Computers, Materials & Continua
DOI:10.32604/cmc.2021.015563
Article

Rotational Effect on the Propagation of Waves in a Magneto-Micropolar Thermoelastic Medium

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Received: 28 November 2020; Accepted: 14 February 2021

Abstract: The present paper aims to explore how the magnetic field, ramp parameter, and rotation affect a generalized micropolar thermoelastic medium that is standardized isotropic within the half-space. By employing normal mode analysis and Lame's potential theory, the authors could express analytically the components of displacement, stress, couple stress, and temperature field in the physical domain. They calculated such manners of expression numerically and plotted the matching graphs to highlight and make comparisons with theoretical findings. The highlights of the paper cover the impacts of various parameters on the rotating micropolar thermoelastic half-space. Nevertheless, the non-dimensional temperature is not affected by the rotation and the magnetic field. Specific attention is paid to studying the impact of the magnetic field, rotation, and ramp parameter of the distribution of temperature, displacement, stress, and couple stress. The study highlighted the significant impact of the rotation, magnetic field, and ramp parameter on the micropolar thermoelastic medium. In conclusion, graphical presentations were provided to evaluate the impacts of different parameters on the propagation of plane waves in thermoelastic media of different nature. The study may help the designers and engineers develop a structural control system in several applied fields.

Keywords: Magnetic field; micropolar; thermoelastic; rotation; ramp parameter; normal mode analysis

Nomenclature

σ_{ij}:	Stress tensor components
m_{ij}:	Couple stress tensor components
ϕ:	Micro–rotation
λ, μ:	Lame's constants
α, β, γ, k:	Micropolar material constants

$\vec{u}$: Components of the displacement vector
$\beta_1 = \alpha_t (3\lambda + 2\mu + k)$
α_t: Linear thermal expansion coefficient
ρ: Density of the medium
e: Cubical dilatation
K^*: Thermal conductivity
$K_1^* = \dfrac{c_E (\lambda + 2\mu)}{4}$
c_E: Specific heat at constant strain
Ω: Angular temperature
$\theta = T - T_0$
T_0: The medium's temperature in its material state supposed to be $\left|\dfrac{\theta}{T_0}\right| << 1$

1 Introduction

Because it is relevant in several applications, researchers have paid due attention to generalized thermoelasticity. Its theories include governing equations of the hyperbolic type and disclose the thermal signals' finite speed. In a standardized thermoelastic isotropic half-space, Abd-Alla et al. [1] investigated how wave propagation is affected by a magnetic field. Abouelregal [2] discussed the improved fractional photo-thermoelastic system for a magnetic field- subjected rotating semiconductor half-space. Singh et al. [3] studied reflecting plane waves from a solid thermoelastic micropolar half-space with impedance boundary circumstances. Said [4] studied the propagation of waves in a two-temperature micropolar magnetothermoelastic medium for a three-phase-lag system. The authors of [5] explored the impact of rotation on a two-temperature micropolar standardized thermoelasticity utilizing a dual-phase lag system. Rupender [6] studied the impact of rotation on a micropolar magnetothermoelastic medium because of thermal and mechanical sources. Bayones et al. [7] investigated the impact of the magnetic field and rotation on free vibrations within a non-standardized spherical within an elastic medium. Bayones et al. [8] discussed the thermoelastic wave propagation within the half-space of an isotropic standardized material affected by initial stress and rotation. Kalkal et al. [9] investigated reflecting plane waves in a thermoelastic micropolar nonlocal medium affected by rotation. Lotfy [10] explored two-temperature standardized magneto-thermoelastic responses within an elastic medium under three models. Zakaria [11] discussed the effect of hall current on a standardized magneto-thermoelasticity micropolar solid under heating of the ramp kind. Ezzata et al. [12] studied the impacts of the fractional order of heat transfer and heat conduction on a substantially long hollow cylinder that conducts perfectly. Morse et al. [13] employed Helmholtz's theorem. The authors of [14] investigated the transient magneto-thermoelasto-diffusive interactions of the rotating media with porous in the absence of the dissipation of energy influenced by thermal shock. Kumar et al. [15] studied the axisymmetric issue within a thermoelastic micropolar standardized half-space. The authors of [16] studied the free surface's reflection of the thermoelastic rotating medium reinforced with fibers with the phase-lag and two temperatures. Deswal et al. [17] studied the law of fractional-order thermal conductivity within a two-temperature micropolar thermo-viscoelastic. Using radial ribs, the authors of [18] examined the improved rigidity of fused circular plates. The authors of [19] investigated the motion equation for a flexible element with one dimension utilized for the dynamical analysis of a multimedia structure.

The paper aims to study the thermoelastic interactions within an elastic standardized micropolar isotropic medium in the presence of rotation affected by a magnetic field. The authors could obtain accurate solutions of the measured factors using the Lame's potential theory and the normal mode analysis. They could obtain and present in graphs the numerical findings of the distributions of stress, displacement, and temperature, for a crystal-like magnesium material.

2 Constitutive Relations and Field Equations

Constitutive relations and field equations are considered within a generalized micropolar magneto-thermoelastic medium in the presence of rotation as:

(i) The Constitutive Relations

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) + k\left(u_{j,i} - \varepsilon_{ijr}\phi_r\right) - \beta_1\theta\delta_{ij}, \tag{1}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \tag{2}$$

(ii) Motion's Stress Equation

$$(\lambda+\mu)\,\underline{\nabla}\left(\underline{\nabla}.\vec{u}\right) + (\mu+k)\,\nabla^2\vec{u} + k\left(\underline{\nabla}x\vec{\phi}\right) - \beta_1\underline{\nabla}\theta + \vec{f} = \rho\left(\frac{\partial^2\vec{u}}{\partial t^2} + \vec{\Omega}x\left(\vec{\Omega}x\vec{u}\right) + 2\vec{\Omega}x\vec{u}\right. \tag{3}$$

(iii) Motion's Couple Stress Equation

$$(\alpha+\beta+\gamma)\,\underline{\nabla}\left(\underline{\nabla}.\vec{\phi}\right) - \gamma\,\underline{\underline{\nabla x(\nabla}}x\vec{\phi}) + k\underline{\nabla}x\vec{u} - 2k\vec{\phi} = \rho J\frac{\partial^2\vec{\phi}}{\partial t^2}. \tag{4}$$

(iv) The Equation of Thermal Conductivity

$$K_1^*\left(1 + K^*\frac{\partial}{\partial t}\right)\nabla^2\theta = \rho c_E(\ddot{\theta} + \beta_1 T_0\ddot{e} \tag{5}$$

For these equations, the authors use the summation convention and utilize the comma to signify material derivative.

