

Stochastic Computational Heuristic for the Fractional Biological Model Based on Leptospirosis

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Abstract: The purpose of these investigations is to find the numerical outcomes of the fractional kind of biological system based on Leptospirosis by exploiting the strength of artificial neural networks aided by scale conjugate gradient, called ANNs-SCG. The fractional derivatives have been applied to get more reliable performances of the system. The mathematical form of the biological Leptospirosis system is divided into five categories, and the numerical performances of each model class will be provided by using the ANNs-SCG. The exactness of the ANNs-SCG is performed using the comparison of the reference and obtained results. The reference solutions have been obtained by using the Adams numerical scheme. For these investigations, the data selection is performed at 82% for training, while the statics for both testing and authentication is selected as 9%. The procedures based on the recurrence, mean square error, error histograms, regression, state transitions, and correlation will be accomplished to validate the fitness, accuracy, and reliability of the ANNs-SCG scheme.

Keywords: Fractional leptospirosis; biological model; artificial neural networks; scale conjugate gradient; numerical performances

1 Introduction

The spreading virus ratio creates several diseases in the population with high or low rates [1]. A few of the mentioned diseases in the world that have been created through the viruses are Dengue,



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Hantavirus and Leptospirosis. The transmittable viruses from one to individual another are known as infectious diseases, while those viruses generated due to the atmospheres are called noninfectious [2]. The famous Leptospirosis is a bacterial virus, mostly found in animals, but occasionally in humans [3]. Leptospirosis is a spreading virus that originated through the soil or water based on the urine of the host animal [4]. Leptospirosis affects humans with a low ratio by using dirty or muddy water, [5–7] and it affects a high ratio when an individual gets a cut in the water [8].

The mathematical modeling provides the basic operators using the good policies of minimizing or eliminating Leptospirosis and predicting its future occurrences [9,10]. There are various differential mathematical systems, that have been performed on population dynamics based on humans and rats. Kongnuy [11] proposed the spreading dynamics based on Leptospirosis using the mathematical formulation between the vector population and humans. Leptospirosis can be controlled by decreasing the transmission ratio (infected to susceptible) in humans. Khan et al. [12] proposed the performance of Leptospirosis from the drenched existence with the disposed population and condensed through the functioning conduct. The system simulations based on the infection ratio are 0.83 reducing Leptospirosis's spreading ratio. Other severe and deathly transmittable viruses have been reported in [13–19].

Newton proposed fractional calculus many years ago, but it obtained enormous fame and significance in recent years. Over the last thirty years, fractional calculus has been widely used in industrial areas. The idea of the fractional order derivative has been modernized through the complexities connected to the typical inhomogeneity. The multilayered media construction based on the diffusion procedure is reported using the operators based on the fractional calculus. A significant apparatus presents the precise performance using the ordinary differential form of the systems. There is a wider range of complicated phenomena that have been proposed by many researchers based on the software models. There are various fractional/integer kinds of schemes reported in [20–26]. Fractional calculus has been studied by various researchers based on the integer kind of calculus using the mathematical form of real-world applications. A new perception of the fractional kind of derivative based on the control scheme of dengue infection has been emphasized efficiently in [27]. The simulations using the analytical/numerical schemes based on the mathematical systems indicate a significant character in describing the formulations of the systems [28–37].

These investigations aim to authenticate the numerical outcomes based on the fractional kind of biological system based on Leptospirosis by exploiting the strength of artificial neural networks (ANNs) aided by scale conjugate gradient (SCG), called ANNs-SCG. The derivatives based on the fractional order have been used to achieve precise mathematical performances. Moreover, the stochastic numerical computing performances based on the ANNs-SCG procedure for the mathematical form based on the biological Leptospirosis model have not been presented before in terms of the fractional form. The solution of this novel fraction order model will be presented first by using the stochastic ANNs-SCG scheme. The stochastic numerical performances have been applied to solve the corneal shape of model [38], human immunodeficiency virus (HIV) system [39], plant virus propagation model [40], coronavirus mathematical model [41], travelling wave solution-based new Riccati equation [42], solar energy model [43], Schrödinger equation [44], dusty plasma model [45] and relation extraction [46,47]. These applications enhance the motivations and inspirations for the authors to perform the exhaustive frameworks in the field of artificial intelligence (AI) through the stochastic computational structure to provide a substitute, stable, precise, robust, and accurate solutions to the mathematical form of the biological Leptospirosis model.

The remaining structure of the paper is given as follows: Section 2 presents the formulation of the fractional Leptospirosis system, Section shows the methodology of the stochastic ANNs-SCG scheme, Section 4 describes the simulations together with some important findings, while the last Section 5 presents the concluding remarks.

2 Fractional Kind of Leptospirosis System

The mathematical kind of the biological Leptospirosis system is divided into five categories (Human and vector). The Susceptible-Infected-Recovered (SIR) population dynamics based on the vector (rat) and human population are shown as [48]:

$$\left\{ \begin{array}{l} \frac{dS_h(\tau)}{d\tau} = A - \mu_h S_h(\tau) + \theta_h R_h(\tau) - \beta_h S_h(\tau) I_r(\tau), \quad (S_h)_0 = a_1, \\ \frac{dI_h(\tau)}{d\tau} = \beta_h S_h(\tau) I_r(\tau) - (\delta_h + r_h + \mu_h) I_h(\tau), \quad (I_h)_0 = a_2, \\ \frac{dR_h(\tau)}{d\tau} = r_h I_h(\tau) - (\mu_h + \theta_h) R_h(\tau), \quad (R_h)_0 = a_3, \\ \frac{dS_r(\tau)}{d\tau} = -\beta_r S_r(\tau) I_h(\tau) - r_r S_r(\tau) + B, \quad (S_r)_0 = a_4, \\ \frac{dI_r(\tau)}{d\tau} = -(\delta_r + r_r) I_r(\tau) + \beta_r S_r(\tau) I_h(\tau), \quad (I_r)_0 = a_5, \end{array} \right. \quad (1)$$

The integer kind of differential Leptospirosis system is presented in Eq. (1). The comprehensive details of the biological model based on Leptospirosis are given in Tab. 1.

