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Article

Managing Delivery of Safeguarding Substances as a Mitigation Against Outbreaks of Pandemics

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Abstract: The optimum delivery of safeguarding substances is a major part of supply chain management and a crucial issue in the mitigation against the outbreak of pandemics. A problem arises for a decision maker who wants to optimally choose a subset of candidate consumers to maximize the distributed quantities of the needed safeguarding substances within a specific time period. A nonlinear binary mathematical programming model for the problem is formulated. The decision variables are binary ones that represent whether to choose a specific consumer, and design constraints are formulated to keep track of the chosen route. To better illustrate the problem, objective, and problem constraints, a real application case study is presented. The case study involves the optimum delivery of safeguarding substances to several hospitals in the Al-Gharbia Governorate in Egypt. The hospitals are selected to represent the consumers of safeguarding substances, as they are the first crucial frontline for mitigation against a pandemic outbreak. A distribution truck is used to distribute the substances from the main store to the hospitals in specified required quantities during a given working shift. The objective function is formulated in order to maximize the total amount of delivered quantities during the specified time period. The case study is solved using a novel Discrete Binary Gaining Sharing Knowledge-based Optimization algorithm (DBGSK), which involves two main stages: discrete binary junior and senior gaining and sharing stages. DBGSK has the ability of finding the solutions of the introduced problem, and the obtained results demonstrate robustness and convergence toward the optimal solutions.

Keywords: Safeguarding substances; mitigation against pandemic outbreaks; supply chain management; metaheuristic optimization

1 Introduction

Reference [1] indicates that the entire world is suffering from a global epidemic that has infected thousands of people in almost all countries. In December 2019, Wuhan in China was the origin of a pneumonia of unknown cause. Cases of the outbreak did not remain limited to this city, and by January 2020, confirmed cases were detected outside Wuhan. Reference [2] indicates that the number of confirmed infected cases across the world shows that this is a vast and evolving situation, and new changes may happen by the hour. Reference [3] indicates that numbers are expected to increase exponentially as new cases are discovered. Reference [4] declared that the outbreak of the COVID-19 pandemic has put humans in all countries facing grave danger. The pandemic has spread across the world, and total number of cases was greater than 67 million as of December 2020. Reference [5] indicates that the growth factor of daily new cases is greater than 1 in many countries, suggesting exponential growth. Reference [6] indicates that the rate of spread of the virus was estimated in the early stages of the disease to be 2.2.

References [7,8] indicate that because the major symptoms of the new pandemic are known, any individual can assess whether they have the symptoms. To safeguard against the disease, many precautions are recommended, such as stay at home except in cases of necessity; clean hands thoroughly and regularly with soap and water or with an alcoholic hand rub solution; regularly clean surfaces; and wear protective masks and gloves.

It is worthwhile to determine the importance of safeguarding substances such as cleaning materials, alcohol, respirator masks, medical gloves, and disinfection fluids both to hospital crews and to patients, especially those suspected to have caught the virus. Countries that have succeeded in slowing the prevalence of the virus have forced their citizens to wear masks in public places. Most of the infected people have contracted the infection in closed environments and in poorly ventilated spaces, such as public places, transportation means, restaurants, cinemas, stores, hospitals, and homes. Therefore, it is essential to provide such places with necessary safeguarding substances.

Reference [9] indicates that the concept of supply chain has gained popularity in the past several years. Many different definitions for it have been presented. The supply chain—a term increasingly used by logistics professionals—encompasses every effort involved in producing and delivering a final product, from the supplier's supplier to the consumer's consumer.

References [10,11] state that the general definition of supply chain is the science and art of obtaining, producing, and delivering materials and products in proper quantities, in the proper place, and at the proper time. So, four basic processes, namely plan, source, make, and deliver, broadly define these efforts. In light of the utmost importance of delivering the safeguarding substances, it is worth noting that the importance of meeting most of the ultimate consumer's demands and improving responsiveness are now considered strategic capabilities. A very significant part of the supply chain management for the safeguarding substances for the pandemic is the optimum delivery phase. With some planning, utilization of the available time for delivering the safeguarding substances can be optimized so that maximum possible quantities can be delivered.

The optimum delivery of the safeguarding substances for mitigating an outbreak in network optimization is defined for scheduling the distribution system with a limited load capacity to a list of consumers with known demanded quantities. The objective is to identify the most effective route for the distribution system as measured by maximizing the total delivered quantities in a certain limited time period. The route starts from a predetermined main store location and proceeds to each chosen consumer exactly once and then returns to the main store.

The rest of this article is organized as follows. Section 2 is devoted to demonstrating the importance of providing hospitals with safeguarding substances to protect medical personnel, patients, and visitors. This section describes the problem under consideration to distribute safeguarding substances to a group of consumers in an optimal way in order to meet the needs of each consumer while increasing the total distributed safeguarding substances during a specified time period.

The mathematical model of the problem is developed in Section 3. The proposed formulation is a nonlinear binary mathematical model with a dimension depending on the number of candidate consumers to be visited. The steps of the solution procedure are also explained.

A real application case study for the optimum delivery of safeguarding substances among candidate hospitals representing an essential type of consumers for such preventive materials is presented in Section 4. In Section 5, a novel discrete binary version of a recently developed Gaining Sharing Knowledge-based Optimization Technique (GSK) is introduced for solving the problem. GSK cannot solve the problem in binary space; therefore, a Discrete Binary GSK Optimization Algorithm (DBGSK) is proposed with two new junior and senior stages. These stages allow DBGSK to inspect the problem search space efficiently.