3 Formulating the Problem

Take into account an isotropic standardized micropolar generalized magneto-thermoelastic in the presence of rotation. The rectangular Cartesian coordinate scheme (x, y, z) is used in which the half-space surface plays the role of the plane $z = 0$ and the $z-axis$ points vertically inwards. There are two extra terms for the displacement motion equation within the rotating frame, i.e., the Coriolis acceleration $2\Omega x\vec{u}$ because of the moving frame of reference and the centripetal acceleration $\vec{\Omega}x\left(\vec{\Omega}x\vec{u}\right)$ because of the motion that varies over time. The analysis is restricted to the $x-z$ plane. Therefore, the quantities within the medium do not depend on the variable y. The angular velocity $\vec{\Omega}$, displacement vector $\vec{u}$, magnetic field vector $\vec{H}$ and micro-rotation vector $\vec{\phi}$ have the components

$$\vec{u} = u(x, z, t), \quad \vec{w} = w(x, z, t), \quad \vec{\Omega} = (0, \Omega, 0), \quad \vec{H} = (0, H_0, 0), \quad \vec{\phi} = (0, \phi_2, 0). \tag{5a}$$

$$\vec{J} = curl\,\vec{h}, \tag{6a}$$

$$curl\,\vec{E} = -\mu_e \frac{\partial \vec{h}}{\partial t}, \tag{6b}$$

$$div\,\vec{h} = 0, \tag{6c}$$

$$div\,\vec{E} = 0, \tag{6d}$$

$$\vec{E} = -\mu_e \left(\frac{\partial \vec{u}}{\partial t} \times \vec{H} \right), \tag{6e}$$

$$\vec{h} = curl\left(\vec{u}\, x \vec{H} \right). \tag{6f}$$

in which $\vec{u}$ represents the displacement vector, $\vec{H}$ represents the magnetic field, μ_e represents the magnetic permeability, $\vec{J}$ represents the density of the electric current, $\vec{E}$ represents the electric intensity, and $\vec{h}$ represents the perturbed magnetic field over the principal magnetic field. We apply the initial magnetic field vector $\vec{H}$ in Cartesian coordinates (x, y, z) to Eq. (1)

$$\vec{u} = (u, 0, w), \quad \vec{H} = (0, H_0, 0), \quad \vec{E} = -\mu_e \left(-H_0 \frac{\partial w}{\partial t}, 0, H_0 \frac{\partial u}{\partial t} \right), \quad \vec{h} = (-H_0 w, 0, H_0 u),$$
$$\vec{j} = -H_0 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right), \quad \vec{f} = \mu_e \left(\vec{j}\, x \vec{H} \right). \tag{7}$$

where $\vec{f}$ is the Lorentz force.

Combining Eqs. (3), (4) and Eqs. (5a), (7) provides

$$(\lambda + \mu) \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) + (\mu + k)\, \nabla^2 u - k \frac{\partial \phi_2}{\partial z} - \beta_1 \frac{\partial \theta}{\partial x} + \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z} \right)$$
$$= \rho \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \right), \tag{8}$$

$$(\lambda + \mu) \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) + (\mu + k)\, \nabla^2 w - k \frac{\partial \phi_2}{\partial x} + \beta_1 \frac{\partial \theta}{\partial z} + \mu_e H_0^2 \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 u}{\partial x \partial z} \right)$$
$$= \rho \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \right), \tag{9}$$

$$\gamma \nabla^2 \phi_2 + k \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) - 2k\phi_2 = \rho j \frac{\partial^2 \phi_2}{\partial t^2}. \tag{10}$$

The following form expresses Eq. (5) of thermal conductivity given expression (5a)

$$(K_1^*\left(1+K^*\frac{\partial}{\partial t}\right)\nabla^2\theta=\frac{\partial^2}{\partial t^2}(\rho c_E\theta+\beta_1^*T_0e). \tag{11}$$

Substituting (5a) into (1) and (2), we can express the arising stress as

$$\sigma_{xx}=(\lambda+2\mu+k)\frac{\partial u}{\partial x}+\lambda\frac{\partial w}{\partial z}-\beta_1\theta, \tag{12}$$

$$\sigma_{zz}=(\lambda+2\mu+k)\frac{\partial w}{\partial z}+\lambda\frac{\partial u}{\partial x}-\beta_1\theta, \tag{13}$$

$$\tau_{zx}=\mu\frac{\partial w}{\partial x}+(\mu+k)\frac{\partial u}{\partial z}-k\phi_2, \tag{14}$$

$$m_{zy}=\gamma\frac{\partial\phi_2}{\partial z},\quad m_{xy}=\gamma\frac{\partial\phi_2}{\partial x}. \tag{15}$$

where

$$e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z},\quad \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2},$$

These variables that are non-dimensional are employed to change the previous equations into non-dimensional structures

$$\left(x',z'\right)=\frac{\omega^*}{c_1}(x,z),\quad t'=\omega^*t,$$

$$\left(u',w'\right)=\frac{\rho\omega^*c_1}{T_0\beta_1}(u,w),\quad \theta'=\frac{\theta}{T_0},\quad \Omega'=\frac{\Omega}{\omega^*},\quad \sigma'_{ij}=\frac{\sigma_{ij}}{T_0\beta_1},$$

$$\phi'_2=\frac{\rho c_1^2}{T_0\beta_1}\phi_2,\quad m'_{ij}=\frac{\omega^*}{c_1T_0\beta_1}m_{ij}. \tag{16}$$

where

$$\omega^*=\frac{\rho c_1^2c_E}{K^*},\quad c_1^2=\frac{(\lambda+2\mu+k)}{\rho},\quad \beta_1=(3\lambda+2\mu+k)\,\alpha_t.$$

