{-05}$, $-1.8e^{-05}$ and $-2.8e^{-05}$. The convergence of the model using the complexity, MSE, training, verification, generations, and testing is provided in Tab. 1.

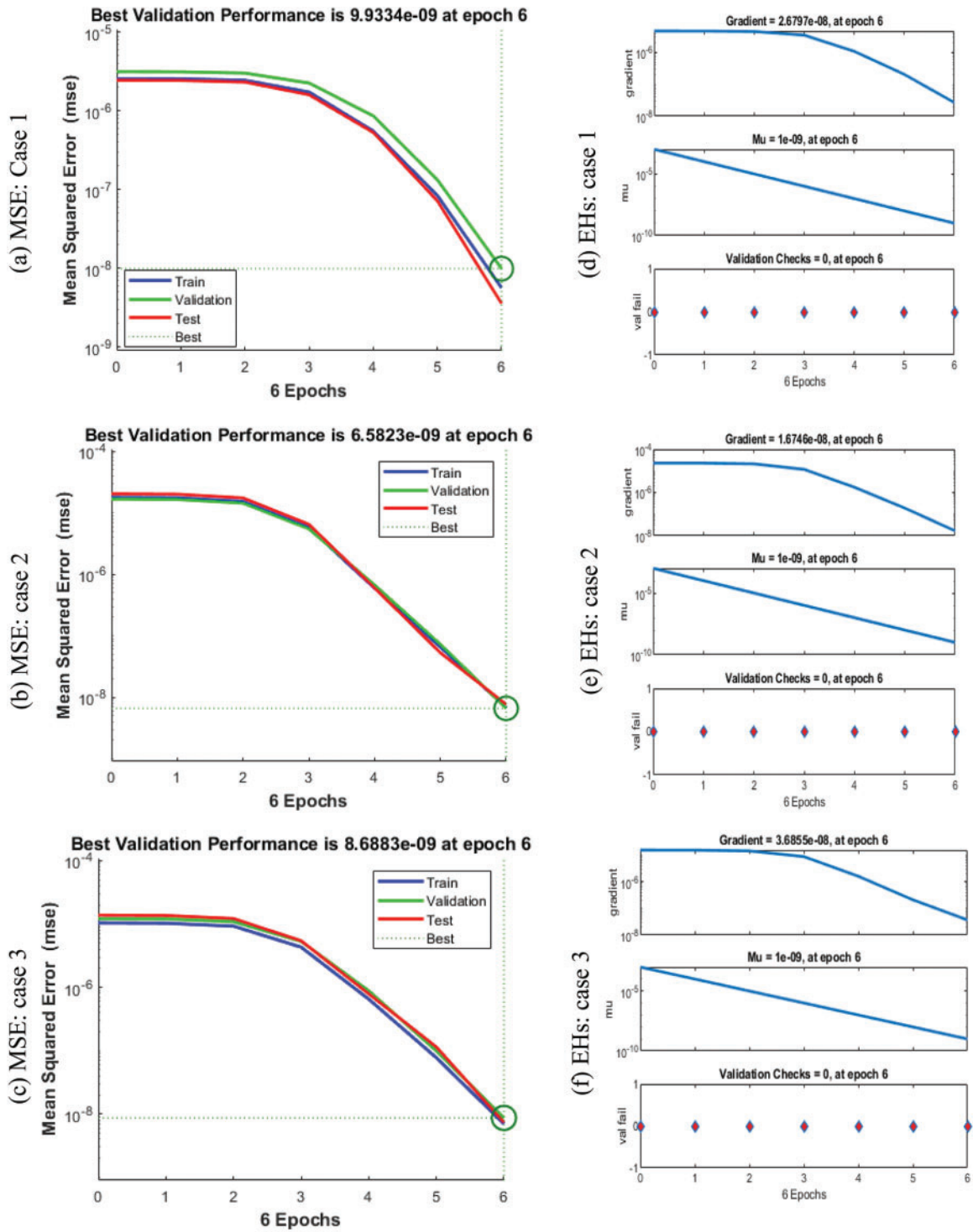


Figure 3: MSE and STs performances for the fractional order system

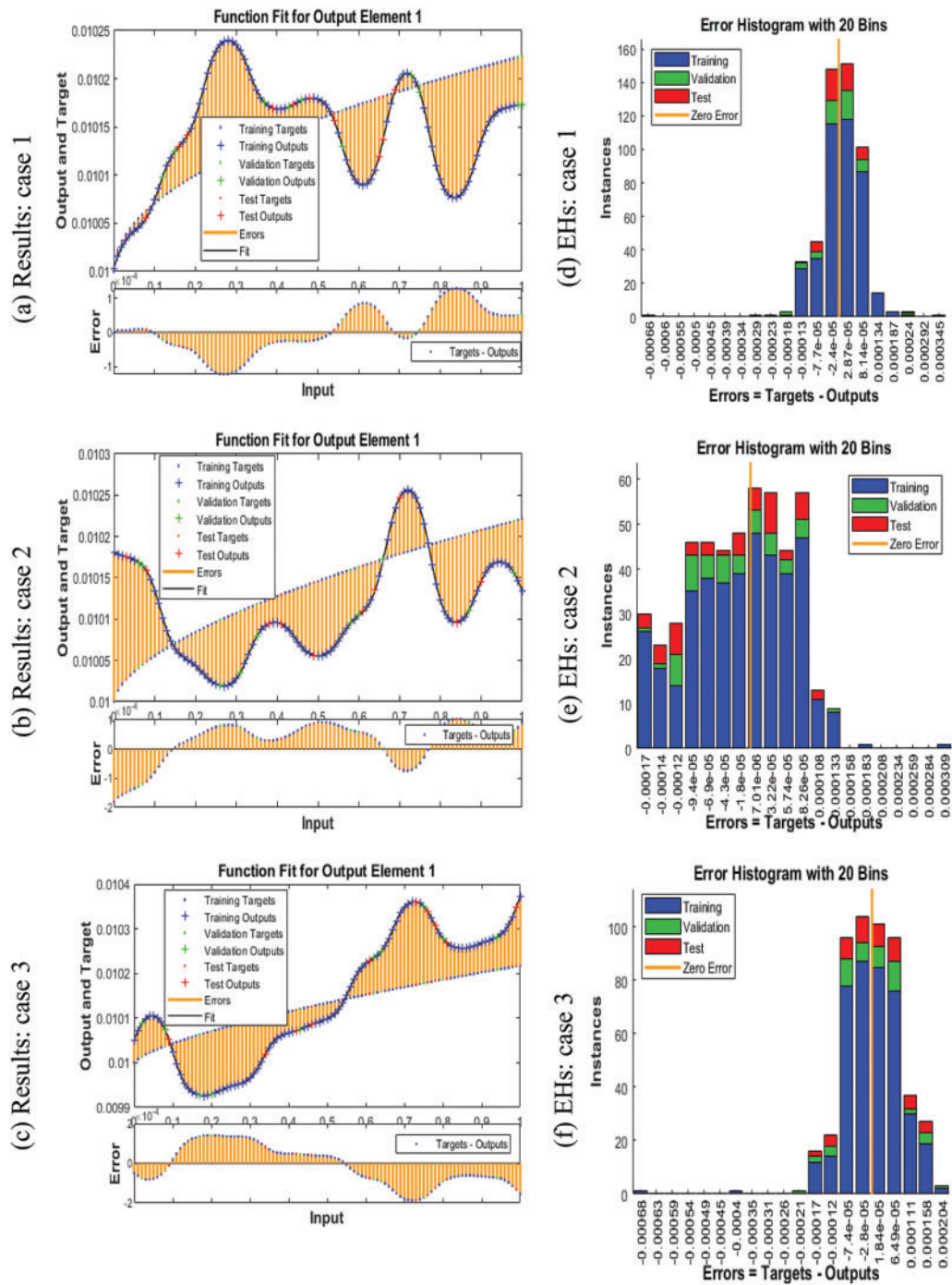


Figure 4: Results and EHs performances for the fractional order system

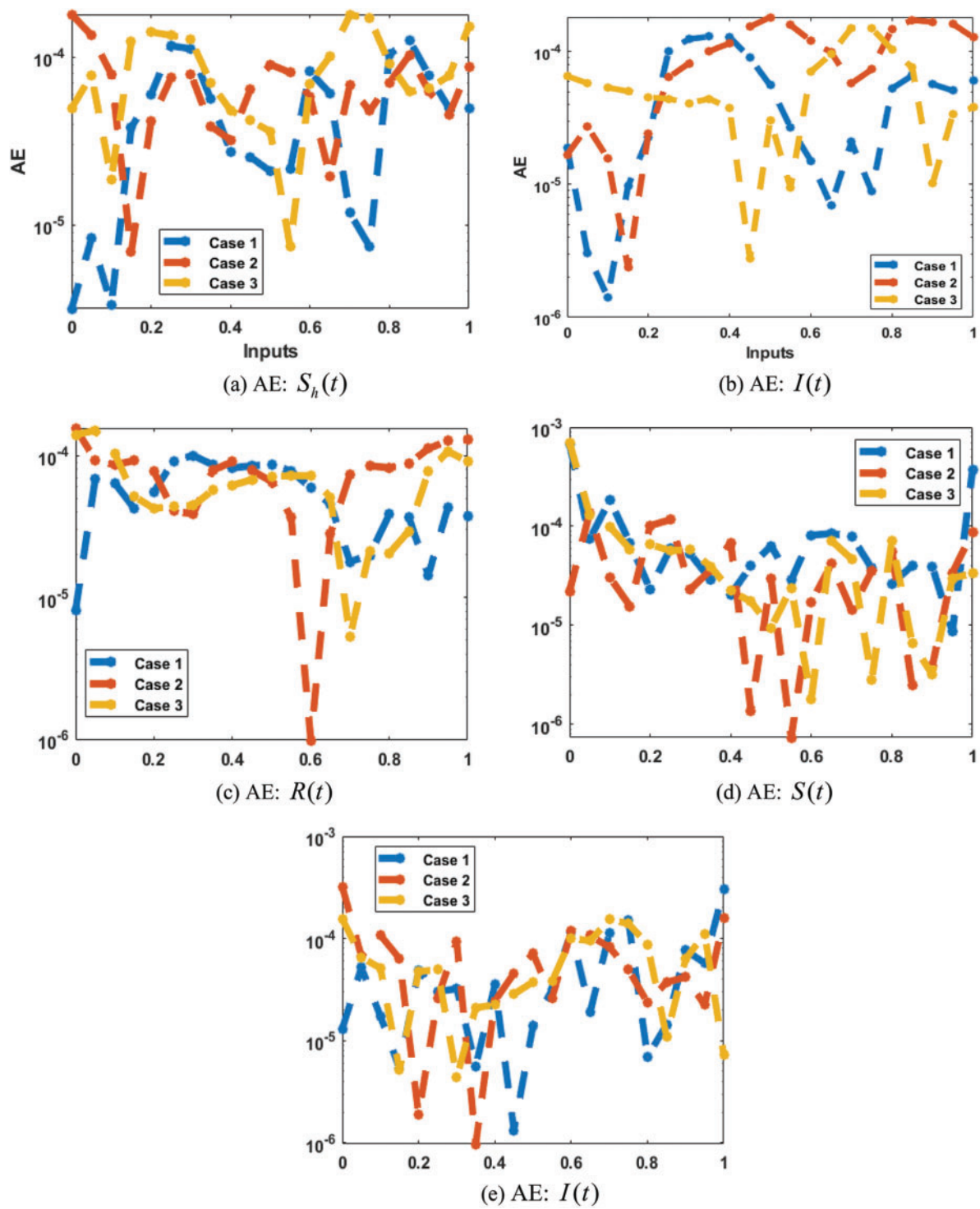


Figure 5: AE for the performances for the fractional order system

Table 1: ANNs-LVMBP method for the FO VHDNS SIRSI-mathematical model

Case	MSE			Performance	Gradient	Mu	Epoch	Time
	[Training]	[Verification]	[Testing]					
1	5.654×10^{-09}	9.933×10^{-09}	3.582×10^{-9}	5.65×10^{-09}	2.68×10^{-08}	1×10^{-09}	6	1
2	6.725×10^{-09}	6.582×10^{-09}	7.614×10^{-9}	6.73×10^{-09}	1.67×10^{-08}	1×10^{-09}	6	1
3	7.000×10^{-09}	8.688×10^{-09}	7.382×10^{-9}	7.00×10^{-09}	3.69×10^{-08}	1×10^{-09}	6	1