Section 6 presents the experimental results of the problem obtained by DBGSK, and Section 7 summarizes the conclusions and the suggested points for future research.

2 Delivery of Safeguarding Substances

Safeguarding substances can disinfect contaminated places and limit the quantities of viruses that travel through respiratory droplets. They also contribute to limiting transmission of infection between people in gatherings, especially in public transport and crowded places. Frontline workers in medical institutions who wore the "N95" respirator mask did not catch the virus despite closely taking care of the infected patients. That is why it is very important to provide public places and hospitals with adequate amounts of these safeguarding substances regularly.

The importance of safeguarding substances and wearing masks in public places becomes clear considering that between 6% and 18% of infected people may not show any symptoms of the disease despite being able to spread the infection. The incubation period of the virus may be up to 14 days before symptoms appear. If everyone, especially the asymptomatic, wears masks, the number of viruses circulating in the air will decrease, and the risk of transmission will be reduced.

About 3,000 drops of spray may come out of the mouth of a person during one sneeze, and some fear that the virus can spread through the spray that comes out of the mouth while speaking. Once the spray comes out of the mouth, the larger droplets settle on surfaces, while the smaller droplets remain suspended in the air for hours, which may be inhaled by a healthy person. After the virus enters the body and multiplies, viral particles exit from the cells and enter the body fluids in the lungs, mouth, and nose. When the person coughs, the tiny droplets that are filled with viruses disperse in the air.

Reference [12] studied the transmission of 2019-nCoV infection from an asymptomatic contact. Reference [13] studied the SARS-CoV-2 viral load in upper respiratory specimens of patients. Reference [14] studied the pre-symptomatic transmission of SARS-CoV-2.

Reference [15] indicates that masks are recommended for people when they are in areas of widespread transmission and they cannot guarantee a distance of at least one meter from others. Reference [16] concluded that masks remarkably reduced the number of microorganisms expelled by volunteers. Reference [17] studied the efficiency of aerosol filtration fabrics in cloth masks. Reference [18] performed a quantitative mechanistic study of home-made masks against spread of respiratory infection. Reference [19] demonstrated the possible benefits of wearing masks and hand hygiene in defending against SARS-CoV-2. References [20,21] discussed the significance of masks for respiratory virus shedding.

The conclusion is that if a virus is given favorable opportunities, it may remain suspended in the air for several hours and still be able to transmit to people who inhale the droplets. The virus appears to be able to circulate through the air particularly in closed environments.

Hospitals and health centers are at the top of the list of public places that need safeguarding substances, and attention should be paid to continuous and timely supply to them of the required amounts. Medical employees are the first line of defense in the fight against the pandemic because they are exposed to patients and visitors who may be infected with the virus.

The problem under consideration is how to distribute safeguarding substances to a group of hospitals—as a good example of public places—in an optimal way. A transport vehicle carrying an amount of these safeguarding substances moves from one store and distributes them to the selected hospitals to meet the needs of each one while maximizing the total distributed quantities. The needed amount for a hospital is determined according to the number of medical crews, attending patients, and number of visitors. The distribution process takes place within a limited time window, namely the shift of the driver and their companions. The problem begins by considering several hospitals to be supplied. Then, the problem is to determine the optimal choice between these hospitals during the limited time shift so that the largest amount of these safeguarding substances is distributed. With the addition of a new list of hospitals including those that were not provided with safeguarding substances in the first round, the process is repeated in the same way until the provision to all hospitals. Fig. 1 represents an essential activity in the supply chain process, which is the delivery of safeguarding substances to the set of chosen consumers in the optimum delivery route. The chosen consumers are a subset or a proper subset of the candidate end consumers.

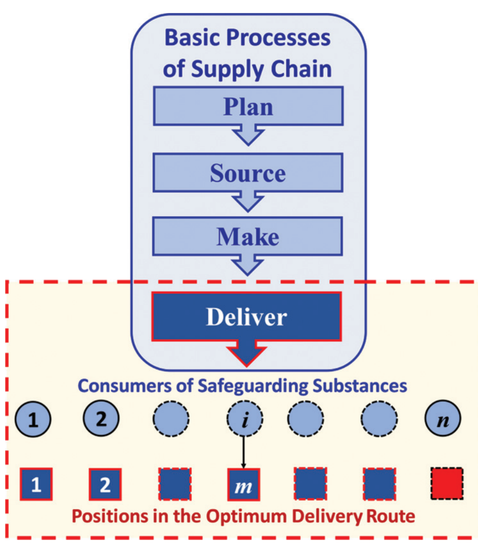

Figure 1: The delivery of safeguarding substances as a part of the supply chain management

3 Formulation of the Mathematical Model

The distribution of safeguarding substances problem is mapped to a graph G with a set V of $(n+1)$ nodes representing n consumers and one additional node denoting the main store location where the distribution system starts the job along with a set of arcs representing the transportation times between each distinct pair of nodes. The required quantity of safeguarding substances for each consumer, the delivery system capacity, the time for delivery of safeguarding substances to each consumer, and the transportation time between each two consumers are specified.

