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ARTICLE

On Some Properties of Neutrosophic Semi Continuous and Almost Continuous Mapping

Bhimraj Basumatary[1,*], Nijwm Wary[1], Jeevan Krishna Khaklary[2] and Usha Rani Basumatary[1]

[1]Department of Mathematical Sciences, Bodoland University, Kokrajhar, 783370, India

[2]Central Institute of Technology, Kokrajhar, 783370, India

[*]Corresponding Author: Bhimraj Basumatary. Email: brbasumatary14@gmail.com

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ABSTRACT

The neutrality's origin, character, and extent are studied in the Neutrosophic set. The neutrosophic set is an essential issue to research since it opens the door to a wide range of scientific and technological applications. The neutrosophic set can find its spot to research because the universe is filled with indeterminacy. Neutrosophic set is currently being developed to express uncertain, imprecise, partial, and inconsistent data. Truth membership function, indeterminacy membership function, and falsity membership function are used to express a neutrosophic set in order to address uncertainty. The neutrosophic set produces more rational conclusions in a variety of practical problems. The neutrosophic set displays inconsistencies in data and can solve real-world problems. We are directed to do our work in semi-continuous and almost continuous mapping on the basis of the neutrosophic set by observing these. Since we are going to study the properties of semi continuous and almost continuous mapping, we present the meaning of $\mathcal{N} \backsim$ semi-open set, $\mathcal{N} \backsim$ semi-closed set, $\mathcal{N} \backsim$ regularly open set, $\mathcal{N} \backsim$ regularly closed set, $\mathcal{N} \backsim$ continuous mapping, $\mathcal{N} \backsim$ open mapping, $\mathcal{N} \backsim$ closed mapping, $\mathcal{N} \backsim$ semi-continuous mapping, $\mathcal{N} \backsim$ semi-open mapping, $\mathcal{N} \backsim$ semi-closed mapping. Additionally, we attempt to demonstrate a portion of their properties and furthermore referred to some examples.

KEYWORDS

$\mathcal{N} \backsim$ regularly open set; $\mathcal{N} \backsim$ regularly closed set; $\mathcal{N} \backsim$ semi-continuous mapping; $\mathcal{N} \backsim$ almost continuous mapping

1 Introduction

After Zadeh [1] created fuzzy set theory (FST), FST was used to define the idea of membership value and explain the concept of uncertainty. Many researchers attempted to apply FST to a variety of other fields of science and technology. Atanassov [2] expanded on the concept of fuzzy set theory and introduced the concept of degree of non-membership, as well as proposing intuitionistic fuzzy set theory (IFST). Chang [3] introduced fuzzy topology (FT), and Coker [4] generalized the concept of FT to intuitionistic fuzzy topology (IFT). Rosenfeld [5] introduced the concept of fuzzy groups and Foster [6] proposed the idea of fuzzy topological groups. Azad [7] went through fuzzy semi-continuity (FSC), fuzzy almost continuity (FAC), and fuzzy weakly continuity (FWC). Smarandache [8,9] suggested neutrosophic set theory (NST) by generalizing FST and IFST and valuing indeterminacy as a separate component. Many researchers have attempted to apply NST to a variety of scientific and technological fields. Kandil et al. [10] studied the fuzzy bitopological spaces. Mwchahary et al. [11] did their work in neutrosophic bitopological space. Neutrosophic topology was proposed by Salama et al. [12,13]. The semi-continuous mapping was investigated by Noiri [14] and the term almost continuous mappings were coined by Singal et al. [15]. The idea of fuzzy neutrosophic groups and a topological group of the neutrosophic set was studied by Sumathi et al. [16,17]. NST was used as a tool in a group discussion framework by Abdel-Basset et al. [18]. Abdel-Basset et al. [19] investigated the use of the base-worst technique to solve chain problems using a novel plithogenic model.

1.1 Motivations

In this current decade, neutrosophic environments are mainly interested by different fields of researchers. In Mathematics also much theoretical research has been observed in the sense of neutrosophic environment. It will be necessary to carry out more theoretical research to establish a general framework for decision-making and to define patterns for complex network conceiving and practical application. Salama et al. [13] studied neutrosophic closed set and neutrosophic continuous functions. The idea of almost continuous functions is done in 1968 [15] in topology. Similarly, the notion of fuzzy almost contra continuous and fuzzy almost contra α-continuous functions was discussed in [20]. Recently, Al-Omeri et al. [21,22] introduced and studied a number of the definitions of neutrosophic closed sets, neutrosophic mapping, and obtained several preservation properties and some characterizations about neutrosophic of connectedness and neutrosophic connectedness continuity. More recently, in [23–26] authors have given how a new trend of Neutrosophic theory is applicable in the field of Medicine and multimedia with a novel and powerful model. From the literature survey, it is noticed that exactly the properties of neutrosophic semi-continuous and almost continuous mapping are not done. To update this research gap, in this research article, we attempt to investigate the neutrosophic semi-continuous and almost continuous mapping and its properties. Also, we study properties of the neutrosophic semi-open set (NSOS), neutrosophic semi-closed set (NSCoS), neutrosophic regularly open set (NROS), neutrosophic regularly closed set (NRCoS), neutrosophic semi-continuous (NSC), and neutrosophic almost continuous mapping (NACM).

2 Methodologies

2.1 Definition [8]

A neutrosophic set (NS) $A^{\mathfrak{N}}$ on X can be expressed as $A^{\mathfrak{N}} = \{< x \in X, \mathfrak{T}_{A^{\mathfrak{N}}}(x), \mathfrak{I}_{A^{\mathfrak{N}}}(x), \mathfrak{F}_{A^{\mathfrak{N}}}(x) >\}$, where $\mathfrak{T}, \mathfrak{I}, \mathfrak{F}\colon X \longrightarrow]^{-}0,\ 1^{+}[$. Note that $0 \preccurlyeq \mathfrak{T}_{A^{\mathfrak{N}}}(x) + \mathfrak{I}_{A^{\mathfrak{N}}}(x) + \mathfrak{F}_{A^{\mathfrak{N}}}(x) \preccurlyeq 3$.

2.2 Definition [8]

Complement of $A^{\mathfrak{N}}$ is expressed as

$$A^{\mathfrak{N}^c}(x) = \{< x \in X, \mathfrak{T}_{A^{\mathfrak{N}^c}}(x) = \mathfrak{F}_{A^{\mathfrak{N}}}(x), \mathfrak{I}_{A^{\mathfrak{N}^c}}(x) = 1 - \mathfrak{I}_{A^{\mathfrak{N}}}(x), \mathfrak{F}_{A^{\mathfrak{N}^c}}(x) = \mathfrak{T}_{A^{\mathfrak{N}}}(x) >\}.$$

2.3 Definition [8]

Let $X \neq \phi$ and $A^{\mathfrak{N}} = \{< x \in X, \mathfrak{T}_{A^{\mathfrak{N}}}(x), \mathfrak{I}_{A^{\mathfrak{N}}}(x), \mathfrak{F}_{A^{\mathfrak{N}}}(x) >\}$ and $B^{\mathfrak{N}} = \{< x \in X, \mathfrak{T}_{B^{\mathfrak{N}}}(x), \mathfrak{I}_{B^{\mathfrak{N}}}(x), \mathfrak{F}_{B^{\mathfrak{N}}}(x) >\}$ are NSs. Then