Eqs. (8)–(11) concerning previous the variables that are non-dimensional decline to (dropping the primes)

$$(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+(\mu+k)\nabla^2u-k\frac{\partial\phi_2}{\partial z}-\frac{\beta_1\rho c_1^2}{\beta_{1e}}\frac{\partial\theta}{\partial x}+\mu_eH_0^2\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2w}{\partial x\partial z}\right)$$

$$=\rho c_1^2\left(\frac{\partial^2u}{\partial t^2}-\Omega^2u+2\Omega\frac{\partial w}{\partial t}\right), \tag{17}$$

$$(\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+(\mu+k)\nabla^2 w+k\frac{\partial \phi_2}{\partial z}-\frac{\beta_1\rho c_1^2}{\beta_{1e}}\frac{\partial \theta}{\partial z}+\mu_e H_0^2\left(\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 u}{\partial x\partial z}\right)$$

$$=\rho c_1^2\left(\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}\right), \tag{18}$$

$$\frac{\gamma}{c_1^2}\nabla^2\phi_2+\frac{k}{\omega^{*2}}\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}-2\phi_2\right)=\rho j\frac{\partial^2\phi_2}{\partial t^2}, \tag{19}$$

$$\left[K_1^*+K^*\omega^*\frac{\partial}{\partial t}\right]\frac{\omega^{*2}T_0}{c_1^2}\nabla^2\theta=\omega^{*2}\frac{\partial^2}{\partial t^2}(\rho c_E T_0\theta+\frac{\beta_1^* T_0^2\beta_{1e}}{\rho c_1^2}e \tag{20}$$

Utilizing Helmholtz decomposition [13] and providing the potential ϕ and $\vec{\psi}$ through the following equation

$$\vec{u}=\nabla\phi+\vec{\nabla}\wedge\vec{\psi},\quad \vec{\psi}=(0,-\psi,0)\,. \tag{21}$$

Using Eq. (5a), the components of displacement u and w take the following form

$$u=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z},\quad w=\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}. \tag{22}$$

where

$\phi\,(x, z, t)$ and $\psi\,(x, z, t)$ represent scalar potential functions and $\vec{\psi}$ represents the vector of the potential function

Substituting Eq. (22) into Eqs. (17)–(20) gives us

$$\left(\nabla^2-\frac{\partial^2}{\partial t^2}+\Omega^2\right)\phi-\left(1+\beta_1\frac{\partial}{\partial t}\right)\theta+2\Omega\frac{\partial\psi}{\partial t}=0, \tag{23}$$

$$\left(a_0+a_2+4\frac{\partial}{\partial t}\right)\psi-2\Omega\frac{\partial\phi}{\partial t}-a_0\phi_2=0, \tag{24}$$

$$\left(\nabla^2-2a_3-a_4\frac{\partial^2}{\partial t^2}\right)\phi_2+a_3\nabla^2\psi=0, \tag{25}$$

$$\left(\left\{(a_1+2a_2)+4\frac{\partial}{\partial t}\right\}-4\frac{\partial^2}{\partial t^2}\right)\theta-4a_5\frac{\partial^2}{\partial t^2}\nabla^2\phi=0. \tag{26}$$

where

$$a_0=\frac{k}{\rho c_1^2},\quad a_1=\frac{\lambda}{\rho c_1^2},\quad a_2=\frac{\mu}{\rho c_1^2},\quad a_3=\frac{kc_1^2}{\gamma\omega^{*2}},$$

$$a_4=\frac{j\rho c_1^2}{\gamma},\quad a_5=\frac{T_0\beta_1^2}{\rho K^*\omega^*}.$$

4 Normal Mode Analysis

For resolving the governing equations, the authors measured physically the decomposed variable concerning the normal modes in the following form:

$$\left(u,\, w,\, \phi,\, \psi,\, \theta,\, \sigma_{ij},\, \phi_2\right)(x,\, z,\, t) = \left(u^*,\, w^*,\, \phi^*,\, \psi^*,\, \theta^*,\, \sigma_{ij}^*,\, \phi_2^*\right)(z)\, e^{(\omega t + imx)}. \tag{27}$$

in which ω represents the complex time constant (frequency), i represents the imaginary unit, m represents the wave number in the x-direction, and u^*, w^*, ϕ^*, ψ^*, θ^*, σ_{ij}^* and ϕ_2^* represent the functions' amplitudes.

Using (27), Eqs. (23)–(25) and Eq. (20) are, as follows

$$\left(D^2 - A_1\right)\phi^*(z) - A_2\theta^*(z) + A_3\psi^*(z) = 0, \tag{28}$$

$$\left(D^2 - B_1\right)\psi^*(z) - B_2\phi^*(z) - B_3\phi_2^*(z) = 0, \tag{29}$$

$$\left(D^2 - C_1\right)\phi_2^*(z) + a_3\left(D^2 - m^2\right)\psi^*(z) = 0, \tag{30}$$

$$\left(D_1\left(D^2 - m^2\right) - D_3\right)\theta^*(z) - D_2\left(D^2 - m^2\right)\phi^*(z) = 0. \tag{31}$$

in which

$$A_1 = m^2 + \frac{\omega^2 - \Omega^2}{\varepsilon_1}, \quad A_2 = \frac{1 + \beta_1\omega}{\varepsilon_1}, \quad A_3 = \frac{2\Omega\omega}{\varepsilon_1}, \quad B_1 = m^2 + \frac{\omega^2 - \Omega^2}{\varepsilon_2},$$

$$B_2 = \frac{2\Omega\omega}{\varepsilon_2}, \quad B_3 = \frac{a_0}{\varepsilon_2}, \quad C_1 = m^2 + 2a_3 + a_4\omega^2, \quad D_1 = (a_1 + 2a_2)(1 + \tau_\nu\omega) + 4\omega(1 + \tau_T\omega),$$

$$D_2 = 4a_5\omega^2, \quad D_3 = 4\omega^2,$$

$$\varepsilon_1 = 1 + \frac{\mu_e H_0^2}{\rho c_1^2}, \quad \varepsilon_2 = a_0 + a_2.$$