Table 1: Parameter details based on the Leptospirosis model

Parameter	Details
$R_h(\tau)$	Recovered human
$S_h(\tau)$	Susceptible human
$I_h(\tau)$	Infected human
$S_r(\tau)$	Susceptible rat
$I_r(\tau)$	Infected rat
θ_h	Individual rate of immune to becoming again susceptible
A	The recruitment rate of the human population
β_h	The transmission rate of Leptospirosis from infected rats to susceptible human
μ_h	The natural rate of death in the human population
δ_h	Infected human die through disease
r_h	Recovered human rate
B	The recruitment rate of the rat population
δ_r	Rate of infection that rat dies
r_r	Natural death of the population of rat
β_r	Transmission of Leptospirosis from infected rat to susceptible rat
τ	Time
a_1, a_2, \dots, a_5	Initial conditions

The current work provides the numerical representations of the mathematical model based on Leptospirosis using artificial intelligence (AI) together through the artificial neural networks (ANNs) along with the scale conjugate gradient (SCG), i.e., ANNs-SCG. As a result, the mathematical model based on Leptospirosis is presented as follows:

$$\left\{ \begin{array}{l} \frac{d^v S_h(\tau)}{d\tau^v} = A - \mu_h S_h(\tau) + \theta_h R_h(\tau) - \beta_h S_h(\tau) I_r(\tau), \quad (S_h)_0 = a_1, \\ \frac{d^v I_h(\tau)}{d\tau^v} = \beta_h S_h(\tau) I_r(\tau) - (\delta_h + r_h + \mu_h) I_h(\tau), \quad (I_h)_0 = a_2, \\ \frac{d^v R_h(\tau)}{d\tau^v} = r_h I_h(\tau) - (\mu_h + \theta_h) R_h(\tau), \quad (R_h)_0 = a_3, \\ \frac{d^v S_r(\tau)}{d\tau^v} = -\beta_r S_r(\tau) I_h(\tau) - r_r S_r(\tau) + B, \quad (S_r)_0 = a_4, \\ \frac{d^v I_r(\tau)}{d\tau^v} = -(\delta_r + r_r) I_r(\tau) + \beta_r S_r(\tau) I_h(\tau), \quad (I_r)_0 = a_5, \end{array} \right. \quad (2)$$

where v represents the fractional kind of Caputo derivative mathematical form of Leptospirosis, and the fraction order values have been chosen between $[0, 1]$. The fractional kinds of the derivatives in the mathematical form of the Leptospirosis shown in Eq. (2) to perform the minute's specifications, i.e., superslow growth as well as superfast performances, which is hard to estimate from the integer kind provided in Eq. (1). Few novels feature to solve the fractional kind of mathematical model based on the Leptospirosis system are presented as:

- The fractional derivatives provide more reliable performances of the mathematical Leptospirosis system.
- The design of the stochastic ANNs-SCG is provided for the numerical investigations of the fractional kind of mathematical model based on Leptospirosis.
- Three different variants of the fraction order have been used to perform the numerical simulations of the fractional kind of mathematical model based on Leptospirosis.
- The correctness of the stochastic ANNs-SCG scheme is performed to compare the obtained results based on the ANNs-SCG and the reference solutions.
- The excellence of the stochastic ANNs-SCG scheme is approved through the obtained negligible absolute error magnitude based fractional kind of mathematical Leptospirosis system.
- The recurrence, mean square error, error histograms, regression, state transitions, and correlation supports the reliability of the ANNs-SCG scheme using the fractional kind of mathematical model based on Leptospirosis.

3 Proposed ANNs-SCG Scheme

The current section provides the stochastic artificial neural networks (ANNs) along with the scale conjugate gradient (SCG), i.e., ANNs-SCG scheme, along with the numerical results procedures to achieve the fractional kind of mathematical model based on the Leptospirosis as shown in Eq. (1). Fig. 1 represents the workflow depictions of the fractional kind of mathematical model based on the Leptospirosis using the ANNs-SCG based three blocks, methodology and obtained solutions.