(i) $A^{\mathfrak{N}} \Cap B^{\mathfrak{N}} = \left\{< x, min\left(\mathfrak{T}_{A^{\mathfrak{N}}}(x), \mathfrak{T}_{B^{\mathfrak{N}}}(x)\right), min\left(\mathfrak{I}_{A^{\mathfrak{N}}}(x), \mathfrak{I}_{B^{\mathfrak{N}}}(x)\right), max\left(\mathfrak{F}_{A^{\mathfrak{N}}}(x), \mathfrak{F}_{B^{\mathfrak{N}}}(x)\right) >\right\}$

(ii) $A^{\mathfrak{N}} \Cup B^{\mathfrak{N}} = \{< x, max\left(\mathfrak{T}_{A^{\mathfrak{N}}}(x), \mathfrak{T}_{B^{\mathfrak{N}}}(x)\right), max\left(\mathfrak{I}_{A^{\mathfrak{N}}}(x), \mathfrak{I}_{B^{\mathfrak{N}}}(x)\right), min\left(\mathfrak{F}_{A^{\mathfrak{N}}}(x), \mathfrak{F}_{B^{\mathfrak{N}}}(x)\right) >\}$

(iii) $\mathrm{A}^{\mathfrak{N}} \preccurlyeq \mathrm{B}^{\mathfrak{N}}$ *if* $\mathfrak{T}_{\mathrm{A}^{\mathfrak{N}}}(\mathrm{x}) \preccurlyeq \mathfrak{T}_{\mathrm{B}^{\mathfrak{N}}}(\mathrm{x}), \mathfrak{I}_{\mathrm{A}^{\mathfrak{N}}}(\mathrm{x}) \preccurlyeq \mathfrak{I}_{\mathrm{B}^{\mathfrak{N}}}(\mathrm{x}), \mathfrak{F}_{\mathrm{A}^{\mathfrak{N}}}(\mathrm{x}) \succcurlyeq \mathfrak{F}_{\mathrm{B}^{\mathfrak{N}}}(\mathrm{x})$, *for* $\mathrm{x} \in \mathrm{X}$.

2.4 Definition [12]

Let $X \neq \phi$, then neutrosophic topology space (NTS) on X is a family $\mathcal{T}_{X_N}$ of neutrosophic subsets of X satisfying the following axiom:

(i) $0_{X_N}, 1_{X_N} \in \mathcal{T}_{X_N}$

(ii) $G_{N_1} \Cap G_{N_2} \in \mathcal{T}_{X_N}$; for $G_{N_1}, G_{N_2} \in \mathcal{T}_{X_N}$

(iii) $\Cup G_{N_i} \in \mathcal{T}_{X_N}, \forall \{G_{N_i}: i \in J\} \preccurlyeq \mathcal{T}_{X_N}$.

Then the pair $(X, \mathcal{T}_{X_N})$ is called a NTS.

2.5 Definition [12]

Let $(X, \mathcal{T}_{X_N})$ be NTS. Then for a NS $A^{\mathfrak{N}} = \{< x, \mu_{N_i}, \sigma_{N_i}, \delta_{N_i} >: x \in X\}$, neutrosophic interior of $A^{\mathfrak{N}}$ can be defined as $\mathcal{N} \backsim Int\left(A^{\mathfrak{N}}\right) = \{< x, \Cup\ \mu_{N_i}, \Cap\ \sigma_{N_i}, \Cap\ \delta_{N_i} >: x \in X\}$.

2.6 Definition [12]

Let $(X, \mathcal{T}_{X_N})$ be NTS. Then for a NS $A^{\mathfrak{N}} = \{< x, \mu_{N_i}, \sigma_{N_i}, \delta_{N_i} >: x \in X\}$, neutrosophic closure of $A^{\mathfrak{N}}$ can be defined as $\mathcal{N} \backsim Cl\left(A^{\mathfrak{N}}\right) = \{< x, \Cap\ \mu_{N_i}, \Cup\ \sigma_{N_i}, \Cup\ \delta_{N_i} >: x \in X\}$.

3 Results and Discussion

3.1 Definition

Let $\mathcal{A}$ be a NS of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$, then $\mathcal{A}$ is called a $\mathcal{N} \backsim$ semi-open set (NSOS) of X if $\exists$ a $\mathcal{B} \in \mathcal{T}_{X_{\mathcal{N}}}$ such that $\mathcal{A} \preccurlyeq \mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{B}))$.

3.2 Definition

Let $\mathcal{A}$ be a NS of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$, then $\mathcal{A}$ is called a $\mathcal{N} \backsim$ semi-closed set (NSCoS) of X if $\exists$ a $\mathcal{B}^c \in \mathcal{T}_{X_N}$ such that $\mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{B})) \preccurlyeq \mathcal{A}$.

3.3 Lemma

Let $\phi: X \longrightarrow Y$ be a mapping and $\{\mathcal{A}_\alpha\}$ be a family of NSs of Y, then

(i) $\phi^{-1}(\Cup\ \mathcal{A}_\alpha) = \Cup\ \phi^{-1}(\mathcal{A}_\alpha)$ and (ii) $\phi^{-1}(\Cap\ \mathcal{A}_\alpha) = \Cap\ \phi^{-1}(\mathcal{A}_\alpha)$.

Prove is Straightforward.

3.4 Lemma

Let $\mathcal{A}, \mathcal{B}$ be NSs of X and Y, then $1_{X_N} - \mathcal{A} \times \mathcal{B} = (\mathcal{A}^c \times 1_{X_N}) \Cup (1_{X_N} \times \mathcal{B}^c)$.

Proof:

Let (p, q) be any element of $X \times Y$, $\left(1_{X_N} - \mathcal{A} \times \mathcal{B}\right)(p, q) = \max\left(1_{X_N} - \mathcal{A}(p), 1_{X_N} - \mathcal{B}(q)\right) = \max\left\{\left(\mathcal{A}^c \times 1_{X_N}\right)(p, q), \left(\mathcal{B}^c \times 1_{X_N}\right)(p, q)\right\} = \left\{\left(\mathcal{A}^c \times 1_{X_N}\right) \Cup \left(1_{X_N} \times \mathcal{B}^c\right)\right\}(p, q)$, for each $(p, q) \in X \times Y$.

3.5 Lemma

Let $\phi_i\colon X_i \longrightarrow Y_i$ and $\mathcal{A}_i$ be NSs of Y_i, $i = 1, 2$; we have $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2) = {\phi_1}^{-1}(\mathcal{A}_1) \times {\phi_2}^{-1}(\mathcal{A}_2)$.

Proof:

For each $(p_1, p_2) \in X_1 \times X_2$, we have

$$\begin{aligned}(\phi_1 \times \phi_2)^{-1}(\mathcal{A}_1 \times \mathcal{A}_2)(p_1, p_2) &= (\mathcal{A}_1 \times \mathcal{A}_2)(\phi_1(p_1), \phi_2(p_2))\\ &= \min\{\mathcal{A}_1\phi_1(p_1), \mathcal{A}_2\phi_2(p_2)\}\\ &= \min\left\{\phi_1^{-1}(\mathcal{A}_1)(p_1), \phi_2^{-1}(\mathcal{A}_2)(p_2)\right\}\\ &= \left({\phi_1}^{-1}(\mathcal{A}_1) \times {\phi_2}^{-1}(\mathcal{A}_2)\right)(p_1, p_2)\end{aligned}$$

3.6 Lemma

Let $\psi\colon X \longrightarrow X \times Y$ be the graph of a mapping $\phi\colon X \longrightarrow Y$. Then, if $\mathcal{A}, \mathcal{B}$ be NSs of X and Y, $\psi^{-1}(\mathcal{A} \times \mathcal{B}) = \mathcal{A} \Cap \phi^{-1}(\mathcal{B})$.