Eliminating $\psi^*(z)$, $\phi^*(z)$, $\phi_2^*(z)$ and $\theta^*(z)$ from Eqs. (28)–(31), the differential equation become

$$\left(D^8 - PD^6 + QD^4 - RD^2 + S\right)\left(\psi^*(z),\, \phi^*(z),\, \phi_2^*(z),\, \theta^*(z)\right) = 0. \tag{32}$$

in which

$$P = A_1 + B_1 + C_1 + m^2 + \varepsilon_3(D_3 + A_2D_2) - a_3B_3,$$

$$Q = A_1B_1 + A_1C_1 + B_1C_1 + \left(m^2 + \varepsilon_3D_3\right)(A_1 + B_1 + C_1) + \varepsilon_3A_2D_2\left(B_1 + C_1 + m^2\right)$$
$$+ A_3B_2 - a_3B_3\left(2m^2 + A_1 + \varepsilon_3(D_3 + A_2D_2)\right),$$

$$R = A_1B_1C_1 + \left(m^2 + \varepsilon_3D_3\right)(A_1B_1 + B_1C_1 + A_1C_1) + \varepsilon_3A_2D_2\left(m^2(B_1 + C_1) + B_1C_1\right)$$
$$+ A_3B_2\left(m^2 + C_1 + \varepsilon_3D_3\right) - a_3B_3\left(m^2A_1 + \left(m^2 + A_1\right)\left(m^2 + \varepsilon_3D_3\right) + 2\varepsilon_3m^2A_2D_2\right),$$

$$S=\left(m^2+\varepsilon_3 D_3\right)(A_1B_1C_1+A_3B_2C_1)+\varepsilon_3 m^2 A_2D_2B_1C_1-a_3B_3\left(m^2A_1\left(m^2+\varepsilon_3 D_3\right)+\varepsilon_3 m^4 A_2D_2\right).$$

in which $\varepsilon_3=\dfrac{1}{D_1}$.

Eq. (27) is represented, as follows

$$\left(\left(D^2-\lambda_1^2\right)\left(D^2-\lambda_2^2\right)\left(D^2-\lambda_3^2\right)\left(D^2-\lambda_4^2\right)\right)\left(\psi^*(z),\phi^*(z),\phi_2^*(z),\theta^*(z)\right)=0. \tag{33}$$

in which λ_n^2 (n = 1, 2, 3, 4) represent the roots of the characteristic Eq. (32).

$$\lambda_1=\frac{1}{2\sqrt{3}}\left(\sqrt{(\sqrt{3}e_1-\sqrt{6}e_2)}\right),\quad \lambda_2=\frac{1}{2\sqrt{3}}\left(\sqrt{(\sqrt{3}e_1+\sqrt{6}e_2)}\right),$$

$$\lambda_3=\frac{1}{2\sqrt{3}}\left(\sqrt{(\sqrt{3}e_1-\sqrt{6}e_2)}\right),\quad \lambda_4=\frac{1}{2\sqrt{3}}\left(\sqrt{(\sqrt{3}e_1+\sqrt{6}e_2)}\right). \tag{34}$$

where

$$\Theta=2Q^3-9PQR+27R^2+27P^2S-72QS,\quad \Theta_1=Q^2-3PR+12S,$$

$$\Theta_2=3Q^3-9Q(PR+8S)+27\left(R^2+P^2S\right),\quad \Theta_3=\left(\Theta+\sqrt{-\Theta_1^3+\Theta_2^2}\right)^{\frac{1}{3}},$$

$$\Theta_4=4\sqrt{3}\left(P^3-4PQ+8R\right),\quad e_1=\sqrt{(3P^2 8Q-2^{\frac{7}{3}}\frac{\Theta_1}{\Theta_3}+2^{\frac{5}{3}}\Theta_3)},$$

$$e_2=\sqrt{(3P^2-8Q-2^{\frac{4}{3}}\frac{\Theta_1}{\Theta_3}-2^{\frac{2}{3}}\Theta_3-e_3)},\quad e_3=\frac{\Theta_4}{e_1}.$$

The solution of Eq. (28) is written as

$$\psi^*(z)=\sum_{n=1}^{4}M_n(m,\omega)e^{-\lambda_n z}, \tag{35}$$

$$\phi^*(z)=\sum_{n=1}^{4}M'_n(m,\omega)e^{-\lambda_n z}, \tag{36}$$

$$\phi_2^*(z)=\sum_{n=1}^{4}M''_n(m,\omega)e^{-\lambda_n z}, \tag{37}$$

$$\theta^*(z)=\sum_{n=1}^{4}M'''_n(m,\omega)e^{-\lambda_n z}. \tag{38}$$

in which M_n, M'_n, M''_n and M'''_n represent some parameters that depend on m and ω. Substitution of Eqs. (35)–(38) into Eqs. (28)–(31) gives the following relations

$$\phi_2^*(z) = \sum_{n=1}^{4} S_{1n} M_n(m, \omega) e^{-\lambda_n z}, \tag{39}$$

$$\phi^*(z) = \sum_{n=1}^{4} S_{2n} M_n(m, \omega) e^{-\lambda_n z}, \tag{40}$$

$$\theta^*(z) = \sum_{n=1}^{4} S_{3n} M_n(m, \omega) e^{-\lambda_n z}. \tag{41}$$

where

$$S_{1n} = \frac{-a_3\left(\lambda_n^2 - m^2\right)}{\left(\lambda_n^2 - C_1\right)}, \quad S_{2n} = \frac{\left(\lambda_n^2 - B_1\right)}{B_2} - \frac{B_3}{B_2} S_{1n}, \quad S_{3n} = \frac{D_3\left(\lambda_n^2 - m^2\right) S_{2n}}{D_1\left(\lambda_n^2 - m^2\right) - D_3}.$$

5 Applications

Take into account a magneto-thermoelastic micropolar solid having a half-space that rotates $z \geq 0$. $M'_n s$, as constants, are defined by imposing the proper boundary settings.