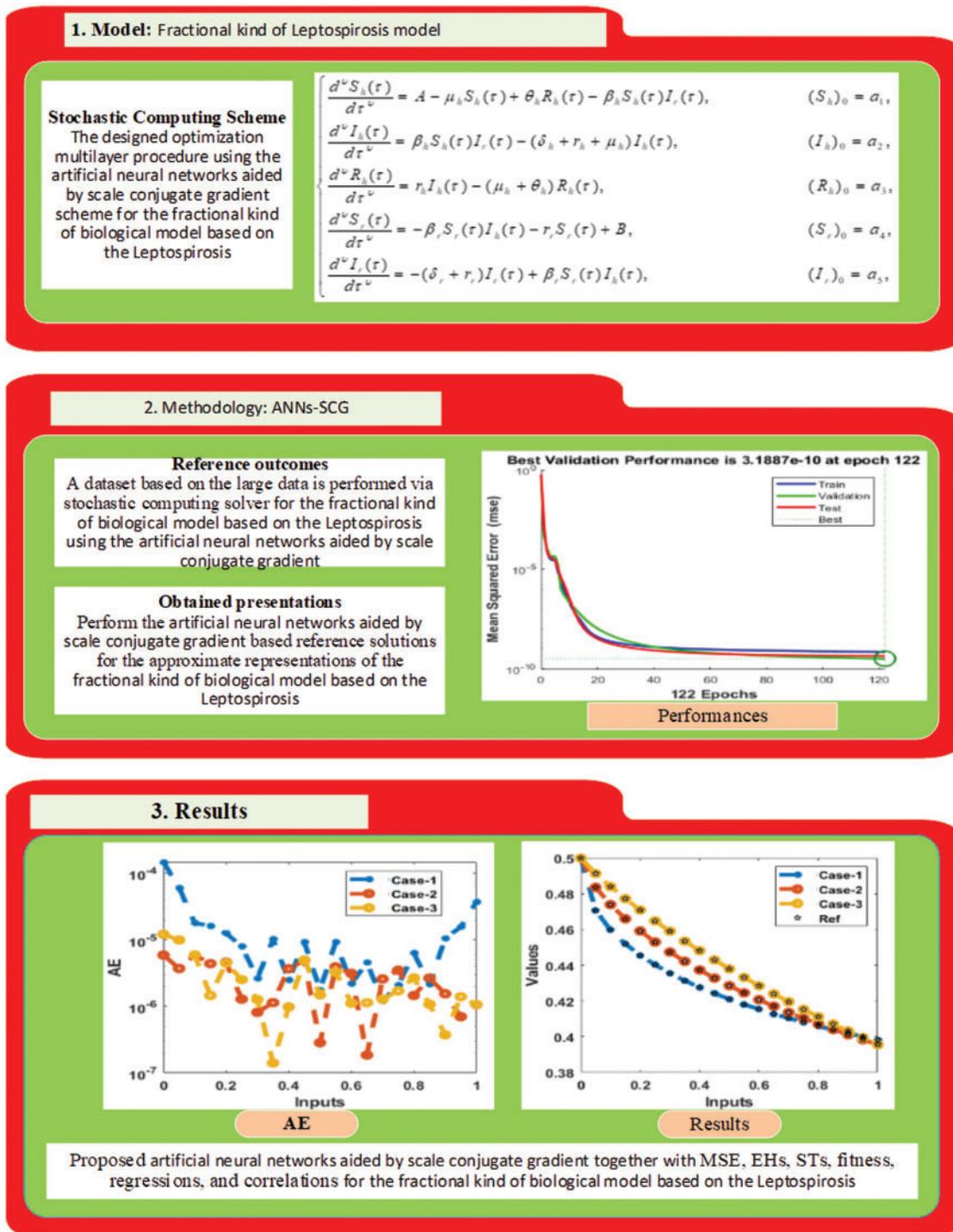


Figure 1: Workflow depictions of the stochastic scheme for the fractional kind of mathematical model based on the Leptospirosis

The computational ANNs-SCG scheme is provided in two steps: the system dataset is generated using the generalized Adams scheme and the significant operators to execute the estimated results of the fractional kind of mathematical model based on the Leptospirosis using the ANNs-SCG. The essential information based on the generalized Adam method can be observed in the reported investigation [48], whereas these numerical performances with default conditions perform the system dataset for different disparities. The statics performances are provided by taking 14 numbers of neurons together with the data is selected as 82% for training and the statics for both testing, and authentication is selected as 9% for the fractional kind of mathematical model based on the Leptospirosis using the ANNs-SCG.

The approximate numerical performances of the mathematical model of Leptospirosis with the artificial intelligence abilities based on the computing framework using the supervised neural networks trained with the scaled conjugate gradient in terms of best negotiation between the measures, including the complexity, premature convergence, underfitting, and overfitting situations.

Moreover, all the network parameters are adjusted after comprehensive simulation investigations, knowledge, care, experience, and minor dissimilarities in these situations to degrade the network's performance.

4 Analysis and Simulations of Results

This section provides three cases-based fraction Leptospirosis systems through the artificial neural networks (ANNs) along with the scale conjugate gradient (SCG), i.e., ANNs-SCG. The mathematical representation of each case (C) is provided as:

C-1: Suppose the mathematical form of the fractional Leptospirosis model is written as:

$$\left\{ \begin{array}{l} \frac{d^{0.6} S_h(\tau)}{d\tau^{0.6}} = 0.0067 R_h(\tau) + 1.6 - 0.0098 I_r(\tau) S_h(\tau) - 0.03 S_h(\tau), \quad (S_h)_0 = 0.05, \\ \frac{d^{0.6} I_h(\tau)}{d\tau^{0.6}} = 0.0098 I_r(\tau) S_h(\tau) - 0.0411 I_h(\tau), \quad (I_h)_0 = 0.1, \\ \frac{d^{0.6} R_h(\tau)}{d\tau^{0.6}} = 0.007 I_h(\tau) - 0.0407 R_h(\tau), \quad (R_h)_0 = 0.15, \\ \frac{d^{0.6} S_r(\tau)}{d\tau^{0.6}} = -0.078 I_h(\tau) S_r(\tau) + 1.2 - 0.17 S_r(\tau), \quad (S_r)_0 = 0.2, \\ \frac{d^{0.6} I_r(\tau)}{d\tau^{0.6}} = 0.078 I_h(\tau) S_r(\tau) - 0.264 I_r(\tau), \quad (I_r)_0 = 0.25. \end{array} \right. \quad (3)$$