Proof:

For each $p \in X$, we have

$$\begin{aligned}\psi^{-1}(\mathcal{A} \times \mathcal{B})(p) = (\mathcal{A} \times \mathcal{B})\,\psi(p) &= (\mathcal{A} \times \mathcal{B})(p, \phi(p))\\ &= \min\{\mathcal{A}(p), \mathcal{B}(\phi(p))\}\\ &= \left(\mathcal{A} \Cap \phi^{-1}(\mathcal{B})\right)(p)\end{aligned}$$

3.7 Lemma

For a family $\{\mathcal{A}\}_\alpha$ of NSs of NTS $\left(X, \mathcal{T}_{X_\mathcal{N}}\right)$, $\Cup N \backsim Cl(\mathcal{A}_\alpha) \preccurlyeq \mathcal{N} \backsim Cl(\Cup (\mathcal{A}_\alpha))$. In case $\mathcal{B}$ is a finite set, $\Cup \mathcal{N} \backsim Cl(\mathcal{A}_\alpha) \preccurlyeq \mathcal{N} \backsim Cl(\Cup (\mathcal{A}_\alpha))$. Also, $\Cup \mathcal{N} \backsim Int(\mathcal{A}_\alpha) \preccurlyeq \mathcal{N} \backsim Int(\Cup (\mathcal{A}_\alpha))$, where a subfamily $\mathcal{B}$ of $\left(X, \mathcal{T}_{X_\mathcal{N}}\right)$ is said to be subbase for $\left(X, \mathcal{T}_{X_\mathcal{N}}\right)$ if the collection of all intersections of members of $\mathcal{B}$ forms a base for $\left(X, \mathcal{T}_{X_\mathcal{N}}\right)$.

3.8 Lemma

For a NS $\mathcal{A}$ of NTS $\left(X, \mathcal{T}_{X_\mathcal{N}}\right)$, (a) $1 - \mathcal{N} \backsim Int(\mathcal{A}) = \mathcal{N} \backsim Cl(1 - \mathcal{A})$, and (b) $1 - \mathcal{N} \backsim Cl(\mathcal{A}) = \mathcal{N} \backsim Int(1 - \mathcal{A})$.

Prove is Straightforward.

3.9 Theorem

The statements below are equivalent:

(i) $\mathcal{A}$ is a NSCoS,
(ii) $\mathcal{A}^c$ is a NSOS,
(iii) $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})) \preccurlyeq \mathcal{A}$, and
(iv) $\mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}^c)) \succcurlyeq \mathcal{A}^c$.

Proof:

(i) and (ii) are equivalent follows from Lemma 3.8, since for a NS $\mathcal{A}$ of NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ such that $1 - \mathcal{N} \backsim Int(\mathcal{A}) = \mathcal{N} \backsim Cl(1 - \mathcal{A})$ and $1 - \mathcal{N} \backsim Cl(\mathcal{A}) = \mathcal{N} \backsim Int(1 - \mathcal{A})$.

(i)⇒(iii). By definition ∃ a NCoS $\mathcal{B}$ such that $\mathcal{N} \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A} \preccurlyeq \mathcal{B}$ and hence $\mathcal{N} \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A} \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{A}) \preccurlyeq \mathcal{B}$. Since $\mathcal{N} \backsim Int(\mathcal{B})$ is the greatest NOS contained in $\mathcal{B}$, we have $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{B})) \preccurlyeq \mathcal{N} \backsim Int(\mathcal{B}) \preccurlyeq \mathcal{A}$.

(iii)⇒(i) follows by taking $\mathcal{B} = \mathcal{N} \backsim Cl(\mathcal{A})$.

(ii)⇔(iv) can similarly be proved.

3.10 Theorem

(i) Arbitrary union of NSOSs is a NSOS, and
(ii) Arbitrary intersection of NSCoSs is a NSCoS.

Proof:

(i) Let $\{\mathcal{A}_\alpha\}$ be a collection of NSOSs of NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$. Then ∃ a $\mathcal{B}_\alpha \in \mathcal{T}_{X_{\mathcal{N}}}$ such that $\mathcal{B}_\alpha \preccurlyeq \mathcal{A}_\alpha \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{B}_\alpha)$, for each α. Thus, $\Cap \mathcal{B}_\alpha \preccurlyeq \Cup \mathcal{A}_\alpha \preccurlyeq \Cup \mathcal{N} \backsim Cl(\mathcal{B}_\alpha) \preccurlyeq \mathcal{N} \backsim Cl(\Cup(\mathcal{B}_\alpha))$ [***Lemma 3.7***], and $\Cup \mathcal{B}_\alpha \in \mathcal{T}_{X_{\mathcal{N}}}$, this shows that $\Cup \mathcal{B}_\alpha$ is a NSOS.

(ii) Let $\{\mathcal{A}_\alpha\}$ be a collection of NSCoSs of NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$. Then ∃ a $\mathcal{B}_\alpha \in \mathcal{T}_{X_{\mathcal{N}}}$ such that $\mathcal{N} \backsim Int(\mathcal{B}_\alpha) \preccurlyeq \mathcal{A}_\alpha \preccurlyeq \mathcal{B}_\alpha$, for each α. Thus, $\mathcal{N} \backsim Int(\Cap(\mathcal{B}_\alpha)) \preccurlyeq \Cap \mathcal{N} \backsim Int(\mathcal{B}_\alpha) \preccurlyeq \Cap \mathcal{A}_\alpha \preccurlyeq \Cap \mathcal{B}_\alpha$ [Lemma 3.7], and $\Cup \mathcal{B}_\alpha \in \mathcal{T}_{X_{\mathcal{N}}}$, this shows that $\Cap \mathcal{B}_\alpha$ is a NSCoS.

3.11 Remark

It is clear that every neutrosophic open set (NOS) (neutrosophic closed set (NCoS)) is a NSOS (NSCoS). The converse is false, it is seen in ***Example 3.12***. It also shows that the intersection (union) of any two NSOSs (NSCoSs) need not be a NSOS (NSCoS). Even the intersection (union) of a NSOS (NSCoS) with a NOS (NCoS) may fail to be a NSOS (NSCoS). It should be noted that the ordinary topological setting the intersection of a NSOS with an NOS is a NSOS.

Further, the closure of NOS is a NSOS and the interior of NCoS is a NSCoS.

3.12 Example

Let $X = \{a, b\}$ and $\mathcal{A}, \mathcal{B}$ be neutrosophic subsets of X such that

$$\mathcal{A} = \left\{ < \frac{a}{(0.6, 0.3, 0.2)} >, < \frac{b}{(0.5, 0.2, 0.3)} > \right\}$$

$$\mathcal{B} = \left\{ < \frac{a}{(0.5, 0.4, 0.3)} >, < \frac{b}{(0.4, 0.2, 0.3)} > \right\}$$

Then, $\mathcal{T}_{X_{\mathcal{N}}} = \{1_{X_{\mathcal{N}}}, 0_{X_{\mathcal{N}}}, \mathcal{A}, \mathcal{B}, \mathcal{A} \Cup \mathcal{B}, \mathcal{A} \Cap \mathcal{B}\}$ is a NTS on X.