The problem is then defined as:

—Each consumer is visited only once to deliver the required safeguarding substances.
—The route starts and ends at the main store location from where the safeguarding substances are supplied.
—The overall goal of the problem is the best utilization of the available time, calculated as maximizing the total amount of distributed safeguarding substances during the predetermined time interval.

3.1 Decision Variables

Let:

$$x_i^m = \begin{cases} 1, & \text{if consumer } i \text{ is approached by the distribution system on position } m, i \text{ and} \\ & \text{m = 1, 2, …, n.} \\ 0, & \text{otherwise.} \end{cases}$$

where $n =$ number of candidate consumers.

3.2 Constraints

(i) Avoid the Trivial Solution:

In order to avoid the trivial solution that the distribution system will be saturated by supplying one consumer only, the following condition should hold: The quantity of safeguarding substances to be supplied to any candidate consumer should fill up completely the maximum carrying capacity of the used distribution system. In case of violation, the used system will travel to that consumer, deliver its total capacity, and obviate the need to perform the scheduling process.

In order to avoid the solution that the distribution system can deliver the needed safeguarding substances to all consumers, the problem in this case will be to minimize the total transportation distances or times (the well-known Travelling Salesman Problem). The following condition should hold: The total quantities of safeguarding substances to be delivered to all the candidate consumers should be greater than the maximum carrying capacity of the used distribution system.

These two conditions should be checked before designing the appropriate mathematical model.

(ii) Positions Constraints:

Each position m in the optimum truck route has at most one consumer; see Eq. (1):

$$\sum_{i=1}^{n} x_i^m \le 1, \quad m = 1, 2, \ldots, n. \tag{1}$$

(iii) Consumer Constraints:

Each consumer i can be in one position of the optimum truck route or not visited; see Eq. (2):

$$\sum_{m=1}^{n} x_i^m \le 1, \quad i = 1, 2, \ldots, n. \tag{2}$$

(iv) Consecutive Positions Constraints:

A position $(m+1)$ cannot exist in the distribution route unless the preceding position m exists. This is achieved by the following set of constraints; see Eq. (3):

$$\sum_{i=1}^{n} x_i^{m+1} \le \sum_{i=1}^{n} x_i^m, \quad m = 1, 2, \ldots, n-1 \tag{3}$$

If $\sum_{i=1}^{n} x_i^{m+1} = 1$, then $\sum_{i=1}^{n} x_i^m = 1$, $m = 1, 2, \ldots, n-1$,

If $\sum_{i=1}^{n} x_i^{m+1} = 0$, then there is no restriction on the value of $\sum_{i=1}^{n} x_i^m$, $m = 1, 2, \ldots, n-1$.

(v) Shift Hours Constraint:

The total time T spent in the whole optimum route by the distribution system is equal to four parts:

$T = T_1 + T_2 + T_3 + T_4$,

where:

$T_1 =$ Time of transportation from the main store to the first consumer in the route;

$T_2 =$ Total intermediate transportation times between two adjacent consumers in the route;

$T_3 =$ Transportation time spent from the last visited consumer to the main store;

$T_4 =$ Total time for delivery of items (counting and inspection) to the visited consumers.

These four parts are calculated as follows: $T_1 = \sum_{i=1}^{n} t_{0,i} x_i^1$, where $t_{0i} =$ transportation time between the main store and consumer i, $\forall i \in V$. $T_2 = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \ne i}}^{n} t_{i,j}.\ (\sum_{m=1}^{n-1} x_i^m \cdot x_j^{m+1})$, where $t_{i,j} =$ transportation time between the two adjacent consumers i and j, $\forall i, j \in V$.

The last visited position in the optimum route is characterized by a unique particularity that it does not have any adjacent subsequent positions. The expression $(x_i^m).\ (1 - \sum_{i=1}^{n} x_i^{m+1}) = 1$ holds only for the last position in the distribution route and equals 0 for all other positions. Then, we have the following expression for T_3:

$T_3 = \sum_{m=1}^{n-1} \sum_{i=1}^{n} (t_{i,0}.x_i^m).\ (1 - \sum_{i=1}^{n} x_i^{m+1}) + \sum_{i=1}^{n} t_{i,0}.x_i^n$, where $t_{i,0} =$ transportation time between consumer i and the main store, $\forall i \in V$. The second term in T_3 is added to cover the case when the optimum route will contain all the candidate n consumers since the first summation in the first term does not include the position n.

The following expression is used to calculate T_4: $T_4 = \sum_{m=1}^{n} \sum_{i=1}^{n} (t_i x_i^m)$.

The total time of the whole optimum route T will have the form: $T = T_1 + T_2 + T_3 + T_4$.

Substituting for T_1, T_2, T_3, and T_4, we obtain Eq. (4):

$$T = \sum_{i=1}^{n} t_{0,i} x_i^1 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} t_{i,j} . \left(\sum_{m=1}^{n-1} x_i^m . x_j^{m+1} \right) + \sum_{m=1}^{n-1} \sum_{i=1}^{n} (t_{i,0} . x_i^m) . \left(1 - \sum_{i=1}^{n} x_i^{m+1} \right)$$
$$+ \sum_{i=1}^{n} t_{i,0} . x_i^n + \sum_{m=1}^{n} \sum_{i=1}^{n} (t_i x_i^m) \leq T_{max}, \tag{4}$$

where T_{max} = time available in the working shift.