C-2: Suppose the mathematical form of the fractional Leptospirosis model is written as:

$$\left\{ \begin{aligned} \frac{d^{0.7}S_h(\tau)}{d\tau^{0.7}} &= 0.0067R_h(\tau) + 1.6 - 0.0098I_r(\tau) S_h(\tau) - 0.03S_h(\tau), & (S_h)_0 &= 0.05, \\ \frac{d^{0.7}I_h(\tau)}{d\tau^{0.7}} &= 0.0098I_r(\tau) S_h(\tau) - 0.0411I_h(\tau), & (I_h)_0 &= 0.1, \\ \frac{d^{0.7}R_h(\tau)}{d\tau^{0.7}} &= 0.007I_h(\tau) - 0.0407R_h(\tau), & (R_h)_0 &= 0.15, \\ \frac{d^{0.7}S_r(\tau)}{d\tau^{0.7}} &= -0.078I_h(\tau) S_r(\tau) + 1.2 - 0.17S_r(\tau), & (S_r)_0 &= 0.2, \\ \frac{d^{0.7}I_r(\tau)}{d\tau^{0.7}} &= 0.078I_h(\tau) S_r(\tau) - 0.264I_r(\tau), & (I_r)_0 &= 0.25. \end{aligned} \right. \quad (4)$$

C-3: Suppose the mathematical form of the fractional Leptospirosis model is written as:

$$\left\{ \begin{aligned} \frac{d^{0.8}S_h(\tau)}{d\tau^{0.8}} &= 0.0067R_h(\tau) + 1.6 - 0.0098I_r(\tau) S_h(\tau) - 0.03S_h(\tau), & (S_h)_0 &= 0.05, \\ \frac{d^{0.8}I_h(\tau)}{d\tau^{0.8}} &= 0.0098I_r(\tau) S_h(\tau) - 0.0411I_h(\tau), & (I_h)_0 &= 0.1, \\ \frac{d^{0.8}R_h(\tau)}{d\tau^{0.8}} &= 0.007I_h(\tau) - 0.0407R_h(\tau), & (R_h)_0 &= 0.15, \\ \frac{d^{0.8}S_r(\tau)}{d\tau^{0.8}} &= -0.078I_h(\tau) S_r(\tau) + 1.2 - 0.17S_r(\tau), & (S_r)_0 &= 0.2, \\ \frac{d^{0.8}I_r(\tau)}{d\tau^{0.8}} &= 0.078I_h(\tau) S_r(\tau) - 0.264I_r(\tau), & (I_r)_0 &= 0.25. \end{aligned} \right. \quad (5)$$

The simulations of the fractional kind of mathematical model based on Leptospirosis using the computational ANNs-SCG approach have been provided by taking 14 numbers of neurons. Fig. 2 shows the stochastic ANNs-SCG scheme of the fractional kind of mathematical model based on Leptospirosis.

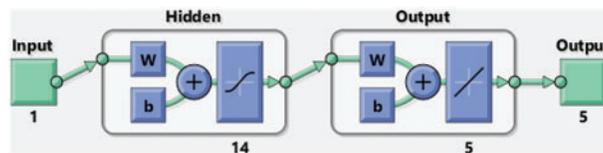


Figure 2: Stochastic ANNs-SCG scheme for the fractional Leptospirosis system

The fractional kind of mathematical Leptospirosis system simulations using the computational ANNs-SCG approach has been provided by taking 14 numbers of neurons together with data selection as 82% for training, and the statics for both testing and authentication is selected as 9%. Fig. 2 shows the stochastic ANNs-SCG scheme of the fractional kind of mathematical model based on Leptospirosis.

Figs. 3–5 represent the numerical outcomes of the fractional Leptospirosis system by using the stochastic ANNs-SCG scheme. Figs. 3 and 4 indicate the mean square error (MSE) values and state

transitions (STs) measures based on the fractional kind of mathematical Leptospirosis system. The best calculations for the fractional kind of Leptospirosis model were performed at epochs 125, 509, and 122, which were performed as 9.46227×10^{-08} , 1.85270×10^{-09} , and 6.90443×10^{-10} . The gradient representations are performed as 7.32×10^{-07} , 9.90×10^{-08} , and 9.92×10^{-08} for the respective cases of the fractional order model based on Leptospirosis. Fig. 4 presents the obtained results and the performances error histograms for solving the fractional mathematical model based on Leptospirosis using the stochastic ANNs-SCG scheme. The obtained best representations based on the training, authentication, and testing outcomes of the fractional order model based on Leptospirosis using the ANNs-SCG scheme have been provided. The EHs performances have also been provided in Fig. 4 which shows the values for C-1, 2 and 3 are 1.51×10^{-04} , 9.33×10^{-06} , and 3.09×10^{-06} . Fig. 5 provides the values of the correlations by using the computational numerical ANNs-SCG approach-based fractional Leptospirosis system. It is depicted that the correlation values perform the perfect model due to 1 in the calculation for the fractional Leptospirosis system. The accuracy of the computational numerical ANNs-SCG approach for the fractional order model based on Leptospirosis is provided based on the substantiation, testing, and training measures. Tab. 2 shows the MSE values based on the fractional kind of model of Leptospirosis by using the computational ANNs-SCG approach.