5.1 Thermal Boundary Conditions

The authors apply a heat shock of the ramp kind to the isothermal boundary of the plane $z = 0$, as follows

$$\theta(x, 0, t) = \phi^* \delta(x) H(t). \tag{42}$$

in which $\delta(x)$ represents Dirac-delta function and ϕ^* represents a constant temperature. Moreover, $H(t)$ represents a Heaviside function defined as

$$H(t) = \begin{cases} 0 & t \leq 0 \\ \dfrac{t}{t_0} & 0 \prec t \leq t_0 \\ 1 & t \geq t_0 \end{cases} \tag{43}$$

In this equation, t_0 represents a ramp parameter that shows the time required for raising the heat.

5.2 Mechanical Boundary Conditions

The boundary plane $z = 0$ is free of traction. In terms of mathematics, the boundary conditions take the following form

$$\sigma_{zz}(x, 0, t) = 0, \tag{44}$$

$$\tau_{zx}(x, 0, t) = 0, \tag{45}$$

$$m_{zy}(x, 0, t) = 0. \tag{46}$$

Applying Eqs. (16)–(27) make the former boundary conditions represented, as follows

$$\theta^*(z) = R_1, \tag{47}$$

$$\sigma^*_{zz}(z) = 0, \tag{48}$$

$$\tau^*_{zx}(z) = 0, \tag{49}$$

$$m^*_{zy}(z) = 0. \quad at\ z = 0 \tag{50}$$

where $R_1 = \phi^* \dfrac{1 - e^{-\omega_0}}{t_0 \omega^2}$.

Utilizing the non-dimensional quantities identified in (16), expressing stress Eqs. (12)–(15) and displacement Eq. (22), as well as the relations (44)–(46) becomes:

$$u^*(z) = \sum_{n=1}^{4} (imS_{2n} - \lambda_n) M_n(m, \omega) e^{-\lambda_n z}, \tag{51}$$

$$w^*(z) = \sum_{n=1}^{4} (-\lambda_n - im) M_n(m, \omega) e^{-\lambda_n z}, \tag{52}$$

$$\sigma^*_{zz}(z) = \sum_{n=1}^{4} S_{4n} M_n(m, \omega) e^{-\lambda_n z}, \tag{53}$$

$$\tau^*_{zx}(z) = \sum_{n=1}^{4} S_{5n} M_n(m, \omega) e^{-\lambda_n z}, \tag{54}$$

$$m^*_{zy}(z) = \sum_{n=1}^{4} S_{6n} M_n(m, \omega) e^{-\lambda_n z}. \tag{55}$$

in which

$$S_{4n} = \Big((a_0 + a_1(1 + \alpha_0\omega) + 2a_2(1 + \alpha_1\omega))\lambda_n^2 - m^2 a_1(1 + \alpha_0\omega) \Big) S_{2n}$$
$$+ im\lambda_n(a_0 + 2a_2(1 + \alpha_1\omega) - (1 + \beta_1\omega) S_{3n},$$

$$S_{5n} = (a_0 + a_2(1 + \alpha_1\omega))\lambda_n^2 - im\lambda_n S_{2n}(a_0 + 2a_2(1 + \alpha_1\omega)) + m^2 a_2(1 + \alpha_1\omega)) - a_0 S_{1n},$$

$$S_{6n} = -\frac{a_0}{a_3}\lambda_n S_{1n}.$$

A four-equation non-homogeneous structure is yielded by the boundary conditions (47)–(50), supported by expressions (51)–(55) and (38) as follows

$$\begin{bmatrix} S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \\ S_{51} & S_{52} & S_{53} & S_{54} \\ S_{61} & S_{62} & S_{63} & S_{64} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} R_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{56}$$

The expressions of M_n, (n = 1, 2, 3, 4) attained by resolving the structure (56), when replaced in Eq. (38) and Eqs. (51)–(55), give these expressions of field variables

$$u(x, z, t) = \left[(imS_{21} - \lambda_1) M_1 e^{-\lambda_1 z} + (imS_{22} - \lambda_2) M_2 e^{-\lambda_2 z} + (imS_{23} - \lambda_3) M_3 e^{-\lambda_3 z} + \left(imS_{24} - \lambda_4\right) M_4 e^{-\lambda_4 z}\right] e^{(\omega t + imx)}, \tag{57}$$

$$w(x, z, t) = -\left[(\lambda_1 S_{21} + im) M_1 e^{-\lambda_1 z} + (\lambda_2 S_{22} + im) M_2 e^{-\lambda_2 z} + (\lambda_3 S_{23} + im) M_3 e^{-\lambda_3 z} + (\lambda_4 S_{24} + im) M_4 e^{-\lambda_4 z}\right] e^{(\omega t + imx)}, \tag{58}$$

$$\theta(x, z, t) = \left[S_{31} M_1 e^{-\lambda_1 z} + S_{32} M_2 e^{-\lambda_2 z} + S_{33} M_3 e^{-\lambda_3 z} + S_{34} M_4 e^{-\lambda_4 z}\right] e^{\omega t + imx}, \tag{59}$$

$$\sigma_{zz}(x, z, t) = \left[S_{41} M_1 e^{-\lambda_1 z} + S_{42} M_2 e^{-\lambda_2 z} + S_{43} M_3 e^{-\lambda_3 z} + S_{44} M_4 e^{-\lambda_4 z}\right] e^{\omega t + imx}, \tag{60}$$