Let $P = \left\{ < \frac{a}{(0.8, 0.2, 0.1)} >, < \frac{b}{(0.7, 0.2, 0.3)} > \right\}$ be any neutrosophic set $X_{\mathcal{N}}$, then $\mathcal{N} \backsim Int(P) = \Cup\{G\colon G \text{ is open set}, G \preccurlyeq P\} = \mathcal{A} \Cup \mathcal{B} = \mathcal{A}$ and $\mathcal{N} \backsim Cl(P) = \Cap\{K \succcurlyeq P\colon K \text{ is closed set in } \mathcal{T}_{X_{\mathcal{N}}}\} = 1_{X_{\mathcal{N}}}$. Therefore, P is a NSOS which is not a NOS and also by ***Theorem 3.9***, P^c is a NSCoS which is not an NCoS.

3.13 Theorem

If $(X, \mathcal{T}_{X_{\mathcal{N}}})$ and $(Y, \mathcal{T}_{Y_{\mathcal{N}}})$ are NTSs and X is product related to Y. Then the product $\mathcal{A} \times \mathcal{B}$ of a NSOS $\mathcal{A}$ of X and a NSOS $\mathcal{B}$ of Y is NSOS of the neutrosophic product space $X \times Y$.

Proof:

Let $\mathcal{P} \preccurlyeq \mathcal{A} \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{P})$ and $\mathcal{Q} \preccurlyeq \mathcal{B} \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{Q})$, where $\mathcal{P} \in \mathcal{T}_{X_{\mathcal{N}}}$ and $\mathcal{Q} \in \mathcal{T}_{Y_{\mathcal{N}}}$. Then $\mathcal{P} \times \mathcal{Q} \preccurlyeq \mathcal{A} \times \mathcal{B} \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{P}) \times \mathcal{N} \backsim Cl(\mathcal{Q})$. For NSs $\mathcal{P}$'s of X and $\mathcal{Q}$'s of Y, we have

(a) $\inf\{\mathcal{P}, \mathcal{Q}\} = \min\{\inf \mathcal{P}, \inf \mathcal{Q}\}$,
(b) $\inf\left\{\mathcal{P} \times 1_{\mathrm{X_N}}\right\} = (\inf \mathcal{P}) \times 1_{\mathrm{X_N}}$, and
(c) $\inf\left\{1_{\mathrm{X_N}} \times \mathcal{Q}\right\} = 1_{\mathrm{X_N}} \times (\inf \mathcal{Q})$.

It is sufficient to prove $\mathcal{N} \backsim Cl(\mathcal{A} \times \mathcal{B}) \succcurlyeq \mathcal{N} \backsim Cl(\mathcal{A}) \times \mathcal{N} \backsim Cl(\mathcal{B})$. Let $\mathcal{P} \in \mathcal{T}_{N_X}$ and $\mathcal{Q} \in \mathcal{T}_{N_Y}$.

Then

$$
\begin{aligned}
\mathcal{N} \backsim Cl\,(\mathcal{A} \times \mathcal{B}) &= \inf\left\{(\mathcal{P} \times \mathcal{Q})^c \,\middle|\, (\mathcal{P} \times \mathcal{Q})^c \succcurlyeq \mathcal{A} \times \mathcal{B}\right\} \\
&= \inf\left\{\left(\mathcal{P}^c \times 1_{\mathrm{X_N}}\right) \Cup \left(1_{\mathrm{X_N}} \times \mathcal{Q}^c\right) \middle| \, (\mathcal{P}^c \times 1_{\mathrm{X_N}}) \Cup (1_{\mathrm{X_N}} \times \mathcal{Q}^c) \succcurlyeq \mathcal{A} \times \mathcal{B}\right\} \\
&= \inf\{(\mathcal{P}^c \times 1_{\mathrm{X_N}}) \Cup (1_{\mathrm{X_N}} \times \mathcal{Q}^c) | \mathcal{P}^c \succcurlyeq \mathcal{A} or \mathcal{Q}^c \succcurlyeq \mathcal{B}\} \\
&= \min\begin{bmatrix} \inf\left\{\left(\mathcal{P}^c \times 1_{\mathrm{X_N}}\right) \Cup \left(1_{\mathrm{X_N}} \times \mathcal{Q}^c\right) | \mathcal{P}^c \succcurlyeq \mathcal{A}\right\}, \\ \inf\left\{(\mathcal{P}^c \times 1_{\mathrm{X_N}}) \Cup (1_{\mathrm{X_N}} \times \mathcal{Q}^c) | \mathcal{Q}^c \succcurlyeq \mathcal{B}\right\} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\text{Since, } \inf\left\{(\mathcal{P}^c \times 1_{\mathrm{X_N}}) \Cup (1_{\mathrm{X_N}} \times \mathcal{Q}^c) | \mathcal{P}^c \succcurlyeq \mathcal{A}\right\} &\succcurlyeq \inf\left\{(\mathcal{P}^c \times 1_{\mathrm{X_N}}) | \mathcal{P}^c \succcurlyeq \mathcal{A}\right\} \\
&= \inf\left\{\mathcal{P}^c | \mathcal{P}^c \succcurlyeq \mathcal{A}\right\} \times 1_{\mathrm{X_N}} \\
&= \mathcal{N} \backsim Cl\,(\mathcal{A}) \times 1_{\mathrm{X_N}}
\end{aligned}
$$

$$
\begin{aligned}
\text{and } \inf\left\{(\mathcal{P}^c \times 1_{\mathrm{X_N}}) \Cup (1_{\mathrm{X_N}} \times \mathcal{Q}^c) | \mathcal{Q}^c \succcurlyeq \mathcal{B}\right\} &\succcurlyeq \inf\left\{(1_{\mathrm{X_N}} \times \mathcal{Q}^c) | \mathcal{Q}^c \succcurlyeq \mathcal{B}\right\} \\
&= 1_{\mathrm{X_N}} \times \inf\left\{\mathcal{Q}^c | \mathcal{Q}^c \succcurlyeq \mathcal{B}\right\} \\
&= 1_{\mathrm{X_N}} \times \mathcal{N} \backsim Cl\,(\mathcal{B})
\end{aligned}
$$

We have, $\mathcal{N} \backsim Cl\,(\mathcal{A} \times \mathcal{B}) \succcurlyeq \min\left\{\mathcal{N} \backsim Cl\,(\mathcal{A}) \times 1_{X_N}, 1_{X_N} \times \mathcal{N} \backsim Cl\,(\mathcal{B})\right\} = \mathcal{N} \backsim Cl\,(\mathcal{A}) \times \mathcal{N} \backsim Cl(\mathcal{B})$. Hence the result.

3.14 Definition

A NS $\mathcal{A}$ of NTS X is called a $\mathcal{N} \backsim$ regularly open set (NROS) of $(X, \mathcal{T}_{X_N})$ if $\mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{A})) = \mathcal{A}$.

3.15 Definition

A NS $\mathcal{A}$ of NTS $(X, \mathcal{T}_{X_N})$ is called a $\mathcal{N} \backsim$ regularly closed set (NRCoS) of X if $\mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{A})) = \mathcal{A}$.