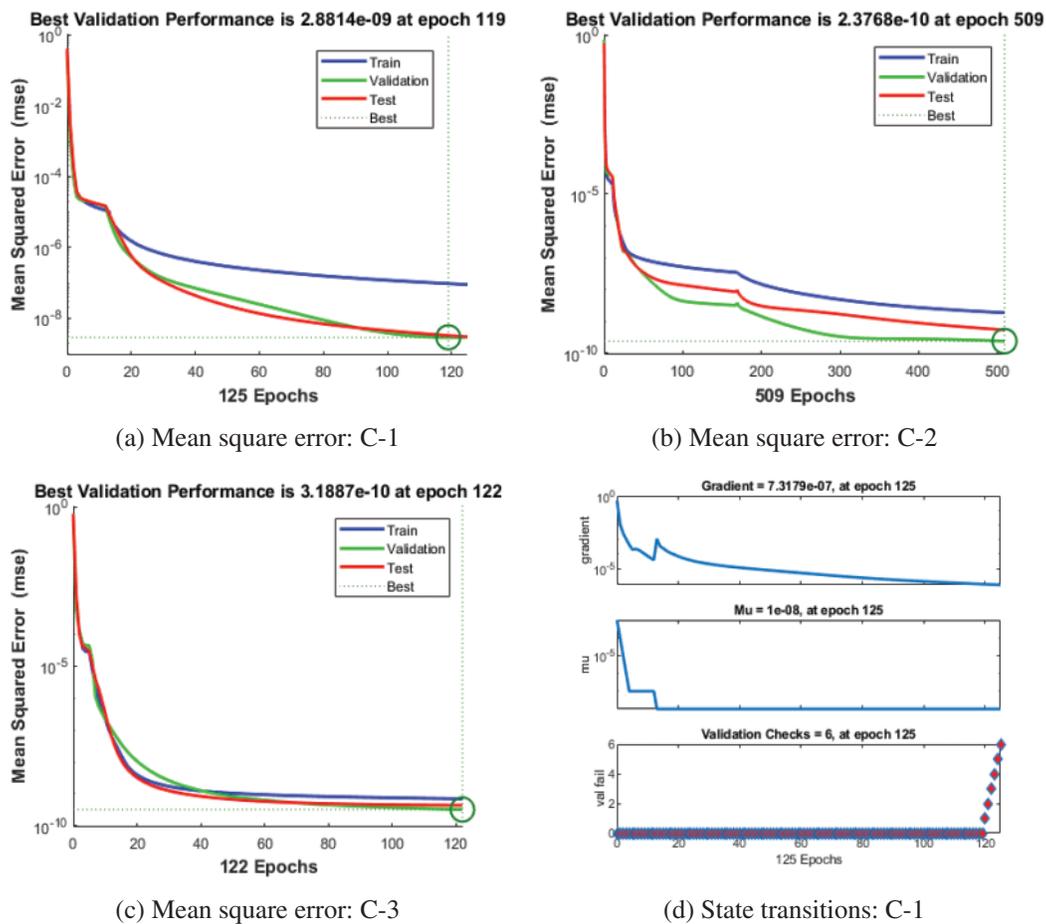
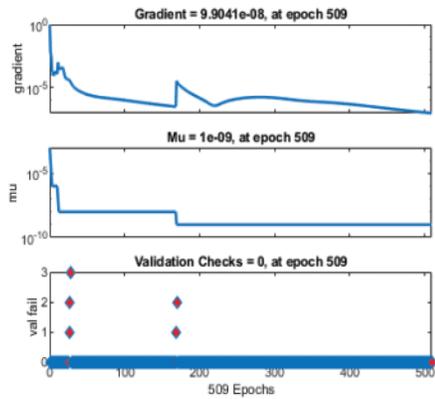
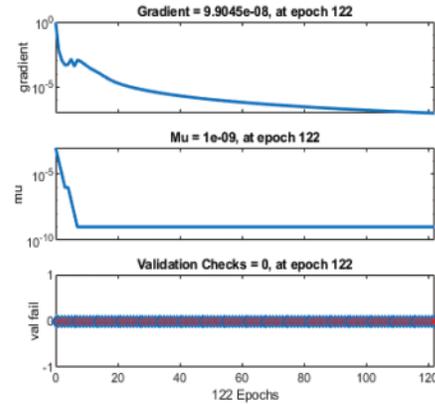


Figure 3: (Continued)

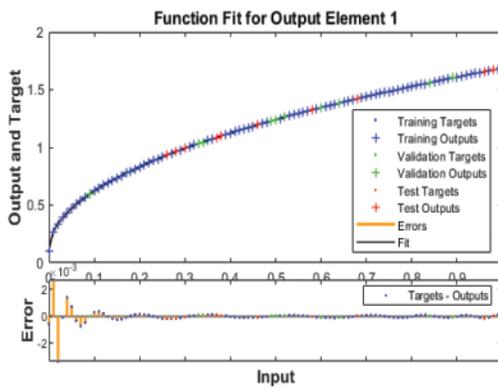


(e) State transitions: C-2

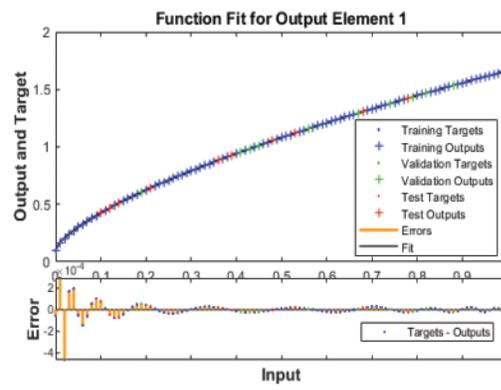


(f) State transitions: C-3

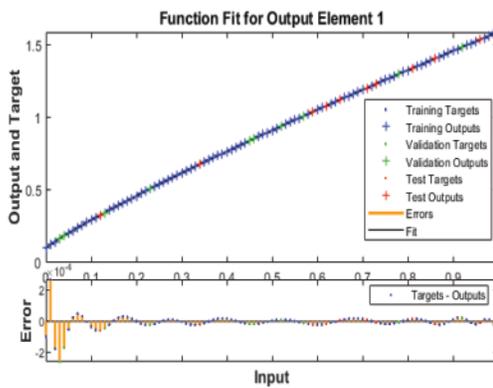
Figure 3: Mean square error measures and State transition performances values based on fractional kind of Leptospirosis system