$$\tau_{zx}(x, z, t) = \left[S_{51} M_1 e^{-\lambda_1 z} + S_{52} M_2 e^{-\lambda_2 z} + S_{53} M_3 e^{-\lambda_3 z} + S_{54} M_4 e^{-\lambda_4 z}\right] e^{\omega t + imx}, \tag{61}$$

$$m_{zy}(x, z, t) = \left[S_{61} M_1 e^{-\lambda_1 z} + S_{62} M_2 e^{-\lambda_2 z} + S_{63} M_3 e^{-\lambda_3 z} + S_{64} M_4 e^{-\lambda_4 z}\right] e^{\omega t + imx}. \tag{62}$$

in which

$$M_1 = \frac{\Delta_1}{\Delta}, \quad M_2 = \frac{\Delta_2}{\Delta}, \quad M_3 = \frac{\Delta_3}{\Delta}, \quad M_4 = \frac{\Delta_4}{\Delta},$$

$$\Delta = S_{31} d_1 - S_{32} d_2 + S_{33} d_3 - S_{34} d_4,$$

$$\Delta_1 = R_1 d_1, \quad \Delta_2 = -R_1 d_2, \quad \Delta_3 = R_1 d_3, \quad \Delta_4 = -R_1 d_4,$$

$$d_1 = S_{42}(S_{53} S_{64} - S_{54} S_{63}) - S_{43}(S_{52} S_{64} - S_{54} S_{62}) + S_{44}(S_{52} S_{63} - S_{53} S_{62}),$$

$$d_2 = S_{41}(S_{53} S_{64} - S_{54} S_{63}) - S_{43}(S_{51} S_{64} - S_{54} S_{61}) + S_{44}(S_{51} S_{63} - S_{53} S_{61}),$$

$$d_3 = S_{41}(S_{52} S_{64} - S_{54} S_{62}) - S_{42}(S_{51} S_{64} - S_{54} S_{61}) + S_{44}(S_{51} S_{62} - S_{61} S_{52}),$$

$$d_4 = S_{41}(S_{52} S_{63} - S_{53} S_{62}) - S_{42}(S_{51} S_{63} - S_{61} S_{53}) + S_{43}(S_{51} S_{62} - S_{61} S_{52}).$$

6 Numerical Results and Discussion

With an aim to highlight the theoretical findings of the previous sections, the authors provide some numerical findings using MATLAB. The material selected for this goal of magnesium crystal whose physical data resemble those provided in [17]:

$$\rho = 1.74 \times 10^3\ kg\,m^{-3}, \quad \lambda = 9.4 \times 10^{10}\ kg\,m^{-1}s^{-2}, \quad \mu = 4.0 \times 10^{10}\ kg\,m^{-1}s^{-2},$$

$$k = 1.0 \times 10^{10}\ kg\,m^{-1}s^{-2},$$

$$\gamma = 0.779 \times 10^{-9}\ kg\,ms^{-2}, \quad j = 0.2 \times 10^{-19}\ m^2, \quad \alpha_t = 2.36 \times 10^{-5}\ K^{-1},$$

$$K^* = 2.510\ W\,m^{-1}K^{-1}, \quad T_0 = 293\ K, \quad C_E = 9.623 \times 10^2\ J\,kg^{-1}K^{-1},$$

$$z = 0.3 \quad \phi^* = 1.0.$$

Taking into account the previous physical data, non-dimensional field variables are estimated, and the findings are presented graphically at various positions of z at $t = 0.1$ and x = 1.0. The motion range is $0 \le z \le 1.0$. Figs. 1–3 show variations, correspondingly.