3.16 Theorem

A NS $\mathcal{A}$ of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$ is a NRO iff $\mathcal{A}^c$ is NRCo.

Proof: It follows from ***Lemma 3.8***.

3.17 Remark

It is obvious that every NROS (NRCoS) is NOS (NCoS). The converse need not be true. For this we cite an example.

3.18 Example

From ***Example 3.12***, it is clear that $\mathcal{A}$ is NOS. Now $\mathcal{N} \backsim Cl(\mathcal{A}) = 1_{X_{\mathcal{N}}}$ and $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})) = 1_{X_{\mathcal{N}}}$. Therefore, $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})) \neq \mathcal{A}$, hence $\mathcal{A}$ is not NROS.

3.19 Remark

The union (intersection) of any two NROSs (NRCoS) need not be a NROS (NRCoS).

3.20 Example

Let $X = \{a, b, c\}$ and $\mathcal{T}_{X_{\mathcal{N}}} = \left\{0_{X_{\mathcal{N}}}, 1_{X_{\mathcal{N}}}, \mathcal{A}, \mathcal{B}, \mathcal{C}\right\}$ be NTS on X, where

$$\mathcal{A} = \left\{< \frac{a}{(0.4, 0.5, 0.6)} >, < \frac{b}{(0.7, 0.5, 0.3)} >, < \frac{c}{(0.5, 0.5, 0.5)} >\right\}$$

$$\mathcal{B} = \left\{< \frac{a}{(0.6, 0.5, 0.4)} >, < \frac{b}{(0.3, 0.5, 0.7)} >, < \frac{c}{(0.5, 0.5, 0.5)} >\right\},$$

$$\mathcal{C} = \left\{< \frac{a}{(0.6, 0.5, 0.4)} >, < \frac{b}{(0.7, 0.5, 0.3)} >, < \frac{c}{(0.5, 0.5, 0.5)} >\right\}.$$

Then $Cl(\mathcal{A}) = \mathcal{B}^c, Int(\mathcal{B}^c) = \mathcal{A}$

Clearly, $Int(Cl(\mathcal{A})) = \mathcal{A}$.

Similarly, $Int(Cl(\mathcal{B})) = \mathcal{B}$.

Now, $\mathcal{A} \bigcup \mathcal{B} = \mathcal{C}$.

But $Cl\left(\mathcal{A} \bigcup \mathcal{B}\right) = 1_{X_{\mathcal{N}}}$ and $Int\left(Cl\left(\mathcal{A} \bigcup \mathcal{B}\right)\right) = 1_{X_{\mathcal{N}}}$.

Hence, $\mathcal{A}$ and $\mathcal{B}$ are two NROSs but $\mathcal{A} \bigcup \mathcal{B}$ is not NROS.

3.21 Theorem

(i) The intersection of any two NROSs is a NROS, and

(ii) The union of any two NRCoSs is a NRCoS.

Proof:

(i) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two NROSs of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$. Since $\mathcal{A}_1 \Cap \mathcal{A}_2$ is NOS [from **Remark 3.17**], we have $\mathcal{A}_1 \Cap \mathcal{A}_2 \preccurlyeq \mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2))$. Now, $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq \mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_1)) = \mathcal{A}_1$ and $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq \mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_1 \Cap \mathcal{A}_2)) \preccurlyeq \mathcal{A}_1 \Cap \mathcal{A}_2$. Hence the theorem.

(ii) Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two NROSs of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$. Since $\mathcal{A}_1 \Cup \mathcal{A}_2$ is NOS [from **Remark 3.17**], we have $\mathcal{A}_1 \Cup \mathcal{A}_2 \succcurlyeq \mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2))$. Now, $\mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2)) \succcurlyeq \mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_1)) = \mathcal{A}_1$ and $\mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2)) \succcurlyeq \mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_2)) = \mathcal{A}_2$ implies that $\mathcal{A}_1 \Cup \mathcal{A}_2 \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{A}_1 \Cup \mathcal{A}_2))$. Hence the theorem.

3.22 Theorem

(i) The closure of a NOS is NRCoS, and

(ii) The interior of a NCoS is NROS.

Proof:

(i) Let $\mathcal{A}$ be a NOS of NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$, clearly, $\mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{A})) \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{A}) \Rightarrow \mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{A}))) \preccurlyeq \mathcal{N} \backsim Cl(\mathcal{A})$. Now, $\mathcal{A}$ is NOS implies that $\mathcal{A} \preccurlyeq \mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{A}))$ and hence $\mathcal{N} \backsim Cl(\mathcal{A}) \preccurlyeq \mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{A})))$. Thus, $\mathcal{N} \backsim Cl(\mathcal{A})$ is NRCoS.

(ii) Let $\mathcal{A}$ be a NCoS of a NTS $\left(X, \mathcal{T}_{X_{\mathcal{N}}}\right)$, clearly, $\mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{A})) \succcurlyeq \mathcal{N} \backsim Int(\mathcal{A}) \Rightarrow \mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{A}))) \succcurlyeq \mathcal{N} \backsim Int(\mathcal{A})$. Now, $\mathcal{A}$ is NCoS implies that $\mathcal{A} \succcurlyeq \mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{A}))$ and hence $\mathcal{N} \backsim Int(\mathcal{A}) \succcurlyeq \mathcal{N} \backsim Int\,(\mathcal{N} \backsim Cl\,(\mathcal{N} \backsim Int\,(\mathcal{A})))$. Thus, $\mathcal{N} \backsim Int(\mathcal{A})$ is NROS.

3.23 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(X, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is called a $\mathcal{N} \backsim$ continuous mapping (NCM), if $\phi^{-1}(\mathcal{A}) \in \mathcal{T}_{X_{\mathcal{N}}}$ for each $\mathcal{A} \in \mathcal{T}_{Y_{\mathcal{N}}}$; or equivalently $\phi^{-1}(\mathcal{B})$ is a NCoS of X for each NCoS $\mathcal{B}$ of Y.

3.24 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{X_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(Y, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is said to be a $\mathcal{N} \backsim$ open mapping (NOM), if $\phi(\mathcal{A}) \in \mathcal{T}_{Y_{\mathcal{N}}}$ for each $\mathcal{A} \in \mathcal{T}_{X_{\mathcal{N}}}$.

3.25 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(Y, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is said to be a $\mathcal{N} \backsim$ closed mapping (NCoM) if $\phi(\mathcal{B})$ is a NCoS of Y for each NCoS $\mathcal{B}$ of X.

3.26 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(X, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is said to be a $\mathcal{N} \backsim$ semi-continuous mapping (NSCM), if $\phi^{-1}(\mathcal{A})$ is a neutrosophic semi-open set of X, for each $\mathcal{A} \in \mathcal{T}_{Y_{\mathcal{N}}}$.

3.27 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(X, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is said to be a $\mathcal{N} \backsim$ semi-open mapping (NSOM), if $\phi(\mathcal{A})$ is a NSOS for each $\mathcal{A} \in \mathcal{T}_{X_{\mathcal{N}}}$.

3.28 Definition

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping from NTS $(X, \mathcal{T}_{X_{\mathcal{N}}})$ to another NTS $(X, \mathcal{T}_{Y_{\mathcal{N}}})$, then ϕ is said to be a $\mathcal{N} \backsim$ semi-closed mapping (NSCoM), if $\phi(\mathcal{B})$ is a NSCoS for each NCoS $\mathcal{B}$ of X.