Fig. 5 presents the AE for the VHDNS model based on the nonlinear FO-SIRSI mathematical system. The AE based on the $S_h(t)$ dynamics are calculated as $10^{-04} - 10^{-05}$, $10^{-04} - 10^{-06}$ and $10^{-05} - 10^{-06}$ for the corresponding cases of the FO-SIRSI model. The AE is based on the $I_h(t)$, R_h , $S_v(t)$ and $I_v(t)$.

6 Conclusions

This study aims to perform the simulations of the vector-host disease nonlinear system using the numerical artificial neural networks scheme along with the support of Levenberg-Marquardt backpropagation. The vector-host disease nonlinear system depends upon five dynamics: susceptible humans $S_h(t)$, infected humans $I_h(t)$, recovered humans $R_h(t)$, infected vectors $I_v(t)$ and susceptible vectors $S_v(t)$. The vector-host disease nonlinear system is generalized into the fractional-order derivative to find more realistic solutions and the calculations have been performed using the SNNs-LMQBP. The correctness of the FO-VHDNS is observed using the comparison performances of the obtained and the reference solutions. This study has taken fifteen neurons and the data selection is 74%, 12% and 14%, for training, certification, and testing. The scheme's exactness is achieved by achieving suitable AE measures for each dynamic of the FO-VHDNS. The AE based on each dynamic of the FO-SIRSI model is calculated as $10^{-05} - 10^{-06}$. Moreover, correlation studies, MSE, EHs, STs and regression have been achieved to observe the accuracy, efficiency, expertise, and aptitude of the computing SNNs-LMQBP. In upcoming studies, the proposed numerical procedure has been used to find the solutions for the fluid mechanic's systems, nonlinear systems, omics studies, lonngren-wave, and data security networks [62–66].

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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