(vi) Maximum Load Capacity of the Distribution System Constraint:

The maximum quantity distributed to all the visited consumers in the considered time shift should not exceed the maximum load capacity of the transportation system, see Eq. (5):

$$\sum_{i=1}^{n} q_i . \left(\sum_{m=1}^{n} x_i^m \right) \leq Q, \tag{5}$$

where q_i = quantity of safeguarding substances required for consumer i; $i = 1, 2, \ldots, n$; Q = maximum load capacity of the distribution system.

(vii) Binary Constraints:

All the decision variables are 0–1; see Eq. (6).

$$x_i^m = 0 \text{ or } 1; \quad i, \ m = 1, 2, \ldots, n. \tag{6}$$

3.3 The Objective Function

The objective function is formulated in order to maximize the total amount of delivered safeguarding substances; see Eq. (7):

$$\text{Maximize Z} = \sum_{i=1}^{n} q_i . \left(\sum_{m=1}^{n} x_i^m \right), \tag{7}$$

where q_i = quantity of safeguarding substances required by consumer number i; $i = 1, 2, \ldots, n$.

Finally, we have a suggested model that contains (n^2) binary variables and ($4n+1$) constraints.

An optimum solution to the problem will produce two distinct situations:

a) If $\sum_{m=1}^{n} x_i^m = 1$, $i = 1, 2, \ldots, n$, then all the n consumers are supplied with the needed safeguarding substances in one shift, and the problem is completed.
b) If $\sum_{m=1}^{n} x_i^m = 0$ for any i, then the corresponding consumer i is not supplied with the needed materials in the considered shift. In this case, it is needed to eliminate the visited consumers, add other candidate ones to be supplied, and then consider one more shift and repeat the procedure once more.

Structured English is used to present the solution procedure. The use of Structured English to describe the steps of the algorithm is clear and unambiguous and can be read from start to finish. The use of Structured English keywords provides a syntax similar to that of a programming language to assist with identifying logical steps necessary to properly describe the algorithm. Structured English aims to provide the benefits of both programming logic and natural language;

program logic helps to attain precision, and natural language helps with the familiarity of the spoken word. See Fig. 2.

```
START
READ Tmax = number of hours per shift,
         tij = travelling times between consumers,
         ti = delivery times,
         qi = needed quantities,
         Q = maximum load of the distribution system.
SET shift no. = 1
DO WHILE there are consumers
      FORMULATE the decision variables
                    the objective function
                    the problem constraints
                    the binary constraints
                    the non-negativity constraints
      SOLVE the mathematical model using an appropriate software
      OUTPUT the optimum solution (visited consumers)
             the optimum value (total supplied materials)
             total time for the route
             arriving time to each consumer
             idle time for the truck
             route sequence
      IF all consumers are supplied THEN
          EXIT
      ELSE
          SET shift no. = shift no. + 1
          DELET the supplied consumers
          ADD any new candidate consumers
      ENDIF
ENDDO
EXIT
```

Figure 2: Steps of the solution procedure of the problem

4 Real Application Case Study

A real example is presented to apply the given mathematical model for very important consumers during an epidemic outbreak. An example of an important consumer of the safe-guarding substances for an outbreak are hospitals that exist in different locations in Al-Gharbia Governorate, Egypt. A small truck with 1.7-ton capacity starts its route from the main store of safeguarding substances located in Tanta, the capital of the Governorate denoted by (STORE), as shown in Fig. 3.

In one special emergency night shift that lasts five hours, five hospitals are identified as candidate consumers, denoted by serial numbers (1, 2, ..., 5). The store is denoted by (0). All are located in Al-Gharbia Governorate. The data is given in Tab. 1, where numbers inside the cells (i, j) represent the transportation times t_{ij} in hours. For simplicity, the time of delivery of the safeguarding substances for each hospital is included in the transportation time.

The mathematical formulation for the given case study is worked out by substituting in the previously described model, that is, formulas 1–7.

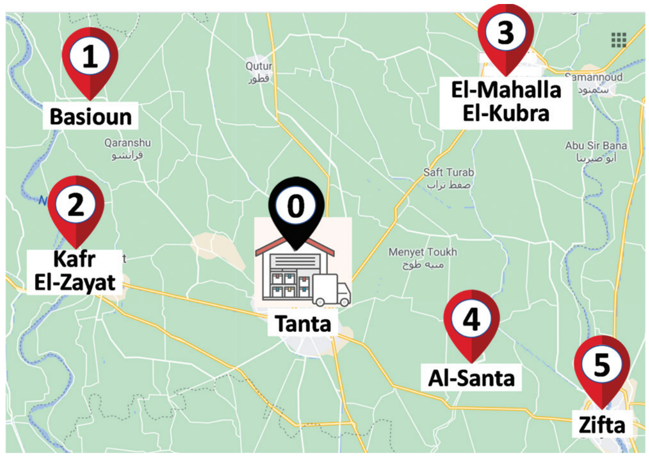

Figure 3: Locations of the main store and consumers

Table 1: Data for the case study example

q_i ($\times$100 kg)	i \ j	1	2	3	4	5
	0	0.75	0.50	1.00	0.75	1.10
3.5	1		1.50	1.25	1. 50	1.75
4	2			1.00	1.10	1.25
5	3				1.00	1. 25
1	4					0.50
2	5					

5 The Proposed Methodology

Metaheuristic approaches have been developed for complex optimization problems with continuous variables. References [22–29] note that these metaheuristic algorithms include Genetic Algorithm, Differential Evolution, Particle Swarm Optimization algorithm, Grey Wolf Optimizer, Water Cycle Algorithm, Teaching Learning based Optimization, Bat Algorithm, and artificial bee colony algorithm. They have been successfully applied to many real-world problems [30–40]. Reference [41] recently proposed a novel GSK based on acquiring knowledge and sharing it with others throughout their lifetimes. The original GSK solves optimization problems over continuous space, but it cannot solve the problem in binary space. So, a new variant of GSK is introduced to solve the proposed problem. DBGSK is proposed over discrete binary space with new binary junior and senior gaining and sharing stages.