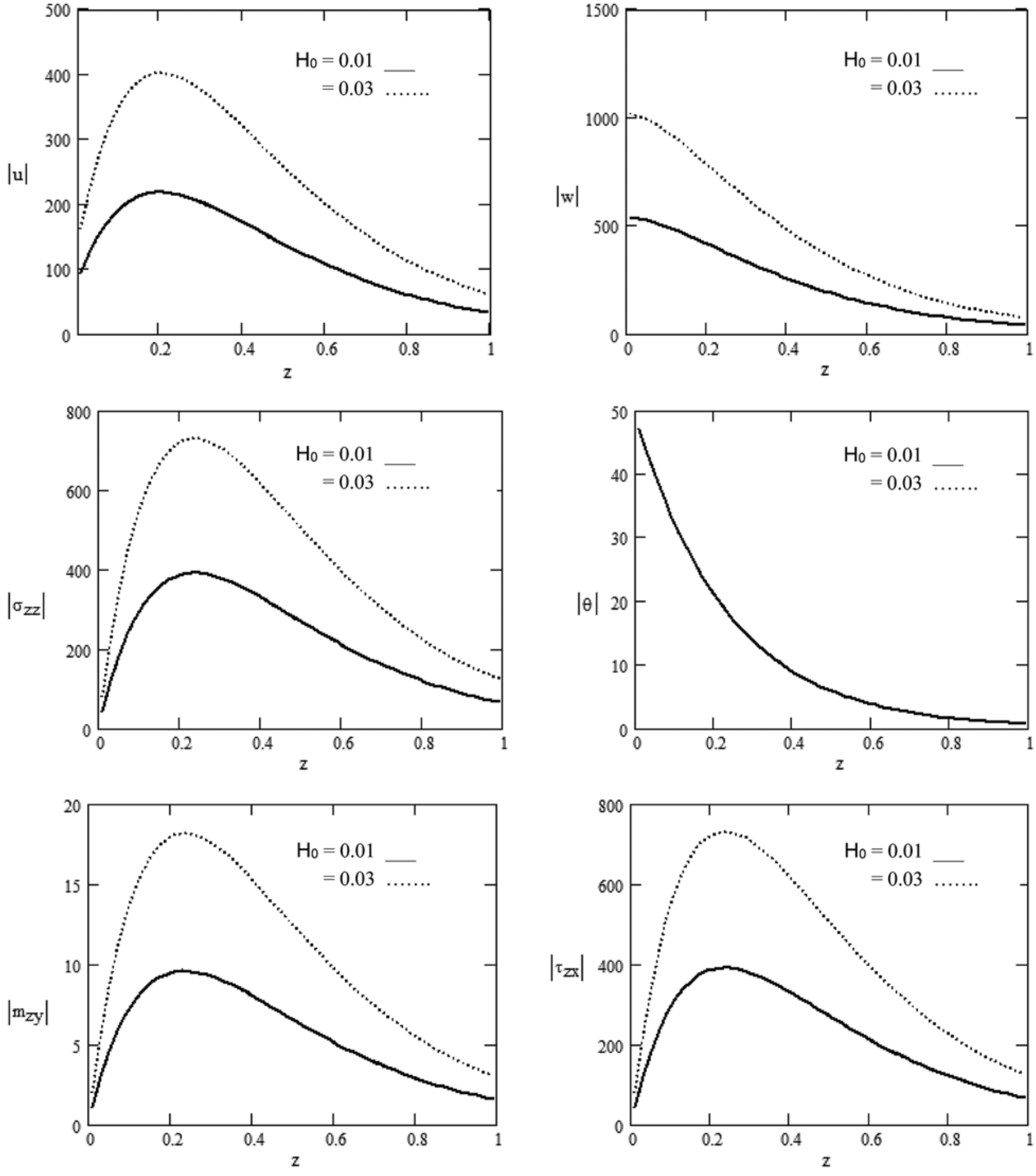

Figure 1: Various values of the magnitude of u, w, σ_{zz}, τ_{zx}, m_{zy}, θ for various values of H_0 in the presence of distance $0 \le z \le 1$

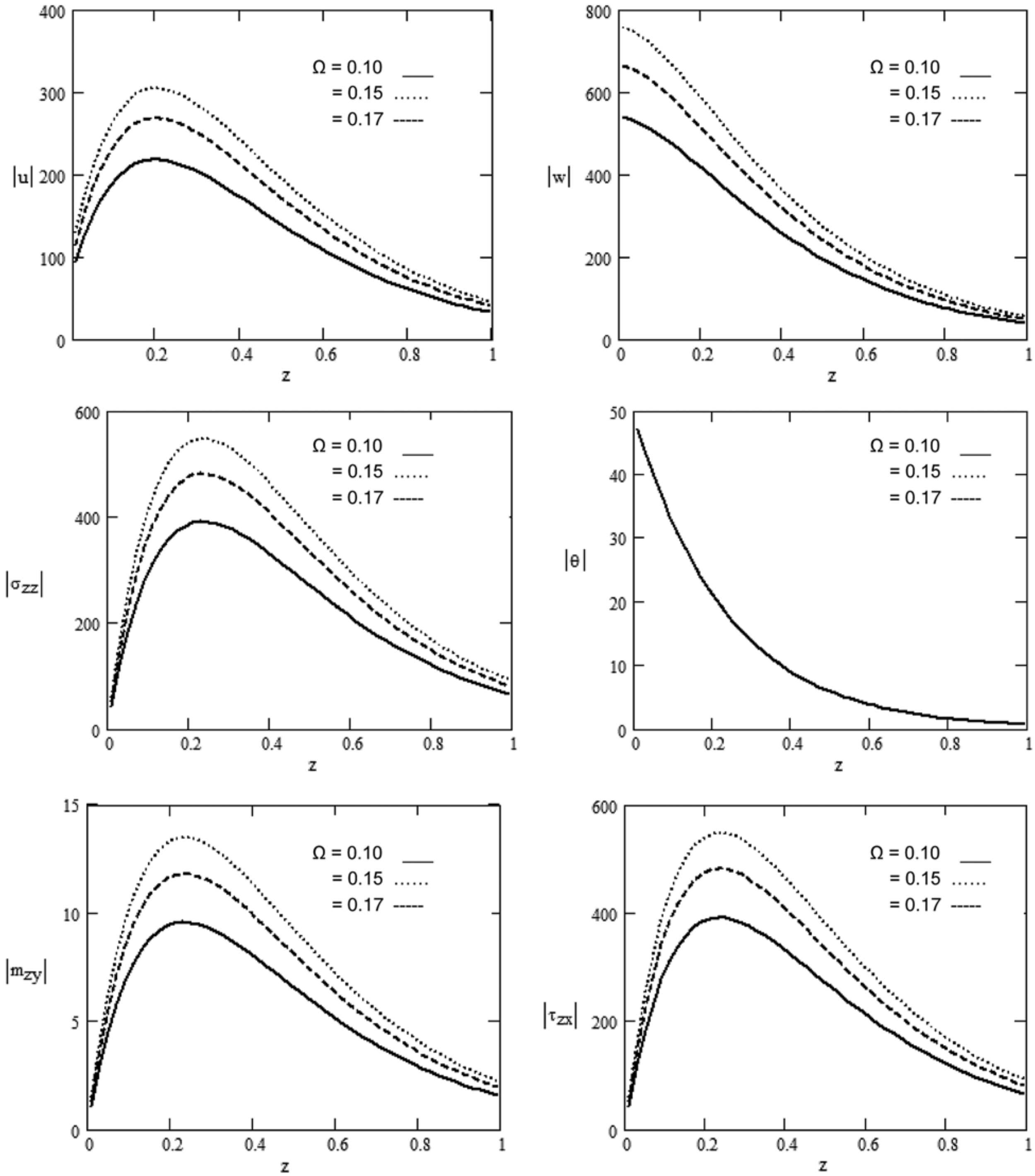

Figure 2: Various values of the magnitude of $u, w, \sigma_{zz}, \tau_{zx}, m_{zy}, \theta$ for various values of Ω in the presence of distance $0 \leq z \leq 1$