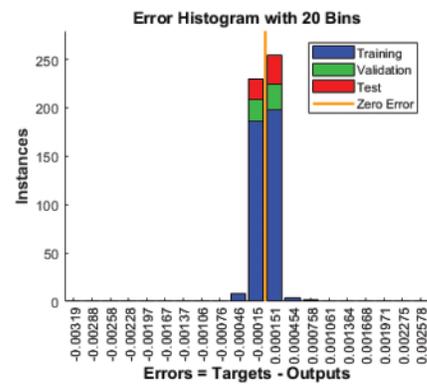
(a) Results: C-1



(b) Results for C-2

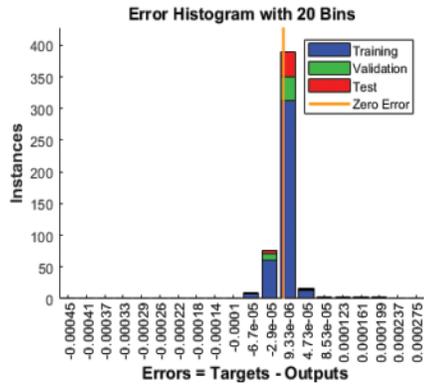


(c) Results: C-3

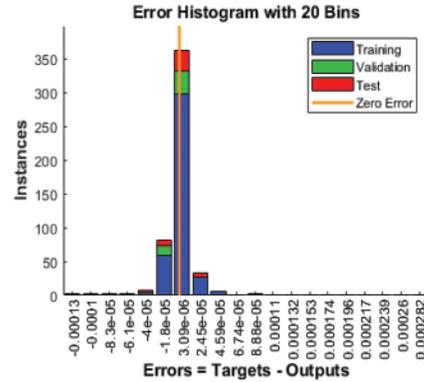


(d) EHs measures: C-1

Figure 4: (Continued)

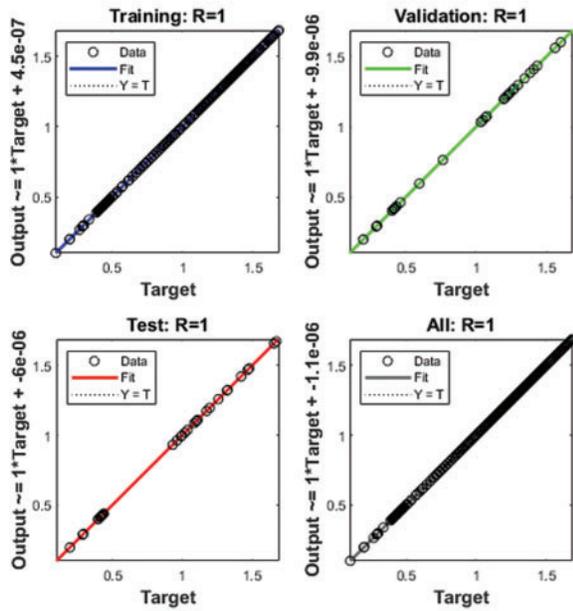


(e) EHs measures: C-2

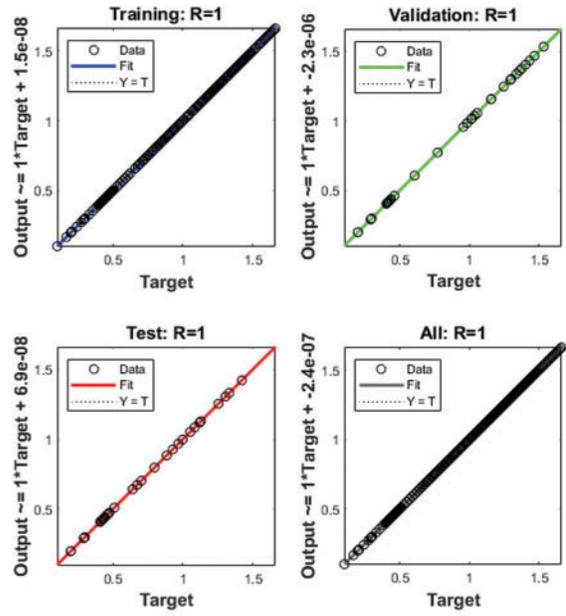


(f) EHs measures: C-3

Figure 4: Assessments of the solutions and error histograms measures based on fractional kind of Leptospirosis system

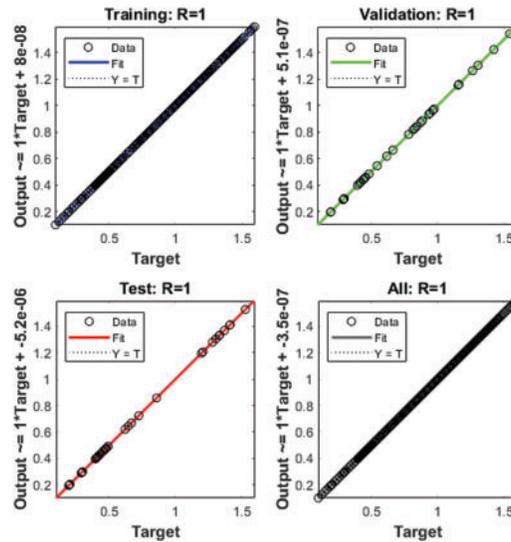


(a) Regression performances: C-1



(b) Regression performances: C-2

Figure 5: (Continued)



(c) Regression performances: C-3

Figure 5: Regression performances based on fractional kind of Leptospirosis

Table 2: MSE based computational ANNs-SCG approach for the fractional order model based on Leptospirosis

C	MSE			Performance	Gradient	Mu	Iteration	Time
	Training	Validation	Testing					
1	9.46×10^{-08}	2.88×10^{-09}	3.23×10^{-09}	7.32×10^{-07}	8.89×10^{-08}	1×10^{-08}	125	3
2	1.85×10^{-09}	2.37×10^{-10}	5.34×10^{-10}	9.90×10^{-08}	1.85×10^{-09}	1×10^{-09}	509	5
3	6.90×10^{-10}	3.18×10^{-10}	4.33×10^{-10}	9.90×10^{-08}	6.90×10^{-10}	1×10^{-09}	122	3