3.29 Remark

From ***Remark 3.11***, a NCM (NOM, NCoM) is also a NSCM (NSOM, NSCoM). But the converse is not true.

3.30 Example

Let $X = \{a, b\}$, $Y = \{x, y\}$, and

$$\mathcal{A} = \left\{ < \frac{a}{(0.6, 0.3, 0.2)} >, < \frac{b}{(0.5, 0.2, 0.3)} > \right\}$$

$$\mathcal{B} = \left\{ < \frac{x}{(0.5, 0.4, 0.3)} >, < \frac{y}{(0.4, 0.2, 0.3)} > \right\},$$

$$\mathcal{C} = \left\{ < \frac{x}{(0.8, 0.2, 0.1)} >, < \frac{y}{(0.7, 0.2, 0.3)} > \right\}.$$

Then $\mathcal{T}_{X_{\mathcal{N}}} = \{0_{X_{\mathcal{N}}}, 1_{X_{\mathcal{N}}}, \mathcal{A}\}$ and $\mathcal{T}_{Y_{\mathcal{N}}} = \{0_{X_{\mathcal{N}}}, 1_{X_{\mathcal{N}}}, \mathcal{B}, \mathcal{C}\}$ are NTSs on X and Y.

Let $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ be a mapping defined as $\phi(a) = y, \phi(b) = x$.

Then $\phi\colon (X, \mathcal{T}_{X_{\mathcal{N}}}) \longrightarrow (Y, \mathcal{T}_{Y_{\mathcal{N}}})$ is NSCM but not NCM.

3.31 Theorem

Let X_1, X_2, Y_1 and Y_2 be NTSs such that X_1 is product related to X_2. Then, the product $\phi_1 \times \phi_2\colon X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ of NSCMs $\phi_1\colon X_1 \longrightarrow Y_1$ and $\phi_2\colon X_2 \longrightarrow Y_2$ is NSCM.

Proof:

Let $\mathcal{A} \equiv \Cup(\mathcal{A}_\alpha \times \mathcal{B}_\beta)$, where $\mathcal{A}_\alpha$'s and $\mathcal{B}_\beta$'s are NOSs of Y_1 and Y_2, respectively, be a NOS of $Y_1 \times Y_2$. By using ***Lemma 3.3***(i) and ***Lemma 3.5***, we have

$(\phi_1 \times \phi_2)^{-1}(\mathcal{A}) = \Cup\left[\phi_1{}^{-1}(\mathcal{A}_\alpha) \times \phi_2{}^{-1}(\mathcal{A}_\beta)\right]$.

That $(\phi_1 \times \phi_2)^{-1}(\mathcal{A})$ is a NSOS follows from ***Theorem 3.13*** and ***Theorem 3.10***(i).

3.32 Theorem

Let X, X_1 and X_2 be NTSs and $p_i\colon X_1 \times X_2 \longrightarrow X_i (i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then, if $\phi\colon X \longrightarrow X_1 \times X_2$ is a NSCM, $p_i\phi$ is also NSCM.

Proof:

For a NOS $\mathcal{A}$ of X_i, we have $(p_i\phi)^{-1}(\mathcal{A}) = \phi^{-1}(p_i{}^{-1}(\mathcal{A}))$. That p_i is a NCM and ϕ is a NSCM imply that $(p_i\phi)^{-1}(\mathcal{A})$ is a NSOS of X.

3.33 Theorem

Let $\phi\colon X \longrightarrow Y$ be a mapping from NTS X to another NTS Y. Then if the graph $\psi\colon X \longrightarrow X \times Y$ of ϕ is NSCM, then ϕ is also NSCM.

Proof:

From ***Lemma 3.6***, $\phi^{-1}(\mathcal{A}) = 1_{X_N} \Cap \phi^{-1}(\mathcal{A}) = \psi^{-1}(1_{X_N} \times \mathcal{A})$, for each NOS $\mathcal{A}$ of Y. Since ψ is a NSCM and $1_{X_N} \times \mathcal{A}$ is a NOS $X \times Y$, $\phi^{-1}(\mathcal{A})$ is a NSOS of X and hence ϕ is a NSCM.

3.34 Remark

The converse of ***Theorem 3.33*** is not true.

3.35 Definition

A mapping $\phi: (X, \mathcal{T}_{X_N}) \longrightarrow (Y, \mathcal{T}_{Y_N})$ from NTS X to another NTS Y is said to be a $\mathcal{N} \backsim$ almost continuous mapping (NACM), if $\phi^{-1}(\mathcal{A}) \in \mathcal{T}_{X_N}$ for each neutrosophic regularly open set $\mathcal{A}$ of Y.

3.36 Theorem

Let $\phi: (X, \mathcal{T}_{X_N}) \longrightarrow (Y, \mathcal{T}_{Y_N})$ be a mapping. Then the statements below are equivalent:

(a) ϕ is a NACM,
(b) $\phi^{-1}(\mathcal{F})$ is a NCoS, for each NRCoS $\mathcal{F}$ of Y,
(c) $\phi^{-1}(\mathcal{A}) \preccurlyeq \mathcal{N} \backsim Int(\phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}))))$, for each NOS $\mathcal{A}$ of Y,
(d) $\mathcal{N} \backsim Cl\left(\phi^{-1}(\mathcal{N} \backsim Cl(\mathcal{N} \backsim Int(\mathcal{F})))\right) \preccurlyeq \phi^{-1}(\mathcal{F})$, for each NCoS $\mathcal{F}$ of Y.

Proof:

Consider that $\phi^{-1}(\mathcal{A}^c) = \left(\phi^{-1}(A)\right)^c$, for any NS $\mathcal{A}$ of Y, (a) $\Leftrightarrow$ (b) follows from ***Theorem 3.16.***

(a) $\Rightarrow$ (c). Since $\mathcal{A}$ is a NOS of Y, $\mathcal{A} \preccurlyeq \mathcal{N} \backsim Int(Cl(\mathcal{A}))$ and hence $\phi^{-1}(\mathcal{A}) \preccurlyeq \phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})))$. From ***Theorem 3.22***(ii), $\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}))$ is a NROS of Y, hence $\phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})))$ is a NOS of X. Thus, $\phi^{-1}(\mathcal{A}) \preccurlyeq \phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}))) = \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})))\right)$.

(c) $\Rightarrow$ (a). Let $\mathcal{A}$ be a NROS of Y, then we have $\phi^{-1}(\mathcal{A}) \preccurlyeq \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})))\right) = \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{A})\right)$. Thus, have $\phi^{-1}(\mathcal{A}) = \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{A})\right)$. This shows that $\phi^{-1}(\mathcal{A})$ is a NOS of X.

(b) $\Leftrightarrow$ (d) similarly can be proved.

3.37 Remark

Clearly, a NCM is NACM. But the converse needs not be true.