References [42–50] state that there are many constraint handling techniques in the literature. References [51,52] used the augmented Lagrangian method in which an unconstrained optimization problem was obtained from a constrained optimization one.

The proposed methodology is described below.

5.1 Gaining-Sharing Knowledge-based Optimization Algorithm

An optimization problem with constraints can be formulated as:

$$Min\ f(X); \quad X = [x_1, x_2, \ldots, x_{Dim}]$$

$$s.to.\ g_i(X) \leq 0; \quad i = 1, 2, \ldots, m$$

$$X \in [\alpha_p, \beta_p]; \quad p = 1, 2, \ldots, Dim$$

where f denotes the objective function; $X = [x_1, x_2, \ldots, x_{Dim}]$ are the decision variables; $g_i(X)$ are the inequality constraints; α_p, β_p are the lower and upper bounds of the decision variables, respectively; and Dim represents the dimension of individuals. If the problem is in maximization form, then consider minimization $= -$maximization.

The human-based GSK algorithm has two stages: junior and senior gaining and sharing stage. All persons acquire knowledge and share their views with others. The people in early stage gain knowledge from their small networks, such as family members, relatives, and neighbours, and want to share their opinions with others who might not be from their networks, because the curiosity about other people. These people may not have the experience to categorize the people. In the same way, people in the middle or later stages enhance their knowledge by interacting with friends, colleagues, and social media friends, among others, and share their views with the most suitable person so that they can improve their knowledge. These people have the experience to judge other people and can categorize them (good or bad). The process mentioned above can be formulated mathematically in the following steps.

Step 1: To get a starting point of the optimization problem, the initial population must be obtained. The initial population is created randomly within the boundary constraints. See Eq. (8):

$$x_{tp}^{0} = \alpha_p + rand_p\left(\beta_p - \alpha_p\right), \tag{8}$$

where t is for the number of populations; $rand_p$ denotes a random number uniformly distributed between *0* and *1*.

Step 2: At this step, the dimensions of junior and senior stages should be computed through the following formula. See Eqs. (9) and (10):

$$Dim_J = Dim \times \left(\frac{Gen^{max} - G}{Gen^{max}}\right)^k, \tag{9}$$

$$Dim_S = Dim - Dim_J, \tag{10}$$

where k (> 0) denotes the learning rate that monitors the experience rate. Dim_J and Dim_S represent the dimensions for the junior and senior stage, respectively. Gen^{max} is the maximum count of generations, and G is the count of generation.

Step 3: Junior gaining sharing knowledge stage: In this stage, the early aged people gain knowledge from their small networks and share their views with other people who may or may not belong to their groups. Thus, individuals are updated as:

i. According to the objective function values, the individuals are arranged in ascending order. For every x_t $(t = 1, 2, \ldots, NP)$, select the nearest best (x_{t-1}) and worst (x_{t+1}) to gain knowledge and also randomly choose (x_r) to share knowledge. The pseudocode to update the individuals is presented in Fig. 4, where k_f (> 0) is the knowledge factor.

Step 4: Senior gaining sharing knowledge stage: This stage comprises the impact and effect of other people (good or bad) on the individual. The updated individuals can be determined as follows:

i. The individuals are classified into three categories (best, middle, and worst) after sorting individuals into ascending order based on the objective function values.

ii. Best individual = 100p% (x_{best}); middle individual = $Dim - 2 * 100p$% (x_{middle}); worst individual = 100p% (x_{worst}).

For every individual x_t, choose the top and bottom 100p% individuals for the gaining part and the third one (middle individual) for the sharing part. The new individuals are updated through the pseudocode shown in Fig. 5, where $p \in [0, 1]$ is the percentage of best and worst classes.

for *t=1:NP*
  for *p=1:Dim*
    if rand$\le k_r$ **(knowledge ratio)**
      if $f(x_t) > f(x_r)$
        $x_{tp}^{new} = \left(x_t + k_f * \left((x_{t-1} - x_{t+1}) + (x_r - x_t)\right)\right)$
        else
        $x_{tp}^{new} = \left(x_t + k_f * \left((x_{t-1} - x_{t+1}) + (x_t - x_r)\right)\right)$
      end
    else $x_{tp}^{new} = x_{tp}^{old}$
    end
  end
end

Figure 4: Pseudocode for junior gaining sharing knowledge stage

for *t=1:NP*
  for *p=1:Dim*
    if rand$\le k_r$ **(knowledge ratio)**
      if $f(x_t) > f(x_{middle})$
        $x_{tp}^{new} = \left(x_t + k_f * \left((x_{pbest} - x_{pworst}) + (x_{middle} - x_t)\right)\right)$
        else
        $x_{tp}^{new} = \left(x_t + k_f * \left((x_{pbest} - x_{pworst}) + (x_t - x_{middle})\right)\right)$
      end
        else $x_{tp}^{new} = x_{tp}^{old}$
    end
  end
end

Figure 5: Pseudocode of senior gaining sharing knowledge stage

5.2 Discrete Binary Gaining Sharing Knowledge-Based Optimization Algorithm

For solving problems in discrete binary space, a novel Discrete Binary Gaining Sharing Knowledge-based Optimization algorithm is proposed. In DBGSK, the new initialization and the working mechanism of both stages (junior and senior gaining sharing stages) are introduced over discrete binary space, and the remaining algorithms remain the same as the previous ones. The working mechanism of DBGSK is presented in the following subsections.