The result comparisons and the values of the AE are performed in Figs. 6 and 7 using the artificial neural networks (ANNs) along with the scale conjugate gradient (SCG), i.e., ANNs-SCG algorithms for the fractional order model based on Leptospirosis. The achieved performances have been performed through the stochastic ANNs-SCG approach for the fractional order model based on Leptospirosis. Fig. 6 presents the results overlapping through the obtained performances based on the stochastic ANNs-SCG approach and reference solutions. These depictions perform the correctness and accuracy of the ANNs-SCG procedure for the fractional kind of system based on Leptospirosis. Fig. 7 depicts the AE descriptions for the fractional kind of model based on the Leptospirosis stochastic ANNs-SCG procedure. The AE measures for the category $S_h(\tau)$ have been performed around 10^{-03} – 10^{-05} , 10^{-04} – 10^{-05} and 10^{-05} – 10^{-06} . For $I_h(\tau)$, the AE values for C (1–3) have been performed as 10^{-04} – 10^{-06} , 10^{-05} – 10^{-06} and 10^{-05} – 10^{-07} . For $R_h(\tau)$, the AE measures have been reported as 10^{-04} – 10^{-05} , 10^{-05} – 10^{-06} and 10^{-06} – 10^{-08} for C (1–3). For the fractional kind of system based on Leptospirosis, the AE for $S_r(\tau)$ is reported as 10^{-03} – 10^{-05} , 10^{-04} – 10^{-06} and 10^{-05} – 10^{-06} . For C (1–3), the AE for $I_r(\tau)$ is found as 10^{-04} – 10^{-06} , 10^{-05} – 10^{-06} and 10^{-05} – 10^{-07} . These best measures calculated through the AE perform the exactness of the stochastic ANNs-SCG procedure for the fractional kind of model based on Leptospirosis.

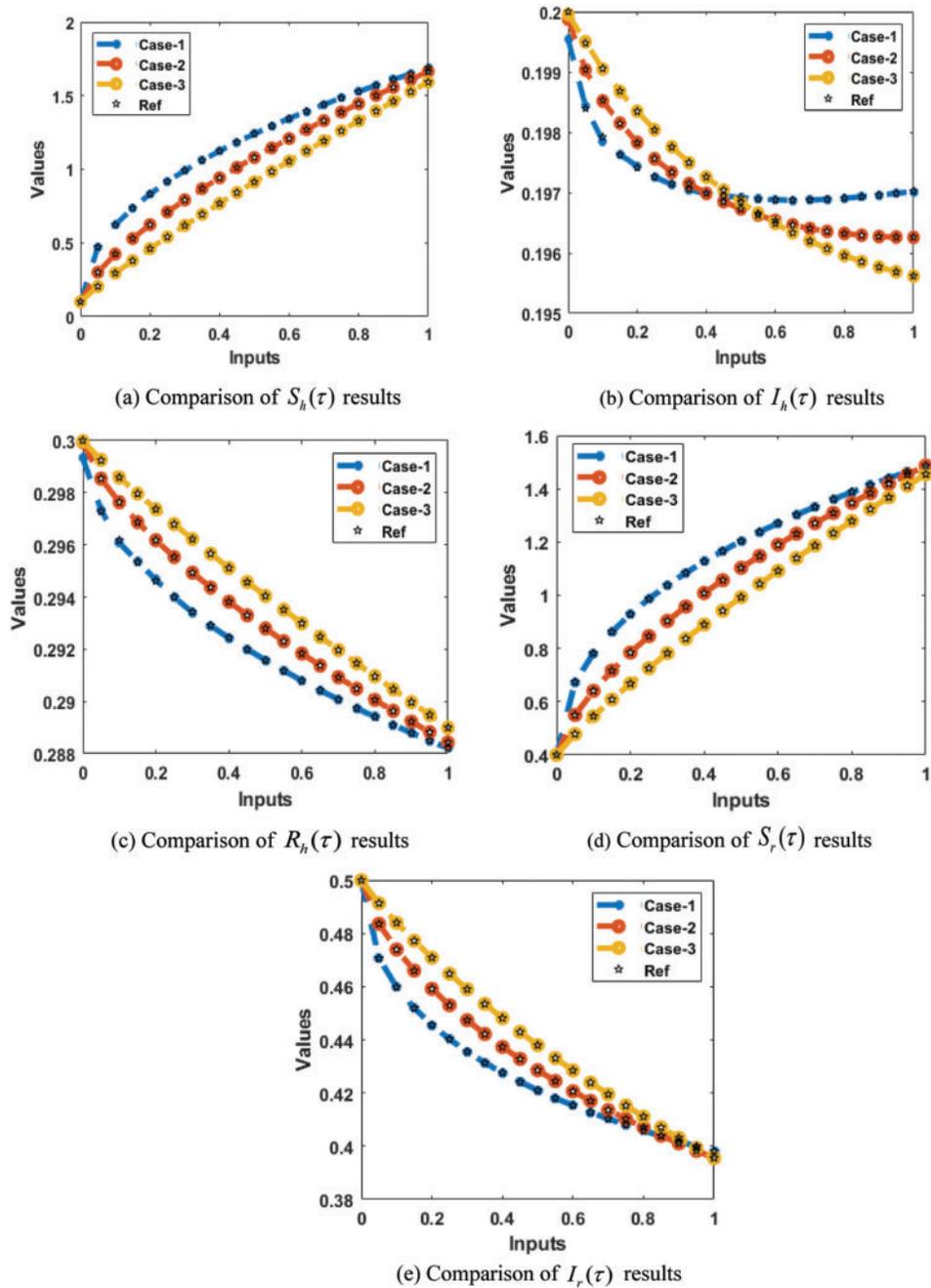


Figure 6: Comparison of the results using the stochastic procedure for the fractional kind of model based on Leptospirosis