3.38 Example

Let $X = \{a, b\}$, $Y = \{x, y\}$, and

$$\mathcal{A} = \left\{ < \frac{a}{(0.6, 0.5, 0.3)} >, < \frac{b}{(0.4, 0.5, 0.5)} > \right\}$$

$$\mathcal{B} = \left\{ < \frac{a}{(0.2, 0.5, 0.7)} >, < \frac{b}{(0.4, 0.5, 0.5)} > \right\},$$

$$\mathcal{C} = \left\{ < \frac{x}{(0.6, 0.5, 0.3)} >, < \frac{y}{(0.4, 0.5, 0.5)} > \right\},$$

$$\mathcal{D} = \left\{ < \frac{x}{(0.2, 0.5, 0.7)} >, < \frac{y}{(0.4, 0.5, 0.5)} > \right\},$$

$$\mathcal{E} = \left\{ < \frac{x}{(0.2, 0.5, 0.5)} >, < \frac{y}{(0.3, 0.5, 0.7)} > \right\}.$$

Then $\mathcal{T}_{X_\mathcal{N}} = \{0_{X_\mathcal{N}}, 1_{X_\mathcal{N}}, \mathcal{A}, \mathcal{B}\}$ and $\mathcal{T}_{Y_\mathcal{N}} = \{0_{X_\mathcal{N}}, 1_{X_\mathcal{N}}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$ are NTSs on X and Y.

Now, let $\phi\colon (X, \mathcal{T}_{X_\mathcal{N}}) \longrightarrow (Y, \mathcal{T}_{Y_\mathcal{N}})$ be a mapping defined as $\phi(a) = y, \phi(b) = x$ and clearly ϕ is NACM.

Here, $0_{X_\mathcal{N}}, 1_{X_\mathcal{N}}, \mathcal{C}, \mathcal{D}$ are open sets in $\mathcal{T}_{Y_\mathcal{N}}$ but $\phi^{-1}(\mathcal{E})$ is not open set in $\mathcal{T}_{X_\mathcal{N}}$ and hence NACM is not NCM.

3.39 Theorem

$\mathcal{N} \backsim$ semi-continuity and $\mathcal{N} \backsim$ almost continuity are independent notions.

3.40 Definition

A NTS $(X, \mathcal{T}_{X_N})$ is said to be a $\mathcal{N} \backsim$ semi-regularly space (NSRS) *iff* the collection of all NROSs of X forms a base for NT $\mathcal{T}_{X_N}$.

3.41 Theorem

Let $\phi\colon (X, \mathcal{T}_{X_N}) \longrightarrow (Y, \mathcal{T}_{Y_N})$ be a mapping from NTS X to a NSRS Y. Then ϕ is NACM iff ϕ is NCM.

Proof:

From ***Remark 3.37***, it suffices to prove that if ϕ is NACM then it is NCM. Let $\mathcal{A} \in \mathcal{T}_{N_Y}$, then $\mathcal{A} = \Cup\, \mathcal{A}_\alpha$, where $\mathcal{A}_\alpha$'s are NROSs of Y. Now, from ***Lemma 3.3***(i), 3.7 and ***Theorem 3.36***(c), we get

$$\phi^{-1}(\mathcal{A}) = \Cup\, \phi^{-1}(\mathcal{A}_\alpha) \preccurlyeq \Cup\, \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{N} \backsim Cl(\mathcal{A}_\alpha))\right) = \Cup\, \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{A}_\alpha)\right).$$

$$\preccurlyeq \mathcal{N} \backsim Int \Cup \left(\phi^{-1}(\mathcal{A}_\alpha)\right) = \mathcal{N} \backsim Int\left(\phi^{-1}(\mathcal{A}_\alpha)\right).$$

which shows that $\phi^{-1}(\mathcal{A}_\alpha) \in \mathcal{T}_{X_N}$.

3.42 Theorem

Let X_1, X_2, Y_1 and Y_2 be the NTSs such that Y_1 is product related to Y_2. Then the product $\phi_1 \times \phi_2\colon X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ of NACMs $\phi_1\colon X_1 \longrightarrow Y_1$ and $\phi_2\colon X_2 \longrightarrow Y_2$ is NACM.

Proof:

Let $\mathcal{A} = \Cup\, (\mathcal{A}_\alpha \times \mathcal{B}_\beta)$, where $\mathcal{A}_\alpha$'s and $\mathcal{B}_\beta$'s are NOSs of Y_1 and Y_2 respectively, be a NOS of $Y_1 \times Y_2$. Following ***Lemma 3.5***, for $(p_1, p_2) \in X_1 \times X_2$, we have

$$(\phi_1 \times \phi_2)^{-1}(\mathcal{A})(p_1, p_2) = (\phi_1 \times \phi_2)^{-1}\left\{\Cup\left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)\right\}(p_1, p_2)$$

$$= \Cup\left\{\left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)(\phi_1(p_1), \phi_2(p_2))\right\}$$
$$= \Cup\left[\min\left\{\mathcal{A}_\alpha\phi_1(p_1), \mathcal{B}_\beta\phi_2(p_2)\right\}\right]$$
$$= \Cup\left[\min\left\{{\phi_1}^{-1}(\mathcal{A}_\alpha)(p_1), {\phi_2}^{-1}\left(\mathcal{B}_\beta\right)(p_2)\right\}\right]$$
$$= \Cup\left[\left({\phi_1}^{-1}(\mathcal{A}_\alpha) \times {\phi_2}^{-1}\left(\mathcal{B}_\beta\right)\right)\right](p_1, p_2)$$

i.e., $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}) = \Cup\left\{{\phi_1}^{-1}(\mathcal{A}_\alpha) \times {\phi_2}^{-1}\left(\mathcal{B}_\beta\right)\right\}$

Now, $(\phi_1 \times \phi_2)^{-1}(\mathcal{A}) = \Cup\left\{{\phi_1}^{-1}(\mathcal{A}_\alpha) \times {\phi_2}^{-1}\left(\mathcal{B}_\beta\right)\right\}$

$$\preccurlyeq \Cup\left[\mathcal{N} \backsim Int\left({\phi_1}^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_\alpha)))\right) \times \mathcal{N} \backsim Int\left({\phi_2}^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right)\right)\right]$$
$$\preccurlyeq \Cup\left[\mathcal{N} \backsim Int\left\{{\phi_1}^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_\alpha))) \times {\phi_2}^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right)\right\}\right]$$
$$\preccurlyeq \Cup\left[\mathcal{N} \backsim Int\left\{{\phi_1}^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A}_\alpha))) \times {\phi_2}^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\mathcal{B}_\beta\right)\right)\right)\right\}\right]$$
$$= \mathcal{N} \backsim Int\left[\Cup(\phi_1 \times \phi_2)^{-1}\left\{\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)\right)\right\}\right]$$
$$\preccurlyeq \mathcal{N} \backsim Int\left[(\phi_1 \times \phi_2)^{-1}\left\{\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\Cup\left(\mathcal{A}_\alpha \times \mathcal{B}_\beta\right)\right)\right)\right\}\right]$$
$$= \mathcal{N} \backsim Int\left[(\phi_1 \times \phi_2)^{-1}(\mathcal{N} \backsim Int(\mathcal{N} \backsim Cl(\mathcal{A})))\right]$$

Thus, by ***Theorem 3.36***(c), $\phi_1 \times \phi_2$ is NACM.

3.43 Theorem

Let X, X_1 and X_2 be NTSs and $p_i\colon X_1 \times X_2 \longrightarrow X_i (i = 1, 2)$ be the projection of $X_1 \times X_2$ onto X_i. Then if $\phi\colon X \longrightarrow X_1 \times X_2$ is a NACM, $p_i\phi$ is also a NACM.