Discrete Binary Initialization:

The initial population is obtained in GSK using Eq. (8). It must be updated using the following equation for binary population; see Eq. (11):

$$x_{tp}^{0} = round(rand(0, 1)), \tag{11}$$

where the round operator is used to convert the decimal number into the nearest binary number.

Discrete Binary Junior Saining and Sharing Stage:

The discrete binary junior gaining and sharing stage is based on the original GSK with $k_f = 1$. The individuals are updated in original GSK using the pseudocode that contains two cases. These two cases are defined for the discrete binary stage as follows.

Case 1. When $f(x_r) < f(x_t)$: There are three different vectors (x_{t-1}, x_{t+1}, x_r), which can take only two values (0 and 1). Therefore, a total of 2^3 combinations are possible, which are listed in Tab. 2. Furthermore, these eight combinations can be categorized into two different subcases [(a) and (b)]; each subcase has four combinations. The results of each possible combination are presented in Tab. 2.

Table 2: Results of the discrete binary junior gaining and sharing stage of Case 1 with $k_f = 1$

	x_{t-1}	x_{t+1}	x_r	Results	Modified results
Subcase (a)	0	0	0	0	0
	0	0	1	1	1
	1	1	0	0	0
	1	1	1	1	1
Subcase (b)	1	0	0	1	1
	1	0	1	2	1
	0	1	0	−1	0
	0	1	1	0	0

Subcase (a): If x_{t-1} is equal to x_{t+1}, the result is equal to x_r.

Subcase (b): When x_{t-1} is not equal x_{t+1}, the result is the same as x_{t-1} by taking −1 as 0 and 2 as 1.

The mathematical formulation of Case 1 is as follows:

$$x_{tp}^{new} = \begin{cases} x_r; & if \;\; x_{t-1} = x_{t+1} \\ x_{t-1}; & if \;\; x_{t-1} \neq x_{t+1} \end{cases}$$

Case 2. When $f(x_r) \geq f(x_t)$: There are four different vectors $(x_{t-1}, x_t, x_{t+1}, x_r)$ that consider only two values (0 and 1). Therefore, there are 2^4 possible combinations that are presented in Tab. 3. The 16 combinations can be divided into two subcases [(c) and (d)] in which (c) and (d) have four and 12 combinations, respectively.

Subcase (c): If x_{t-1} is not equal to x_{t+1} and x_{t+1} is equal to x_r, the result is equal to x_{t-1}.

Subcase (d): If any of the conditions arise $x_{t-1} = x_{t+1} \neq x_r$ or $x_{t-1} \neq x_{t+1} \neq x_r$ or $x_{t-1} = x_{t+1} = x_r$, the result is equal to x_t by considering −1 and −2 as 0 and 2 and 3 as 1.

The mathematical formulation of Case 2 is:

$$x_{tp}^{new} = \begin{cases} x_{t-1}; & if \;\; x_{t-1} \neq x_{t+1} = x_r \\ x_t; & Otherwise \end{cases}$$

Discrete Binary Senior Gaining and Sharing Stage: The working mechanism of the discrete binary senior gaining and sharing stage is the same as that of the binary junior gaining and sharing stage with value of $k_f = 1$. The individuals are updated in the original senior gaining sharing stage using pseudocode (Fig. 7) with two cases. The two cases are further modified for binary senior gaining and sharing stage in the following manner.

Table 3: Results of the discrete binary junior gaining and sharing stage of Case 2 with $k_f = 1$

	x_{t-1}	x_t	x_{t+1}	x_r	Results	Modified results
Subcase (c)	1	1	0	0	3	1
	1	0	0	0	1	1
	0	1	1	1	0	0
	0	0	1	1	−2	0
Subcase (d)	0	0	0	0	0	0
	0	1	0	0	2	1
	0	0	1	0	−1	0
	0	0	0	1	−1	0
	1	0	1	0	0	0
	1	0	0	1	0	0
	0	1	1	0	1	1
	0	1	0	1	1	1
	1	1	1	0	2	1
	1	0	1	1	−1	0
	1	1	0	1	2	1
	1	1	1	1	1	1

Case 1. $f(x_{middle}) < f(x_t)$: It contains three different vectors (x_{best}, x_{middle}, x_{worst}), and they can assume only binary values (0 and 1). Thus, total eight combinations are possible to update the individuals. These eight combinations can be classified into two subcases [(a) and (b)], and each subcase contains only four different combinations. The obtained results for this case are presented in Tab. 4.

Subcase (a): If x_{best} is equal to x_{worst}, then the obtained results are equal to x_{middle}.

Subcase (b): If x_{best} is not equal to x_{worst}, then the results are equal to x_{best} assuming −1 or 2, equivalent to their nearest binary values (0 and 1, respectively).