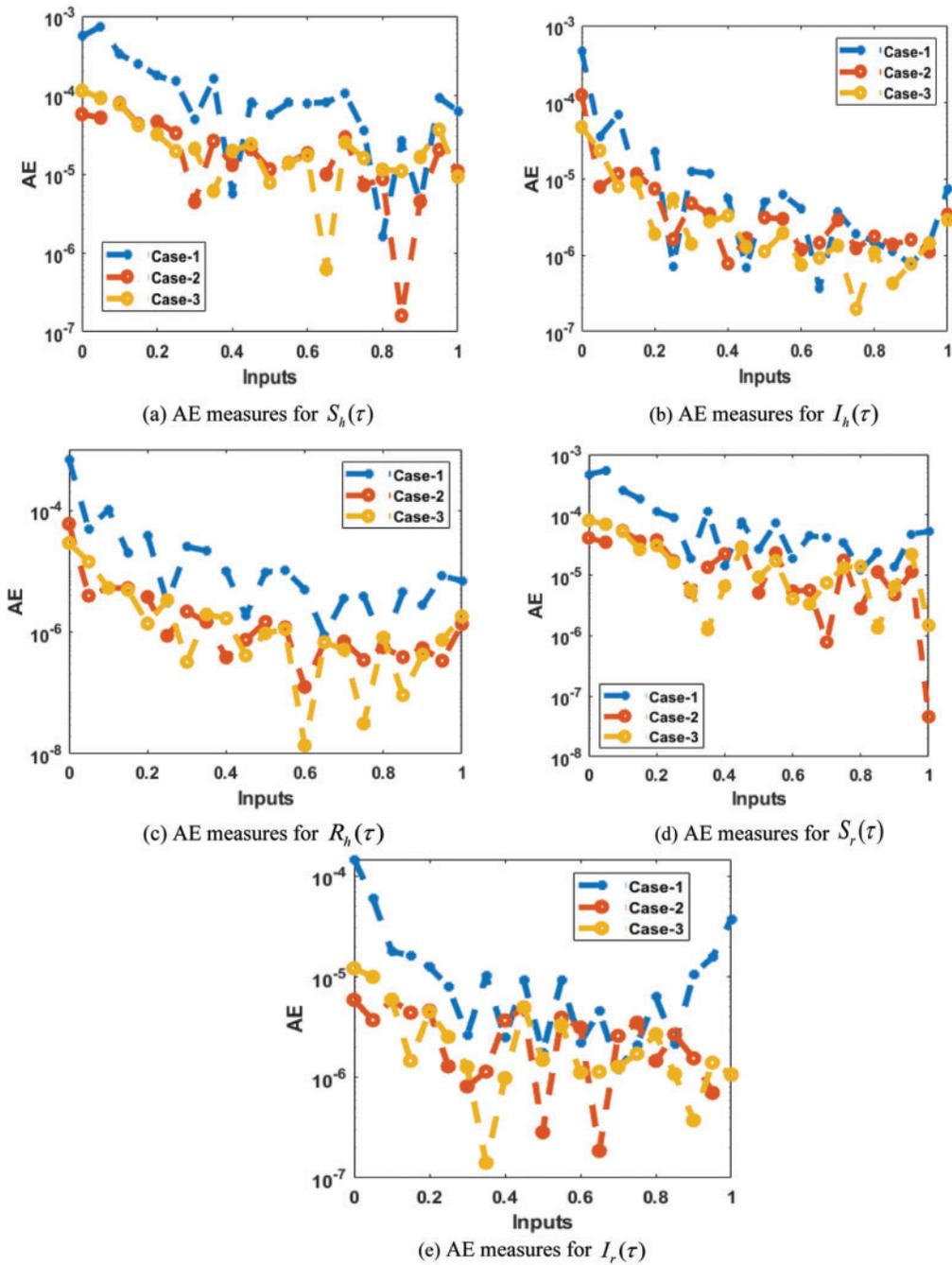


Figure 7: Values of the absolute error using the stochastic procedure for the fractional kind of model based on Leptospirosis

5 Conclusion

In this work, the solutions of the fractional kind of biological model based on Leptospirosis have been provided by exploiting the strength of artificial neural networks aided by the scaled conjugate

gradient. The reliable and accurate results of the model have been achieved by using fractional kinds of derivatives. The biological Leptospirosis mathematical form is categorized into five classes. The numerical performances of each category of the biological Leptospirosis mathematical model have been presented through the stochastic artificial neural networks (ANNs) along with the scale conjugate gradient (SCG), i.e., the ANNs-SCG procedure. The correctness of the computing ANNs-SCG procedure is performed by comparing the achieved and reference solutions (Adam numerical method). The simulations of the fractional kind of mathematical model based on Leptospirosis using the computational ANNs-SCG approach have been provided by taking 14 numbers of neurons together with the selection of the data as 82% for training. At the same time the statics for both testing and authentication is selected as 9%. The AE has been calculated for the respective cases of the fractional kind of system are found as 10^{-06} to 10^{-08} . The numerical procedures based on the recurrence, error histograms, regression, mean square error, state transitions, and correlation have been accomplished to validate the fitness, capability, accuracy, steadiness and reliability of the proposed ANNs-SCG method.

In upcoming work, the proposed ANNs-SCG numerical procedure can be implemented to solve different applications of extreme interest to researchers [49–52].

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Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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