Proof:

Since p_i is NCM ***Definition 3.23***, for any NS $\mathcal{A}$ of X_i, we have (i) $\mathcal{N} \backsim Cl\left({p_i}^{-1}(\mathcal{A})\right) \preccurlyeq {p_i}^{-1}(\mathcal{N} \backsim \mathrm{Cl}(\mathcal{A}))$ and (ii) $\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right) \succcurlyeq {p_i}^{-1}(\mathcal{N} \backsim Int(\mathcal{A}))$. Again, since (i) each p_i is a NOM, and (ii) for any NS $\mathcal{A}$ of X_i (a) $\mathcal{A} \preccurlyeq {p_i}^{-1}p_i(\mathcal{A})$, and (b) ${p_i}^{-1}p_i(\mathcal{A}) \preccurlyeq \mathcal{A}$, we have $p_i\left(\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right)\right) \preccurlyeq p_i{p_i}^{-1}(\mathcal{A}) \preccurlyeq \mathcal{A}$ and hence $p_i\left(\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right)\right) \preccurlyeq \mathcal{N} \backsim Int(\mathcal{A})$. Thus, $\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right) \preccurlyeq {p_i}^{-1}p_i\left(\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right)\right) \preccurlyeq \left({p_i}^{-1}(\mathcal{N} \backsim Int(\mathcal{A})\right)$ establishes that $\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{A})\right) \preccurlyeq {p_i}^{-1}(\mathcal{N} \backsim Int(\mathcal{A}))$. Now, for any NOS $\mathcal{A}$ of X_i,

$$(p_i\phi)^{-1}(\mathcal{A}) = \phi^{-1}\left({p_i}^{-1}(\mathcal{A})\right)$$
$$\preccurlyeq \mathcal{N} \backsim Int\left\{\phi^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left({p_i}^{-1}(\mathcal{A})\right)\right)\right)\right\}$$
$$\preccurlyeq \mathcal{N} \backsim Int\left\{\phi^{-1}\left(\mathcal{N} \backsim Int\left({p_i}^{-1}(\mathcal{N} \backsim Cl(\mathcal{A}))\right)\right)\right\}$$

$$= \mathcal{N} \backsim Int \left\{ \phi^{-1} \left({p_i}^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{A}))) \right) \right\}$$

$$= \mathcal{N} \backsim Int(p_i \phi)^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{A})))$$

3.44 Theorem

Let X and Y be NTSs such that X is product related to Y and let $\phi \colon X \longrightarrow Y$ be a mapping. Then, the graph $\psi \colon X \longrightarrow X \times Y$ of ϕ is NACM iff ϕ is NACM.

Proof:

Consider that ψ is a NACM and $\mathcal{A}$ is a NOS of Y. Then using ***Lemma 3.6*** and ***Theorem 3.36***(c), we have

$$\begin{aligned}
\phi^{-1} (\mathcal{A}) &= 1 \Cap \phi^{-1} (\mathcal{A}) \\
&= \psi^{-1} (1 \times \mathcal{A}) \preccurlyeq \mathcal{N} \backsim Int \left(\psi^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (1 \times \mathcal{A}))) \right) \\
&= \mathcal{N} \backsim Int \left(\psi^{-1} (1 \times \mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{A}))) \right) \\
&= \mathcal{N} \backsim Int \left(\psi^{-1} (\mathcal{N} \backsim Int (1 \times \mathcal{N} \backsim Cl (\mathcal{A}))) \right) \\
&= \mathcal{N} \backsim Int \left(\psi^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{A}))) \right)
\end{aligned}$$

Thus, by ***Theorem 3.36***(c), ϕ is NACM.

Conversely, let ϕ be a NACM and $\mathcal{B} = \Cup \, (\mathcal{B}_\alpha \times \mathcal{A}_\beta)$, where $\mathcal{B}_\alpha$'s and $\mathcal{A}_\beta$'s are NOSs of X and Y, respectively, be a NOS of $X \times Y$.

Since $\mathcal{B}_\alpha \Cap \mathcal{N} \backsim Int \left(\phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right) \right)$ is a NOSs of X contained in

$$\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{B}_\alpha)) \Cap \phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right),$$

$$\mathcal{B}_\alpha \Cap \mathcal{N} \backsim Int \left(\phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right) \right)$$

$$\preccurlyeq \mathcal{N} \backsim Int \left[\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{B}_\alpha)) \Cap \phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right) \right]$$

and hence using ***Lemmas 3.3***(i), 3.6 and ***3.7*** and ***Theorems 3.36***(c), we have

$$\begin{aligned}
\phi^{-1} (\mathcal{B}) &= \phi^{-1} \left(\Cup \left(\mathcal{B}_\alpha \times \mathcal{A}_\beta \right) \right) \\
&= \Cup \left[\mathcal{B}_\alpha \Cap \phi^{-1} \left(\mathcal{A}_\beta \right) \right] \\
&\preccurlyeq \Cup \left[\mathcal{B}_\alpha \Cap \mathcal{N} \backsim Int \left(\phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right) \right) \right] \\
&\preccurlyeq \Cup \left[\mathcal{N} \backsim Int (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{B}_\alpha))) \Cap \phi^{-1} \left(\mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right) \right] \\
&\preccurlyeq \mathcal{N} \backsim Int \left[\Cup \psi^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{B}_\alpha))) \times \mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right] \\
&\preccurlyeq \mathcal{N} \backsim Int \left[\Cup \psi^{-1} (\mathcal{N} \backsim Int (\mathcal{N} \backsim Cl (\mathcal{B}_\alpha))) \times \mathcal{N} \backsim Int \left(\mathcal{N} \backsim Cl \left(\mathcal{A}_\beta \right) \right) \right]
\end{aligned}$$

$$\preccurlyeq \mathcal{N} \backsim Int\left[\psi^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\Cup\left(\mathcal{B}_{\alpha} \times \mathcal{A}_{\beta}\right)\right)\right)\right)\right]$$
$$= \mathcal{N} \backsim Int\left[\psi^{-1}\left(\mathcal{N} \backsim Int\left(\mathcal{N} \backsim Cl\left(\mathcal{B}\right)\right)\right)\right].$$

indentThus, by ***Theorem 3.36***(c), ψ is NACM.

4 Conclusion

The truth membership function, indeterminacy membership function, and falsity membership function are all employed in the Neutrosophic Set to overcome uncertainty. First, we developed the definitions of $\mathcal{N} \backsim$ semi-open set, $\mathcal{N} \backsim$ semi-closed, $\mathcal{N} \backsim$ regularly open set, $\mathcal{N} \backsim$ regularly closed set, $\mathcal{N} \backsim$ continuous mapping, $\mathcal{N} \backsim$ open mapping, $\mathcal{N} \backsim$ closed mapping, $\mathcal{N} \backsim$ semi-continuous mapping, $\mathcal{N} \backsim$ semi-open mapping, $\mathcal{N} \backsim$ semi-closed mapping, set in order to propose the definition of $\mathcal{N} \backsim$ almost continuous mapping. Some properties of $\mathcal{N} \backsim$ almost continuous mapping have been demonstrated. We expect that our study may spark some new ideas for the construction of the neutrosophic almost continuous mapping. It will be necessary to carry out more theoretical research to establish a general framework for decision-making and to define patterns for complex network conceiving and practical application. In the future, we would like to extend our work to study some properties in the neutrosophic semi and almost topological group with the help of the neutrosophic semi and almost continuous mapping.

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