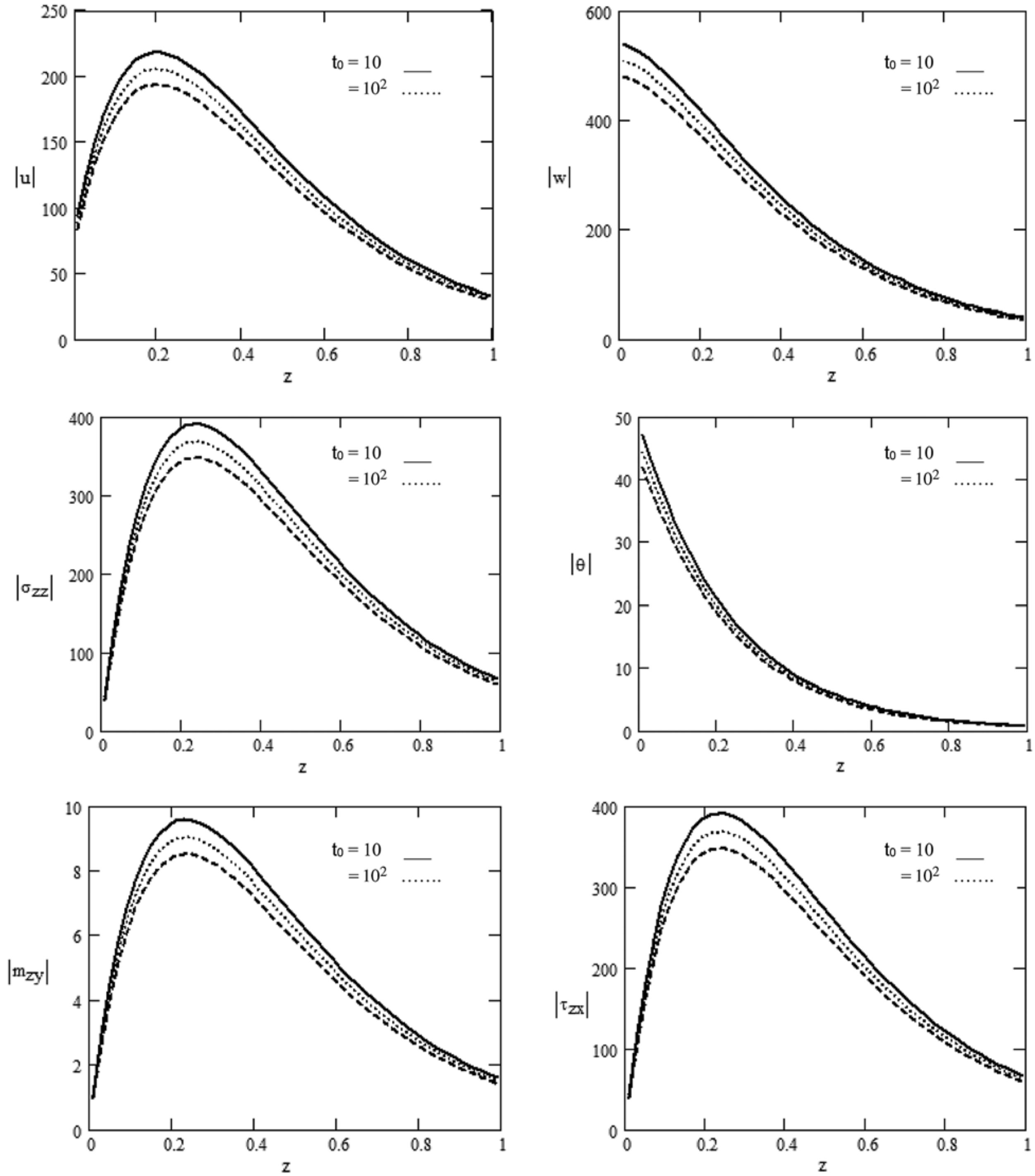

Figure 3: Various values of the magnitude of u, w, σ_{zz}, τ_{zx}, m_{zy}, θ for various values of t_0 in the presence of distance $0 \leq z \leq 1$

Fig. 1: displays the various values of the components of displacement $|u|$ and $|w|$, stress $|\sigma_{zz}|$, $|\tau_{xz}|, |m_{zy}|$, as well as temperature $|\theta|$ in the presence of distance z for the various values of the magnetic field H_0. A zero value is the beginning. It agrees entirely with the boundary conditions. The magnetic field demonstrates significant growing and declining impacts on the components of stress and displacement. These impacts vanish when moving away from the point of the application of the source. The temperature is not affected by the magnetic field. Nevertheless, the qualitative performance is almost equal in the two values.

Variations in displacement components $|u|, |w|$, stress components $|\sigma_{zz}|$, $|\tau_{xz}|, |m_{zy}|$ and temperature $|\theta|$ with spatial coordinates z for various values of rotation Ω are shown in Fig. 2 that begins with the value of zero, which agrees totally with the boundary conditions. The highest impact zone of rotation is about $z = 0.3$. Nevertheless, the temperature is not affected by the rotation. The components of temperature, stress, and displacement increase numerically for $0 \leq z \leq 1$. Then, they decrease and moves to minimum value at $z = 1$.

In Fig. 3, we have depicted displacement components $|u|, |w|$, stress components $|\sigma_{zz}|$, $|\tau_{xz}|, |m_{zy}|$ and temperature $|\theta|$ in the presence of distance z to explore the impacts of ramp parameter t_0. The ramp parameter's highest impact zone is about $z = 0.3$. Furthermore, all curves share a corresponding zero value stating point to satisfy the boundary conditions.

7 Conclusion

The study concludes that:

a. The magnetic field and rotation play an influential part in distributing the physical quantities whose amplitude varies (rise or decline) as the rotation and magnetic field increase. With the rotation and the magnetic field, the quantities to surge near or away from the application source point are restricted.
b. All physical quantities satisfy the boundary conditions.
c. The theoretical findings of the study can give stimulating information for seismologists, researchers, and experimental scientists who are interested in the field.

Funding Statement: The authors express their gratitude to the Academy of Scientific Research and Technology, Egypt for funding the present study under Science Up Grant No. (6459).

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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