Case 1 can be mathematically formulated in the following way:

$$x_{tp}^{new} = \begin{cases} x_{middle}; & if \;\; x_{best} = x_{worst} \\ x_{best}; & if \;\; x_{best} \neq x_{worst} \end{cases}$$

Table 4: Results of the discrete binary senior gaining and sharing stage of Case 1 with $k_f = 1$

	x_{best}	x_{worst}	x_{middle}	Results	Modified results
Subcase (a)	0	0	0	0	0
	0	0	1	1	1
	1	1	0	0	0
	1	1	1	1	1
Subcase (b)	1	0	0	1	1
	1	0	1	2	1
	0	1	0	−1	0
	0	1	1	0	0

Case 2. $f(x_{middle}) > f(x_t)$: It consists of four different binary vectors ($x_{best}, x_{middle}, x_{worst}, x_t$), and with the values of each vector, a total of 16 combinations are possible. The 16 combinations are also divided into two subcases [(c) and (d)]. The subcases (c) and (d) further contain four and 12 combinations, respectively. The subcases are explained in detail in Tab. 5.

Table 5: Results of the discrete binary senior gaining and sharing stage of Case 2 with $k_f = 1$

	x_{best}	x_t	x_{worst}	x_{middle}	Results	Modified results
Subcase (c)	1	1	0	0	3	1
	1	0	0	0	1	1
	0	1	1	1	0	0
	0	0	1	1	−2	0
Subcase (d)	0	0	0	0	0	0
	0	1	0	0	2	1
	0	0	1	0	−1	0
	0	0	0	1	−1	0
	1	0	1	0	0	0
	1	0	0	1	0	0
	0	1	1	0	1	1
	0	1	0	1	1	1
	1	1	1	0	2	1
	1	0	1	1	−1	0
	1	1	0	1	2	1
	1	1	1	1	1	1

Subcase (c): When x_{best} is not equal to x_{worst} and x_{worst} is equal to x_{middle}, then the obtained results are equal to x_{best}.

Subcase (d): If any case arises other than (c), then the obtained results are equal to x_t by taking −2 and −1 as 0 and 2 and 3 as 1.

The mathematical formulation of Case 2 is given as:

$$x_{tp}^{new} = \begin{cases} x_{best}; & if \;\; x_{best} \neq x_{worst} = x_{middle} \\ x_t; & Otherwise \end{cases}$$

The flow chart of DBGSK is presented in Fig. 6.

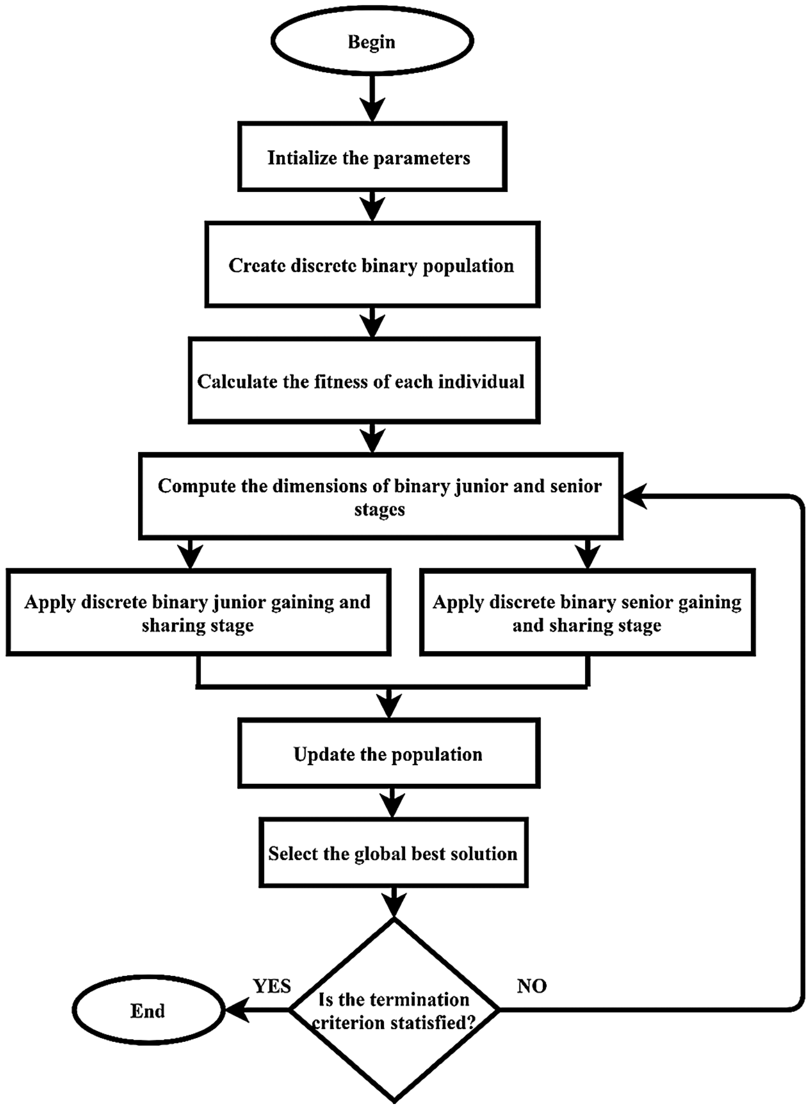

Figure 6: Flow chart of DBGSK

6 Experimental Results

The problem is handled by using the proposed novel DBGSK algorithm. The used parameters are presented in Tab. 6.

DBGSK was run on a personal computer with Intel® Core™ i5-7200U CPU@2.50 GHz and 4 GB RAM and coded on MATLAB R2015a. To get the optimal solutions, 30 independent runs were completed. The obtained statistics are provided in Tab. 7, including the best, median, average, and worst solutions and the DBGSK standard deviations.

Fig. 7 shows the convergence graph of the solutions using DBGSK. From the figure, it can be observed that after few iterations, it converges to the global optimal solution (14.50), which shows the robustness of DBGSK.

Table 6: Numerical values of the parameters

Parameters of DBGSK	Considered values
NP	800
k	10
k_r	0.9
p	0.1
k_f	1
Maximum number of iterations	200

Table 7: Statistical results using DBGSK

Algorithm	Best (Maximum)	Median	Average	Worst (Minimum)	Standard deviation
DBGSK	14.50	14.50	14.50	14.50	0.00

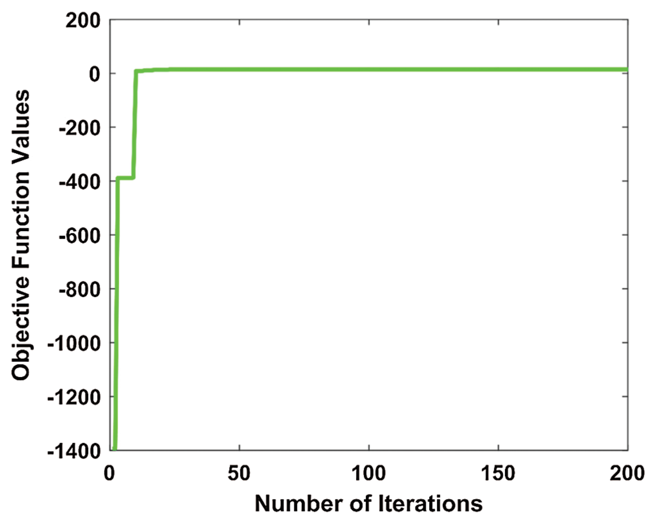

Figure 7: Convergence graph of DBGSK

The route provided by the optimum solution can be seen in Fig. 8. The route begins in the store location (0) at Tanta and then visits four hospitals having numbers 1, 3, 5, and 2, and finally returns to the store. The total amounts of the safeguarding substances supplied to the four hospitals = 1,450 kg. The total time for the optimum route is five hours, which means that all the available working shift time is completely utilized. The remaining unsupplied hospital (number 4) will be added to the new list of candidate hospitals, and the procedure is repeated once more for the next shift.

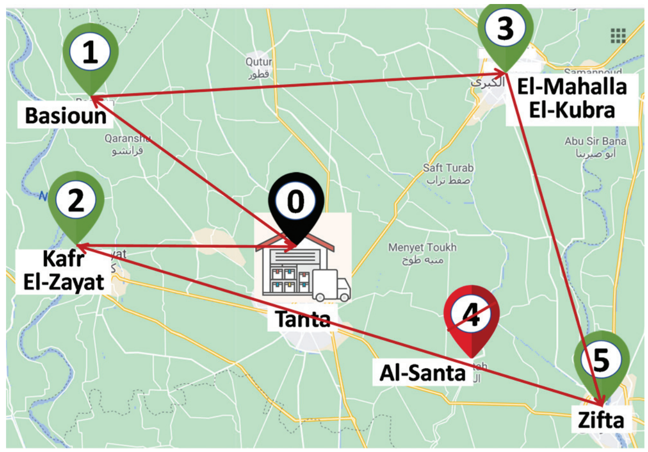

Figure 8: Optimum solution for the case study example

7 Conclusions and Points for Future Researchers Researches

The main conclusions for this paper can be summarized as follows:

i) An optimum distribution of safeguarding substances in the context of the huge danger of a pandemic is presented. The objective is to achieve the maximum total amount of the distributed safeguarding substances to consumers in a specific time shift.
ii) A nonlinear integer constrained mathematical programming model is formulated for the given problem, which is hard to be solved using exact algorithms, especially in large dimensions.
iii) The mathematical model and the solution method are used to solve a real application case study for five hospitals located in El-Gharbia Governorate in Egypt.
iv) The proposed model of the case study is solved by a novel Discrete Binary Gaining Sharing Knowledge-based optimization algorithm, which involves two main stages: discrete binary junior and senior gaining and sharing stages with knowledge factor $k_f = 1$. DBGSK is a discrete binary variant of the general GSK.
v) DBGSK has the ability of finding the solutions of the introduced problem, and the obtained results demonstrate the robustness and convergence of DBGSK toward the optimal solutions.

The suggestions for future research are as follows:

i) To apply the same procedure for other Governorates, other regions in the country, and other countries.
ii) To apply the same problem formulation to other similar fields, such as industry, agriculture, business, education, telecommunications, investing, quality assurance, social and community services, pollution, medical, tourism, marketing, sales, advertising, sports, arts, cooking, and others.
iii) To check the performance of the DBGSK approach in solving different complex optimization problems.
iv) Other problems can be investigated by the extension of DBGSK with different kinds of constraint handling methods.

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